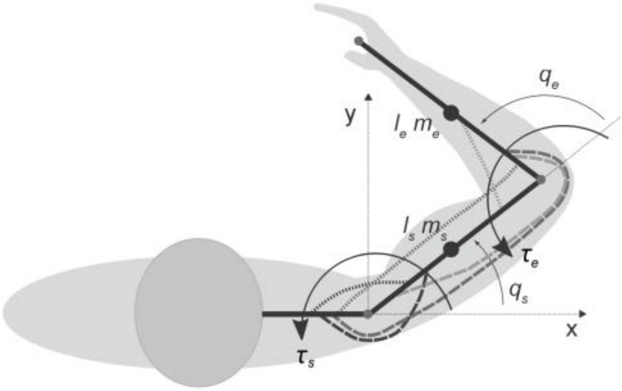
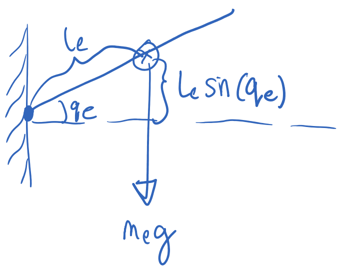
Q1

20 February 2023 11:36

1a)



Elbow in isolation:



NB: We use I rather than a mass matrix since we have 1 single joint.

$$L = T - U$$

$$= \frac{1}{2} I \dot{q}_e^2 - m_e g l_m \sin(q_e)$$

$$\tau = \frac{d}{dt}\left(\frac{dL}{d\dot{q}_e}\right) - \left(\frac{dL}{dq_e}\right)$$

$$= \frac{d}{\quad}\left(I\dot{q}_e\right) + m_e g \cdot l_e \cdot d\left[\sin(q_e)\right]$$

$$= \frac{d}{dt}\left(I\dot{q}_e\right) + M_e g \cdot l_e \cdot \frac{d}{dq_e}\left[\sin(q_e)\right]$$

$$= I\ddot{q}_e + M_e g l_e \cdot \cos(q_e)$$

(This makes sense - we expect no $G(q_e)$ term since we're moving in a horizontal plane)

Subbing in $I_e = 0.0188$, $M_e = 2.04$, $g = 9.81$
$l_e = 0.34$:

$$\tau_e = 0.0188\,\ddot{q}_e + 6.804216 \cdot \cos(q_e)$$

$\therefore$ due to the cos term, the system is non-linear.

1b)

Perturbation Dynamics during Motion

- Muscles move the arm and interact with the environment, yielding the task dynamics $\boldsymbol{\tau}_T(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u) \equiv \boldsymbol{\tau}_B(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u) - \mathbf{J}^T\mathbf{F}_E$
- An unexpected perturbation $\triangle\mathbf{F}_E$ will modify this task dynamics to
$$\boldsymbol{\tau}'_T(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \equiv \boldsymbol{\tau}_T(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) - \triangle\mathbf{F}_E$$
and shift the planed trajectory $\mathbf{q}_u$ to the actual trajectory $\mathbf{q}$
- In general, muscle viscoelasticity and reflexes make the interaction stable, and keep the difference $\mathbf{e} = \mathbf{q}_u - \mathbf{q}$ small
- We use a linear approximation of the restoring force as a function of $\mathbf{e}$ yielding $\underbrace{\mathbf{J}_\mu^T\boldsymbol{\mu}'_T(\boldsymbol{\lambda}, \dot{\boldsymbol{\lambda}}, \mathbf{u})}_{\text{muscles' tension}} = \underbrace{\boldsymbol{\tau}_T(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u)}_{\text{planned motion dynamics}} + \underbrace{\mathbf{K}\mathbf{e} + \mathbf{D}\dot{\mathbf{e}}}_{\text{restoring torque}}$

$$\mathbf{K} \equiv \left(-\frac{\partial \tau_i}{\partial q_j} - \sum_k \frac{\partial \tau_i}{\partial u_k}\frac{\partial u_k}{\partial q_j}\right) \quad \mathbf{D} \equiv \left(-\frac{\partial \tau_i}{\partial \dot{q}_j} - \sum_k \frac{\partial \tau_i}{\partial u_k}\frac{\partial u_k}{\partial \dot{q}_j}\right)$$

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(b) Starting at $q_e(0) = 30°$, the elbow is subjected to a stimulus

$$\tau_e(t) = \begin{cases} 0.02Nm - 0.1\dot{q}_e(t) & 0 \le t < 2s \\ -0.1\,\dot{q}_e(t) & 2s \le t \le 20s \end{cases} \quad (1)$$

Simulate the response of the joint angle $q_e(t)$ in MATLAB (using $q_s(0) = 0°$) and plot the evolution of $q_e(t)$ with respect to time t using a sampling time $0.01s$.

