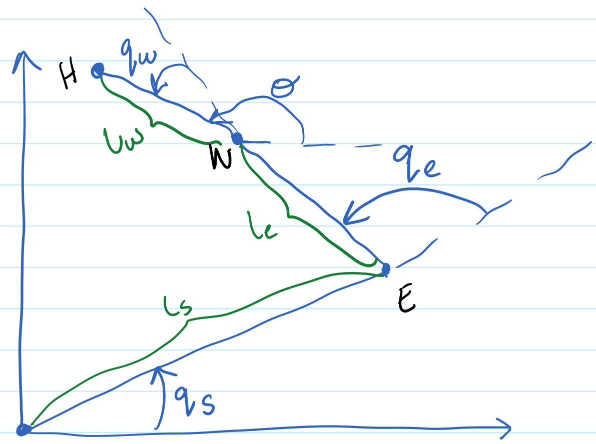
Q1a

03 February 2023 14:35

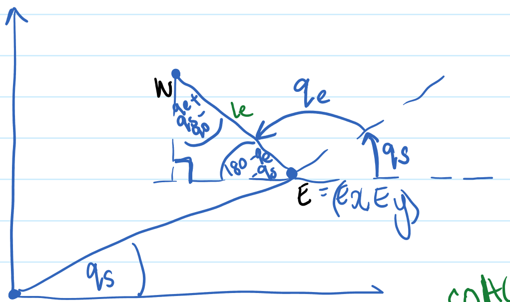
a) $\boxed{\text{Direct kinematics}}$



Using trig to find the position of the elbow

$$① \; E = (l_s \cdot \cos q_s \,,\; l_s \cdot \sin q_s)$$

②



Using trig to find the position of the wrist

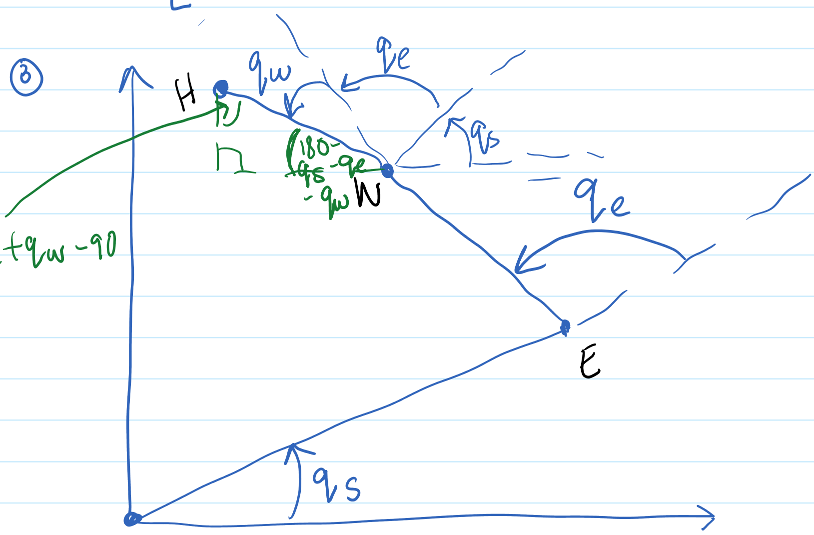
SOHCAHTOA

$$W = [E_x - l_e \cdot \sin(q_e + q_s - 90),\; E_y + l_e \cdot \cos(q_e + q_s - 90)]$$

Subbing in the x, y coordinates for the elbow

$$W = [l_s \cdot \cos q_s - l_e \cdot \sin(q_e + q_s - 90),\; l_s \cdot \sin q_s + l_e \cdot \cos(q_e + q_s - 90)]$$

③



$= q_s + q_e + q_w - 90$

Using trig to find the position of the hand

$$H = [W_x - l_w \cdot \sin(q_s + q_e + q_w - 90),\; W_y + l_w \cdot \cos(q_s + q_e + q_w - 90)]$$

$$H = \left[W_x - l_w \cdot \sin(q_s + q_e + q_w - 90),\quad W_y + l_w \cdot \cos(q_s + q_e + q_w - 90) \right]$$

