

## Chapter 12

# Identification of the dynamic parameters

### 12.1. Introduction

Most advanced control schemes formulated in the recent literature for robots require dynamic models (Chapter 14). The precision, performance, stability, and robustness of these schemes depend, to a large extent, on the accuracy of the parameters that describe the dynamic model. Adaptive and robust schemes can tolerate some errors in the dynamic parameters, while other schemes aimed at achieving perfect feedback linearization, such as the computed torque technique, assume precise knowledge of the dynamic parameters. In view of this, *a priori* precise determination of the dynamic parameters is useful to most schemes and is crucial to some others. Furthermore, these values are necessary to simulate the dynamic equations.

Accurate values of the dynamic parameters are typically unknown, even to the robot manufacturers. In this chapter, we will exploit the fact that the dynamic model and the energy model are linear in these parameters in order to identify them. The problem will be reduced to the least-squares solution of an overdetermined linear system of equations.

We assume *a priori* knowledge of the geometric parameters (Chapter 11). The dynamic parameters of link  $j$  and actuator  $j$  are composed of the inertial parameters of the link, the actuator rotor inertia, and the friction parameters (Chapter 9). We combine these parameters in the vector  $\chi^j$ :

$$\chi^j = [XX_j \ XY_j \ XZ_j \ YY_j \ YZ_j \ ZZ_j \ MX_j \ MY_j \ MZ_j \ M_j \ I_{aj} \ F_{cj} \ F_{vj}]^T \quad [12.1]$$

The dynamic parameters of a robot with  $n$  mobile links are represented by the vector  $\chi$  such that:

$$\chi = [\chi^1{}^T \chi^2{}^T \dots \chi^n{}^T]^T \quad [12.2]$$

To simplify the notations, we will address the case of serial robots only. The extension to tree structure or closed-chain robots can easily be carried out using the results of Chapter 10.

## 12.2. Estimation of inertial parameters

There are three main methods for estimating the inertial parameters of a robot:

i) *physical experiments*: if we could disassemble the robot to isolate each link, the following parameters could be obtained by physical experiment [Armstrong 86]:

- the mass could be weighed directly;
- the coordinates of the center-of-mass could be estimated by determining counterbalanced points of the link;
- the diagonal elements of the inertia tensor could be obtained by pendular motions.

This method is very tedious and should be realized by the manufacturer before assembling the robot;

ii) *using CAD/CAM models*: all robotics CAD/CAM packages provide tools to calculate the inertia parameters from 3D models. This method is prone to errors due to the fact that the geometry of the links is complicated to define precisely, and that certain parts such as bearings, bolts, nuts, and washers are generally neglected;

iii) *identification*: this approach is based on the analysis of the "input/output" behavior of the robot on some planned motion and on estimating the parameter values by minimizing the difference between a function of the real robot variables and its mathematical model. This method has been used extensively and was found to be the best in terms of ease of experimentation and precision of the obtained values. In this chapter, we consider off-line identification methods, for which we collect all the input-output data prior to analysis. The on-line identification will be treated in Chapter 14 when presenting the adaptive control techniques.

## 12.3. Principle of the identification procedure

Several schemes have been proposed in the literature to identify the dynamic parameters [Ferreira 84], [Mayeda 84], [An 85], [Khosla 85], [Atkeson 86], [Gautier 86], [Olsen 86], [Aldon 86], [Kawasaki 88], [Bouzouia 89], [Raucent 90],

[Aubin 91], [Prüfer 94], [Gautier 95], [Restrepo 96]. These methods present the following common features:

- the use of a linear model in the dynamic parameters (dynamic model, energy model, power model, model of the wrench exerted on the base of the robot);
- the construction of an overdetermined linear system of equations by applying the identification model at a sufficient number of points along some trajectories of the robot. In general, a constant sampling rate is used between the different points;
- the estimation of the parameter values using linear regression techniques (ordinary least-squares solution or any other alternative method).

### 12.3.1. Resolution of the identification equations

All the identification models can be written in the following general form:

$$y(\Gamma, \dot{q}) = w(q, \dot{q}, \ddot{q}) \chi \quad [12.3]$$

Applying the identification model at a sufficient number of points on some trajectories, we construct the following overdetermined linear system of equations in  $\chi$ :

$$Y(\Gamma, \dot{q}) = W(q, \dot{q}, \ddot{q}) \chi + \rho \quad [12.4]$$

where  $W$  is an  $(r \times c)$  observation matrix, or regressor,  $r$  is the total number of equations,  $c$  is the number of parameters such that  $r \gg c$ , and  $\rho$  is the residual error vector.

The identification handbooks provide a large variety of deterministic and stochastic methods to estimate  $\chi$  from the previous system of equations. The use of ordinary least-squares solution of linear overdetermined system of equations, such as those based on the SVD or QR decomposition (Appendix 4), gives good results if some care is taken in processing the data measured and the elements of the matrices  $Y$  and  $W$  as we will show in this chapter. Note that the use of scientific software packages such as Matlab or Mathematica facilitates the application of the proposed data processing. Consequently, the estimated values  $\hat{\chi}$  can be obtained from equation [12.4] such that:

$$\hat{\chi} = \underset{\chi}{\text{Min}} \|\rho\|^2$$

If  $\mathbf{W}$  is of full rank, the explicit solution of this problem is given by (Appendix 4):

$$\hat{\chi} = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \mathbf{Y} = \mathbf{W}^+ \mathbf{Y} \quad [12.5]$$

where  $\mathbf{W}^+$  is the pseudoinverse matrix of  $\mathbf{W}$ .

