Cross Section

An Introduction to Nuclear and Particle Physics Scattering Experiments

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Chapter 1

Introduction

1.1 Scattering Experiment

Here we will look into scattering cross sections and how the relate to scattering experiments.

1.1.1 Experimental Setup

A typical experiment usually involves a beam of particles which is incident on some sort of target material as shown in figure 1.1.

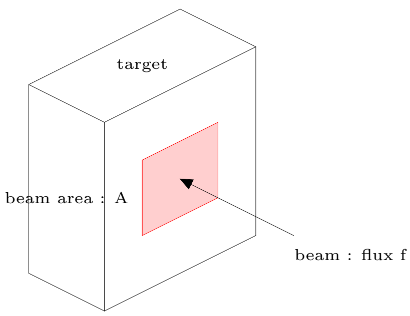

Figure 1.1: Typical scattering experiment setup. A target with a beam of particles incident on it.

The Target

The target material will have n''' target particles per unit volume. If the target material is of some known composition[1] then we can calculate the number of particles per unit mass of the target material.

$$\tilde{n} = \frac{N_A}{\check{m}} \tag{1.1.1}$$

where $\check{m}$ is the mass of a mole of the target material and N_A is the Avogadro constant, which is the number of particles per mole[2].

[1] Either elemental or some known substance

[2] In the notation used in this book, we should really use the symbol $\check{n}$ as this is the number of particles per mole of substance. However due to this being a constant with an already agreed upon symbol, namely N_A, we stall stick to using that here.

An Example: Consider a target made of pure $^{265}_{92}$U. The mass per mole is just the atomic weight or $\check{m} = 265 \times 10^{-3} kg$. So to calculate the number of particles per unit mass we just divide N_A by the atomic weight. In this case $\tilde{n} = \frac{N_A}{\check{m}} = -\frac{6.02214076\times10^{23}}{265\times10^{-3}} kg^{-1} = 2.272505947 \times 10^{24} kg{-}1$

This can be linked to the density of the target material by

$$\text{density} = \rho = m''' = \frac{n'''}{\tilde{n}} \tag{1.1.2}$$

Note that in equation -refeq:density we have used the standard symbol ρ for density. Included is the notation used by this text also. It is more typical to use the density of the material to calculate the number density by multiplying the density and the spesific number together.

Example Continued : The density of $^{265}_{92}$U is $1.91 \times 10^4 kg.m^{-3}$, using this we can calculate the number density of our target. $n''' = \rho\tilde{n} = 1.91 \times 10^2 kg.m^{-3} \times 2.272505947 \times 10^{24} kg^{-1} = 4.340486359 \times 10^{28} m^{-3}$

The Beam

The beam will typically have a known flux, f or using the book notation $\dot{n}''$ per unit area per unit time. If the area does not come into play, then the beam rate might be given instead.

1.1.2 cross section

The cross section for the whole target is given by

$$\text{cross section} = \frac{\text{rate}}{\text{flux}} \tag{1.1.3}$$

As you can see, it is a fairly common poke factor type equation. ie, something we can measure (here the rate of a particular interaction) is given by some poke factor times by something we can control (here the beam flux) and we poke the system with the thing we can control. So as you can see, if we double the beam flux then the rate will also double as a response.

An Example: Sticking with our Uranium target from previous examples. If we have a beam of neutons with a flux of $10^6 m^{-2}.s^{-1}$ incident on the target which results in 10^2 fission events per second, we have a cross section of $\sigma = \frac{\text{rate}}{\text{flux}} = \frac{100s^{-1}}{1000000m^{-2}.s^{-1}} = 10^{-4}m^2 = 1cm^2$

When we talk about cross section normally though, we are more conserned with the cross section per target particle $\bar{\sigma}$. Unless the target is a single atom thick, we have to consider the attenuation of the beam through the target material.

1.1.3 Beam Attenuation

As the beam passes through the target material, the particles in the beam will undergo colisions with the target particles. This will lead to the beam flux decreasing as it goes through the material, which will also have to factor into any calculations for the cross section.

