Filters

Notes on passive electric filter circuits

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1 RC Low Pass Filter

1.1 Filter Attenuation

A simple low pass filter can be formed from a resistor and a capacitor in series.

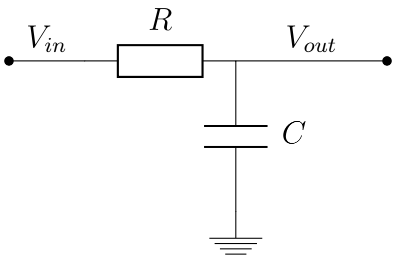

Figure 1: Circuit diagram for a RC low pass filter.

We can see from figure 1, that the output voltage is the same as the voltage across the capacitor. So we have

$$\begin{aligned} V_{in} &= V_R + V_C \\ &= IR + V_{out} \end{aligned} \tag{1.1}$$

Using the fact that the current through the RC section of the circuit is given by

$$I = \frac{V_{in}}{R - iX_C} \tag{1.2}$$

Leading to the output voltage being

$$\begin{aligned} V_{out} &= V_{in} - IR \\ &= V_{in} - \frac{V_{in}R}{R - iX_C} \\ &= V_{in}\left[1 - \frac{V_{in}R}{R - iX_C}\right] \end{aligned} \tag{1.3}$$

which leads to a ratio of the output voltage to the input voltage of

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= 1 - \frac{R}{R - iX_C} \\ &= \frac{-iX_C}{R - iX_C} \\ &= \frac{-iX_C(R + iX_C)}{R^2 + {X_C}^2} \end{aligned} \tag{1.4}$$

Now if we let

$$u = \frac{R}{X_c} = \omega RC \tag{1.5}$$

we can see that $R = uX_C$, and putting this in equation 1.4 leads to

$$\frac{V_{out}}{V_{in}} = \frac{-i{X_C}^2(u + i)}{u^2{X_C}^2 + {X_C}^2} \tag{1.6}$$

$$= \frac{1 - iu}{1 + u^2} \tag{1.7}$$

We can work out the magnitude and the phase angle of the attenuation through the filter as follows.

$$\begin{aligned}\left|\frac{V_{out}}{V_{in}}\right| &= \sqrt{\frac{1 - iu}{1 + u^2}\frac{1 + iu}{1 + u^2}} \\ &= \frac{\sqrt{1 + u^2}}{1 + u^2} \\ &= \frac{1}{\sqrt{1 + u^2}} \end{aligned} \tag{1.8}$$

and for the phase factor

$$\begin{aligned}\phi &= \arctan\left(\frac{\frac{-u}{1+u^2}}{\frac{1}{1+u^2}}\right) \\ &= -\arctan u \end{aligned} \tag{1.9}$$

Summary

We looked at the classic example of a low pass RC filter circuit, and discovered the relationship between the voltage into the filter and the voltage out of the filter is given by

$$\frac{V_{out}}{V_{in}} = \frac{1 - iu}{1 + u^2}$$

Or in terms of magnitude and phase angle

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + u^2}}$$
$$\phi = -\arctan u$$

where $u = \frac{R}{X_C} = \omega RC$.

1.2 Cutoff Frequency

Letting the attenuation $a = \left|\frac{V_{out}}{V_{in}}\right|$ and rearranging will give us

$$\begin{aligned} a &= \frac{1}{\sqrt{1 + u^2}} \\ 1 + u^2 &= \frac{1}{a^2} \\ u &= \frac{\sqrt{1 - a^2}}{a} \end{aligned} \tag{1.10}$$

Equation 1.10 and 1.5 can be used together to calculate component values if a particular attenuation is required at a particular frequency. However an interesting result is the attenuation when

$u = 1$.

$$\begin{aligned}
u &= 1 \\
a &= \frac{1}{\sqrt{1+u^2}} \\
&= \frac{1}{\sqrt{1+1}} \\
&= \frac{1}{\sqrt{2}} \qquad (1.11)
\end{aligned}$$

When looking at equation 1.5 and considering what it means when $u = 1$, you will realise this is when the resistance of the capacitor and the reactance of the capacitor are equal. Also the phase as given by equation 1.9 is $\phi = -\arctan 1 = -\frac{\pi}{4} = -45^\circ$

The frequency when $u = 1$ is known as the cutoff frequency of the filter, and is calculated as follows

$$\begin{aligned}
u &= 1 \\
2\pi RCf &= 1 \\
f &= \frac{1}{2\pi RC} \qquad (1.12)
\end{aligned}$$

Summary

For the RC filter there is a special frequency called the cuttof frequency, which is given by.

We looked at the classic example of a low pass RC filter circuit, and discovered the relationship between the voltage into the filter and the voltage out of the filter is given by

$$f = \frac{1}{2\pi RC}$$

At this frequency both the resistance of the resistor and the reactance of the capacitor are equal, which leads to a value of

$u = 1$. This leads to an attenuation $\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}}$ and a phase of $\phi = -\frac{\pi}{4} = -45°$.