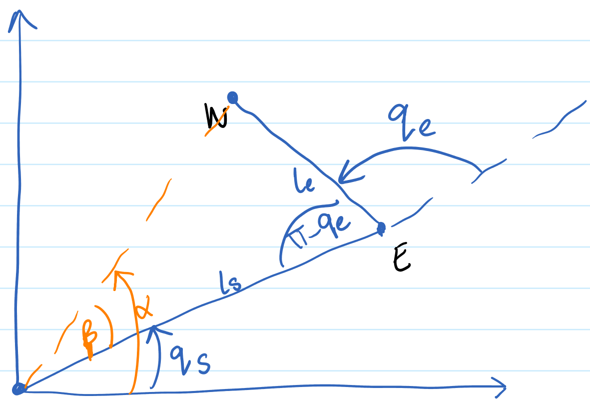
Subbing in the x, y coordinates for the wrist

$$H = \left[l_s \cdot \cos q_s - l_e \cdot \sin(q_e + q_s - 90) - l_w \sin(q_s + q_e + q_w - 90), \right.$$

$$\left. l_s \cdot \sin q_s + l_e \cdot \cos(q_e + q_s - 90) + l_w \cdot \cos(q_s + q_e + q_w - 90) \right]$$

INVERSE KINEMATICS

NB: Derive angles for w.
Add on θ



① $W_x = l_s \cdot \cos q_s - l_e \cdot \sin(q_e + q_s - 90)$

$W_y = l_s \cdot \sin q_s + l_e \cdot \cos(q_e + q_s - 90)$

② $a^2 = b^2 + c^2 - 2bc \cos A$

$\Rightarrow W_x^2 + W_y^2 = l_s^2 + l_e^2 - 2 l_s l_e \cos(\pi - q_e)$

$\Rightarrow q_e = \cos^{-1}\left(\dfrac{W_x^2 + W_y^2 - l_s^2 - l_e^2}{2 l_s l_e} \right)$

③ $l_e^2 = W_x^2 + W_y^2 + l_s^2 - 2 l_1 \sqrt{W_x + W_y} \cdot \cos\beta$

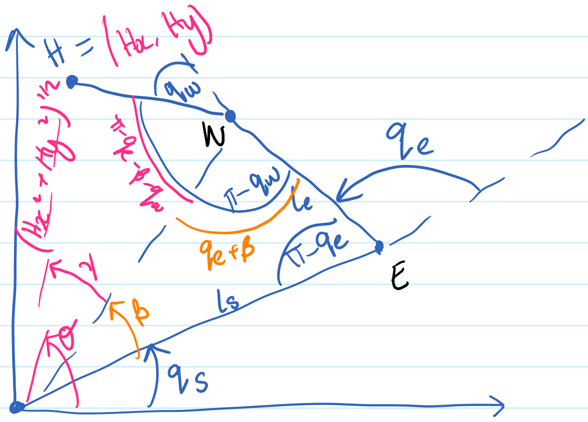
$\Rightarrow \beta = \cos^{-1}\left(\dfrac{W_x^2 + W_y^2 + l_s^2 - l_e^2}{2 L_1 \sqrt{W_x^2 + W_y^2}} \right)$

④ $q_s = \alpha - \beta$

$\alpha = \arctan 2 \left(\dfrac{W_y}{W_x} \right)$

$$\theta = \arctan 2\left(\frac{W_y}{W_x}\right)$$

(5)



(1) $H_x = l_s \cdot \cos q_s - l_e \cdot \sin(q_e + q_s - 90) - l_w \cdot \sin(q_s + q_e + q_w - 90)$

$H_y = l_s \cdot \sin q_s + l_e \cdot \cos(q_e + q_s - 90) + l_w \cdot \cos(q_s + q_e + q_w - 90$

(2) cosine rule. $a^2 = b^2 + c^2 - 2bc \cos A$

$$H_x^2 + H_y^2 = (W_x^2 + W_y^2) + l_w^2 - 2 \cdot l_w \cdot \sqrt{W_x^2 + W_y^2} \cdot \cos(\pi - q_e - \beta - q_w)$$

$$\Rightarrow q_w = \pi - \beta - q_e - \cos^{-1}\left(\frac{H_x^2 + H_y^2 - W_x^2 - W_y^2 - l_w^2}{-2 \cdot l_w \cdot \sqrt{W_x^2 + W_y^2}}\right)$$

(3) $l_w^2 = H_x^2 + H_y^2 + W_x^2 + W_y^2 - 2 \cdot \sqrt{W_x^2 + W_y^2} \cdot \sqrt{H_x^2 + H_y^2} \cdot \cos \gamma$

$$\Rightarrow \gamma = \cos^{-1}\left(\frac{H_x^2 + H_y^2 + W_x^2 + W_y^2 - l_w^2}{2 \cdot \sqrt{W_x^2 + W_y^2} \cdot \sqrt{H_x^2 \cdot H_y^2}}\right)$$

(4) $\theta = \gamma + \beta + q_s$

$\theta = \arctan 2\left(\frac{H_y}{H_x}\right)$