1. *If V changes during the episode, then (6.6) only holds approximately; what would the difference be between the two sides? Let V_t denote the array of state values used at time t in the TD error (6.5) and in the TD update (6.2). Redo the derivation above to determine the additional amount that must be added to the sum of TD errors in order to equal the Monte Carlo error.*

$$\begin{aligned}
G_t - V_t(S_t) &= R_{t+1} + \gamma G_{t+1} - V_t(S_t) + \gamma V_t(S_{t+1}) - \gamma V_t(S_{t+1}) + \gamma V_{t+1}(S_{t+1}) \\
&\quad - \gamma V_{t+1}(S_{t+1}) \\
&= (R_{t+1} + \gamma V_t(S_{t+1}) - V_t(S_t)) + (\gamma G_{t+1} - \gamma V_{t+1}(S_{t+1})) \\
&\quad + (\gamma V_{t+1}(S_{t+1}) - \gamma V_t(S_{t+1})) \\
&= \delta_t + \gamma(G_{t+1} - V_{t+1}(S_{t+1})) + \gamma\alpha\delta_{t+1} \\
&= \delta_t + \gamma\alpha\delta_{t+1} + \gamma\delta_{t+1} + \gamma^2\alpha\delta_{t+2} + \gamma^2(G_{t+2} - V_{t+2}(S_{t+2})) = \dots \\
&= \sum_{k=t}^{T-1} \gamma^{k-t}(1+\alpha)\delta_k
\end{aligned}$$

2. *This is an exercise to help develop your intuition about why TD methods are often more efficient than Monte Carlo methods. Consider the driving home example and how it is addressed by TD and Monte Carlo methods. Can you imagine a scenario in which a TD update would be better on average than a Monte Carlo update? Give an example scenario—a description of past experience and a current state—in which you would expect the TD update to be better. Here's a hint: Suppose you have lots of experience driving home from work. Then you move to a new building and a new parking lot (but you still enter the highway at the same place). Now you are starting to learn predictions for the new building. Can you see why TD updates are likely to be much better, at least initially, in this case? Might the same sort of thing happen in the original scenario?*

   I can't really see it.

   I have lots of experience, so the estimates that are made after I reached the highway are quite accurate, there will be only small updates for those. For the estimate before I reach the highway, the two update rules just give approximately the same. I can't see a big difference before that, either.

3. *From the results shown in the left graph of the random walk example it appears that the first episode results in a change in only V(A). What does this tell you about what happened on the first episode? Why was only the estimate for this one state changed? By exactly how much was it changed?*

   The first episode must have terminated on the left with a reward of 0.

   The other state values didn't change because the error in those cases was $R_{t+1} + \gamma V(S_{t+1}) - V(S_t) = 0 + 1 \cdot 0.5 - 0.5 = 0$.

   $v(A)$ changed by $\alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)) = 0.1(0 + 0 - 0.5) = -0.05$.

4. *The specific results shown in the right graph of the random walk example are dependent on the value of the step-size parameter, α. Do you think the conclusions about which algorithm is better would be affected if a wider range of α values were used? Is there a different, fixed value of α at which either algorithm would have performed significantly better than shown? Why or why not?*

   I don't think a wider range of α values would have a significant effect on the results.

   - **Monte Carlo methods.** The Monte Carlo version with $\alpha = 0.4$ starts to oscillate pretty fast and doesn't seem to decrease. I expect a Monte Carlo method with a bigger α value to oscillate around an error level which is at least as high as the error in the $\alpha = 0.4$ case. The smallest α for the MC method among the examples is 0.1. That is the only MC method which we can't see oscillate. This version does not achieve the performance of the TD methods. A smaller α value would make the learning progress even slower. It's possible that the error using Monte Carlo update with $\alpha =< 0.1$ goes under the TD error eventually, but it would still be pretty slow.
   - **TD methods.** The TD version with $\alpha = 1.5$ plateaued fast at a relatively high error. I expect bigger α values to make the convergence faster, but plateaues at a higher value. The version with $\alpha = 0.5$ is the best among all shown algorithms. A TD method with a smaller α value might reach an even lower error, but the $\alpha = 0.5$ case is already pretty good.

