Q1c

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$$K = \begin{bmatrix} K_{s+} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{s-} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{b+} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{b-} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{e+} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{e-} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{w+} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{w-} \end{bmatrix}$$

Stiffness Transformation

from

$$\mathbf{K} \equiv \left(\frac{\partial \tau_i}{\partial q_j}\right) = \left(\frac{\partial(\mathbf{J}^T\mathbf{F})_i}{\partial q_j}\right)$$

$$= \left(\sum_k \frac{\partial(\mathbf{J}^T)_{ik}}{\partial q_j} F_k\right) + \mathbf{J}^T\left(\frac{\partial F_i}{\partial q_j}\right)$$

$$= \left(\sum_k \frac{\partial(\mathbf{J}^T)_{ik}}{\partial q_j} F_k\right) + \mathbf{J}^T \sum_k \left(\frac{\partial F_i}{\partial x_k}\right)\left(\frac{\partial x_k}{\partial q_j}\right)$$

follows

$$\mathbf{K} = \frac{d\mathbf{J}^T}{d\mathbf{q}}\mathbf{F} + \mathbf{J}^T \mathbf{K}_x \mathbf{J}$$

similarly

$$\mathbf{K} = \cancel{\frac{d\mathbf{J}_\mu^T}{d\mathbf{q}}}^{\,0}\boldsymbol{\mu} + \mathbf{J}_\mu^T \mathbf{K}_\mu \mathbf{J}_\mu$$

→ this Jacobian is muscle space!

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goes to 0 since we are considering no muscle tension!

Jacobian from Q1b)

not this Jacobian.

$$\cancel{\begin{bmatrix} -l_e \cdot \sin(q_e+q_s-90) + l_w \cdot \sin(90-q_s-q_e-q_w) & -l_s \cdot \sin(q_s) - l_e \cdot \sin(q_e+q_s-90) + l_w \cdot \sin(90-q_s-q_e-q_w) & l_w \cdot \sin(90-q_s-q_e-q_w) \\ -l_e \cdot \cos(q_e+q_s-90) + l_w \cdot \cos(90-q_s-q_e-q_w) & l_s \cos(q_s) - l_e \cos(q_e+q_s-90) + l_w \cdot \cos(90-q_s-q_e-q_w) & l_w \cdot \cos(90-q_s-q_w-q_e) \\ 1 & 1 & 1 \end{bmatrix}}$$

$$K = J_\mu^T \cdot K_\mu \cdot J_\mu$$

$$= \begin{bmatrix} \rho_{s+} & -\rho_{s-} & \rho_{bs+} & -\rho_{bs-} & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{be+} & -\rho_{be-} & \rho_{e+} & -\rho_{e-} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{w+} & -\rho_{w-} \end{bmatrix} \begin{bmatrix} K_{s+} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{s-} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{b+} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{b-} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{e+} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{e-} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{w+} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{w-} \end{bmatrix} \begin{bmatrix} \rho_{s+} & 0 & 0 \\ -\rho_{s-} & 0 & 0 \\ \rho_{bs+} & \rho_{be+} & 0 \\ -\rho_{bs-} & -\rho_{be-} & 0 \\ 0 & \rho_{e+} & 0 \\ 0 & -\rho_{e-} & 0 \\ 0 & 0 & \rho_{w+} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & K_{w+} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{w-} \end{bmatrix} \begin{bmatrix} 0 & -P_{e-} & 0 \\ 0 & 0 & P_{w+} \\ 0 & 0 & -P_{w-} \end{bmatrix}$$

$$= \begin{bmatrix} P_{s+} \cdot K_{s+} & -P_{s-} \cdot K_{s-} & P_{bs+} \cdot K_{b+} & -P_{bs-} \cdot K_{b-} & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{be+} \cdot K_{b+} & -P_{be-} \cdot K_{b-} & P_{e+} \cdot K_{e+} & -P_{e-} \cdot K_{e-} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{w+} \cdot K_{w+} & -P_{w-} \cdot K_{w-} \end{bmatrix} \begin{bmatrix} P_{s+} & 0 & 0 \\ -P_{s-} & 0 & 0 \\ P_{bs+} & P_{be+} & 0 \\ -P_{bs-} & -P_{be-} & 0 \\ 0 & P_{e+} & 0 \\ 0 & -P_{e-} & 0 \\ 0 & 0 & P_{w+} \\ 0 & 0 & -P_{w-} \end{bmatrix}$$

$$= \begin{bmatrix} P_{s+}^2 K_{s+} + P_{s-}^2 \cdot K_{s-} + P_{bs+}^2 \cdot K_{b+} + P_{bs-}^2 \cdot K_{b-} & P_{be+} \cdot P_{bs+} \cdot K_{b+} + P_{be-} \cdot P_{bs-} \cdot K_{b-} & 0 \\ P_{be+} \cdot P_{bs+} \cdot K_{b+} + P_{be-} \cdot P_{bs-} \cdot K_{b-} & P_{be+}^2 \cdot K_{b+} + P_{be-}^2 \cdot K_{b-} + P_{e+}^2 \cdot K_{e+} + P_{e-}^2 \cdot K_{e-} & 0 \\ 0 & 0 & P_{w+}^2 \cdot K_{w+} + P_{w-}^2 \cdot K_{w-} \end{bmatrix}$$

Can simplify more - each of these moment arms equal the same constant.

let the moment arm constant be p

$$= \begin{bmatrix} p^2 K_{s+} + p^2 K_{s-} + p^2 \cdot K_{b+} + p^2 K_{b-} & p^2 \cdot K_{b+} + p^2 \cdot K_{b-} & 0 \\ p^2 \cdot K_{b+} + p^2 \cdot K_{b-} & p^2 \cdot K_{b+} + p^2 \cdot K_{b-} + p^2 \cdot K_{e-} + p^2 \cdot K_{e+} & 0 \\ 0 & 0 & p^2 \cdot K_{w+} + p^2 \cdot K_{w-} \end{bmatrix}$$