# Tutorial 3: Dynamics and Control of Planar Arm Movements

Due: Thursday 23th Feb 23:59

# Two-joint planar arm movement

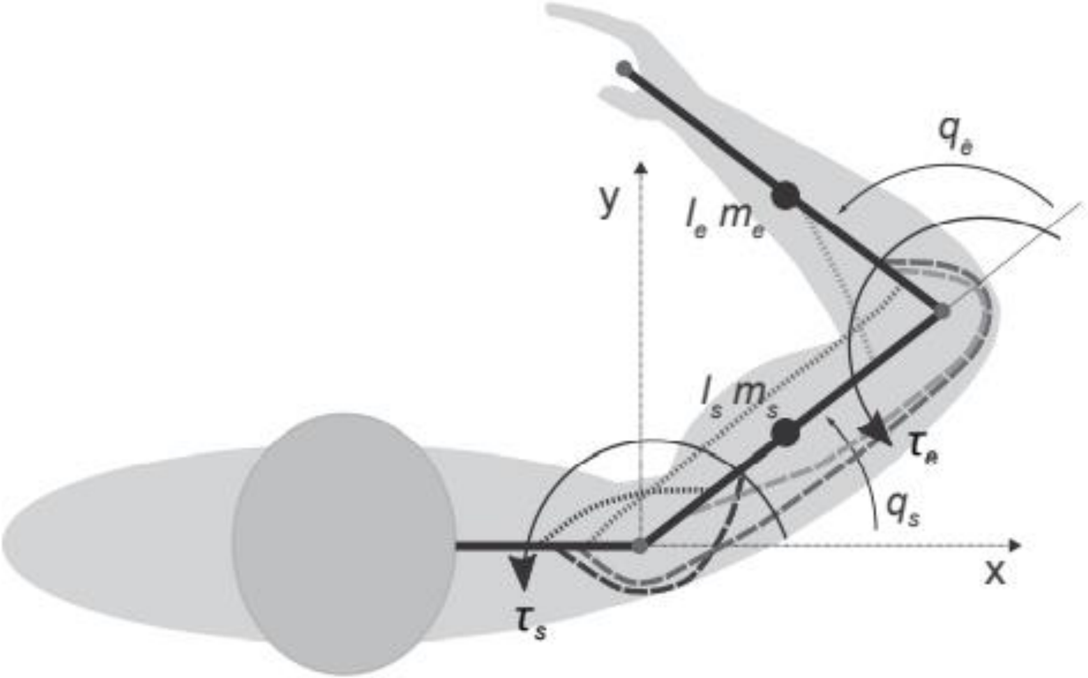



Figure 1: Two-joint model of planar arm movement. $q_s$ and $q_e$ denote the angles of the shoulder and elbow joints, respectively, $\tau_s$ and $\tau_e$ the torques applied to these joints. The following parameters are used: shoulder and elbow limbs mass: $m_s = 1.93kg$ and $m_e = 2.04kg$, length: $l_s = 0.31m$ and $l_e = 0.34m$, distance to the centre of mass: $l_{ms} = 0.165m$ and $l_{me} = 0.2m$ and moment of inertia: $J_s = 0.0141kg\,m^2$ and $J_e = 0.0188kg\,m^2$.

# Two-joint planar arm movement

For this tutorial you are given two main *Matlab* scripts that rely on several functions:

- Tutorial3 Question1.m can be used to solve both questions 1b and 1c.
- Tutorial3 Question2.m can be used to solve questions 2a, 2b and 2c.

In both scripts, the sections that you need to complete are marked with comments in capital letters and given default values

# Question 1a:

The dynamics of the two-joint arm model are given by the equations

$$\tau = \Psi(q, \dot{q}, \ddot{q})\, p\,, \quad \Psi_{11} = \Psi_{21} = \ddot{q}_s + \ddot{q}_e\,, \quad \Psi_{12} = (2\ddot{q}_s + \ddot{q}_e)\cos(q_e) - \dot{q}_e(2\,\dot{q}_s + \dot{q}_e)\sin(q_e)\,,$$
$$\Psi_{22} = \ddot{q}_s \cos(q_e) + \dot{q}_s^2 \sin(q_e)\,, \quad \Psi_{13} = \ddot{q}_s\,, \quad \Psi_{23} = 0$$
$$p_1 \equiv J_e + m_e\, l_{me}^2 = 0.1004\, kg\, m^2, \quad p_2 \equiv m_e\, l_s\, l_{me} = 0.12\, kg\, m^2, \quad p_3 \equiv J_s + m_s\, l_{ms}^2 + m_e\, l_s^2 = 0.263\, kg\, m^2$$

First, suppose that the shoulder joint is blocked, and the elbow is moving the forearm horizontally.

