EPR Paradox and Bell's Theorem

Reference: Griffiths 12.1–12.2
McIntyre 4.1

By now, we can all agree on what quantum mechanics says. But what does it mean about reality and the future of the theory?

EPR thought experiment (originally proposed by Einstein, Podolsky & Rosen in 1935)
A simplified version is due to David Bohm, which I present here.

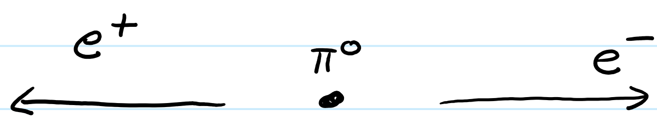
Consider a spin 0 particle which decays into two spin $\frac{1}{2}$ particles.

$$\pi^0 \to e^+ e^- .$$

The pion has spin 0, so its spin state is $|0,0\rangle$. Since angular momentum is conserved, the electron/positron must be in singlet configuration.

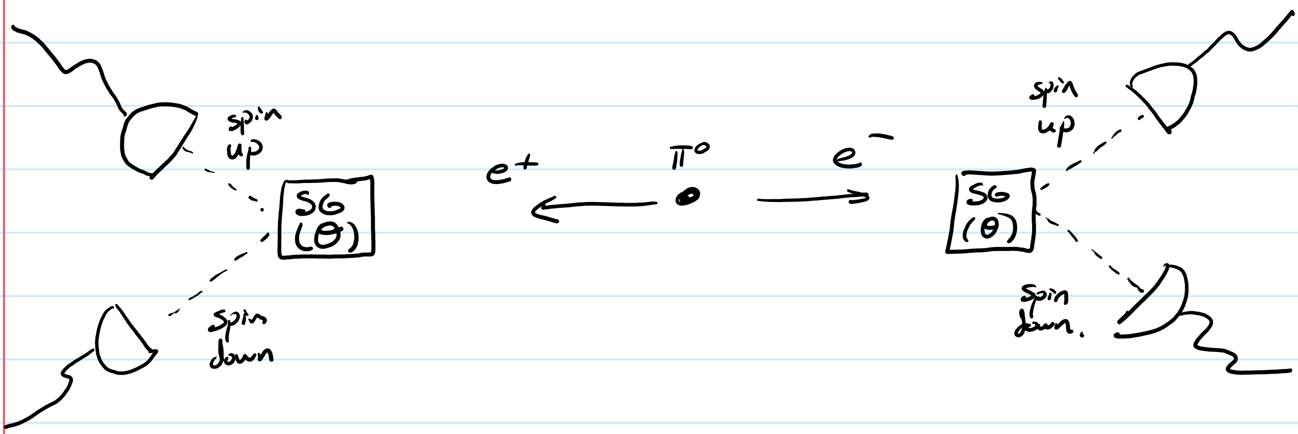
$$|0,0\rangle = \frac{1}{\sqrt{2}}\left[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,\right].$$

Suppose you have a π^0 at rest in your lab. It decays into e^+e^- and they fly off back to back.



We then (optionally) can send each one through a Stern-Gerlach apparatus to measure its spin. We assume that any S-G apparatuses can be set to any desired orientation. Repeat this many times.

Experiment II Ensure that both S-G apparatuses are at the same angle θ. Use 4-detectors to measure the spin of each particle.



Result: 50% of the time the electron is spin up
50% of the time the electron is spin down,
and same for the positron.
BUT the measurements are anti-correlated.
everytime the e^- is spin up, the positron is spin down.

Trial	Spin (e^-)	Spin (e^+)
1	up	down
2	up	down
3	down	up
4	up	down
5	down	up
6	down	up.

These patterns persist no matter which orientation the SG apparatus are set to (as long as their orientations are the same).

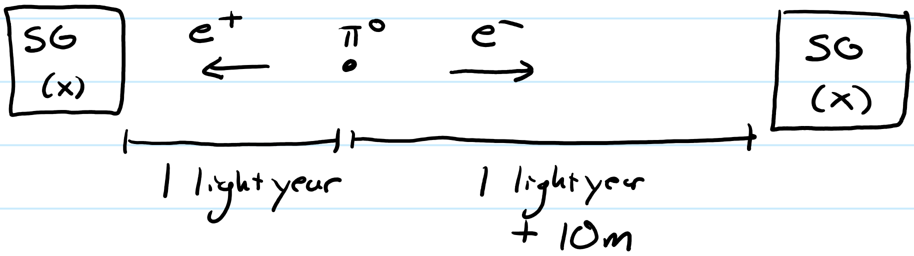
The spins are "entangled"

EPR Paradox and Bell's Theorem

Einsten - Podolsky and Rosen then made the following argument:

If I set the two detectors very far apart and I measure the positron's spin. I could then predict the measurement of the e^- measurement with 100% accuracy. EPR argued that if I can predict something with 100% accuracy, it must be a real property of the particle

As an extreme example:



According to the experiment, if I measure the positron to have spin $|+\rangle_x$, I know for sure the electron will have spin $|-\rangle_x$. This is true even though there's no way the first measurement can infuence the second since a signal would take 2 years to get there.