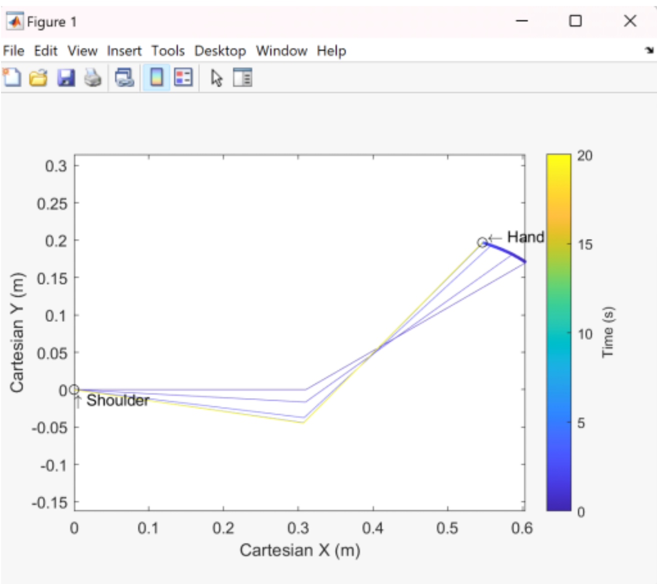
[14 marks]

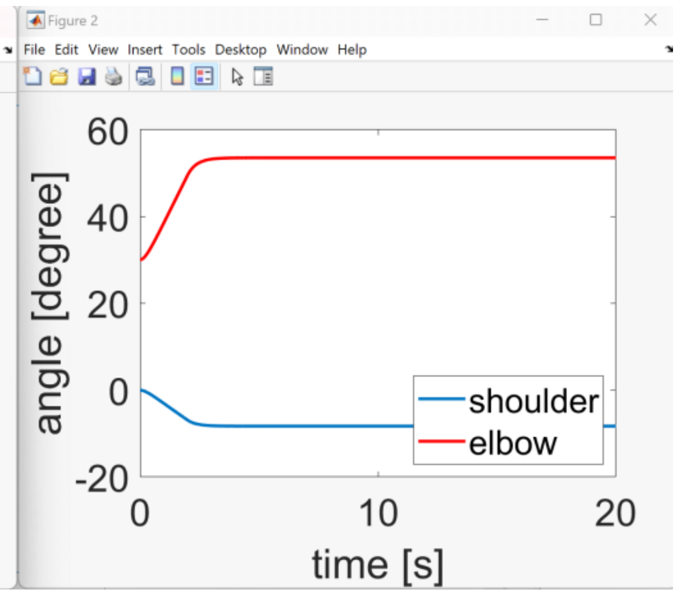
Adding in the perturbation given in the question, we get:

$$T_e(t) = \begin{cases} 0.0188\,\ddot{q}_e - 0.1\,\dot{q}_e(t) + 6.804216 \cdot \cos(q_e) + 0.02 & 0 \leq t < 2 \\ 0.0188\,\ddot{q}_e - 0.1\,\dot{q}_e(t) + 6.804216 \cdot \cos(q_e) & 2 < t \leq 20 \end{cases}$$

