Appendix 5

Numerical computation of the base parameters

A5.1. Introduction

The base parameters constitute the minimum set of parameters that characterize completely a given system. They also represent the identifiable parameters of the system. They are obtained from the standard parameters by eliminating those that have no effect on the model and by grouping some others in linear combinations. The determination of the base inertial parameters has been carried out in Chapters 9 and 10 by the use of straightforward symbolic methods for serial and tree structured robots. However, the symbolic approach cannot give all the base parameters for robots containing closed loops. This problem can be solved by the use of the numerical method presented in this appendix. In addition, the numerical method can also be applied to determine the base parameters for the geometric calibration of robots.

The symbolic approach of computing the base parameters is based on determining the independent elements of the energy functions represented by the row vector $\mathbf{h}$ (equation [9.41]), or by determining the independent columns of the $\mathbf{D}$ matrix of the dynamic model (equation [9.36]). Numerically this problem is equivalent to the study of the space span by the columns of a matrix $\mathbf{W}$ formed from $\mathbf{h}$ (or $\mathbf{D}$) using r random values of $\mathbf{q}$, $\dot{\mathbf{q}}$ (or $\mathbf{q}$, $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$). This study can be carried out using the singular value decomposition (SVD) or the QR decomposition of $\mathbf{W}$ [Gautier 91]. In this appendix, we develop the numerical method that is based on the QR decomposition of a matrix $\mathbf{W}$, which is derived from the energy functions. Both cases of tree structured robots and closed loop robots are treated.

A5.2. Base inertial parameters of serial and tree structured robots

The total energy of the system H is linear in terms of the standard inertial parameters. It is given by the following equation:

$$\mathrm{H} = \mathbf{h}\,\mathbf{K} \qquad [A5.1]$$

where:

- $\mathbf{K}$ represents the (11nx1) vector of the standard inertial parameters of the links and of the rotors of actuators;
- $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$ is the (1x11n) row vector composed of the energy functions;
- $\mathbf{q}$ and $\dot{\mathbf{q}}$ are the (nx1) joint position and joint velocity vectors respectively.

To determine the base parameters, we construct a matrix $\mathbf{W}$ by calculating the energy row $\mathbf{h}$ for r random values of joint positions and velocities such that:

$$\mathbf{W} = \begin{bmatrix} \mathbf{h}(1) \\ \mathbf{h}(2) \\ \cdots \\ \mathbf{h}(r) \end{bmatrix} \qquad [A5.2]$$

with $\mathbf{h}(i) = \mathbf{h}[\mathbf{q}(i), \dot{\mathbf{q}}(i)]$, $i = 1, ..., r$, and $r \gg 11n$.

An inertial parameter has no effect on the dynamic model if the elements of its corresponding column in $\mathbf{W}$ have the same value, i.e. its function in $\mathbf{h}$ is constant and independent of $\mathbf{q}(i), \dot{\mathbf{q}}(i)$. By eliminating such parameters and the corresponding columns, the matrix $\mathbf{W}$ is reduced to c columns and r rows.

We note that:

- the number of the base inertial parameters b is equal to the rank of $\mathbf{W}$;
- the base parameters are those corresponding to b independent columns of $\mathbf{W}$;
- the grouping equations are obtained by calculating the relationship between the independent columns and the dependent columns of $\mathbf{W}$.

The application of the foregoing statements can be achieved by the use of the QR decomposition of $\mathbf{W}$ (§ A4.4), which is given by:

$$\mathbf{Q}^{\mathrm{T}}\,\mathbf{W} = \begin{bmatrix} \mathbf{R} \\ \mathbf{0}_{(r-c)xc} \end{bmatrix} \qquad [A5.3]$$

where **Q** is an (rxr) orthogonal matrix, **R** is a (cxc) upper-triangular matrix, and $\mathbf{0}_{ixj}$ is the (ixj) matrix of zeros.

Theoretically, the non-identifiable parameters are those whose corresponding elements on the diagonal of the matrix **R** are zero [Forsythe 77], [Golub 83]. Let τ be the numerical zero:

$$\tau = r\,\varepsilon \max_{i}(|R_{ii}|) \qquad [A5.4]$$

where $|R_{ii}|$ is the absolute value of R_{ii}, and ε is the computer precision.