It should be noted that this least-squares estimation is biased because the observation matrix  $\mathbf{W}$  is random, and because  $\mathbf{W}$  and  $\mathbf{p}$  are realizations of random and correlated variables [Mendel 73], [Eykhoff 74], [de Larminat 77], [Gautier 86]. Furthermore, the elements of the matrix  $\mathbf{W}$  are nonlinear functions in  $\mathbf{q}$  and  $\dot{\mathbf{q}}$ , which leads one to assume some statistical properties of the noise in order to calculate the quality of the estimation process (bias and standard deviation) [Armstrong 89], [Raucent 90]. Consequently, it is important to verify the accuracy of the values obtained using appropriate validation procedures (§ 12.7).

The standard deviations of the estimated values are calculated by assuming that  $\mathbf{W}$  is deterministic, and  $\mathbf{p}$  is a zero mean additive independent noise, with standard deviation  $\sigma_p$ . As stated in § 11.4.3, the variance-covariance matrix  $\mathbf{C}_p$  is given by:

$$\mathbf{C}_p = E(\mathbf{p} \mathbf{p}^T) = \sigma_p^2 \mathbf{I}_r \quad [12.6]$$

where  $E$  is the expectation operator, and  $\mathbf{I}_r$  is the ( $r \times r$ ) identity matrix.

An unbiased estimation of  $\sigma_p$  can be calculated using the following equation:

$$\sigma_p^2 = \frac{\|\mathbf{Y} - \mathbf{W} \hat{\chi}\|^2}{(r - c)} \quad [12.7]$$

The variance-covariance matrix of the estimation error is given by [de Larminat 77]:

$$\mathbf{C}_{\hat{\chi}} = E[(\chi - \hat{\chi})(\chi - \hat{\chi})^T] = \mathbf{W}^+ \mathbf{C}_p (\mathbf{W}^+)^T = \sigma_p^2 (\mathbf{W}^T \mathbf{W})^{-1} \quad [12.8]$$

The standard deviation on the  $j^{\text{th}}$  parameter is obtained from the  $(j, j)$  element of  $\mathbf{C}_{\hat{\chi}}$ :

$$\sigma_{\hat{\chi}_j} = \sqrt{C_{\hat{\chi}}(j, j)} \quad [12.9]$$

This interpretation has been proposed by Raucent [Raucent 90], but we should be careful with the results obtained because the corresponding assumptions are not verified.

The relative standard deviation can be used as a criterion to measure the quality of the identification value for each parameter (§ 12.7). It is obtained as:

$$\sigma_{\hat{\chi}_{jr}} \% = 100 \frac{\sigma_{\hat{\chi}_j}}{|\hat{\chi}_j|} \quad [12.10]$$

For example, if the relative standard deviation of a parameter is greater than ten times the minimum relative standard deviation value, this parameter can be considered as poorly identified.

### 12.3.2. *Identifiability of the dynamic parameters*

The dynamic parameters can be classified into three groups: fully identifiable, identifiable in linear combinations, and completely unidentifiable. Consequently, the observation matrix  $\mathbf{W}$  corresponding to the set of parameters  $\chi$  is rank deficient (some columns of  $\mathbf{W}$  are linearly dependent whatever the values of  $\mathbf{q}$ ,  $\dot{\mathbf{q}}$  and  $\ddot{\mathbf{q}}$ ). In order to obtain a unique solution, we have to determine a set of independent *identifiable parameters*, which are also called *base dynamic parameters* or *minimum dynamic parameters*.

We have shown how to calculate the base inertial parameters using symbolic methods (Chapters 9 and 10) or numerical methods (Appendix 5). It is easy to demonstrate that the columns corresponding to the Coulomb and viscous friction parameters are independent. The determination of the base parameters is a prerequisite for the identification algorithms. It should be noted that the grouping equations do not need to be computed since the identification will give directly the grouped values.

To simplify the notations, we assume in the following that  $\chi$  represents the base inertial parameters and the friction parameters, and that  $\mathbf{W}$  is composed of the corresponding columns.

### 12.3.3. *Estimation of the friction parameters*

Using the classical friction model at non-zero velocity, which is represented by viscous and Coulomb frictions, we can write the friction torque on joint  $j$  as (§ 9.3.4):

$$\Gamma_{fj} = F_{cj} \text{sign}(\dot{q}_j) + F_{vj} \dot{q}_j \quad [12.11]$$

Two approaches can be used to identify the joint friction parameters:

*i) global identification of the inertial and friction parameters:* in this approach, we consider the estimation of the inertial parameters together with the friction parameters;

*ii) separate identification of the friction parameters:* in this approach, we identify at first the friction parameters using constant velocity motion on an axis-by-axis basis [Spetch 88], [Held 88]. These parameters are then considered to be known for the identification of the inertial parameters. This simple method induces the risk of error accumulation between the two steps.

#### 12.3.4. Trajectory selection

In order to improve the convergence rate and the noise immunity of the least-squares estimation, the trajectory used in the identification must be carefully selected. Such a trajectory is known as a *persistently exciting trajectory*. To obtain an exciting trajectory, two schemes are generally used:

- calculation of a trajectory satisfying some optimization criteria;
- use of sequential sets of special test motions, where each motion will excite some dynamic parameters. As the number of the parameters to be identified is reduced with respect to the global problem, it is easier in this case to find an exciting trajectory.