EPR would say: "Since I know the outcome of the measurement, the e^- had that property prior to measurement. Because the first measurement cannot influence the second, it must have had that property all along."

QM would say: "The e^- does not have any definite properties before you measure it. The measurement itself creates the property. Once I measure the positron, the e^- is instantaneously affected even if it is halfway across the universe."

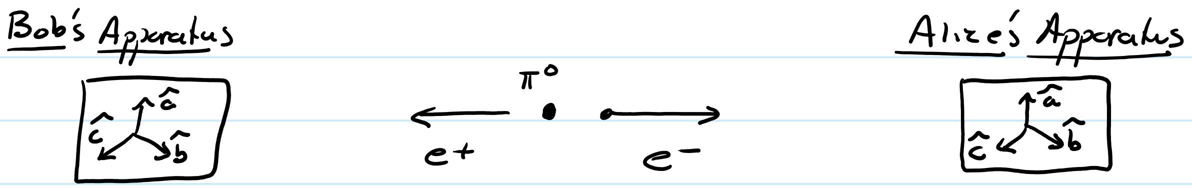
This "spooky action at a distance" was unacceptable to Einstein.

EPR Paradox and Bell's Theorem

Experiment II - aka the Alain Aspect Experiment

Done in 1980-1982

The setup is as above, but now the two SG apparatuses can be set to one of 3 angles: $\hat{a} = 0°$, $\hat{b} = 120°$, $\hat{c} = 240°$
the SG apparatuses are set randomly and independently.



Before each measurement, Bob & Alice each roll a 6-sided die. they don't tell each other what they got. Depending on the results of the die roll, they set the SG apparatus.

Roll	Angle
1 or 2	$\hat{a}$ (0°)
3 or 4	$\hat{b}$ (120°)
5 or 6	$\hat{c}$ (240°)

They then collect data which might look like this

Bob's data

Trial	Angle	Spin
1	$\hat{a}$	+
2	$\hat{c}$	−
3	$\hat{a}$	−
4	$\hat{b}$	+
5	$\hat{c}$	+

Alice's data

Trial	Angle	Spin
1	$\hat{b}$	+
2	$\hat{c}$	+
3	$\hat{b}$	−
4	$\hat{b}$	−
5	$\hat{a}$	−

Notice that when Bob & Alice have their apparatuses aligned, they get opposite results (that's consistent with experiment I).

Explanation from QM

$$|\chi_0\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right] = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right]$$

Now, Bob measures spin in direction θ_1 and Alice measures spin along direction θ_2. What are the possible results and the corresponding probabilities?

$$P_{++} = \left|\left(\langle +n_1|\langle +n_2|\right)\left(|\chi_0\rangle\right)\right|^2$$

Recall [HW#5]
$$|+n_1\rangle = \begin{pmatrix}\cos(\theta_1/2)\\ \sin(\theta_1/2)e^{i\phi}\end{pmatrix} \qquad |+n_2\rangle = \begin{pmatrix}\cos(\theta_2/2)\\ \sin(\theta_2/2)e^{i\phi}\end{pmatrix}$$

$$P_{++} = \left|\frac{1}{\sqrt{2}}\left[\langle +n_1|\uparrow\rangle\langle +n_2|\downarrow\rangle - \langle +n_1|\downarrow\rangle\langle +n_2|\uparrow\rangle\right]\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}\left[\cos(\theta_1/2)\sin(\theta_2/2)e^{-i\phi} - \sin(\theta_1/2)\cos(\theta_2/2)e^{-i\phi}\right]\right|^2$$
$$= \frac{1}{2}\left[\sin\left(\frac{\theta_2-\theta_1}{2}\right)\right]^2 = \frac{1}{2}\sin^2\left(\frac{\Delta\theta}{2}\right)$$

where $\Delta\theta$ = angle between Bob's & Alice's SG apparatus.

