Q1b

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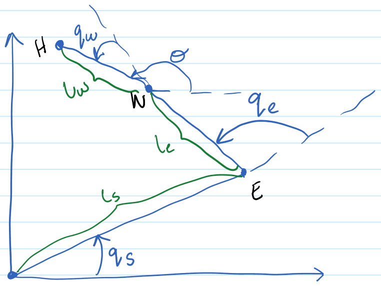

δx
δy
$\delta\theta$

From 1a:

$$H_x = l_s \cdot \cos q_s + l_e \cdot \cos(q_e + q_s - 90) + l_w \cdot \cos(90 - q_s - q_e - q_w)$$

$$H_y = l_s \cdot \sin q_s - l_e \cdot \sin(q_e + q_s - 90) - l_w \cdot \sin(90 - q_s - q_e - q_w)$$

$$H_\theta = \cancel{q_s + q_e + q_w} \rightarrow$$ is θ parallel to z axis?

Note that the Jacobian looks as follows:

3 joints = 3 columns
First 3 rows = linear velocities
last 3 rows = angular.

$$J_F(x_1, x_2, x_3) = \begin{bmatrix} \frac{\delta y_1}{\delta x_1} & \frac{\delta y_1}{\delta x_2} & \frac{\delta y_1}{\delta x_3} \\ \frac{\delta y_2}{\delta x_1} & \frac{\delta y_2}{\delta x_2} & \frac{\delta y_2}{\delta x_3} \\ \frac{\delta y_3}{\delta x_1} & \frac{\delta y_3}{\delta x_2} & \frac{\delta y_3}{\delta x_3} \end{bmatrix}$$

$\boxed{NB}$ $\dot{x} = \frac{\delta x}{dt} = \frac{\delta x_i}{\delta q_i} \cdot \frac{\delta q_i}{\delta t} = \frac{\delta x_i}{\delta q_i} \cdot \dot{q}_i$

$$\frac{\delta H_x}{\delta q_e} = -l_e \cdot \sin(q_e + q_s - 90) + l_w \cdot \sin(90 - q_s - q_e - q_w)$$

$$\frac{\delta H_x}{\delta q_s} = -l_s \cdot \sin(q_s) - l_e \cdot \sin(q_e + q_s - 90) + l_w \sin(90 - q_s - q_e - q_w)$$

$$\frac{\delta H_x}{\delta q_w} = l_w \cdot \sin(90 - q_s - q_e - q_w)$$

$$\Rightarrow \dot{H}_x = \left[-l_e \cdot \sin(q_e + q_s - 90) + l_w \cdot \sin(90 - q_s - q_e - q_w)\right]\dot{q}_e + \left[-l_s \cdot \sin(q_s) - l_e \cdot \sin(q_e + q_s - 90) + l_w \sin(90 - q_s - q_e - q_w)\right]\dot{q}_s + \left[l_w \cdot \sin(90 - q_s - q_e - q_w)\right]\dot{q}_w$$

$$\frac{\delta H_y}{\delta q_e} = -l_e \cdot \cos(q_e + q_s - 90) + l_w \cdot \cos(90 - q_s - q_e - q_w)$$

$$\frac{\delta H_y}{\delta q_s} = l_s \cos(q_s) - l_e \cos(q_e + q_s - 90) + l_w \cdot \cos(90 - q_s - q_e - q_w)$$

$$\frac{\delta H_y}{\delta q_w} = + l_w \cdot \cos(90 - q_s - q_w - q_e)$$

$$\Rightarrow \dot{H}_y = \left[-l_e \cdot \cos(q_e + q_s - 90) + l_w \cdot \cos(90 - q_s - q_e - q_w)\right]\dot{q}_e + \left[l_s \cos(q_s) - l_e \cos(q_e + q_s - 90) + l_w \cdot \cos(90 - q_s - q_e - q_w)\right]\dot{q}_s + \left[l_w \cdot \cos(90 - q_s - q_w - q_e)\right]\dot{q}_w$$

$$\frac{\delta H_\theta}{\delta q_e} = 1$$

$$\frac{\delta H_\theta}{\delta q_s} = 1$$

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$$\frac{\delta H_\theta}{\delta q_s} = 1$$

$$\frac{\delta H_\theta}{\delta q_w} = 1$$

$$\Rightarrow \delta \dot{H}_\theta = \dot{q}_e + \dot{q}_s + \dot{q}_w$$

$$\Rightarrow \begin{bmatrix} \dot{H}_x \\ \dot{H}_y \\ \dot{H}_\theta \end{bmatrix} = \underbrace{\begin{bmatrix} -l_e \cdot \sin(q_e + q_s - 90) + l_w \cdot \sin(90 - q_s - q_e - q_w) & -l_s \cdot \sin(q_s) - l_e \cdot \sin(q_e + q_s - 90) + l_w \cdot \sin(90 - q_s - q_e - q_w) & l_w \cdot \sin(90 - q_s - q_e - q_w) \\ -l_e \cdot \cos(q_e + q_s - 90) + l_w \cdot \cos(90 - q_s - q_e - q_w) & l_s \cos(q_s) - l_e \cos(q_e + q_s - 90) + l_w \cdot \cos(90 - q_s - q_e - q_w) & l_w \cdot \cos(90 - q_s - q_w - q_e) \\ 1 & 1 & 1 \end{bmatrix}}_{\text{Jacobian}} \begin{bmatrix} \dot{q}_s \\ \dot{q}_e \\ \dot{q}_w \end{bmatrix}$$

Checking answer using symbolic MATLAB

J =

- le*sin(qe + qs - pi/2) - lw*sin(qe + qs + qw - pi/2)	- le*sin(qe + qs - pi/2) - lw*sin(qe + qs + qw - pi/2) - ls*sin(qs)	-lw*sin(qe + qs + qw - pi/2)
lw*cos(qe + qs + qw - pi/2) - le*cos(qe + qs - pi/2)	lw*cos(qe + qs + qw - pi/2) - le*cos(qe + qs - pi/2) + ls*cos(qs)	lw*cos(qe + qs + qw - pi/2)

For joint space to muscle space

Now we have 8 muscles & 3 joints

$$J_u(\rho) \equiv \left(\frac{\delta \lambda_i}{\delta q_j}\right) = \begin{bmatrix} \rho_{s+} & 0 & 0 \\ -\rho_{s-} & 0 & 0 \\ \rho_{bs+} & \rho_{be+} & 0 \\ -\rho_{bs-} & -\rho_{be-} & 0 \\ 0 & \rho_{e+} & 0 \\ 0 & -\rho_{e-} & 0 \\ 0 & 0 & \rho_{w+} \\ 0 & 0 & -\rho_{w-} \end{bmatrix}$$