##### 12.3.4.1. Trajectory optimization

The sensitivity of the least-squares solution with respect to the modeling errors and noise can be measured by the condition number of the observation matrix. Thus, the planification of an exciting trajectory can be formulated by calculating a trajectory whose points give a "good" conditioned observation matrix. This is a nonlinear optimization problem whose degrees of freedom are the starting point, the intermediate points, the maximum joint velocities and accelerations, etc. In the literature, the following criteria have been used to define the exciting condition [Armstrong 89], [Lu 93], [Gautier 92a], [Benhlila 93]:

*i) the condition number of the matrix  $W$ , which is defined using the 2-norm, as:*

$$\text{cond}(W) = \frac{\sigma_{\max}}{\sigma_{\min}} \geq 1 \quad [12.12]$$

where  $\sigma_{\max}$  and  $\sigma_{\min}$  denote the maximum and minimum singular values of  $\mathbf{W}$  (Appendix 4). The optimization problem consists of determining the trajectory, which provides a condition number of  $\mathbf{W}$  that is close to 1;

ii) the sum of the condition number of  $\mathbf{W}$  with a parameter equilibrating the values of the elements of  $\mathbf{W}$  such that they will be of the same order of magnitude [Gautier 90a]:

$$C = \text{cond}(\mathbf{W}) + k_1 \frac{\max |W_{i,j}|}{\min |W_{i,j}|} \quad \text{with } \min |W_{i,j}| \neq 0 \quad [12.13]$$

where  $|W_{i,j}|$  is the absolute value of the  $(i, j)$  element of  $\mathbf{W}$ , and  $k_1 > 0$  is a weighting scalar parameter;

iii) the sum of the condition number of  $\mathbf{W}$  and the inverse of the smallest singular value of  $\mathbf{W}$ :

$$C = \text{cond}(\mathbf{W}) + k_2 \frac{1}{\sigma_{\min}} \quad [12.14]$$

This criterion prevents a trajectory of good condition number but with small singular values being obtained [Pressé 93]. It equilibrates the standard deviation  $\sigma_{\Delta}$  on the different parameters, but the relative standard deviation of the parameters with small values will be too high;

iv) the condition number of a weighted observation matrix. If we have *a priori* information about the order of magnitude of the dynamic parameters, the following cost function will equilibrate the contribution of each parameter on the identification model. This will result in equilibrating the relative standard deviation of the different parameters [Pressé 93]:

$$C = \text{cond}(\mathbf{W} \text{diag}(\mathbf{Z})) \quad [12.15]$$

where  $\text{diag}(\mathbf{Z})$  is the diagonal matrix formed by the elements of the vector  $\mathbf{Z}$  representing the  $(b \times 1)$  vector of the *a priori* absolute values of the dynamic parameters.

The generation of an exciting trajectory by an optimization procedure for the identification of the dynamic parameters presents the following difficulties:

- there is no analytical expression for the cost functions;
- the algorithm must take into account the joint limits, the maximum velocities, and the maximum accelerations;

- the number of degrees of freedom of the optimization problem is very large.

The generation of an exciting trajectory for the energy identification model (§ 12.6) is easier than that for the dynamic model because the energy model expression is simpler and does not need the joint accelerations [Gautier 92b], [Pressé 94]. In any case, if we cannot determine an exciting trajectory using an optimization procedure, we make use of a random trajectory and verify the corresponding cost criterion.

#### 12.3.4.2. *Sequential identification*

The most widespread approaches propose to use a set of different trajectories where each trajectory excites some parameters. For instance, we can move some joints while locking some others [Mayeda 84], [Olsen 86], [Atkeson 86], [Ha 89], [Aubin 91], [Gaudin 92]. This technique simplifies the identification equations. However, an accumulation of errors may occur since the values of some estimated parameters will be assumed to be known in subsequent identification.

Vandanjon [Vandanjon 95] has proposed to avoid this drawback by generating four different trajectories to excite four different physical phenomena, which are: inertial effect, centrifugal coupling, inertial coupling and gravity effect. The trajectories are periodic between two points (except for gravity). During an experiment, a limited number of joints move while the others are locked. The experiments are designed in order to ensure optimal condition number of the observation matrix. These trajectories are then combined in a global identification system of equations.

#### 12.3.5. *Calculation of the joint velocities and accelerations*

The observation matrix elements are functions of  $\mathbf{q}$ ,  $\dot{\mathbf{q}}$  and also  $\ddot{\mathbf{q}}$  in the case of the dynamic identification model. Industrial robots are generally equipped with position sensors with good accuracy although they can be corrupted by high frequency noise due to quantization errors. On the contrary, velocity sensors provide noisy information, and the acceleration sensors are not used in industrial robots. Consequently, the joint velocities and accelerations have to be obtained by numerically differentiating the joint positions. However, the derivative, and especially the second derivative, of the joint positions amplifies the high frequency noise because the differentiation process behaves like a high-pass filter.

A solution to this problem is to filter the joint position readings using a low-pass filter prior to compute the derivatives [Khosla 86], [Bouzouia 89], [Benhlila 93], [Gautier 95]. Such a strategy has been successfully used while identifying the dynamic parameters of the Acma SR400 robot [Restrepo 96]: the position filtering is



carried out with a non-causal zero-phase digital filter by processing the input data through an IIR low-pass Butterworth filter in both the forward and reverse direction, thanks to the "filtfilt" procedure of Matlab. The numerical derivation is carried out using a central difference algorithm to avoid phase shift. Thus, the velocity is obtained from:

$$\dot{q}(k) = [q(k+1) - q(k-1)] / 2T \quad [12.16]$$

where  $q(k)$  indicates the joint positions at the  $k^{\text{th}}$  sample, and  $T$  is the sampling period.