5. *In the right graph of the random walk example, the RMS error of the TD method seems to go down and then up again, particularly at high α's. What could have caused this? Do you think this always occurs, or might it be a function of how the approximate value function was initialized?*

   I think what causes this is the following. Suppose our current estimates are the real values and e.g. we reach state E. Depending on our next step, $V_t(E)$ might take a small step in the direction of $V_{t-1}(D)$ or 1. But you can only reach E from D, so $V_t(D)$ will be bigger than $V_{t-1}(D)$. This means that the expected value of $V_{t+1}(E)$ will be closer to 1 than to $V_{t-1}(D)$. This means that the equilibrium for $V_t(E)$ is a bit bigger than the real $V(E)$. The smaller the α is, the closer the equilibrium is to the real value.

   If we have started with a higher initial value for D and E than the true value and a lower initial value for A and B, then I wouldn't expect to see this increase in the error.

6. *In Example 6.2 we stated that the true values for the random walk example are $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$, for states A through E. Describe at least two different ways that these could have been computed. Which would you guess we actually used? Why?*

   A fast method is the following. $v(S) = 0.5 \cdot v(S_l) + 0.5 \cdot v(S_r)$, where S_l is the left neighbor of S and S_r is the right neighbor of S. This means that $v(S) - v(S_l) = v(S_r) - v(S)$. Hence, the numbers $0 \leq v(A) \leq v(B) \leq v(C) \leq v(D) \leq v(E) \leq 1$ are equidistant, so each difference is $\frac{1}{6}$.

   A slightly different method is to write down these five equations and solve the resulting linear equation system.

7. *Design an off-policy version of the TD(0) update that can be used with arbitrary target policy π and covering behavior policy b, using at each step t the importance sampling ratio $\rho_{t:t}$ (5.3).*

   I would use the update rule:

$$V(S_t) \leftarrow V(S_t) + \alpha\rho_{t:t}(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)).$$

8. *Show that an action-value version of (6.6) holds for the action-value form of the TD error $\delta_t = R_{t+1} + Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)$, again assuming that the values don't change from step to step.*

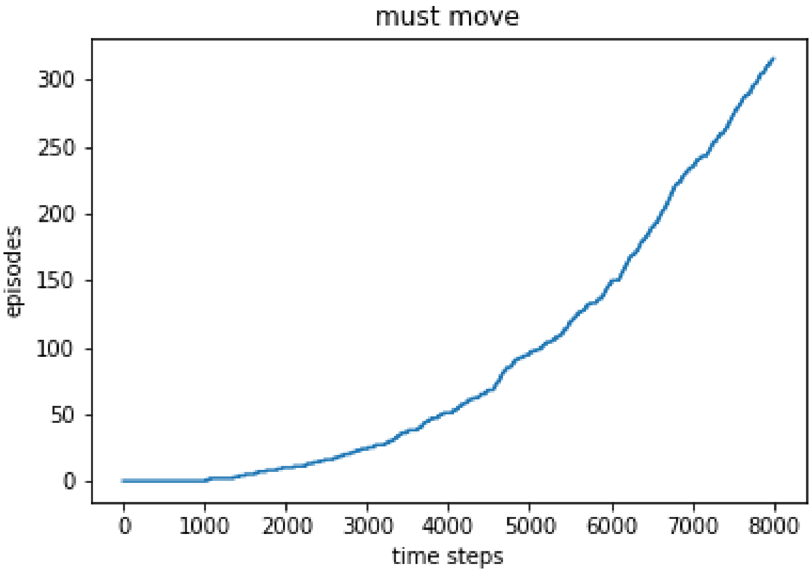
   It's basically the same, we just change $V(S_t)$ to $Q(S_t, A_t)$:

