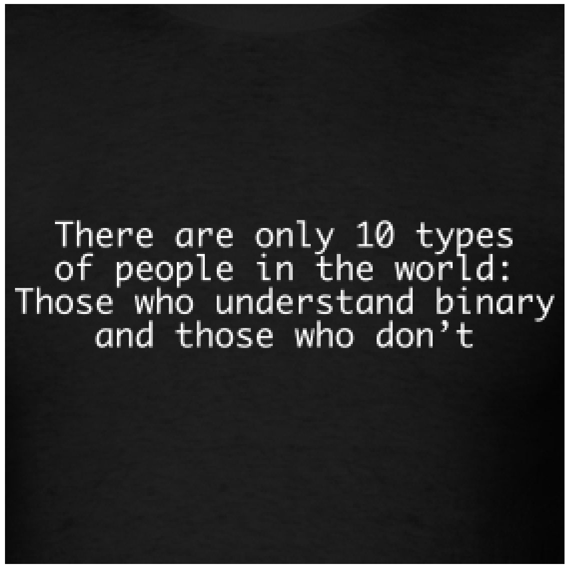
Homework 2: Binary!



Despite being a great party trick to impress all your friends with your computer speaking abilities, binary is critical to your study and knowledge of computer science… it's the language of computers! It's important that we understand this early.

We said humans interpret numbers in decimal, or base 10. Let's look at 365 for example:

$$365 = 5 * 10^0 + 6 * 10^1 + 3 * 10^2 = 5 + 60 + 300$$

Since computers can only interpret two states on a wire -- high or low, electrons or no electrons -- we need to use the binary, or base 2, representation of numbers.

If I want to convert the number 365 to binary, I first need to decide how many bits I need. As we said in class, the range of numbers you can represent with a binary number depends on the number of bits you use. If we use 1 bit, we can represent 2^1 = 2 numbers, 0 and 1. If we use 2 bits, we can represent 2^2 = 4 numbers (0, 1, 2, and 3). If we use 10 bits, we can represent 2^{10} = 1024 numbers (0, 1, 2, … , 1023). And so on. So how many bits do we need for 365? 9! Because 2^9 = 512, and with 9 bits we can represent numbers 0 through 511. So what powers of 2 add up to 365? Well, 256 + 64 + 32 + 8 + 4 + 1. So we write 365 as:

$$\begin{gathered} 101101101 \\ = 1 * 2^8 + 0 * 2^7 + 1 * 2^6 + 1 * 2^5 + 0 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 \\ = 256 + 0 + 64 + 32 + 0 + 8 + 4 + 0 + 1 \\ = 365 \end{gathered}$$

If you don't feel great about this yet, try watching [this "Introduction to number systems and binary"]() video from Khan Academy.

Part 00: How many bits do I need?

For each decimal number below, write the number of bits required to represent that number.

1. 7
   $\log_2(7) = 2.807\ldots \rightarrow 3$ bits
2. 3,894
   $\log_2(3894) = 11.927\ldots \rightarrow 12$ bits
3. 256
   $\log_2(256) = 8$, but with 8 bits we represent numbers 0 to 255, so → 9 bits
4. 24
   $\log_2(24) = 4.584\ldots \rightarrow 5$ bits
5. 1027
   $\log_2(1027) = 10.004\ldots \rightarrow 11$ bits

Part 01: Converting binary to decimal.

Convert each binary number to its decimal form. Show all of your work. (I know how to use Google, too.)

1. 1100111
   $1*2^6 + 1*2^5 + 0*2^4 + 0*2^3 + 1*2^2 + 1*2^1 + 1*2^0$
   $= 64 + 32 + 4 + 2 + 1$
   $= 103$
2. 101
   $1*2^2 + 0*2^1 + 1*2^0$
   $= 4 + 1$
   $= 5$
3. 00000000
   $= 0$
4. 11111111
   $1*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 1*2^2 + 1*2^1 + 1*2^0$
   $= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$
   $= 255$
5. 1
   $1*2^0$
   $= 1$

Part 10: Converting decimal to binary.

Convert each decimal number to its binary form. Show all of your work. (I still know how to use Google.)

1. 75
   1001011
2. 237
   11101101
3. 1860
   11101000100
4. 45,363
   1011000100110011
5. 5
   101

Part 11: Counting in binary.

Count to 20 in binary!

We need 5 bits to represent numbers 0 to 20.
00000 (0)
00001 (1)
00010 (2)
00011 (3)
00100 (4)
00101 (5)
00110 (6)
00111 (7)
01000 (8)
01001 (9)
01010 (10)
01011 (11)
01100 (12)
01101 (13)
01110 (14)
01111 (15)
10000 (16)
10001 (17)
10010 (18)
10011 (19)
10100 (20)