The others are done similarly

$$P_{--}(\Delta\theta) = \frac{1}{2}\sin^2\left(\frac{\Delta\theta}{2}\right)$$

$$P_{+-}(\Delta\theta) = P_{-+}(\Delta\theta) = \frac{1}{2}\cos^2\left(\frac{\Delta\theta}{2}\right)$$

Note that if $\Delta\theta = 0$ and both SG apparatuses are set to the same angle, $P_{+-}(0) = P_{-+}(0) = 0$ which is consistent with experiment II.

Explanation from a local hidden variable theory

How would a realist explain the randomness in these experiments? They would say there are other properties of the particle which we just don't know about that are determining these outcomes.

Think of these "plans" or "instruction sets" like genes in biology. Before we knew about genes, whether someone was born with blue eyes or brown eyes seemed random.

Now that we know about genes, we know it's not completely random, there were these hidden instructions which we didn't know about before. Could a similar thing be going on here? Do the particles have "plans" or "instructions" which we just don't know about & need a better theory than QM to determine?

A realist would say "there are actually several types of particles.. each has a different set of instructions when it encounters each apparatus." An example of such a set of plans is: $(\hat{a}_+, \hat{b}_-, \hat{c}_+)$ meaning a particle with these instructions would be measured to have spin up when SG is set to $\hat{a}$, spin down if SG-apparatus is set to $\hat{b}$, and spin up when SG is set to $\hat{c}_+$ Each particle must have a set of instructions and the instructions must be opposite for the two particles to allow for the anti-correlations of experiment I. There are 8 possible sets:

Population	Particle 1	Particle 2
N_1	$(\hat{a}+, \hat{b}+, \hat{c}+)$	$(\hat{a}-, \hat{b}-, \hat{c}-)$
N_2	$(\hat{a}+, \hat{b}+, \hat{c}-)$	$(\hat{a}-, \hat{b}-, \hat{c}+)$
N_3	$(\hat{a}+, \hat{b}-, \hat{c}+)$	$(\hat{a}-, \hat{b}+, \hat{c}-)$
N_4	$(\hat{a}+, \hat{b}-, \hat{c}-)$	$(\hat{a}-, \hat{b}+, \hat{c}+)$
N_5	$(\hat{a}-, \hat{b}+, \hat{c}+)$	$(\hat{a}+, \hat{b}-, \hat{c}-)$
N_6	$(\hat{a}-, \hat{b}+, \hat{c}-)$	$(\hat{a}+, \hat{b}-, \hat{c}+)$
N_7	$(\hat{a}-, \hat{b}-, \hat{c}+)$	$(\hat{a}+, \hat{b}+, \hat{c}-)$
N_8	$(\hat{a}-, \hat{b}-, \hat{c}-)$	$(\hat{a}+, \hat{b}+, \hat{c}+)$

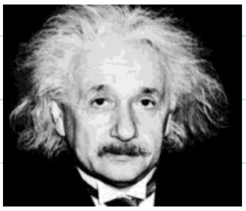
So a realist would explain Bob's & Alice's data as follows:

Bob's data			Alice's data			"Instruction" Type
Trial	Angle	Spin	Trial	Angle	Spin	
1	$\hat{a}$	+	1	$\hat{b}$	+	3, 4, 5, 6
2	$\hat{c}$	−	2	$\hat{c}$	+	any
3	$\hat{a}$	−	3	$\hat{b}$	+	1, 2, 7, 8
4	$\hat{b}$	+	4	$\hat{b}$	−	any
5	$\hat{c}$	+	5	$\hat{a}$	−	1, 3, 6, 8

So who is right? Can we know if the "spooky" action at a distance is really the way nature behaves? Or might the particles have hidden properties which determine these measurements? Or, is this an unknowable question to leave to philosophers?

"We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it."

-A. Pais



"I am convinced the He (God) does not play dice"

- A. Einstein

The standard "Copenhagen" interpretation advocated by Bohr was what I've been teaching in this class.

"Observations not only disturb what has to be measured, they produce it….We compel [the electron] to assume a definite position…. We ourselves produce the results of measurements." -P. Jordan

"...Nor is it our business to prescribe to God how He should run the world" - N. Bohr

Finally, there are those "agnostics" who think asking these sorts of questions are unimportant.

"One should no more rack one's brain about the problem of whether something one cannot know anything about exists all the same, than about the ancient question of how many angels are able to sit on the point of a needle." -W. Pauli

EPR Paradox and Bell's Theorem

J.S. Bell 1928-1990

The truly remarkable observation by John Bell was that this question can be answered experimentally.