(a) Write the dynamic equation/s of the single-joint elbow movement. Are these dynamics linear or nonlinear?

[8 mark]

# Question 1b:

$\boxed{\text{The shoulder is still blocked}}$

(b) Starting at $q_e(0) = 30°$, the elbow is subjected to a stimulus

$$\tau_e(t) = \begin{cases} 0.02Nm - 0.1\dot{q}_e(t) & 0 \le t < 2s \\ -0.1\,\dot{q}_e(t) & 2s \le t \le 20s \end{cases} \tag{1}$$

Simulate the response of the joint angle $q_e(t)$ in MATLAB (using $q_s(0) = 0°$) and plot the evolution of $q_e(t)$ with respect to time $t$ using a sampling time $0.01s$.

[14 marks]

Hint: *Matlab* code uses

$$\boldsymbol{\tau}_B = \underline{\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}} = \boldsymbol{\Psi}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\,\mathbf{p}$$

# Question 1c:

The shoulder is no longer blocked

(c) Suppose now that the shoulder is no longer blocked. Simulate the joint angles $q_s(t)$ and $q_e(t)$, starting at $q_s(0) = 0°$ and $q_e(0) = 30°$, when the elbow is stimulated as in Eq.(1) and $\tau_s = -0.1\dot{q}_s$. Plot the evolution of $q_s(t)$ and $q_e(t)$ with respect to time $t$, next to the results of (b).

Describe the differences in the elbow movement to that in Question 1(b), and explain the reason(s) for the differences.

[21 marks]

# Question 2a:

(a) Use the initial joint angles ($q_s(0) = 90°$, $q_e(0) = 130°$) and the planned hand trajectory

$$\begin{aligned} x^*(t_n) &= -0.2605 + 0.11\,g(t/T)\,, \quad g(t_n) \equiv t_n^3\,(6\,t_n^2 - 15\,t_n + 10) \\ y^*(t_n) &= 0.0915 + 0.5\,g(t/T) \end{aligned} \tag{2}$$

where $t \in [0, T]$ is time, $T = 1s$ is the movement duration and $\Delta t = 0.01s$ is the sampling time. Using the inverse differential kinematics, this information can be used to compute the planned joint angles.

To apply torques to the dynamic equation of the two-joint arm, use a linear PD controller (without any feed forward component) at each joint. Use the control gains $K_p = 100\ Nm/rad$ and $K_d = 10\ Nms/rad$ for both shoulder and elbow. Plot the evolution of $q_s(t)$ and $q_e(t)$ with respect to time $t$ against the evolution of the planned angles $q_s^*(t)$ and $q_e^*(t)$ respectively. Plot also the resultant hand movement (evolution of $x(t)$ and $y(t)$, each with respect to time $t$), against the planned movement ($x^*(t)$ and $y^*(t)$). Finally, plot the resultant hand trajectory against the planned trajectory.

[21 marks]

Hint: Careful with the torque dimensions, use the same as the default given value

# Questions 2b and 2c:

(b) Change both the $K_d$ and $K_p$ values independently and discuss the resulting control and how it compares with your expectation based on the physical meaning of each term. In particular, explore the effect of large $K_d$. Finally, compare the results with (a).