Another solution to avoid the calculation of joint accelerations consists of using an identification model devoid of joint accelerations such as the filtered dynamic model (§ 12.5.2), the energy model (§ 12.6.1), the power model (§ 12.6.2), or by using a stochastic filter such as the extended Kalman filter [Guglielmi 87], [Gautier 93].

### 12.3.6. Calculation of joint torques

The dynamic identification methods are based on estimating the parameters minimizing a cost function of the difference between the real robot variables and its mathematical model. Most of the proposed cost functions require joint torques. Since torque sensors are not used in industrial robots, the actuator torque may be estimated from the reference current of the amplifier current loop. Owing to the high bandwidth of the current loop, the relation between the actuator torque and the reference current can be represented by a constant gain in the operating range of the robot. For joint  $j$ , this relation can be written as (Figure 12.1):

$$\Gamma_j = G_{Tj} u_j \quad [12.17a]$$

where:

$$G_{Tj} = N_j K_{aj} K_{Tj} \quad [12.17b]$$

and where  $G_{Tj}$  is the torque gain of the drive chain of joint  $j$ ,  $u_j$  is the reference current,  $N_j$  is the gear transmission ratio,  $K_{aj}$  is the current amplifier gain, and  $K_{Tj}$  is the actuator torque constant.

The parameters of equations [12.17] can be obtained from the manufacturers data sheets. A global estimation of  $G_{Tj}$  can be obtained using specific experimentation [Restrepo 95].

Note that the dynamic identification methods can be reformulated to be independent of the drive chain constant  $G_{Tj}$  [Khalil 93], [Gautier 94]. In this case, a new set of dynamic parameters is defined.

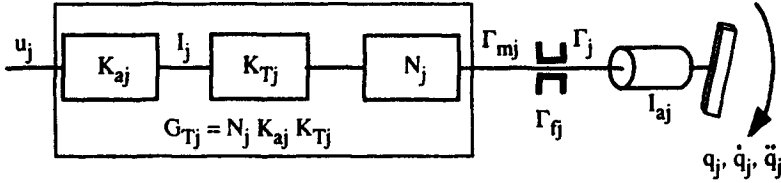


Figure 12.1. Drive chain of joint  $j$

#### 12.4. Dynamic identification model

The dynamic model is linear in the dynamic parameters. It is given by the following equation:

$$\Gamma = \Phi(q, \dot{q}, \ddot{q}) \chi \quad [12.18]$$

where  $\Phi$  is an  $(n \times b)$  matrix, and  $b$  is the number of the base dynamic parameters.

From the above equation, we deduce that the  $i^{\text{th}}$  column of  $\Phi$ , denoted by  $\Phi^i$ , is equal to:

$$\Phi^i = \Gamma(q, \dot{q}, \ddot{q} \text{ with } \chi_i = 1, \chi_j = 0 \text{ for } j \neq i) \quad [12.19]$$

Consequently,  $\Phi^i$  can be computed using a specialized version of the inverse dynamic model in which the dynamic parameters are assumed to be  $\chi_i = 1$ ,  $\chi_j = 0$  for  $j \neq i$ . To increase the efficiency of this algorithm, we use the customized symbolic technique, taking into account that the forward recursive equations are the same for all the columns  $\Phi^i$ . Moreover, we note that this customized symbolic technique is convenient for the computation of the observation matrix using an array multiply operator (`.*` of Matlab). Collecting  $(q, \dot{q}, \ddot{q}, \Gamma)_{(i)}$  for a sufficient large number of points  $i = 1, \dots, e$  on a given trajectory, and using equation [12.19], we can construct the following overdetermined system of equations:

$$Y(\Gamma) = W(q, \dot{q}, \ddot{q}) \chi + \rho \quad [12.20]$$

with:

$$Y = \begin{bmatrix} \Gamma(1) \\ \dots \\ \Gamma(e) \end{bmatrix}, W = \begin{bmatrix} \Phi(1) \\ \dots \\ \Phi(e) \end{bmatrix} \quad [12.21]$$

such that:

- $r = nxe \gg b$
- $\Phi(i) = \Phi[(q, \dot{q}, \ddot{q})_{(i)}]$
- $(q, \dot{q}, \ddot{q})_{(i)} = q(t_i), \dot{q}(t_i), \ddot{q}(t_i)$
- $\Gamma(i) = \Gamma(t_i)$

In order to eliminate high frequency noise out of torque signals, we filter the vector  $Y$  and the columns of the observation matrix  $W$ . The values obtained are then decimated at a low rate. This procedure is known as *parallel filtering* [Richalet 98]. For the identification of the Acma SR400 robot [Restrepo 96], the authors used a low-pass Tchebychev filter of order 8 with a cut-off frequency of 40 Hz. The filtered signal was then decimated at order 10. The "decimate" procedure of Matlab (Signal Processing ToolBox) can be used to carry out these two steps.

## 12.5. Other approaches to the dynamic identification model

In this section, we present two different approaches to develop the identification model. The first is sequential and is based on a link-by-link identification starting from the terminal link. The second approach makes use of a filtered dynamic model, which is a function of  $q$  and  $\dot{q}$ , and no more of  $\ddot{q}$ . The two approaches can be combined to obtain a filtered sequential dynamic identification model.

