
BEZIER CURVES

NOTES ON THE DERIVATION OF BEZIER CURVES CURVE

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Chapter 1

Bezier Curves

There are many different kinds of Bezier curve, however they all share some similar properties and can be characterised by their order. For example a zeroth order Bezier curve is just a point, first order is a line, second order is a quadratic curve and third order is a cubic curve. Higher order Bezier curves do exist but don't tend to get used much. All Bezier curves have parameters that are given by "control points". The number of control points is one more than the order of the curve.

1.1 Examples

1.1.1 Zeroth Order



Figure 1.1: Example of a zeroth order Bezier curve

The zeroth order Bezier curve is just a point. It has just one control point which is just where the point is positioned. The equation for this curve is very simple, as shown in 1.1.1.

$$C_0(\vec{A}) = 1 \times \vec{A} \quad (1.1.1)$$

1.1.2 First Order

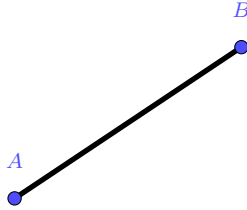


Figure 1.2: Example of a first order Bezier curve

The first order Bezier curve is a straight line between two control points. You might also see this called a linear interpolation or sometimes lerp. The basic equation for this line is shown in 1.1.2.

$$C_1(\vec{A}, \vec{B}, t) = (1 - t)\vec{A} + t\vec{B} \quad (1.1.2)$$

1.1.3 Second Order

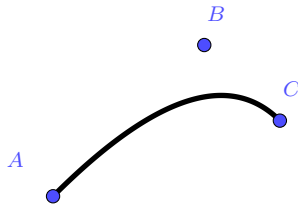


Figure 1.3: Example of a second order Bezier curve

The second order Bezier curve is a quadratic curve that starts at the first control point and ends on the third, with the middle control point controlling the gradient at the two end points.

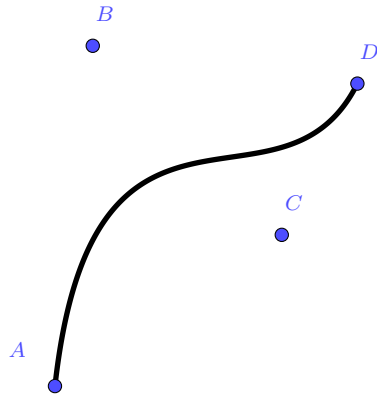


Figure 1.4: Example of a third order Bezier curve

1.1.4 Third Order

The third order Bezier curve is a cubic curve that starts at the first control point and ends on the fourth, with the middle two control point controlling the gradient at the two end points separately.

Chapter 2

Methods of Calculating Bezier Curves

2.1 De Casteljau's algorithm

This method involves building a higher order Bezier curve from first order curves. Starting at a first order curve (ie a line), we can calculate new points for more lines and eventually build up our curve.

Second Order

Third Order