Imagine a slice of through the target material perpendicular to the beam at a point a distance x into the material and δx thick. $\dot{N}(x)$ beam particles pass enter through area A per unit time, and $\dot{N}(x + \delta x)$ particles leave through area A per unit time through the back face at $x + \delta x$. The

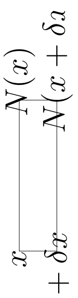

Figure 1.2: a cross section of the target material perpendicular to the beam

volume of material which the beam passes through is $\delta V = A\delta x$ which will contain $\delta n = n'''\delta V$ target particles. The rate of interaction for the slice is $r = \bar{\sigma}\delta n$. This means that

$$\dot{N}(x+\delta x) - \dot{N}(x) = -r \tag{1.1.4a}$$
$$\dot{N}(x+\delta x) - \dot{N}(x) = -\bar{\sigma}\delta n \tag{1.1.4b}$$
$$= -\bar{\sigma}n'''\delta V \tag{1.1.4c}$$
$$= -\bar{\sigma}n'''A\delta x \tag{1.1.4d}$$
$$\frac{\dot{N}(x+\delta x) - \dot{N}(x)}{A\delta x} = -\bar{\sigma}n''' \tag{1.1.4e}$$
$$\frac{\frac{\dot{N}}{A}(x+\delta x) - \frac{\dot{N}}{A}(x)}{\delta x} = -\bar{\sigma}n''' \tag{1.1.4f}$$
$$\lim_{\delta x\to 0}\frac{f(x+\delta x) - f(x)}{\delta x} = -\bar{\sigma}n''' \tag{1.1.4g}$$
$$\frac{df}{dx} = -\bar{\sigma}n''' \tag{1.1.4h}$$

$f(x)$ is the beam flux at the depth x into the target material. Equation 1.1.4h has the solution

$$f(x) = f(0)e^{-\bar{\sigma}n'''x} \tag{1.1.5}$$

Where the term $e^{-\bar{\sigma}n'''x}$ is considered to be the attenuation factor of the target material.

A note on alternative units

It might be that for a particular problem, you are not given a target number density etc and instead may be given different units for things. For example you may have the number of particles per square metre or some way of calculating it. In this case we can use that $n'' = n'''x$

An Example: Going back to our ${}^{265}_{92}\mathrm{U}$ taget from earlier. It has a thickness of $m'' = 10^{-1}kg.m^{-2}$. It also has a beam of neutrons incident on it. The total cross section for interactions with neutrons is $\bar{\sigma}_t = 2.7002 \times 10^{-26}m^2$ We can work out the attenuation factor as follows. Firstly we note that we can calculate the spesific number (as we did before). $\tilde{n} = \frac{N_A}{\breve{m}}$ where $\breve{m} = 2.65^{-2}kg.mol^{-1}$. Using this we can get the number of target particles per unit area $n'' = \tilde{n}m''$. Then we get the attenuation factor by $e^{\bar{\sigma}_t n''} = e^{2.7002\times 10^{-26}m^2 \frac{6.02214076\times 10^{23}}{2.65\times 10^{-2}kg} 10^{-1}kg.m^{-2}}$

Appendix A

Notation

Due to having many material properties used in calculations, it is useful to be able to track the type of property the symbol represents. To aid in keeping track the following the following notation (unless stated otherwise) will be used throughout this text. In the case of space and time based

Quantity	Description	Example
Specific quanity or a quanity per unit mass	Tilde above symbol	$\tilde{n}$
Quantity per unit length	Dash following symbol	a'
Quantity per unit area	Double dash following symbol	a''
Quantity per unit volume	Triple dash following symbol	a'''
Quantity per particle	Bar above symbol	$\bar{a}$
Quantity per mole	Check above symbol	$\check{a}$
Quantity per unit time	Dot above the symbol	$\dot{a}$

Table A.1: Notation used in this book

symbols, you might see two used together to represent a quantity that is both spread through the material as well as through time. For example the flux of particles in a beam would be denoted as $\dot{n}''$ as it is the number of particles per unit area per unit time.