### 12.5.1. Sequential formulation of the dynamic model

Since the torque of joint  $j$  is independent of the dynamic parameters of links  $1, \dots, j-1$  (property d, § 9.3.3.3), we can write the dynamic model [12.18] such that the matrix  $\Phi$  is upper-block triangular:

$$\begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \dots & \Phi_{1n-1} & \Phi_{1n} \\ 0 & \Phi_{22} & \dots & \Phi_{2n-1} & \Phi_{2n} \\ \vdots & 0 & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \Phi_{nn} \end{bmatrix} \begin{bmatrix} \chi^1 \\ \chi^2 \\ \vdots \\ \chi^n \end{bmatrix} \quad [12.22]$$

where  $\chi^j$  contains the identifiable parameters of both link and actuator  $j$ , and  $\Phi_{ij}$  is the row vector to be multiplied by  $\chi^j$  in the equation for  $\Gamma_i$ .

This form of the dynamic equation can be exploited to estimate the dynamic parameters of each link individually, starting sequentially from link  $n$  and proceeding back to link 1. Thus, the dynamic parameters of link  $j$  are known when considering the torque equation of link  $j - 1$ . This sequential procedure reduces the number of the dynamic parameters that must be estimated at each step. The method can be summarized as follows:

- identify the base parameters of link  $n$ ,  $\chi^n$ , using the dynamic equation of joint  $n$  on a sufficient number of points:

$$\Gamma_n = \Phi_{nn}(q, \dot{q}, \ddot{q}) \chi^n \quad [12.23]$$

- then, identify  $\chi^{n-1}$  using the torque equation of joint  $n - 1$  assuming  $\chi^n$  to be known:

$$\Gamma_{n-1} - \Phi_{n-1,n}(q, \dot{q}, \ddot{q}) \hat{\chi}^n = \Phi_{n-1,n-1}(q, \dot{q}, \ddot{q}) \chi^{n-1} \quad [12.24]$$

- and so on until the estimation of the parameters of link 1.

Since the vector  $\Phi_{jj}$  is independent of the positions, velocities and accelerations of joints  $j + 1, \dots, n$ , we can lock these joints while identifying the parameters of link  $j$ . Furthermore, the number of points needed for the identification is significantly reduced with respect to the global method, owing to the reduced number of parameters. Consequently, the problem of planning an exciting trajectory is greatly simplified.

The main drawback of this method is the possible accumulation of errors from one step to the next. To overcome this problem, a weighted least-squares solution can be used [Gautier 97].

### 12.5.2. Filtered dynamic model (reduced order dynamic model)

We can avoid calculating the joint accelerations by applying to the dynamic equations a low-pass filter of at least second order with unit gain at zero frequency [Aubin 91]. This approach has also been used for adaptive control schemes [Slotine 87], [Middleton 88]. To develop the new model, let us expand equation [12.18] to isolate the elements that are functions of the joint accelerations:

$$\Gamma = [A^1(q) \ddot{q} \quad \dots \quad A^m(q) \ddot{q}] K + H_2(q, \dot{q}) K + \text{diag}(\text{sign}(\dot{q})) F_c + \text{diag}(\dot{q}) F_v \quad [12.25]$$

where  $K$  is the  $(m \times 1)$  vector of the base inertial parameters with  $m = b - 2n$ ;  $A^j$  is the  $(n \times n)$  matrix of the coefficients of the inertial parameter  $K_j$  in the robot inertia matrix  $A$ ;  $H_2$  is the  $(n \times m)$  matrix of the coefficients of the inertial parameters in the centrifugal, Coriolis and gravity terms;  $\text{diag}(u)$  is a diagonal matrix whose diagonal elements are composed of the elements of the vector  $u$ ;  $F_c = [F_{c1} \dots F_{cn}]^T$  and  $F_v = [F_{v1} \dots F_{vn}]^T$  are the friction parameters.

The vector  $A^j \ddot{q}$  can be written as:

$$A^j \ddot{q} = \frac{d}{dt} (A^j \dot{q}) - \dot{A}^j \dot{q} \quad [12.26]$$

Using equations [12.25] and [12.26], we obtain:

$$\Gamma = \left[ \frac{d}{dt} B1(q, \dot{q}) + B2(q, \dot{q}) \right] K + \text{diag}(\text{sign}(\dot{q})) F_c + \text{diag}(\dot{q}) F_v \quad [12.27]$$

with:

$$B1(q, \dot{q}) = \left[ \frac{d}{dt} (A^1 \dot{q}) \quad \frac{d}{dt} (A^2 \dot{q}) \quad \dots \quad \frac{d}{dt} (A^m \dot{q}) \right]$$

$$B2(q, \dot{q}) = [-\dot{A}^1 \dot{q} \quad -\dot{A}^2 \dot{q} \quad \dots \quad -\dot{A}^m \dot{q}] + H_2(q, \dot{q})$$

Applying the Laplace transformation of the filter  $F(s)$  to equation [12.27] yields:

$$F(s) \Gamma = [sF(s) B1(q, \dot{q}) + F(s) B2(q, \dot{q})] K + F(s) \text{diag}(\text{sign}(\dot{q})) F_c + F(s) \text{diag}(\dot{q}) F_v \quad [12.28]$$

In [Aubin 91], the following second order filter was proposed:

$$F(s) = \frac{a^2}{(s+a)^2} \quad [12.29]$$

The cut-off frequency value "a" should be chosen to be sufficiently large to maintain the dynamic behavior of the robot while rejecting the high frequency components ( $10 \leq a \leq 40$ ).

