

Appendix 7

Dynamic model of the Stäubli RX-90 robot

In this appendix, we present the simplified Newton-Euler inverse dynamic model of the Stäubli RX-90 robot. This model is obtained automatically using the software package SYMORO+ [Khalil 97]. The inertial parameters correspond to the case of symmetric links, which are given in Table 9.4. The components of the force and moments exerted by the end-effector on the environment are denoted by FX_6 , FY_6 , FZ_6 , CX_6 , CY_6 , and CZ_6 . The joint friction forces are neglected. The velocity, acceleration and torque of joint j are denoted by QP_j , QDP_j and GAM_j respectively. The acceleration of gravity is denoted by G_3 . As already mentioned, S_j and C_j denote $\sin(\theta_j)$ and $\cos(\theta_j)$ respectively.

Noting that the equations with an asterisk (*) on their left are constants and can be evaluated off-line, the computational cost of this model is 160 multiplications and 113 additions.

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No31=QDP1*ZZ1R
WI12=QP1*S2
WI22=C2*QP1
WP12=QDP1*S2 + QP2*WI22
WP22=C2*QDP1 - QP2*WI12
DV222=-WI22**2
DV332=-QP2**2
DV122=WI12*WI22
DV132=QP2*WI12
DV232=QP2*WI22
U112=DV222 + DV332
U212=DV122 + QDP2
U312=DV132 - WP22
VP12=- G3*S2
VP22=- C2*G3
PIS22=XXR2 - ZZR2
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$No12 = WP12 * XXR2 + DV232 * ZZR2$
 $No22 = DV132 * PIS22$
 $No32 = -DV122 * XXR2 + QDP2 * ZZR2$
 $WI13 = C3 * WI12 + S3 * WI22$
 $WI23 = -S3 * WI12 + C3 * WI22$
 $W33 = QP2 + QP3$
 $WP13 = QP3 * WI23 + C3 * WP12 + S3 * WP22$
 $WP23 = -QP3 * WI13 - S3 * WP12 + C3 * WP22$
 $WP33 = QDP2 + QDP3$
 $DV113 = -WI13 ** 2$
 $DV333 = -W33 ** 2$
 $DV123 = WI13 * WI23$
 $DV133 = W33 * WI13$
 $DV233 = W33 * WI23$
 $U123 = DV123 - WP33$
 $U223 = DV113 + DV333$
 $U323 = DV233 + WP13$
 $VSP13 = d3 * U112 + VP12$
 $VSP23 = d3 * U212 + VP22$
 $VSP33 = d3 * U312$
 $VP13 = C3 * VSP13 + S3 * VSP23$
 $VP23 = -S3 * VSP13 + C3 * VSP23$
 $F13 = MYR3 * U123$
 $F23 = MYR3 * U223$
 $F33 = MYR3 * U323$
 $*PIS23 = XXR3 - ZZR3$
 $No13 = WP13 * XXR3 + DV233 * ZZR3$
 $No23 = DV133 * PIS23$
 $No33 = -DV123 * XXR3 + WP33 * ZZR3$
 $WI14 = -S4 * W33 + C4 * WI13$
 $WI24 = -C4 * W33 - S4 * WI13$
 $W34 = QP4 + WI23$
 $WP14 = QP4 * WI24 + C4 * WP13 - S4 * WP33$
 $WP24 = -QP4 * WI14 - S4 * WP13 - C4 * WP33$
 $WP34 = QDP4 + WP23$
 $DV124 = WI14 * WI24$
 $DV134 = W34 * WI14$
 $DV234 = W34 * WI24$
 $VSP14 = RL4 * U123 + VP13$
 $VSP24 = RL4 * U223 + VP23$
 $VSP34 = RL4 * U323 + VSP33$
 $VP14 = C4 * VSP14 - S4 * VSP34$
 $VP24 = -S4 * VSP14 - C4 * VSP34$