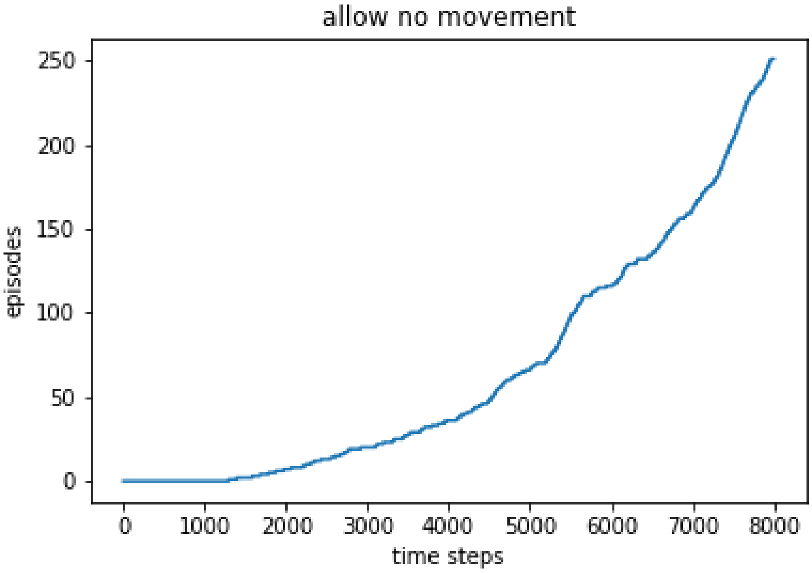
$$\begin{aligned} G_t - Q(S_t, A_t) &= R_{t+1} + \gamma G_{t+1} - Q(S_t, A_t) + \gamma Q(S_{t+1}, A_{t+1}) - \gamma Q(S_{t+1}, A_{t+1}) \\ &= \delta_t + \gamma(G_{t+1} - Q(S_{t+1}, A_{t+1}) = \dots \\ &= \sum_{k=t}^{T-1} \gamma^{k-t}\delta_k \end{aligned}$$

9. *Windy Gridworld with King's Moves. (Programming.) Re-solve the windy gridworld assuming eight possible actions, including the diagonal moves, rather than four. How much better can you do with the extra actions? Can you do even better by including a ninth action that causes no movement at all other than that caused by the wind?*

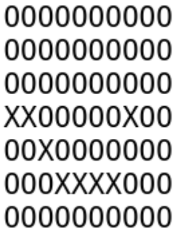
   The ninth action doesn't help, the minimum number of steps is also available with eight options. As there are more possibilities, it makes the learning a bit slower.

   The code can be found at https://github.com/hannagabor/SBRL/blob/master/6.9/windy_gridworld.ipynb.





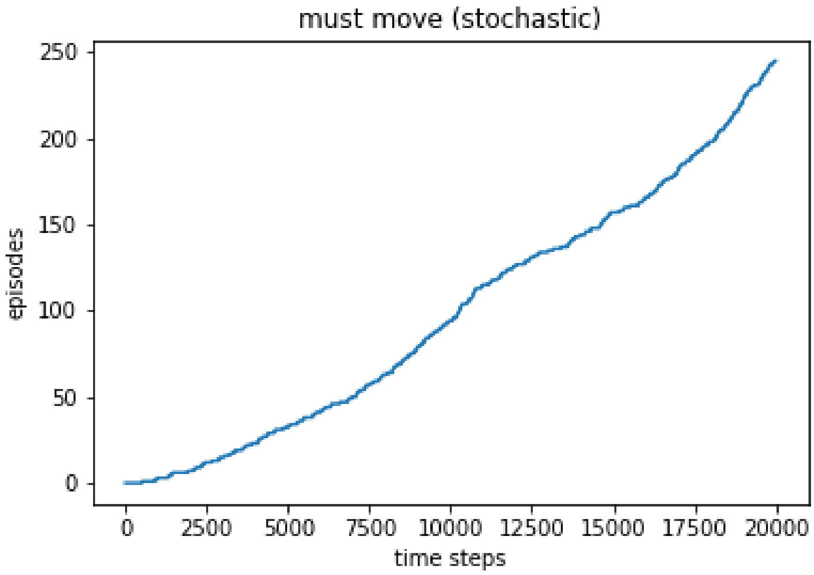
Example trajectory:

