Chapter 2 solutions **Hanna Gábor**

1. *In ϵ-greedy action selection, for the case of two actions and $\epsilon = 0.5$, what is the probability that the greedy action is selected?*

   $\mathbb{P}(\text{greedy action is selected}) = 0.5 + 0.5/2 = 0.75$.

2. *Bandit example. Consider a k-armed bandit problem with $k = 4$ actions, denoted 1, 2, 3, and 4. Consider applying to this problem a bandit algorithm using ϵ-greedy action selection, sample-average action-value estimates, and initial estimates of $Q_1(a) = 0$, for all a. Suppose the initial sequence of actions and rewards is $A_1 = 1$, $R_1 = 1$, $A_2 = 2$, $R_2 = 1$, $A_3 = 2$, $R_3 = 2$, $A_4 = 2$, $R_4 = 2$, $A_5 = 3$, $R_5 = 0$. On some of these time steps the ϵ case may have occurred, causing an action to be selected at random. On which time steps did this definitely occur? On which time steps could this possibly have occurred?*

   Any action can be an exploratory move.

   What were the greedy options in different time steps?
   Step 1: all actions have 0 estimated values. Every action is a greedy choice.
   Step 2: $Q_1(1) = 1$. The greedy choice now is 1. $A_2 = 2$ must have been an explorative move.
   Step 3: $Q_2(2) = 1$. The greedy choice is either 1 or 2.
   Step 4: $Q_3(2) = 1.5$. The greedy choice is 2.
   Step 5: $Q_4(2) = 1.67$. The greedy choice is 2. $A_5 = 3$ must have been an explorative move.

   On time steps 2 and 5 a random action must have been selected.

3. *In the comparison shown in Figure 2.2, which method will perform best in the long run in terms of cumulative reward and probability of selecting the best action? How much better will it be? Express your answer quantitatively.*

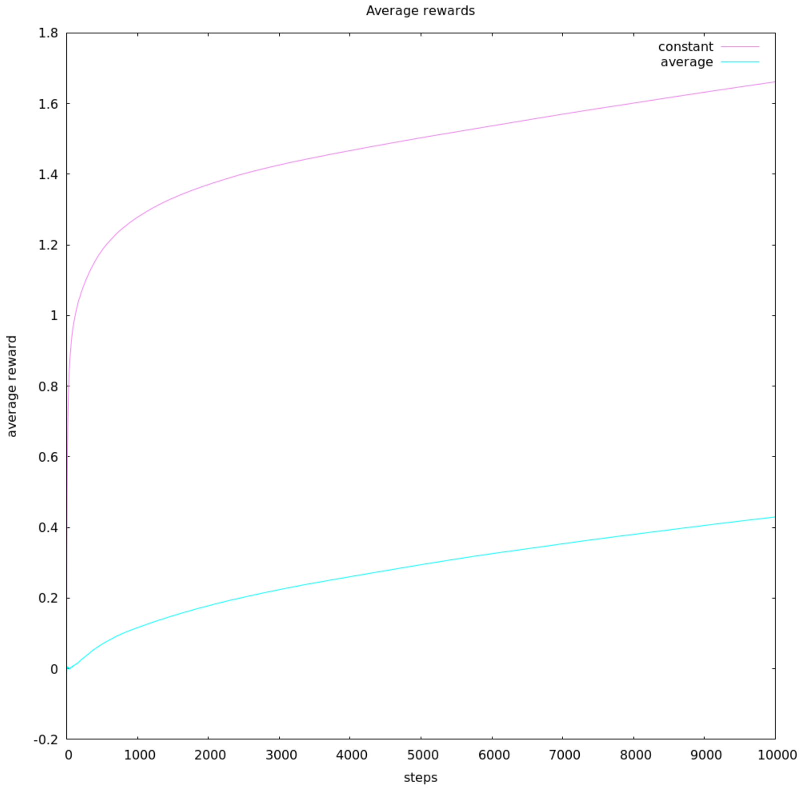
   On the long run, the $\epsilon = 0.01$ method will perform the best in both sense. The non-greedy methods will eventually find the best action, but only choose it with probability $1 - \epsilon + 0.01\epsilon$. In the $\epsilon = 0.01$ case this means the method chooses the best action with probability 0.991, while in the $\epsilon = 0.1$ case, this probability is 0.91.