Using equation [12.27] on a sufficient number of points leads to the overdetermined system of equations:

$$Y = \left[ \left( \frac{d}{dt} W1 + W2 \right) \quad W3 \quad W4 \right] \begin{bmatrix} K \\ F_c \\ F_v \end{bmatrix} + \rho \quad [12.30]$$

In practice, the application of  $F(s)$  is carried out in equation [12.30] as follows:

- numerically differentiating  $W1$  using a central difference algorithm;
- using the "decimate" function of Matlab on  $\dot{W}1$ ,  $Y$ ,  $W2$ ,  $W3$ , and  $W4$ .

The least-squares solution is applied to the filtered model to estimate the values of  $\hat{K}$ ,  $\hat{F}_c$  and  $\hat{F}_v$ .

The computation of equation [12.28] requires the symbolic expressions of  $A^j$  and  $\dot{A}^j$ , for  $j = 1, \dots, m$ . For a robot with more than three degrees of freedom, these expressions are dramatically complex. To overcome this difficulty, we propose to develop the filtered dynamic model starting from the Lagrange equation [9.4] [Khalil 96a]. In fact, taking into account the friction effects, the Lagrange equation can be written as:

$$\Gamma = \frac{d}{dt} \frac{\partial E}{\partial \dot{q}} - \frac{\partial E}{\partial q} + \frac{\partial U}{\partial q} + \text{diag}(\text{sign}(\dot{q})) F_c + \text{diag}(\dot{q}) F_v \quad [12.31]$$

Since the energy is linear in the inertial parameters (§ 9.4), we can write:

$$\begin{cases} E = e(q, \dot{q}) K \\ U = u(q) K \end{cases} \quad [12.32]$$

where  $E$  and  $U$  are the kinetic energy and potential energy of the robot respectively.

The elements of the row vector  $e$  are obtained using equations [9.44] and [9.25]. Consequently, we can write:

$$\left\{ \begin{array}{l} e_{XXj} = \frac{1}{2} \omega_{1j} \omega_{1j} \\ e_{XYj} = \omega_{1j} \omega_{2j} \\ e_{XZj} = \omega_{1j} \omega_{3j} \\ e_{YYj} = \frac{1}{2} \omega_{2j} \omega_{2j} \\ e_{YZj} = \omega_{2j} \omega_{3j} \\ e_{ZZj} = \frac{1}{2} \omega_{3j} \omega_{3j} \\ e_{MXj} = \omega_{3j} V_{2j} - \omega_{2j} V_{3j} \\ e_{MYj} = \omega_{1j} V_{3j} - \omega_{3j} V_{1j} \\ e_{MZj} = \omega_{2j} V_{1j} - \omega_{1j} V_{2j} \\ e_{Mj} = \frac{1}{2} j V_j^T j V_j \\ e_{Iaj} = \frac{1}{2} \dot{q}_j^2 \end{array} \right. \quad [12.33]$$

where  $j\omega_j = [\omega_{1j} \ \omega_{2j} \ \omega_{3j}]^T$  and  $jV_j = [V_{1j} \ V_{2j} \ V_{3j}]^T$ .

The elements of the row vector  $u$  are obtained from equation [9.25] as:

$$\left\{ \begin{array}{l} u_{XXj} = u_{XYj} = \dots = u_{ZZj} = 0 \\ u_{MXj} = -{}^0g^T {}^0s_j \\ u_{MYj} = -{}^0g^T {}^0n_j \\ u_{MZj} = -{}^0g^T {}^0a_j \\ u_{Mj} = -{}^0g^T {}^0P_j \\ u_{Iaj} = 0 \end{array} \right. \quad [12.34]$$

where  ${}^0s_j$ ,  ${}^0n_j$ ,  ${}^0a_j$  and  ${}^0P_j$  are the  $(3 \times 1)$  vectors appearing in the transformation matrix  ${}^0T_j$  and  $g$  is the acceleration of gravity.

Thus, equation [12.31] can be written as:

$$\Gamma = \left[ \frac{d}{dt} B1(q, \dot{q}) + B2(q, \dot{q}) \right] K + \text{diag}(\text{sign}(\dot{q})) F_c + \text{diag}(\dot{q}) F_v \quad [12.35]$$

with:

$$B1(q, \dot{q}) = \frac{\partial e}{\partial \dot{q}} \quad [12.36]$$

$$\mathbf{B2}(\mathbf{q}, \dot{\mathbf{q}}) = \frac{\partial \mathbf{u}}{\partial \mathbf{q}} - \frac{\partial \mathbf{e}}{\partial \dot{\mathbf{q}}} \quad [12.37]$$

Since the expressions for the velocity vectors  $\mathbf{J}\mathbf{V}_j$  and  $\mathbf{J}\dot{\omega}_j$  are not so complicated, even for a six degree-of-freedom robot, we can symbolically compute **B1** and **B2**. We can also make use of efficient recursive algorithms to compute them using customized symbolic techniques [Stepanenko 93], [Khalil 96a]. However, the computational burden of the filtered dynamic model is still higher than that of the dynamic model.

## 12.6. Energy (or integral) identification model

In order to avoid the calculation of the joint accelerations, a model based on energy theorem has been proposed [Gautier 88]. In addition, this model has the following advantages:

- it is linear in the dynamic parameters, and the corresponding base dynamic parameters are the same as those of the dynamic model;
- the planning of an exciting trajectory is easier than for the dynamic model;
- the computation of the observation matrix is easier than that using the dynamic model.

### 12.6.1. Principle of the energy model

The energy (or integral) model is based on the energy theorem, which states that the total mechanical energy applied to the robot is equal to the sum of potential and kinetic energy contained in the system.

