**Structural Econometrics in Labor and IO.**
**Problem Set: Preliminaries.**

## Description

The aim of this problem set is to refresh (or make you familiar with) some basic concepts – notably GMM estimation, numerically solving systems of nonlinear equations, and the intuition for a contraction mapping - that will be essential in subsequent sessions and problem sets (and in large parts of the literature).

Main reference: Berry, Steven T. (1994), "Estimating Discrete Choice Models of Product Differentiation," *Rand Journal of Economics*, 25 (2), 242-262.

## Problems

*Together with your answers, please submit your computer code for the questions below.*

1. **A simple demand estimation example.** We assume consumer $i$ chooses one unit of product $j \in J$ or an outside good (e.g. no purchase) to obtain utility

$$u_{ijt} = x_{jt}\beta + \alpha p_{jt} + \xi_{jt} + \varepsilon_{ijt} = \delta_{jt} + \varepsilon_{ijt} \tag{1}$$

where $(x_{jt}, p_{jt})$ are observable characteristics and price, $\xi_{jt}$ is an unobservable characteristic, and an idiosyncratic error term $\varepsilon_{ijt}$ assumed i.i.d. extreme value type 1. The utility of the outside good is normalized such that $\delta_{0t} = 0$. The assumption of utility-maximizing consumers and the distribution of $\varepsilon_{ijt}$ yields the logit choice probabilities:

$$s_{jt}(\delta_t) = \frac{\exp(\delta_{jt})}{1 + \sum_{l=1}^{J} \exp(\delta_{lt})}. \tag{2}$$

(a) Simulate product-level data based on the described model assuming the following

- $J = 10$ products are sold in $T = 25$ markets (size $L_t = 1$) by single-product firms.
- Two observable product characteristics $x_{jt} = (1, x_{jt}^1)$, with $x_{jt}^1 \sim U(1,2)$.
- Marginal cost $c_{jt} = x_{jt}\gamma_1 + w_{jt}\gamma_2 + \omega_{jt}$.
- Three observable cost shifters $w_{jt} = (w_{jt}^1, w_{jt}^2, w_{jt}^3)$, all i.i.d. $U(0,1)$.
- Marginal cost parameters: $\gamma_1 = (0.7, 0.7)$ and $\gamma_2 = (1,1,1)$.
- Unobserved demand and cost characteristic $(\xi_{jt}, \omega_{jt}) \sim N(0, \sigma_c)$ with $\sigma_c = \begin{bmatrix} 1 & 0.7 \\ 0.7 & 1 \end{bmatrix}$.
- Assume perfect competition on price so that $p_{jt} = c_{jt}$.
- Preference parameters $\beta = (2,2), \alpha = -2$.

! To simulate the data, you must code a function computing a $(T \times J) \times 1$ vector containing market shares using equation (2).

(b) The simulated data at hand, forget parameter values and, following Berry (1994),

- estimate $\{\alpha, \beta\}$ using OLS and report the results.
- estimate $\{\alpha, \beta\}$ by GMM using as instrumental variables the observable characteristics $x_{jt}$ and cost shifters $w_{jt}$, and report the results.

2. **Solving for a static industry equilibrium.** Now consider an imperfectly competitive industry, that is $p_{jt} \neq c_{jt}$. Take the market share equation, cost and demand parameters from above as given. Assume that single-product firms j maximize profits given by

$$\pi_{jt} = (p_{jt} - c_{jt}) s_{jt} L_t, \tag{3}$$

where $c_{jt}$ are marginal cost and $L_t$ market size, so that the system of FOC for a Nash equilibrium:

$$s_{jt} + (p_{jt} - c_{jt}) \frac{\partial s_{jt}}{\partial p_{jt}} = 0 \tag{4}$$

With multi-product firms, it is useful to write expression (4) in vector notation as $s_t + \Delta_t(p_t - c_t) = 0$, where $\Delta_t(j,k)$ denotes a diagonal matrix of own-price derivatives and off-diagonal elements according to market structure. With single-product firms, off-diagonal elements are equal to zero so that $\Delta_t$ can be collapsed to a vector. If marginal cost are known, we obtain the supply side by solving the system for $c_t$:

$$p_t + \Delta_t^{-1} s_t = c_t \tag{5}$$

For single-product firms, $\partial s_{jt}/\partial p_{jt}$ is given by $\alpha s_{jt}(1 - s_{jt})$.

(a) Generate a $(T \times J) \times 1$ vector $\Delta$ containing market share derivatives with respect to own-price price, and

(b) compute a $(T \times J) \times 1$ vector $p$ containing (Bertrand-Nash) equilibrium prices, using equation (5) and Matlab's root-finding function *fsolve*, and report the mean, minimum, and maximum industry price.

3. **Contraction Mapping.** Newton's method is an important tool in nonlinear optimization.[1] It is used to find roots of a function $f(x)$. Write a function finding the number $\sqrt{a}$ by iterating $x_{t+1} = x_t - \frac{f(x_t)}{f'(x_t)}$. Hint: $\sqrt{a}$ is the positive root of $f(x) = a - x^2$.

[1]By the Contraction Mapping Principle (Banach Fixed Point Theorem) this method is a contraction under the conditions that 1) $f(x)$ has a continuous second derivative, 2) $f'(x) \neq 0 \; \forall \; x \in \mathbb{R}$, and 3) a $q \in (0,1)$ exists such that $|f(x)f''(x)| \leq q|f'(x)|^2 \; \forall \; x \in \mathbb{R}$.