The argument is amazingly simple!

Take experiment II measure the probability that both particles are measured to have the same spin or opposite spin, regardless of the direction of the SG apparatus. So the data table acquires an extra column.

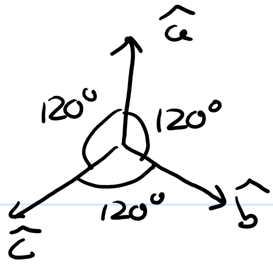
Bob's data			Alice's data			Spin sign
Trial	Angle	Spin	Trial	Angle	Spin	same or opposite?
1	$\hat{a}$	+	1	$\hat{b}$	+	same
2	$\hat{c}$	−	2	$\hat{c}$	+	opposite
3	$\hat{a}$	−	3	$\hat{b}$	+	opposite
4	$\hat{b}$	+	4	$\hat{b}$	−	opposite
5	$\hat{c}$	+	5	$\hat{a}$	−	opposite.

QM makes a definite prediction here.

$$P_{same}(\Delta\theta) = P_{++}(\Delta\theta) + P_{--}(\Delta\theta) = \frac{1}{2}\sin^2\left(\frac{\Delta\theta}{2}\right) + \frac{1}{2}\sin^2\left(\frac{\Delta\theta}{2}\right) = \sin^2\left(\frac{\Delta\theta}{2}\right)$$

In the experiment II, the SG apparatuses are oriented randomly.

EPR Paradox and Bell's Theorem



possibilities:

Bob	Alice	$\Delta\theta$	Probability
$\hat{a}$	$\hat{a}$	0	1/9
$\hat{a}$	$\hat{b}$	120°	1/9
$\hat{a}$	$\hat{c}$	120°	1/9
$\hat{b}$	$\hat{a}$	120°	1/9
$\hat{b}$	$\hat{b}$	0°	1/9
$\hat{b}$	$\hat{c}$	120°	1/9
$\hat{c}$	$\hat{a}$	120°	1/9
$\hat{c}$	$\hat{b}$	120°	1/9
$\hat{c}$	$\hat{c}$	0°	1/9

Average over all possibilities:

$$P_{same} = \frac{3}{9}\cdot\left[P_{++}(0°) + P_{--}(0°)\right] + \frac{6}{9}\left[P_{++}(120°) + P_{--}(120°)\right]$$

$$P_{same} = \frac{1}{3}\cdot\sin^2\left(\frac{0°}{2}\right) + \frac{2}{3}\cdot\sin^2\left(\frac{120°}{2}\right)$$

$$= 0 + \frac{2}{3}\cdot\left(\frac{\sqrt{3}}{2}\right)^2 = 1/2$$

So, averaged over all trials, QM predicts 50% same sign spin
50% opposite sign spin.

EPR Paradox and Bell's Theorem

Hidden Variable Predictions

Population	Particle 1	Particle 2
N_1	$(\hat{a}+, \hat{b}+, \hat{c}+)$	$(\hat{a}-, \hat{b}-, \hat{c}-)$
N_2	$(\hat{a}+, \hat{b}+, \hat{c}-)$	$(\hat{a}-, \hat{b}-, \hat{c}+)$
N_3	$(\hat{a}+, \hat{b}-, \hat{c}+)$	$(\hat{a}-, \hat{b}+, \hat{c}-)$
N_4	$(\hat{a}+, \hat{b}-, \hat{c}-)$	$(\hat{a}-, \hat{b}+, \hat{c}+)$
N_5	$(\hat{a}-, \hat{b}+, \hat{c}+)$	$(\hat{a}+, \hat{b}-, \hat{c}-)$
N_6	$(\hat{a}-, \hat{b}+, \hat{c}-)$	$(\hat{a}+, \hat{b}-, \hat{c}+)$
N_7	$(\hat{a}-, \hat{b}-, \hat{c}+)$	$(\hat{a}+, \hat{b}+, \hat{c}-)$
N_8	$(\hat{a}-, \hat{b}-, \hat{c}-)$	$(\hat{a}+, \hat{b}+, \hat{c}+)$

$N_{TOT} = N_1 + N_2 + N_3 + \ldots N_8$ = Total # of trials.

