

HW 3 written 27/3/20

27. $y' = 2\sqrt{y}$ B $y(0) = 0$

$$y(x) = \begin{cases} 0 & x \leq c \\ (x-c)^2 & x > c \end{cases}$$

a) $y'(x) = \begin{cases} 0 & x \leq c \\ 2x-2c & x > c \end{cases}$

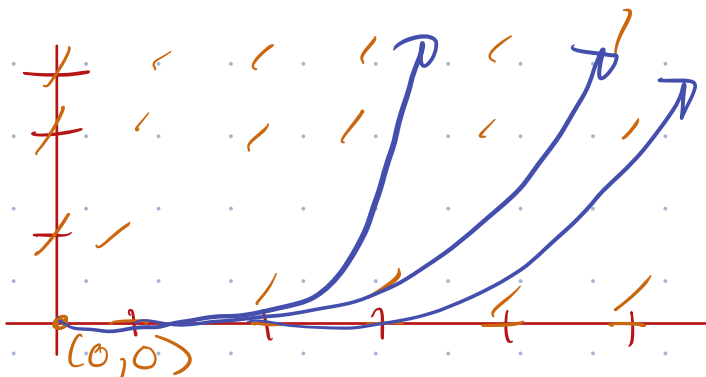
this y satisfies DE for $x \leq c$ $\left[\begin{array}{l} y' = 2\sqrt{y} \\ 0 = 2\sqrt{0} \end{array} \right\}$ subst in 0 from $x \leq c$

$$y' = 2\sqrt{y}$$

$$2x-2c = 2\sqrt{(x-c)^2}$$

$$2x-2c = 2x-2c$$

✓ satisfies given DE for $x > c$



b) initial value problem has
no solution for $b < 0$

$$\rightarrow y(a) = 0$$

the initial value problem has
a unique solution for $b > 0$

$$\rightarrow y(a) = b$$

30. $y' = -\sqrt{1-y^2} \quad y(a) = b$

$$y(x) = \begin{cases} 1 & x \leq c \\ \cos(x-c) & c < x < c+\pi \\ -1 & x \geq c+\pi \end{cases}$$

$$y'(x) = \begin{cases} 0 & x \leq c \\ -\sin(x-c) & c < x < c+\pi \\ 0 & x \geq c+\pi \end{cases}$$

$x \leq c$: $y' = -\sqrt{1-y^2}$
 $0 = -\sqrt{1-1^2}$

$$0 = 0$$

✓ satisfies DE

$c < x < c+\pi$: $y' = -\sqrt{1-y^2}$

$$-\sin(x-c) = -\sqrt{1-(\cos(x-c))^2}$$

$$= -\sqrt{1-\cos^2}$$

$$-\sin(x-c) = -\sin(x-c) \quad \checkmark \text{ satisfies DE}$$

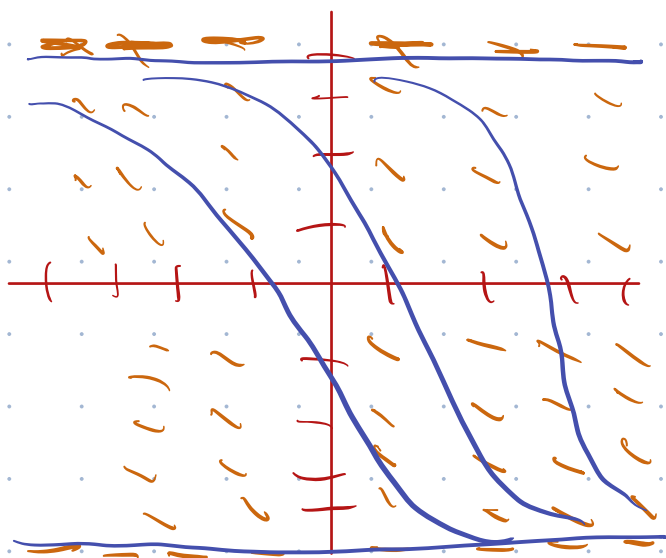
$$x \geq C + \pi :$$

$$y' = -\sqrt{1-y^2}$$

$$0 = -\sqrt{1-1^2}$$

$$0=0$$

\checkmark satisfies DE



IVP has no solution if $y(b)$
 $|b| > 1$

also, IVP has unique solution if
 $|b| < 1$

IVP has infinite sol if $|b| = 1$