Filters

Notes on passive electric filter circuits

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Contents

1 RC Low Pass Filter **2**
1.1 Filter Attenuation 2
1.1.1 Cutoff Frequency 2
1.1.2 Attenuation revisted 3
1.2 Log-Log Form 5

List of Tables

List of Figures

1 Circuit diagram for a RC low pass filter. 2

1 RC Low Pass Filter

1.1 Filter Attenuation

A simple low pass filter can be formed from a resistor and a capacitor in series.

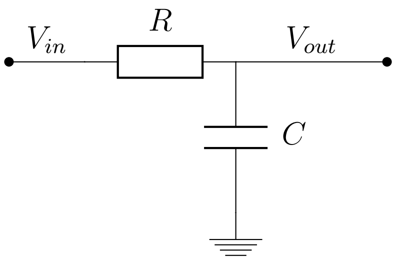

Figure 1: Circuit diagram for a RC low pass filter.

We can see from figure 1, that the circuit forms a potential divider just with a reactive element instead of purely resistive. The attenuation is then given by the standard potential divider result

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-iX_{\text{C}}}{R - iX_{\text{C}}} \tag{1.1}$$

1.1.1 Cutoff Frequency

Let's introduce a new variable called u, where

$$\begin{aligned} u &= \frac{R}{X_{\text{C}}} \\ &= \omega RC \end{aligned} \tag{1.2}$$

where $\omega = 2\pi f$. If we look at the frequency when the resulting $u = 1$, which we will label f_0 or ω_0

$$\begin{aligned} \omega_0 RC &= 1 \\ \omega_0 &= \frac{1}{RC} \end{aligned} \tag{1.3}$$

We call the frequency when $u = 1$ the *cutoff frequency*, for reasons that will be clear later on. This frequency is when the resistance of the resistor is equal to the reactance of the capacitor[1] You can see that we can use the cutoff frequency as a replacement for our RC value, in equation 1.2.

$$\begin{aligned} u &= \omega RC \\ &= \frac{\omega}{\omega_0} = \frac{f}{f_0} \end{aligned} \tag{1.4}$$

1.1.2 Attenuation revisted

Now we have some understanding of the variable we introduced u, we can substitute it into our equation for the attenuation (equation 1.1), by noting that from equation 1.2 $R = uX_\mathrm{C}$

$$\begin{aligned} \frac{V_\mathrm{out}}{V_\mathrm{in}} &= \frac{-iX_\mathrm{C}}{R - iX_\mathrm{C}} \\ &= \frac{-iX_\mathrm{C}}{uX_\mathrm{C} - iX_\mathrm{C}} \\ &= \frac{-i}{u - i} \\ &= \frac{1 - iu}{u^2 + 1} \end{aligned} \tag{1.5}$$

Normally, we don't consider the attenuation as a complex value, instead we care more about the magnitude and phase shift of an attenuation.

$$\begin{aligned} \left|\frac{V_\mathrm{out}}{V_\mathrm{in}}\right| &= \frac{\sqrt{1 + u^2}}{1 + u^2} \\ &= \frac{1}{\sqrt{1 + u^2}} \\ &= \frac{1}{\sqrt{1 + \left(\frac{f}{f_0}\right)^2}} \end{aligned} \tag{1.6}$$

[1] by equal here, we mean the magnitudes are equal. If not the phase shift.

Where we have used equation 1.4 in place of u. For the phase shift of the filter,

$$\phi = -\arctan u = -\arctan \frac{f}{f_0} \tag{1.7}$$

Sumary

In the last section we discovered the cutoff frequency was given by

$$f_0 = \frac{1}{2\pi RC}$$

and that the ratio of resistance to reactance can be given by

$$u = \frac{R}{X_\mathrm{C}} = 2\pi RCf = \frac{f}{f_0}$$

and that the attenuation of the filter is given by

$$\begin{aligned}\frac{V_\mathrm{out}}{V_\mathrm{in}} &= \frac{-iX_\mathrm{C}}{R - iX_\mathrm{C}} \\ &= \frac{1 - iu}{u^2 + 1}\end{aligned}$$

or in terms of magnitude and phase shift

$$\begin{aligned}\left|\frac{V_\mathrm{out}}{V_\mathrm{in}}\right| &= \frac{1}{\sqrt{1 + u^2}} \\ &= \frac{1}{\sqrt{1 + \left(\frac{f}{f_0}\right)^2}}\end{aligned}$$

$$\phi = -\arctan u = -\arctan \frac{f}{f_0}$$

1.2 Log-Log Form

You won't often see attenuation given in the form seen earlier. It is more likely to be seen in Log-Log form, due to wanting to see the behaviour over a large range of frequencies and the fact the attenuation itself can get very small very fast. However it helps to look at the logirthm of u before looking at the attenuation straight away.

$$\ln u = \ln \frac{f}{f_0} = \ln f - \ln f_0 = F - F_0 \tag{1.8}$$

where we have used $F = \ln f$ and $F_0 = \ln f_0$