Consider a pair with instruction set #2. The possible outcomes are:

All are equally likely since SG apparatus are set randomly.

particle 1	particle 2	spin same or opp?
$\hat{a}+$	$\hat{a}-$	opp
$\hat{a}+$	$\hat{b}-$	opp
$\hat{a}+$	$\hat{c}+$	same
$\hat{b}+$	$\hat{a}-$	opp
$\hat{b}+$	$\hat{b}-$	opp
$\hat{b}+$	$\hat{c}+$	same
$\hat{c}-$	$\hat{a}-$	same
$\hat{c}-$	$\hat{b}-$	same
$\hat{c}-$	$\hat{c}+$	opp.

For instruction set #2: $P_{++}(2) + P_{--}(2) = 4/9$

You can do the same for all other instruction sets

$$P_{++}(i) + P_{--}(i) = \begin{cases} 0 & (\text{set } \#1 \text{ or } \#8) \\ 4/9 & (\text{set } \#2 - \#7) \end{cases}$$

Averaged over all instruction sets:

probability of particles to have instruction set #1.

$$P(\text{same}) = \frac{N_1}{N_{TOT}} \cdot 0 + \frac{N_2}{N_{TOT}} \cdot \frac{4}{9} + \frac{N_3}{N_{TOT}} \cdot \frac{4}{9} + \cdots \frac{N_7}{N_{TOT}} \cdot \frac{4}{9} + \frac{N_8}{N_{TOT}} \cdot 0$$

$$\boxed{P(\text{same}) = \frac{4}{9}\Bigg[\underbrace{\frac{N_2+N_3+N_4+N_5+N_6+N_7}{N_{TOT}}}_{<1}\Bigg] \leq \frac{4}{9}}$$

predictions from Hidden Variables.

since $N_{TOT} = \sum_i N_i$

So: the predictions are: $P(\text{same}) = \frac{1}{2}$ (QM)

$P(\text{same}) \leq \frac{4}{9}$ (EPR)

"Hidden variables"

The inequality $P(\text{same}) \leq \frac{4}{9}$ is called a "Bell Inequality".

The experiments show a conclusive result. $P(\text{same}) > \frac{4}{9}$ which violates Bells Inequality. The EPR explanation is ruled out.

Bells Theorem: Quantum Mechanics is incompatible with any local hidden variable theory.

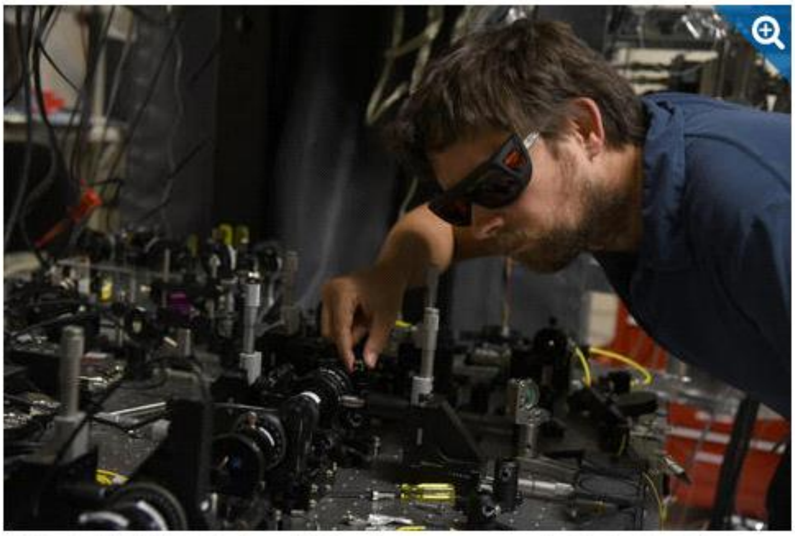
This means that the particles do not have plans or instructions. This "spooky" action at a distance is real. QM is inherently non-local.

NIST Team Proves 'Spooky Action at a Distance' is Really Real

November 10, 2015

BOULDER, Colo.—Einstein was wrong about at least one thing: There are, in fact, "spooky actions at a distance," as now proven by researchers at the National Institute of Standards and Technology (NIST).

Einstein used that term to refer to quantum mechanics, which describes the curious behavior of the smallest particles of matter and light. He was referring, specifically, to entanglement, the idea that two physically separated particles can have correlated properties, with values that are uncertain until they are measured. Einstein was dubious, and until now, researchers have been unable to support it with near-total confidence.



NIST physicist Krister Shalm with the photon source used in the Bell test that strongly supported a key prediction of quantum mechanics: There are in fact spooky actions at a distance.
Credit: J. Burrus/NIST