HNCL Tutorial 3: Dynamics and control of planar arm movements

For this tutorial you are given two main Matlab scripts that rely on several functions:

- Tutorial3_Question1.m can be used to solve both questions 1b and 1c.
- Tutorial3_Question2.m can be used to solve questions 2a, 2b and 2c.

In both scripts, the sections that you need to complete are marked with comments in capital letters and given default values.

Please provide a solution with all the required plots and explanations. Submit a Zip file (NameSurnameTutorial3.zip) containing:

- Solutions for Question 1 and Question 2 (InitialsT3.pdf)
- Your modified version of Tutorial3_Question1.m. The script should work for both parts b and c, either in separate sections or in a conditional statement (InitialsT3Q1.m)
- Your modified version of Tutorial3_Question2.m. The script should work for part c (InitialsT3Q2.m)

Consider the two-joint arm model of horizontal movement illustrated in Figure 1. Using this model you will learn the effect of coupled and nonlinear dynamics on arm movements, and implement linear feedback control. MATLAB will be required to simulate two-joint planar arm movements and plot various variables during movement.

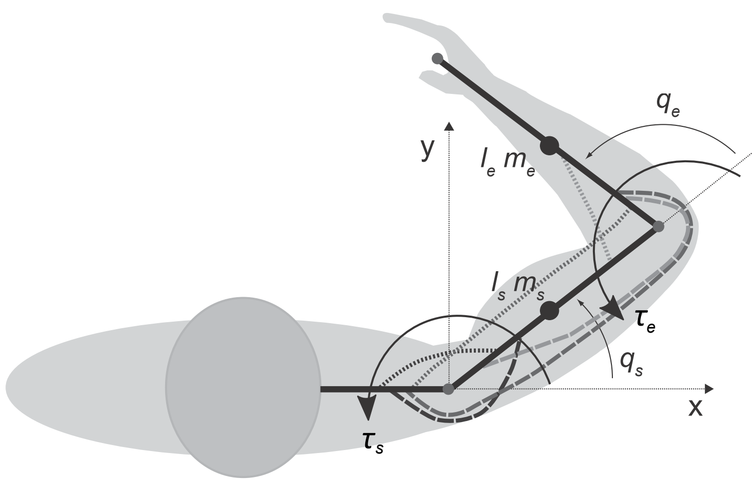

Figure 1: Two-joint model of planar arm movement. q_s and q_e denote the angles of the shoulder and elbow joints, respectively, τ_s and τ_e the torques applied to these joints. The following parameters are used: shoulder and elbow limbs mass: $m_s = 1.93kg$ and $m_e = 2.04kg$, length: $l_s = 0.31m$ and $l_e = 0.34m$, distance to the centre of mass: $l_{ms} = 0.165m$ and $l_{me} = 0.2m$ and moment of inertia: $J_s = 0.0141kg\,m^2$ and $J_e = 0.0188kg\,m^2$.

Question 1: Coupled dynamics

The dynamics of the two-joint arm model are given by the equations

$$\tau = \Psi(q, \dot{q}, \ddot{q})\, p\,, \quad \Psi_{11} = \Psi_{21} = \ddot{q}_s + \ddot{q}_e\,, \quad \Psi_{12} = (2\ddot{q}_s + \ddot{q}_e)\cos(q_e) - \dot{q}_e(2\,\dot{q}_s + \dot{q}_e)\sin(q_e)\,,$$
$$\Psi_{22} = \ddot{q}_s\cos(q_e) + \dot{q}_s^2\sin(q_e)\,, \quad \Psi_{13} = \ddot{q}_s\,, \quad \Psi_{23} = 0$$
$$p_1 \equiv J_e + m_e\, l_{me}^2 = 0.1004\, kg\, m^2, \quad p_2 \equiv m_e\, l_s\, l_{me} = 0.12\, kg\, m^2, \quad p_3 \equiv J_s + m_s\, l_{ms}^2 + m_e\, l_s^2 = 0.263\, kg\, m^2$$

First, suppose that the shoulder joint is blocked, and the elbow is moving the forearm horizontally.

(a) Write the dynamic equation/s of the single-joint elbow movement. Are these dynamics linear or nonlinear?

[8 mark]

(b) Starting at $q_e(0) = 30°$, the elbow is subjected to a stimulus

$$\tau_e(t) = \begin{cases} 0.02Nm - 0.1\dot{q}_e(t) & 0 \leq t < 2s \\ -0.1\,\dot{q}_e(t) & 2s \leq t \leq 20s \end{cases} \tag{1}$$

Simulate the response of the joint angle $q_e(t)$ in MATLAB (using $q_s(0) = 0°$) and plot the evolution of $q_e(t)$ with respect to time t using a sampling time $0.01s$.

[14 marks]

(c) Suppose now that the shoulder is no longer blocked. Simulate the joint angles $q_s(t)$ and $q_e(t)$, starting at $q_s(0) = 0°$ and $q_e(0) = 30°$, when the elbow is stimulated as in Eq.(1) and $\tau_s = -0.1\dot{q}_s$. Plot the evolution of $q_s(t)$ and $q_e(t)$ with respect to time t, next to the results of (b).

Describe the differences in the elbow movement to that in Question 1(b), and explain the reason(s) for the differences.

[21 marks]

Question 2: Nonlinear dynamics and linear feedback control

(a) Use the initial joint angles ($q_s(0) = 90°,\ q_e(0) = 130°$) and the planned hand trajectory

$$\begin{aligned} x^*(t_n) &= -0.2605 + 0.11\, g(t/T)\,, \quad g(t_n) \equiv t_n^3\,(6\,t_n^2 - 15\,t_n + 10) \\ y^*(t_n) &= 0.0915 + 0.5\, g(t/T) \end{aligned} \tag{2}$$

where $t \in [0, T]$ is time, $T = 1s$ is the movement duration and $\Delta t = 0.01s$ is the sampling time. Using the inverse differential kinematics, this information can be used to compute the planned joint angles.

To apply torques to the dynamic equation of the two-joint arm, use a linear PD controller (without any feed forward component) at each joint. Use the control gains $K_p = 100\ Nm/rad$ and $K_d = 10\ Nms/rad$ for both shoulder and elbow. Plot the evolution of $q_s(t)$ and $q_e(t)$ with respect to time t against the evolution of the planned angles $q_s^*(t)$ and $q_e^*(t)$ respectively. Plot also the resultant hand movement (evolution of $x(t)$ and $y(t)$, each with respect to time t), against the planned movement ($x^*(t)$ and $y^*(t)$). Finally, plot the resultant hand trajectory against the planned trajectory.

[21 marks]

(b) Change both the K_d and K_p values independently and discuss the resulting control and how it compares with your expectation based on the physical meaning of each term. In particular, explore the effect of large K_d. Finally, compare the results with (a).

[21 marks]

(c) What happens when the arm has to move five times as fast as specified in (a), i.e. when $T \equiv 1s$ changes to $T \equiv 0.2s$ in the planned trajectory (x^*, y^*) of Eq. (2)? Compare with (a) and provide an interpretation of the results.

[15 marks]