# Appendix 3

# Dyalitic elimination

Let us consider the following system of equations in the two unknowns x, y:

$$\begin{cases} a\,x^2\,y^2 + b\,xy = c\,y + d \\ e\,x^2\,y^2 + f\,xy + g = 0 \end{cases} \tag{A3.1}$$

where the coefficients a, b, ..., g are constants with arbitrary values. The so-called *dyalitic elimination technique* [Salmon 1885] consists of:

*i) transforming the system [A3.1] as a linear system such that:*

$$\begin{bmatrix} ax^2 & bx-c & -d \\ ex^2 & fx & g \end{bmatrix} \begin{bmatrix} y^2 \\ y \\ 1 \end{bmatrix} = \mathbf{0} \tag{A3.2}$$

where $y^2$, y and 1 are termed *power products*;

*ii) increasing the number of equations:* by multiplying both equations by y, we obtain two new equations that form, together with those of [A3.2], a homogeneous system consisting of four equations in four unknowns (power products):

$$\mathbf{M\,Y} = \mathbf{0} \tag{A3.3}$$

where **M** is a function of x:

$$\mathbf{M} = \begin{bmatrix} 0 & ax^2 & bx-c & -d \\ 0 & ex^2 & fx & g \\ ax^2 & bx-c & -d & 0 \\ ex^2 & fx & g & 0 \end{bmatrix} \text{ and } \mathbf{Y} = [\, y^3 \;\; y^2 \;\; y \;\; 1 \,]^T$$

Since one of the elements of **Y** is 1, the system [A3.3] is compatible if, and only if, it is singular, which implies that the determinant of **M** is zero. Applying this condition to the example leads to a fourth degree equation in x. For each of the four roots, we obtain a different matrix **M**. By choosing three equations out of the system [A3.3], we obtain a system of three linear equations of type $\mathbf{A}\,\mathbf{Y}' = \mathbf{B}$ where $\mathbf{Y}' = [\, y^3 \;\; y^2 \;\; y \,]^T$. Doing that, each value of x provides a single value of y.

To summarize, the method requires four steps:

- construct the power product equation in order to minimize the number of unknowns;
- add equations to obtain a homogeneous system;
- from this system, compute a polynomial in a single unknown using the fact that the system is necessarily singular;
- compute the other variables by solving a system of linear equations.