
FILTERS

NOTES ON PASSIVE ELECTRIC FILTER CIRCUITS

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Contents

1	RC Low Pass Filter	2
1.1	Filter Attenuation	2

List of Tables

List of Figures

1	Circuit diagram for a RC low pass filter.	2
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1 RC Low Pass Filter

1.1 Filter Attenuation

A simple low pass filter can be formed from a resistor and a capacitor in series.

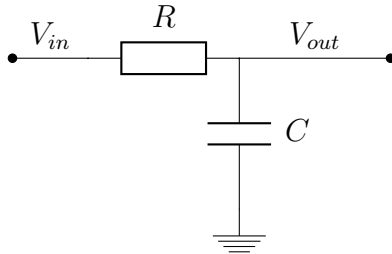


Figure 1: Circuit diagram for a RC low pass filter.

We can see from figure 1, that the output voltage is the same as the voltage across the capacitor. So we have

$$\begin{aligned} V_{in} &= V_R + V_C \\ &= IR + V_{out} \end{aligned} \tag{1.1}$$

Using the fact that the current through the RC section of the circuit is given by

$$I = \frac{V_{in}}{R - iX_C} \tag{1.2}$$

Leading to the output voltage being

$$\begin{aligned} V_{out} &= V_{in} - IR \\ &= V_{in} - \frac{V_{in}R}{R - iX_C} \\ &= V_{in} \left[1 - \frac{V_{in}R}{R - iX_C} \right] \end{aligned} \tag{1.3}$$

which leads to a ratio of the output voltage to the input voltage of

$$\begin{aligned}
 \frac{V_{out}}{V_{in}} &= 1 - \frac{R}{R - iX_C} \\
 &= \frac{-iX_C}{R - iX_C} \\
 &= \frac{-iX_C(R + iX_C)}{R^2 + X_C^2}
 \end{aligned} \tag{1.4}$$

Now if we let

$$u = \frac{R}{X_C} = \omega RC \tag{1.5}$$

we can see that $R = uX_C$, and putting this in equation 1.4 leads to

$$\frac{V_{out}}{V_{in}} = \frac{-iX_C^2(u + i)}{u^2X_C^2 + X_C^2} \tag{1.6}$$

$$= \frac{1 - iu}{1 + u^2} \tag{1.7}$$

We can work out the magnitude and the phase angle of the attenuation through the filter as follows.

$$\begin{aligned}
 \left| \frac{V_{out}}{V_{in}} \right| &= \sqrt{\frac{1 - iu}{1 + u^2} \frac{1 + iu}{1 + u^2}} \\
 &= \frac{\sqrt{1 + u^2}}{1 + u^2} \\
 &= \frac{1}{\sqrt{1 + u^2}}
 \end{aligned} \tag{1.8}$$

and for the phase factor

$$\begin{aligned}
 \phi &= \arctan \left(\frac{\frac{-u}{1+u^2}}{\frac{1}{1+u^2}} \right) \\
 &= -\arctan u
 \end{aligned} \tag{1.9}$$

Summary

We looked at the classic example of a low pass RC filter circuit, and discovered the relationship between the voltage into the filter and the voltage out of the filter is given by

$$\frac{V_{out}}{V_{in}} = \frac{1 - iu}{1 + u^2}$$

Or in terms of magnitude and phase angle

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{1 + u^2}}$$
$$\phi = -\arctan u$$

where $u = \frac{R}{X_C} = \omega RC$.