Let the total energy of the system, also termed *Hamiltonian*, be denoted by  $H = E + U$ . From the energy theorem, we can write (§ 14.5.2):

$$dH = \mathbf{T}^T d\mathbf{q} \quad [12.38]$$

$$dH = \mathbf{T}^T \dot{\mathbf{q}} dt \quad [12.39]$$

where:

$$\mathbf{T} = \mathbf{\Gamma} - \text{diag}(\text{sign}(\dot{\mathbf{q}})) \mathbf{F}_c - \text{diag}(\dot{\mathbf{q}}) \mathbf{F}_v \quad [12.40]$$

After integration:



$$y = \int_{t_a}^{t_b} \mathbf{T}^T \dot{\mathbf{q}} dt = H(t_b) - H(t_a) = \Delta H \quad [12.41]$$

The total energy of the system can be written as equation [9.41]:

$$H = [h_1 \dots h_n] [K_1^T \dots K_n^T]^T = \mathbf{h} \mathbf{K} \quad [12.42]$$

where  $\mathbf{K}_j$  is the vector of the base inertial parameters of link  $j$ , and  $\mathbf{h}_j$  is the row vector of the energy functions such that  $\mathbf{h}_j = \mathbf{e}_j + \mathbf{u}_j$ .

Hence, the energy equation is linear in the dynamic parameters of the robot. The expressions of the elements of  $\mathbf{h}_i$  are given by equations [12.33] and [12.34]. Substituting equation [12.42] into equation [12.41] yields the scalar identification model:

$$y = \int_{t_a}^{t_b} \Gamma^T \dot{\mathbf{q}} dt = [\Delta \mathbf{h} \quad \Delta \mathbf{f}_c \quad \Delta \mathbf{f}_v] \begin{bmatrix} \mathbf{K} \\ \mathbf{F}_c \\ \mathbf{F}_v \end{bmatrix} \quad [12.43]$$

with:

$$\Delta \mathbf{h} = \mathbf{h}(t_b) - \mathbf{h}(t_a) \quad [12.44]$$

$$\Delta \mathbf{f}_{cj} = \int_{t_a}^{t_b} \dot{q}_j \text{sign}(\dot{q}_j) dt \quad [12.45]$$

$$\Delta \mathbf{f}_{vj} = \int_{t_a}^{t_b} \dot{q}_j^2 dt \quad [12.46]$$

$$\Delta \mathbf{f}_c = [\Delta f_{c1} \dots \Delta f_{cn}] \quad [12.47]$$

$$\Delta \mathbf{f}_v = [\Delta f_{v1} \dots \Delta f_{vn}] \quad [12.48]$$

Applying equation [12.43] to a sufficient number of intervals  $ab(i)$  and collecting the corresponding  $(\Gamma, \mathbf{q}, \dot{\mathbf{q}})_{ab(i)}$ , we obtain a linear system of  $r$  equations in  $b$  unknowns such that  $r \gg b$ , where  $b$  is the number of base dynamic parameters:

$$\mathbf{Y}(\Gamma, \dot{\mathbf{q}}) = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}) \begin{bmatrix} \mathbf{K} \\ \mathbf{F}_c \\ \mathbf{F}_v \end{bmatrix} + \rho \quad [12.49]$$

with:

$$\bullet \mathbf{Y} = \begin{bmatrix} y(1) \\ \dots \\ y(r) \end{bmatrix}, \mathbf{W} = \begin{bmatrix} \Delta \mathbf{h}(1) & \Delta \mathbf{f}_c(1) & \Delta \mathbf{f}_v(1) \\ \dots & \dots & \dots \\ \Delta \mathbf{h}(r) & \Delta \mathbf{f}_c(r) & \Delta \mathbf{f}_v(r) \end{bmatrix}$$

$$\bullet \Delta \mathbf{h}(i) = \mathbf{h}[(\mathbf{q}, \dot{\mathbf{q}})_{b(i)}] - \mathbf{h}[(\mathbf{q}, \dot{\mathbf{q}})_{a(i)}]$$

$$\bullet (\mathbf{q}, \dot{\mathbf{q}})_{a(i)} = [\mathbf{q}(t_{a(i)}), \dot{\mathbf{q}}(t_{a(i)})]$$

$$\bullet (\mathbf{q}, \dot{\mathbf{q}})_{b(i)} = [\mathbf{q}(t_{b(i)}), \dot{\mathbf{q}}(t_{b(i)})]$$

$$\bullet y(i) = \int_{t_{a(i)}}^{t_{b(i)}} \Gamma^T \dot{\mathbf{q}} dt$$

The least-squares solution of equation [12.49] gives the estimated dynamic parameters  $\hat{\mathbf{K}}$ ,  $\hat{\mathbf{F}}_c$  and  $\hat{\mathbf{F}}_v$ .