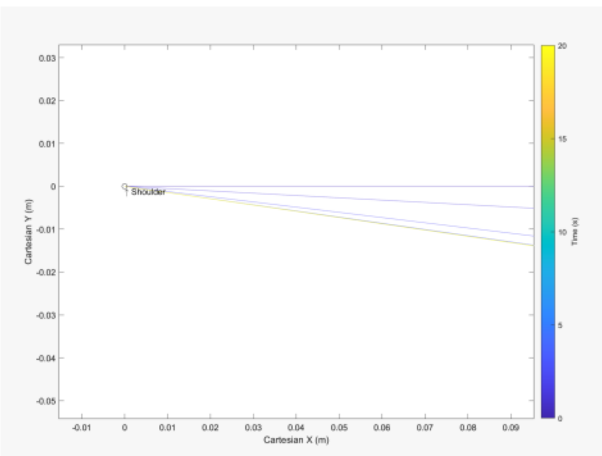
1c)

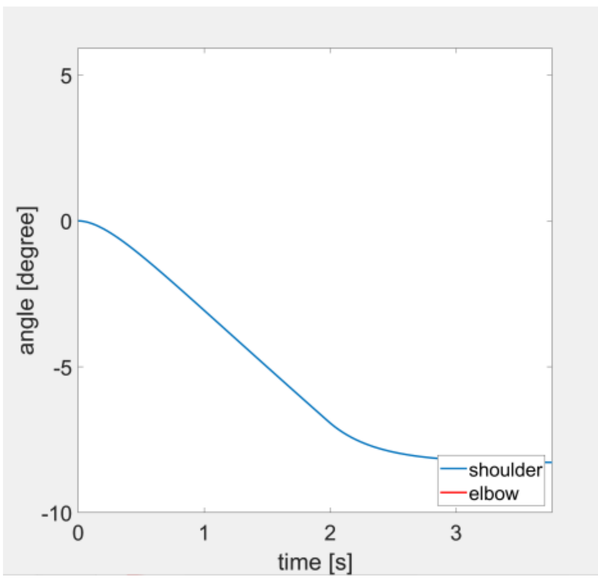
Shoulder blocked:

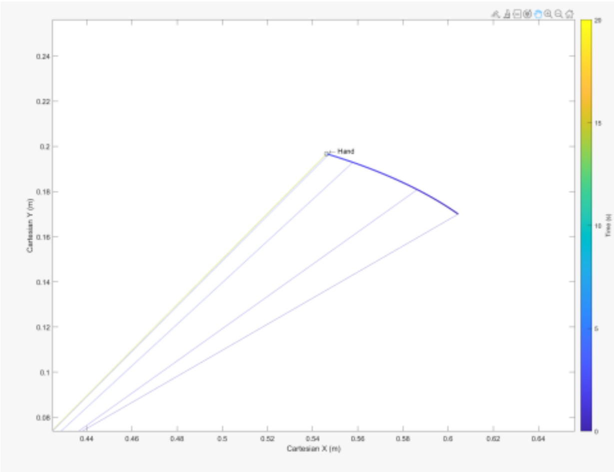




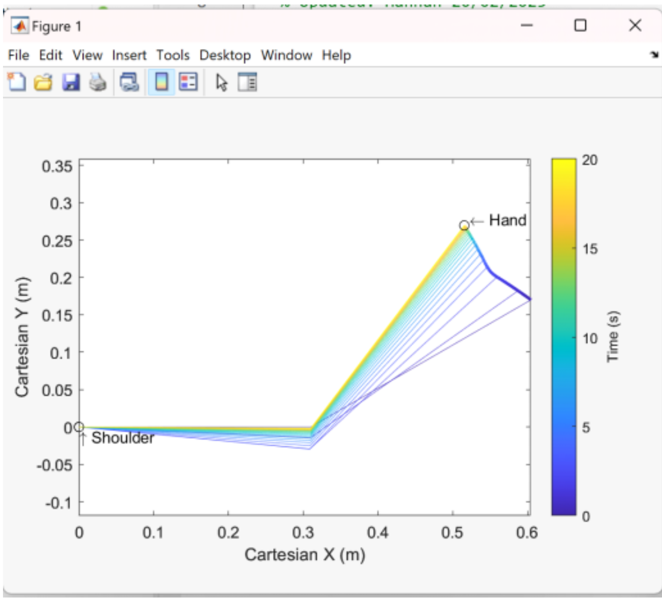
(zooming in)