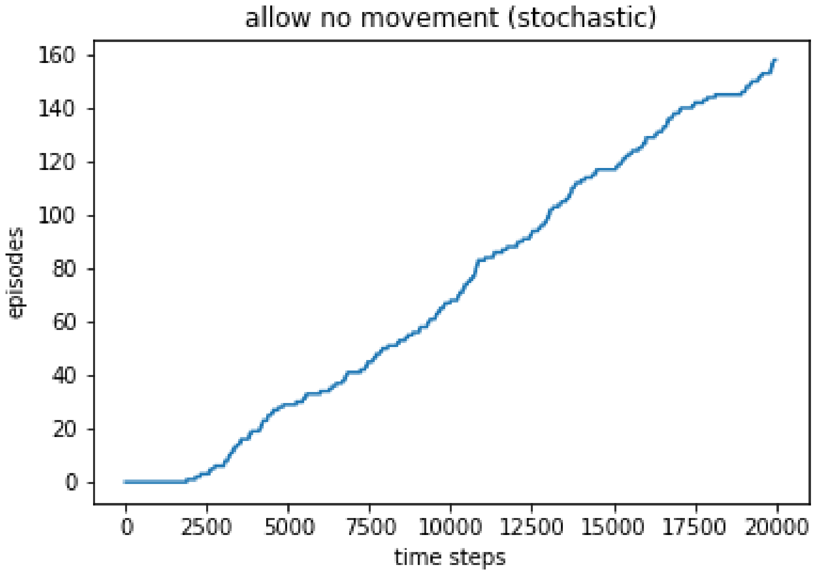


```
0000000000
0000000000
0000000000
XX00000X00
00X0000000
000XXXX000
0000000000
```

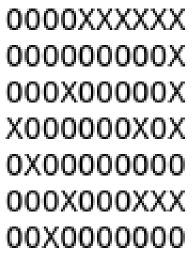
10. *Stochastic Wind. (Programming.) Re-solve the windy gridworld task with King's moves, assuming that the effect of the wind, if there is any, is stochastic, sometimes varying by 1 from the mean values given for each column. That is, a third of the time you move exactly according to these values, as in the previous exercise, but also a third of the time you move one cell above that, and another third of the time you move one cell below that. For example, if you are one cell to the right of the goal and you move left, then one-third of the time you move one cell above the goal, one-third of the time you move two cells above the goal, and one-third of the time you move to the goal.*

    Apparently, this makes the problem much more difficult. The code can be found at `https://github.com/hannagabor/SBRL/blob/master/6.9/windy_gridworld.ipynb`.





An example trajectory:



```
0000XXXXXX
000000000X
000X00000X
X000000X0X
0X00000000
000X000XXX
00X0000000
```

11. *Why is Q-learning considered an off-policy control method?*

    Q reflects the values for the greedy policy based on the current estimates, regardless of the policy that generates the trajectory.

12. *Suppose action selection is greedy. Is Q-learning then exactly the same algorithm as Sarsa? Will they make exactly the same action selections and weight updates?*

Almost. An exception is when S_t equals S_{t+1}. It might happen that before the update $Q(S_t, A_t) = \max_a Q(S_t, a)$ and after the update $Q(S_t, A_t) < Q(S_t, a')$. Then we will choose $A_{t+1} = A_t$ for Sarsa and choose $A_{t+1} = a'$ for Q-learning.

13. *What are the update equations for Double Expected Sarsa with an ϵ-greedy target policy?*

    Let π_1 be the ϵ-greedy policy with regard to Q_1. Then one of the Double Expected Sarsa updates is:

    $$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha(R_{t+1} + \gamma \sum_a \pi_1(a|S_{t+1})Q_2(S_{t+1}, a) - Q_1(S_t, A_t))$$

14. *Describe how the task of Jack's Car Rental (Example 4.2) could be reformulated in terms of afterstates. Why, in terms of this specific task, would such a reformulation be likely to speed convergence?*

    Instead of estimating the value for pairs like (state in the end of the day, action), we could estimate values for "afterstates", that is, for the state after the relocation happens.

    This would be beneficial, as we can arrive to the same afterstate from many different states by adjusting the number of cars we transfer from one location to the other. With estimates for afterstates, if we arrive to a specific afterstate, we immediately get an update for all the states that can appear just before the afterstate.