1) Show that $m(a+bX) = a + b \cdot m(X)$

$$m(a+bX) = \frac{1}{N} \cdot \sum_{i=1}^{N} (a+bX)$$

$$m(a+bX) = \frac{1}{N} \cdot \left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} bX \right)$$

$$m(a+bX) = \frac{1}{N} \sum_{i=1}^{N} a + \frac{1}{N} \sum_{i=1}^{N} bX$$

$$m(a+bX) = (a+b) \left[\frac{1}{N} \sum_{i=1}^{N} X \right]$$

$$m(a+bX) = (a+b) \cdot mX$$

2) Show That $cov(X, a+bY) = b \cdot cov(X, Y)$

$$cov(X, a+bY) = \frac{1}{N} \sum_{i=1}^{N} (x - m(X))(y - m(Y))$$

$$cov(X, a+bY) = \frac{1}{N} \cdot \sum_{i=1}^{N} (x - m(X)) \cdot (a+by - (a + b \cdot m(Y)))$$

$$cov(X, a+bY) = \frac{1}{N} \sum_{i=1}^{N} (x - m(X))(by - b \cdot m(Y))$$

$$cov(X, a+bY) = b \left[\frac{1}{N} \sum_{i=1}^{N} (x - m(X))(y - m(Y)) \right]$$

$$cov(X, a+bY) = b \cdot cov(X, Y)$$

3) Show that $cov(a+bX, a+bX) = b^2 cov(X,X)$, specifically that $cov(X,X) = s^2$

Using the answer from the second proof:

Found that $cov(X, a+bY) = b \cdot cov(X,Y)$

If $X=Y$, $cov(X, a+bX) = b \cdot cov(X,X)$

Now, start replacing $a+bX$ for X.

$$cov(a+bX, a+bX) = b \cdot cov(X, a+bX)$$

Substitute in $cov(X, a+bX) = b \cdot cov(X,X)$

$$cov(a+bX, a+bX) = b \cdot (b \cdot cov(X,X))$$

$$cov(a+bX, a+bX) = b^2 cov(X,X)$$

Now, prove $cov(X,X) = s^2$

$$cov(X,X) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))(x_i - m(X))$$

$$cov(X,X) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))^2$$

$$cov(X,X) = s^2$$

SO!

$$\boxed{cov(a+bX, a+bX) = b^2 s^2}$$