Abstract Maths Cheat Sheet

1 Ket-vector Axioms

1. The sum of any two ket-vectors is also a ket-vector:
$$|A\rangle + |B\rangle = |C\rangle \tag{1.1}$$
2. Vector addition is commutative:
$$|A\rangle + |B\rangle = |B\rangle + |A\rangle \tag{1.2}$$
3. Vector addition is associative:
$$(|A\rangle + |B\rangle) + |C\rangle = |A\rangle + (|B\rangle + |C\rangle) \tag{1.3}$$
4. There is a unique vector 0 such that when you add it to any ket, it gives the same ket back:
$$|A\rangle + 0 = |A\rangle \tag{1.4}$$
5. Given any ket $|A\rangle$, there is a unique ket $-|A\rangle$ such that
$$|A\rangle + (-|A\rangle) \tag{1.5}$$
6. Given any $|A\rangle$ and any complex number z, you can multiply them to get a new ket. Also, multiplication by a scalar is linear:
$$|zA\rangle = z|A\rangle = |B\rangle \tag{1.6}$$
7. The distributive property holds:
$$z(|A\rangle + |B\rangle) = z|A\rangle + z|B\rangle \tag{1.7}$$
$$(z + w)|A\rangle = z|A\rangle + w|A\rangle \tag{1.8}$$

Taken together 6 and 7 are often call linearity

2 Bra-vector Axioms

1. The sum of any two bra-vectors is also a bra-vector:
$$\langle A| + \langle B| = \langle C| \tag{2.1}$$
2. Vector addition is commutative:
$$\langle A| + \langle B| = \langle B| + \langle A| \tag{2.2}$$
3. Vector addition is associative:
$$(\langle A| + \langle B|) + \langle C| = \langle A| + (\langle B| + \langle C|) \tag{2.3}$$
4. There is a unique vector 0 such that when you add it to any bra, it gives the same bra back:
$$\langle A| + 0 = \langle A| \tag{2.4}$$
5. Given any bra $\langle A|$, there is a unique bra $-\langle A|$ such that
$$\langle A| + (-\langle A|) \tag{2.5}$$
6. Given any $\langle A|$ and any complex number z, you can multiply them to get a new bra. Also, multiplication by a scalar is linear:
$$\langle zA| = z\langle A| = \langle B| \tag{2.6}$$
7. The distributive property holds:
$$z(\langle A| + \langle B|) = z\langle A| + z\langle B| \tag{2.7}$$
$$(z + w)\langle A| = z\langle A| + w\langle A| \tag{2.8}$$

Taken together 6 and 7 are often call linearity

3 Bras and Kets

As we have seen, the complex numbers have a dual version: in the form of complex conjugate numbers. In the same way, a complex vector space has a dual version that is essentially the complex conjugate vector space.

For every ket-vector $|A\rangle$, there is a bra-vector in the dual space, denoted by $\langle A|$. Bra and Ket vectors together form inner products of bras and kets, using expressions like ¡B—A¿ to form bra-kets or brackets.

Inner products are extremely important in the mathematical machinery of quantum mechanics,and for characterizing vector spaces in general.

Bra-vectors satisfy the same axioms as the ket-vectors, but there are two things to keep in mind about the correspondence between kets and bras:

Suppose ¡A— is the bra corresponding to the ket —A¿, and ¡B— is the bra corresponding to the ket —B¿. Then the bra corresponding to —A¿ + —B¿ is ¡A— + ¡B—

If z is a complex number, then it is not true that the bra corresponding to z—A¿ is ¡A—z. You have to remember to complex-conjugate. Thus, the bra corresponding to z—A¿ is ¡A—z*

4 Inner Products

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