Thus, if $|R_{ii}| < \tau$, then the i^{th} parameter is not identifiable. On the contrary, if $|R_{ii}| > \tau$, then the corresponding column in **W** is independent and constitutes a base of the space span by **W**. Let the b independent columns be collected in the matrix **W1**, and the corresponding parameters be collected in the vector **K1**. The other columns and parameters are represented by **W2** and **K2** respectively, such that:

$$\mathbf{W\,K} = [\,\mathbf{W1}\;\;\mathbf{W2}\,]\begin{bmatrix}\mathbf{K1}\\ \mathbf{K2}\end{bmatrix} \qquad [A5.5]$$

The matrix **W2** can be written in terms of **W1** as follows:

$$\mathbf{W2} = \mathbf{W1}\,\beta \qquad [A5.6]$$

Consequently:

$$\mathbf{W\,K} = [\,\mathbf{W1}\;\;\mathbf{W2}\,]\begin{bmatrix}\mathbf{K_B}\\ \mathbf{0}\end{bmatrix} = \mathbf{W1\,K_B} \qquad [A5.7]$$

where the base parameter vector $\mathbf{K_B}$ is given by:

$$\mathbf{K_B} = \mathbf{K1} + \beta\,\mathbf{K2} \qquad [A5.8]$$

Thus, the matrix β allows us to obtain the grouping equations of the parameters **K2** with **K1**. In order to determine β, we compute the QR decomposition of the matrix $[\,\mathbf{W1}\;\;\mathbf{W2}\,]$, which is written as:

$$[\,\mathbf{W1}\;\;\mathbf{W2}\,] = [\,\mathbf{Q1}\;\;\mathbf{Q2}\,]\begin{bmatrix}\mathbf{R1} & \mathbf{R2}\\ \mathbf{0}_{(r-b)xb} & \mathbf{0}_{(r-b)x(c-b)}\end{bmatrix} = [\,\mathbf{Q1R1}\;\;\mathbf{Q1R2}\,] \qquad [A5.9]$$

where **R1** is an (rxr) regular upper-triangular matrix, and **R2** is a (bx(c−b)) matrix.

From equation [A5.9], we obtain:

$$\mathbf{Q1} = \mathbf{W1}\,\mathbf{R1}^{-1} \qquad [A5.10]$$

$$\mathbf{W2} = \mathbf{Q1}\,\mathbf{R2} = \mathbf{W1}\,\mathbf{R1}^{-1}\,\mathbf{R2} \qquad [A5.11]$$

and finally, using equation [A5.6]:

$$\beta = \mathbf{R1}^{-1}\,\mathbf{R2} \qquad [A5.12]$$

A5.3. Base inertial parameters of closed loop robots

The geometric description of closed loop robots is given in Chapter 7. The joint position vector of the equivalent tree structure is given by:

$$\mathbf{q}_{ar} = \begin{bmatrix} \mathbf{q}_a \\ \mathbf{q}_p \end{bmatrix} \qquad [A5.13]$$

where $\mathbf{q}_a$ is the (Nx1) vector of the active joint variables, and $\mathbf{q}_p$ is the ((n – N)x1) vector of the passive joint variables.

The energy functions of the inertial parameters of the closed loop structure are the same as those of the equivalent tree structure. This means that we can apply the algorithm of the tree structured robots to the closed loop structures with the difference that the matrix $\mathbf{W}$ is calculated using random values for the independent active variables $\mathbf{q}_a(i)$ and $\dot{\mathbf{q}}_a(i)$ for $i = 1, ..., r$. The corresponding passive variables $\mathbf{q}_p$ and $\dot{\mathbf{q}}_p$ are evaluated from the constraint equations of the loops.

A5.4. Generality of the numerical method

The numerical method can be used for the determination of the minimum parameters of other applications such as:

- determination of the identifiable parameters for the geometric calibration of the parameters (Chapter 11) [Khalil 91a];
- calculation of the minimum parameters of flexible structures [Pham 91a].

This numerical method is easy to implement, thanks to a software package such as SYMORO+ [Khalil 97] for the automatic computation of the symbolic expressions of the energy functions (to determine the elements of **h**) and thanks to scientific software packages of matrix computation such as Matlab and Mathematica.