$*PIS24 = XXR4 - ZZR4$
 $No14 = WP14 * XXR4 + DV234 * ZZR4$
 $No24 = DV134 * PIS24$
 $No34 = -DV124 * XXR4 + WP34 * ZZR4$
 $WI15 = S5 * W34 + C5 * WI14$
 $WI25 = C5 * W34 - S5 * WI14$
 $W35 = QP5 - WI24$
 $WP15 = QP5 * WI25 + C5 * WP14 + S5 * WP34$
 $WP25 = -QP5 * WI15 - S5 * WP14 + C5 * WP34$
 $WP35 = QDP5 - WP24$
 $DV115 = -WI15 ** 2$
 $DV335 = -W35 ** 2$
 $DV125 = WI15 * WI25$
 $DV135 = W35 * WI15$
 $DV235 = W35 * WI25$
 $U125 = DV125 - WP35$
 $U225 = DV115 + DV335$
 $U325 = DV235 + WP15$
 $VP15 = C5 * VP14 + S5 * VSP24$
 $F15 = MYR5 * U125$
 $F25 = MYR5 * U225$
 $F35 = MYR5 * U325$
 $*PIS25 = XXR5 - ZZR5$
 $No15 = WP15 * XXR5 + DV235 * ZZR5$
 $No25 = DV135 * PIS25$
 $No35 = -DV125 * XXR5 + WP35 * ZZR5$
 $WI16 = -S6 * W35 + C6 * WI15$
 $WI26 = -C6 * W35 - S6 * WI15$
 $W36 = QP6 + WI25$
 $WP16 = QP6 * WI26 + C6 * WP15 - S6 * WP35$
 $WP36 = QDP6 + WP25$
 $DV126 = WI16 * WI26$
 $DV136 = W36 * WI16$
 $DV236 = W36 * WI26$
 $*PIS26 = XXR6 - ZZ6$
 $No16 = WP16 * XXR6 + DV236 * ZZ6$
 $No26 = DV136 * PIS26$
 $No36 = -DV126 * XXR6 + WP36 * ZZ6$
 $N16 = CX6 + No16$
 $N26 = CY6 + No26$
 $N36 = CZ6 + No36$
 $FDI16 = C6 * FX6 - FY6 * S6$
 $FDI36 = -C6 * FY6 - FX6 * S6$

$$E15=F15 + FDI16$$

$$E25=F25 + FZ6$$

$$E35=F35 + FDI36$$

$$N15=C6*N16 + No15 - N26*S6 - MYR5*VP24$$

$$N25=N36 + No25$$

$$N35=-(C6*N26) + No35 - N16*S6 - MYR5*VP15$$

$$FDI15=C5*E15 - E25*S5$$

$$FDI35=C5*E25 + E15*S5$$

$$N14=C5*N15 + No14 - N25*S5$$

$$N24=-N35 + No24$$

$$N34=C5*N25 + No34 + N15*S5$$

$$FDI14=C4*FDI15 + E35*S4$$

$$FDI34=C4*E35 - FDI15*S4$$

$$E13=F13 + FDI14$$

$$E23=F23 + FDI35$$

$$E33=F33 + FDI34$$

$$N13=C4*N14 + No13 + FDI34*RL4 - N24*S4 + MYR3*VSP33$$

$$N23=N34 + No23$$

$$N33=-(C4*N24) + No33 - FDI14*RL4 - N14*S4 - MYR3*VP13$$

$$FDI23=C3*E23 + E13*S3$$

$$N12=C3*N13 + No12 - N23*S3$$

$$N22=-(d3*E33) + C3*N23 + No22 + N13*S3$$

$$N32=d3*FDI23 + N33 + No32 - MY2*VP12 + MXR2*VP22$$

$$N31=C2*N22 + No31 + N12*S2$$

$$GAM1=N31$$

$$GAM2=N32$$

$$GAM3=N33 + IA3*QDP3$$

$$GAM4=N34 + IA4*QDP4$$

$$GAM5=N35 + IA5*QDP5$$

$$GAM6=N36 + IA6*QDP6$$