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Super-duper Thesis Title

Diploma Thesis

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Budapest, 2018

Contents

Introduction

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Chapter 1

Coupon coloring in planar graphs

1.1 ...

We will examine the so-called total domatic number of graphs. Let $G = (V, E)$ be a graph without isolated vertices.

Definition 1.1.1. *$S \subseteq V$ is a total dominating set if every vertex has a neighbor in S. The total domatic number of G is the maximum number of disjoint total dominating sets.*

Sometimes it's more convenient to look at total dominating sets as color classes.

Definition 1.1.2. *A coloring of the vertices is called a k-coupon coloring if every vertex has a neighbor from each color class. The coupon coloring number of G is the maximum k for which a k-coupon coloring exists. The coupon coloring number is denoted by $\chi_c(G)$.*

It turns out that determining the total domatic number (or equivalently the coupon coloring number) of a graph is rather hard.

Theorem 1.1.1. *It's NP-complete to decide whether the total domatic number of a graph is at least* 2.

Proof. ... (https://pdfs.semanticscholar.org/d44c/a18638eadeeb17b69f657cfa06c313646b8c.pdf or for bipartite: https://people.cs.clemson.edu/ goddard/papers/twoTotalDominationAugment.pdf) □

1.2 Degree restrictions

A natural question is whether graphs with an appropriately big minimum degree always have a total domatic number of at least 2.

Theorem 1.2.1. *For every d there exists a graph with minimum degree d and without 2 disjoint total dominating sets.*

Proof. ... □

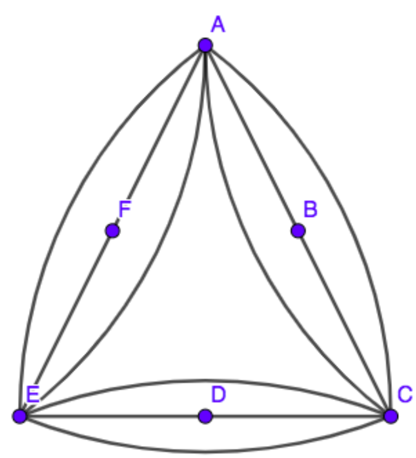
...Other degree stuff (k-regular, maxdeg-mindeg small enough)...

1.3 A conjecture of Goddard and Henning

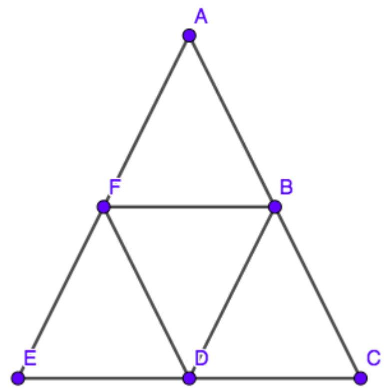
From now on we will focus on 2-coupon colorings and planar graphs. A conjecture of Goddard and Henning is the following.

Conjecture 1.3.1. *If G is a simple triangulated planar graph of order at least 4, then the total domatic number of G is at least 2.*

Remark 1.3.1. *The simplicity of the graph is necessary. Suppose the graph below has a 2-coupon coloring. Then A and C must have different colors, because they are the only neighbors of B. Similarly, C and E must have different colors, as well as E and A. That's a contradiction, since A, C and E form a triangular.*



Remark 1.3.2. *Allowing triangulated disks (i.e. planar graphs with at most one face greater than 3), the conjecture doesn't hold. For example, the graph below doesn't have a 2-coupon coloring from similar reasons as the previous one. We will show later that this graph is a member of a bigger graph family without 2 disjoint dominating sets.*



Theorem 1.3.1. *Let G be a triangulated planar graph. If all the vertices of G have an odd degree, then there exists a coupon coloring with 2 colors.*

Proof. ... □

Theorem 1.3.2. *Outerplanar graphs ...*

Theorem 1.3.3. *Every triangulated graph with a Hamiltonial circle admits 2 disjoint dominating sets.*

Hypergraph connection...

Chapter 2

Restricted 2-factors

2.1 Connection

2.2 Barnette's conjecture

Conjecture 2.2.1. *Every 3-connected cubic planar bipartite graph is Hamiltonian.*