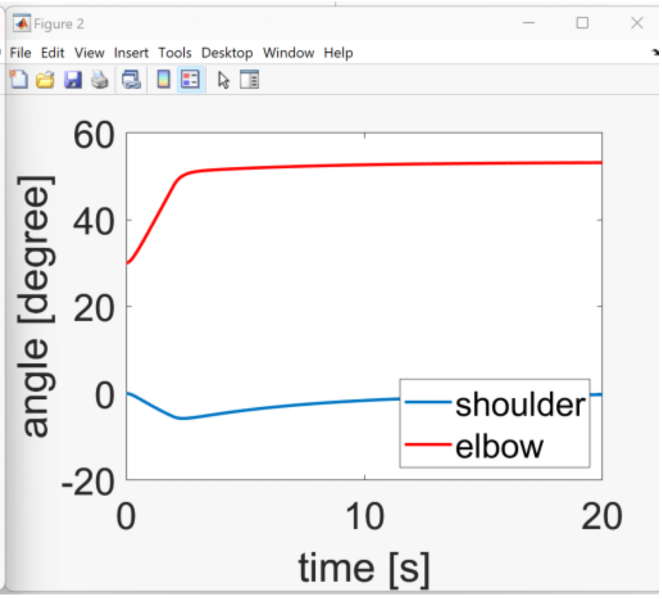


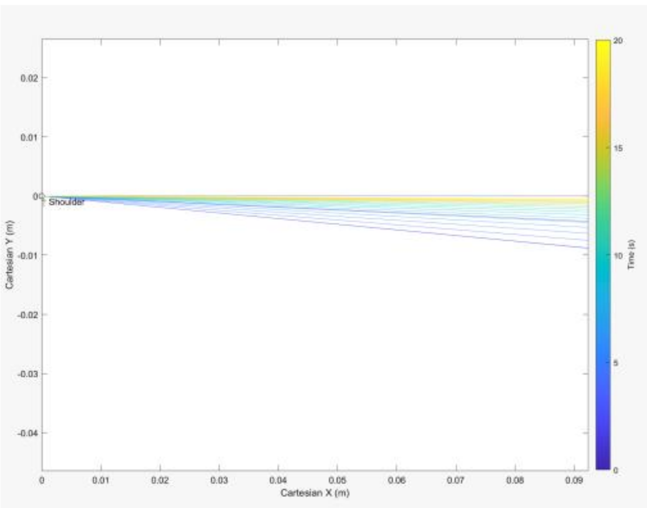


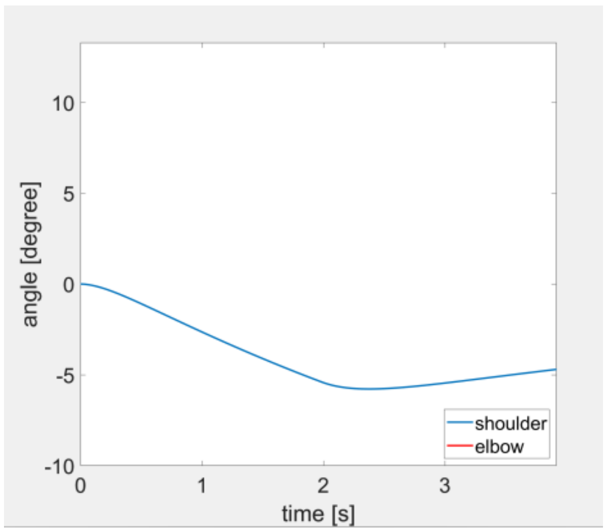


Shoulder not blocked:









Thoughts:

When the shoulder is blocked, damping is increased which is shown by the model not overshooting in the angle vs time graph. When the shoulder is not blocked, damping is decreased and hence there is overshooting. Damping is as a result of higher viscosity.

Similarly, when the shoulder is not blocked, the inertia of the model increases which increases the amount of overshooting as shown again in the angle vs time graph.

Shoulder's angular velocity has a smaller magnitude when not blocked (i.e. the slope of the graph is shallower).

As a result, when the shoulder is not blocked, the hand has a wider range of motion and takes longer to reach its end-position.

This paper does a similar experiment and has some interesting findings:
https://www.ncbi.nlm.nih.gov/pmc/articles/PMC7186382/