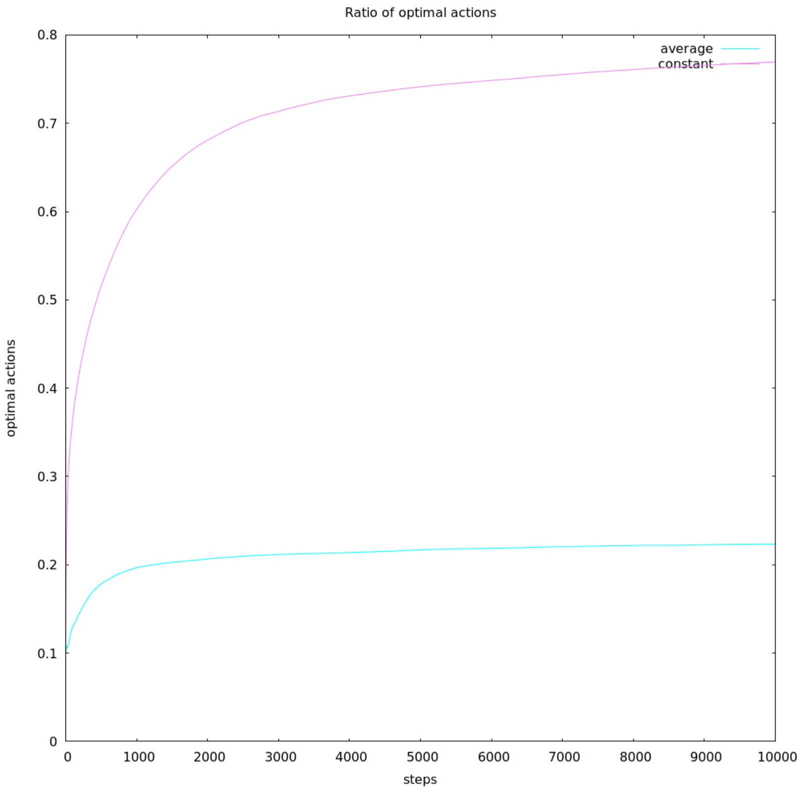
4. *If the step-size parameters, α_n, are not constant, then the estimate Q_n is a weighted average of previously received rewards with a weighting different from that given by (2.6). What is the weighting on each prior reward for the general case, analogous to (2.6), in terms of the sequence of step-size parameters?*

$$\begin{aligned}
Q_{n+1} =& Q_n + \alpha(R_n - Q_n) = \alpha_n R_n + (1-\alpha_n)Q_n \\
=& \alpha_n R_n + (1-\alpha_n)(Q_{n-1} + \alpha_{n-1}(R_{n-1} - Q_{n-1})) = \dots \\
=& \Big(\prod_{j=1}^{n}(1-\alpha_j)\Big)Q_1 + \sum_{i=1}^{n}\Big(\prod_{j=i+1}^{n}(1-\alpha_j)\Big)\alpha_i R_i
\end{aligned}$$

5. *(programming) Design and conduct an experiment to demonstrate the difficulties that sample-average methods have for nonstationary problems. Use a modified version of*

the 10-armed testbed in which all the $q_*(a)$ *start out equal and then take independent random walks (say by adding a normally distributed increment with mean zero and standard deviation 0.01 to all the* $q_*(a)$ *on each step). Prepare plots like Figure 2.2 for an action-value method using sample averages, incrementally computed, and another action-value method using a constant step-size parameter,* $\alpha = 0.1$*. Use* $\epsilon = 0.1$ *and longer runs, say of 10,000 steps.*





6. *Mysterious Spikes. The results shown in Figure 2.3 should be quite reliable because they are averages over 2000 individual, randomly chosen 10-armed bandit tasks. Why, then, are there oscillations and spikes in the early part of the curve for the optimistic method? In other words, what might make this method perform particularly better or worse, on average, on particular early steps?*

   With very high probability, the first 10 choices will cover all the possible actions once. After the 10^{th} action, we have an estimation of the action values based on actually trying them, but the optimal starting value still dominates these estimates. The 11^{th} step is chosen based on the rewards, not on the optimisic starting values, this causes a big spike at step 11. In steps $11 - 20$ (with high probability) we will choose all the actions once, starting with the action that has the highest estimated action value and continuing in this order. After 20 steps, we will have tried all the actions twice and at step 21, we choose an action based on these estimates. This will cause a spike at step 21. The starting values will have less and less effect, it won't be true that in steps $10k - 10(k + 1)$ we will try all the actions once, so these spikes will disappear.