Appendix 2

The inverse robot

The n degree-of-freedom robot whose set of geometric parameters are (σ_j', α_j', d_j', θ_j', r_j') is defined as the inverse of the robot (σ_j, α_j, d_j, θ_j, r_j) if the transformation matrix ${}^0\mathbf{T}_n(\sigma_j', \alpha_j', d_j', \theta_j', r_j')$ is equal to ${}^0\mathbf{T}_n^{-1}(\sigma_j, \alpha_j, d_j, \theta_j, r_j)$.

Table A2.1 gives the geometric parameters of a general six degree-of-freedom robot. Table A2.2 gives those of the corresponding inverse robot. Indeed, let us write the transformation matrix ${}^0\mathbf{T}_6$ under the following form:

$$ {}^0\mathbf{T}_6 = \mathbf{Rot}(z,\theta_1)\ \mathbf{Trans}(z,r_1)\ \mathbf{Rot}(x,\alpha_2)\ \mathbf{Trans}(x,d_2)\ \mathbf{Trans}(z,r_2)\ \mathbf{Rot}(z,\theta_2)\ \ldots\ \mathbf{Rot}(x,\alpha_6)\ \mathbf{Trans}(x,d_6)\ \mathbf{Trans}(z,r_6)\ \mathbf{Rot}(z,\theta_6) \qquad [A2.1] $$

Table A2.1. *Geometric parameters of a general six degree-of-freedom robot*

j	σ_j	α_j	d_j	θ_j	r_j
1	σ_1	0	0	θ_1	r_1
2	σ_2	α_2	d_2	θ_2	r_2
3	σ_3	α_3	d_3	θ_3	r_3
4	σ_4	α_4	d_4	θ_4	r_4
5	σ_5	α_5	d_5	θ_5	r_5
6	σ_6	α_6	d_6	θ_6	r_6

The inverse transformation matrix ${}^6\mathbf{T}_0$ can be written as:

$$ {}^6\mathbf{T}_0 = \mathbf{Rot}(z,-\theta_6)\ \mathbf{Trans}(z,-r_6)\ \mathbf{Trans}(x,-d_6)\ \mathbf{Rot}(x,-\alpha_6)\ \mathbf{Rot}(z,-\theta_5)\ \mathbf{Trans}(z,-r_5)\ \ldots\ \mathbf{Trans}(x,-d_2)\ \mathbf{Rot}(x,-\alpha_2)\ \mathbf{Rot}(z,-\theta_1)\ \mathbf{Trans}(z,-r_1) \qquad [A2.2] $$

The parameters of Table A2.2 result from comparing equations [A2.1] and [A2.2]. The corresponding elementary transformation matrices are denoted by ${}^{j-1}T_j'$ such that:

$$ {}^0T_6' = {}^0T_1' \, {}^1T_2' \ldots {}^5T_6' = {}^0T_6^{-1} \qquad [A2.3] $$

Table A2.2. *Geometric parameters of the six degree-of-freedom inverse robot*

j	σ_j'	α_j'	d_j'	θ_j'	r_j'
1	σ_6	0	0	$-\theta_6$	$-r_6$
2	σ_5	$-\alpha_6$	$-d_6$	$-\theta_5$	$-r_5$
3	σ_4	$-\alpha_5$	$-d_5$	$-\theta_4$	$-r_4$
4	σ_3	$-\alpha_4$	$-d_4$	$-\theta_3$	$-r_3$
5	σ_2	$-\alpha_3$	$-d_3$	$-\theta_2$	$-r_2$
6	σ_1	$-\alpha_2$	$-d_2$	$-\theta_1$	$-r_1$