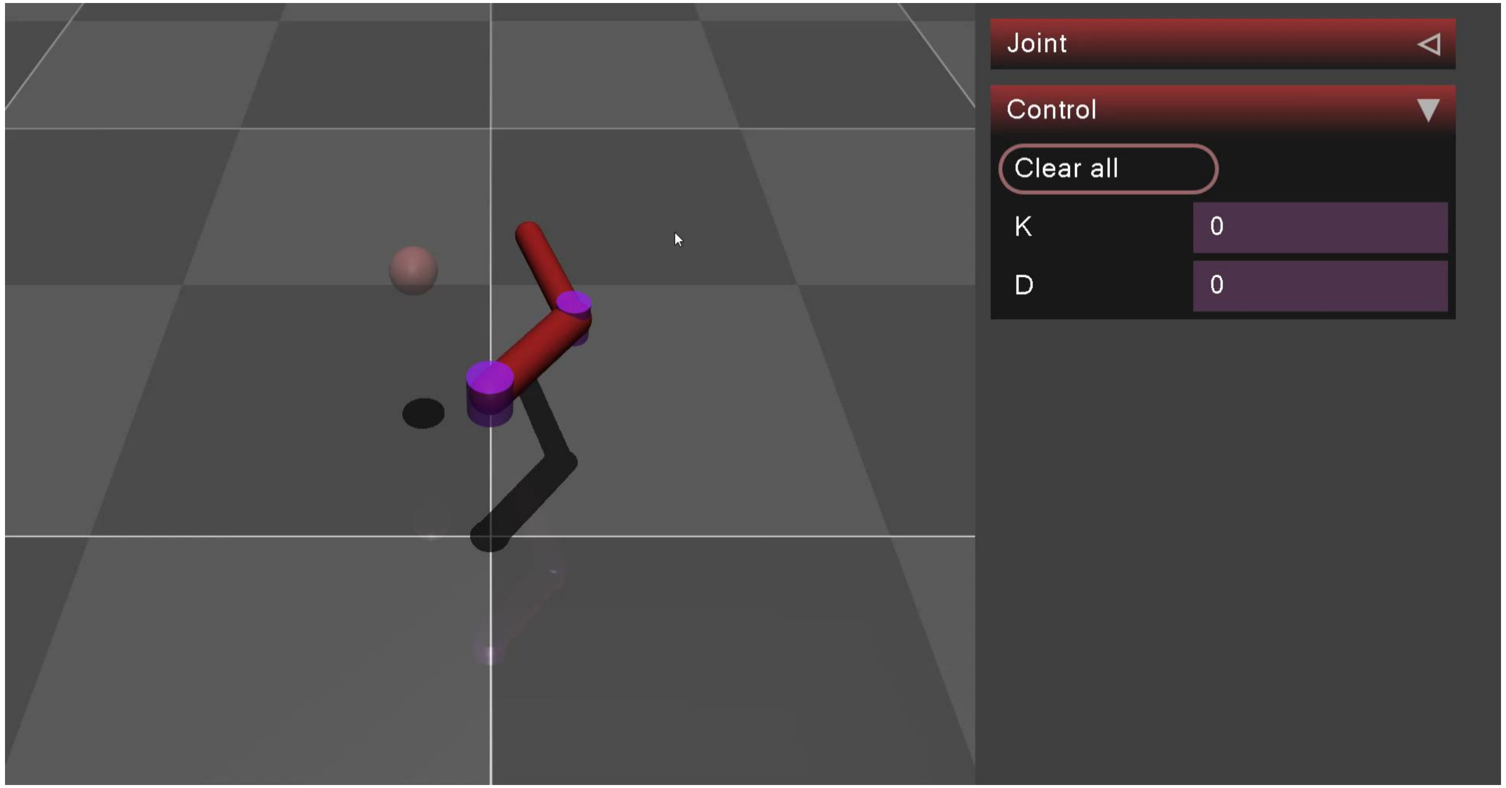
[21 marks]

(c) What happens when the arm has to move five times as fast as specified in (a), i.e. when $T \equiv 1s$ changes to $T \equiv 0.2s$ in the planned trajectory $(x^*, y^*)$ of Eq. (2)? Compare with (a) and provide an interpretation of the results.

[15 marks]

# Optional: Simulated movement

If you have some existing Python scripting skills, we'll provide near-complete scripts for rendering the PD arm control problem of Q2 in real-time tuneable and interactive simulations. These scripts are not graded and are fully optional. You should not include python scripts in your submission. They are intended only to help your understanding and verify your implementation.



# Submission

Due: Thursday 23$^{th}$ Feb 23:59

This week's tutorial has a shorter submission deadline (1 week as opposed to 2)!

- Typed out explanations preferred. Equations are preferred to be typeset (through Word or LaTeX) but legible handwriting is accepted.
- We ask your folders to be self-containing, i.e. all dependent files, functions should be present in the folder. We should be able to run your program without having to add more files, dependencies etc. from our end.

Submit a Zip file (NameSurname_Tutorial3.zip) containing :

- One PDF with the math derivation, joint angle plots and their discussion for Question 1 and 2 (InitialsT3.pdf)
- MATLAB code with concise explanation comments for added/edited lines