### 12.6.2. Power model

The integrator appearing in the energy model is an infinite-gain filter at zero frequency. This means that small low-frequency errors such as offsets can produce large errors. To overcome this problem, we can make use of the differential equation [12.39] instead of the integral one [Gautier 96]. This leads to the power model, which is written as:

$$\Gamma^T \dot{\mathbf{q}} = \frac{d}{dt} (\mathbf{h} \mathbf{K}) + \dot{\mathbf{q}}^T [\mathbf{diag}(\text{sign}(\dot{\mathbf{q}})) \mathbf{F}_c + \mathbf{diag}(\dot{\mathbf{q}}) \mathbf{F}_v] \quad [12.50]$$

or, in a linear form with respect to the dynamic parameters:

$$\Gamma^T \dot{\mathbf{q}} = \left[ \frac{d}{dt} \mathbf{h} \quad \dot{\mathbf{q}}^T \mathbf{diag}(\text{sign}(\dot{\mathbf{q}})) \quad \dot{\mathbf{q}}^T \mathbf{diag}(\dot{\mathbf{q}}) \right] \begin{bmatrix} \mathbf{K} \\ \mathbf{F}_c \\ \mathbf{F}_v \end{bmatrix} \quad [12.51]$$

Using a sufficient number of points, we obtain the system of linear equations:

$$\mathbf{Y} = \begin{bmatrix} \frac{d}{dt} \mathbf{W1} & \mathbf{W2} & \mathbf{W3} \end{bmatrix} \begin{bmatrix} \mathbf{K} \\ \mathbf{F}_c \\ \mathbf{F}_v \end{bmatrix} + \rho \quad [12.52]$$

As we did for the filtered dynamic model (§ 12.5.2), we process the columns  $Y$ ,  $W2$  and  $W3$  using a low-pass filter  $F(s)$ , while the columns of  $W1$  are filtered by  $sF(s)$ . In practice, this process can be carried out using a central difference algorithm to obtain the time derivative of  $W1$ , then by using the "decimate" function of Matlab to filter all the model. The least-squares solution can then be used to estimate the dynamic parameters.

Another advantage of the power model with respect to the integral model comes from the fact that it is calculated for each point of the trajectory such that the problem of determining the integration period  $[t_b - t_a]$  is avoided.

Before closing this section, we note that the energy and the power identification methods do not minimize the torque errors. Thus, minimum equation errors with these models do not guarantee that the torques will be correctly evaluated using the identified parameters. Therefore, when using these methods, the results must be validated by comparing the real torques and the prediction torques (inverse dynamic model) on a test trajectory.

## 12.7. Recommendations for experimental application

For experimental application of the identification algorithms, the following recommendations and remarks should be taken into account:

- the dynamic model consists of as many equations as the number of joints, while the energy (or power) model is composed of a single one. Thus, the dynamic model is basically more exciting and the energy model is more sensitive to the use of exciting trajectories;
- the energy identification method is robust with respect to high frequency perturbation, thus less sensitive to the filtering parameters (cut-off frequency, order of the filter). On the contrary, the dynamic identification model is more sensitive to the filtering parameters and to the quality of estimating the joint velocities and accelerations;
- after filtering the joint positions, the joint velocities and accelerations should be obtained with a central difference algorithm to avoid phase lag;
- when using the dynamic identification model, we must filter the vector of measured torques in order to reject the high frequency ripples. The same filter must be applied to the columns of the observation matrix. This process is called *parallel filtering*. We note that parallel filtering is automatically incorporated in the filtered dynamic model (reduced order dynamic model);
- it may appear that the filtered dynamic model has the advantages of the dynamic model (number of equations equal to the number of joints) and that of the energy model (no joint accelerations are needed). However, if the

dynamic model is correctly processed, it gives equivalent results to the filtered dynamic model [Restrepo 96];

- in the case of robots with several degrees of freedom, six for instance, it is recommended to carry out the identification sequentially in two steps: first, identify the parameters of the wrist links, then, identify the parameters of the shoulder links while locking the wrist joints and assuming that the wrist parameters are known. This procedure is especially efficient because the dimensions of the wrist links are generally not in the same order of magnitude as those of the shoulder;
- the number of equations must be at least 500 times the number of parameters to identify;
- the filtering parameters (cut-off frequency, order of the filter, ...) can be determined by simulating the identification method;
- the relative standard deviation given by equation [12.10] can be used as a criterion to measure the identification quality of a parameter. If the relative standard deviation of a parameter is greater than ten times the minimum relative standard deviation value, this parameter can be considered as poorly identified. Thus, either this parameter has not been sufficiently excited or its contribution to the model is negligible. If the same result is obtained with different trajectories, and if the value of this parameter is relatively small with respect to the other parameters, we can cancel this parameter and define a new set of essential parameters that can be better identified [Pham 91b];
- in order to validate the results obtained, the following tests can be carried out:
  - direct validation on the identification trajectory, by calculating the error vector;
  - cross validation on a new trajectory that has not been used for the identification;
  - verify that the inertia matrix of the robot computed with the estimated parameters is positive definite [Yoshida 00];
  - identification of the dynamic parameters twice: without load, and with a known load, to verify if the load parameter values can be correctly estimated;
  - carrying out the identification using different methods – dynamic model, filtered dynamic model and power model – to compare the results;
  - realizing a simulator of the robot using the identified parameters and comparing the response of the real robot with that of the simulator.

## 12.8. Conclusion

In this chapter, we have addressed the problem of identification of the dynamic parameters of robots. We have proposed several methods that are linear in these

parameters and have the same set of base dynamic parameters. The filtered dynamic model, the energy model and the power model do not need the joint accelerations. Other identification methods, which have not been treated in this chapter, can be found in the literature, namely the method based on the minimization of the error between the measured and calculated reaction wrench (forces and moments) on the base of the robot [West 89], [Raucent 92], [Geffard 00] and the method based on the use of the extended Kalman filter [Guglielmi 87], [Poignet 00].

We have presented different criteria to measure the excitation of a given trajectory. We have pointed out that the dynamic and filtered dynamic models are basically more exciting than the energy and power models.

Having laid the foundation to identify the dynamic parameters, we can now proceed further with the generation of reference trajectories and the control schemes to track them.