Filters

Notes on passive electric filter circuits

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June 2, 2020

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1 RC Low Pass Filter

A simple low pass filter can be formed from a resistor and a capacitor in series.

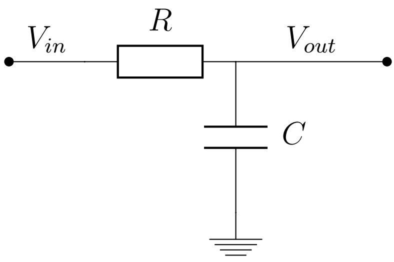

Figure 1: Circuit diagram for a RC low pass filter.

We can see from figure 1, that the output voltage is the same as the voltage across the capacitor. So we have

$$\begin{aligned} V_{in} &= V_R + V_C \\ &= IR + V_{out} \end{aligned} \tag{1.1}$$

Using the fact that the current through the RC section of the circuit is given by

$$I = \frac{V_{in}}{R - iX_C} \tag{1.2}$$

Leading to the output voltage being

$$\begin{aligned} V_{out} &= V_{in} - IR \\ &= V_{in} - \frac{V_{in}R}{R - iX_C} \\ &= V_{in}\left[1 - \frac{V_{in}R}{R - iX_C}\right] \end{aligned} \tag{1.3}$$

which leads to a ratio of the output voltage to the input voltage of

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= 1 - \frac{R}{R - iX_C} \\ &= \frac{-iX_C}{R - iX_C} \\ &= \frac{-iX_C(R + iX_C)}{R^2 + {X_C}^2} \qquad (1.4)\end{aligned}$$

Now if we let

$$u = \frac{R}{X_c} = \omega RC \qquad (1.5)$$

we can see that $R = uX_C$, and putting this in equation 1.4 leads to

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= \frac{-i{X_C}^2(u + i)}{u^2{X_C}^2 + {X_C}^2} \qquad (1.6) \\ &= \frac{1 - iu}{1 + u^2} \qquad (1.7)\end{aligned}$$

Summary

We looked at the classic example of a low pass RC filter circuit, and discovered the relationship between the voltage into the filter and the voltage out of the filter is given by

$$\frac{V_{out}}{V_{in}} = \frac{1 - iu}{1 + u^2}$$

where $u = \frac{R}{X_C} = \omega RC$.

We can work out the magnitude and the phase angle of the attenuation through the filter as follows.

$$\left|\frac{V_{out}}{V_{in}}\right| = \sqrt{\frac{1-iu}{1+u^2}\frac{1+iu}{1+u^2}} = \frac{\sqrt{1+u^2}}{1+u^2} = \frac{1}{\sqrt{1+u^2}} \tag{1.8}$$