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Extra Credit 1.19

1.19

```
(define (fib n)
  (fib-iter 1 0 0 1 n))
(define (fib-iter a b p q count)
  (cond ((= count 0) b)
        ((even? count)
         (fib-iter a
                   b
                   (+ (* q q) (* p p))      ; compute p'
                   (+ (* 2 p q) (* q q))    ; compute q'
                   (/ count 2)))
        (else (fib-iter (+ (* b q) (* a q) (* a p))
                        (+ (* b p) (* a q))
                        p
                        q
                        (- count 1)))))
```

Thought Process Below

1.19 Thought Process

1) The effect of a single transformation $T_{p'q'}$ is the same as T_{pq} twice

$\rightarrow T_{p'q'} = T_{pq} (T_{pq})$

$T_{p'q'} = a \leftarrow bq' + aq' + ap'$
$\qquad b \leftarrow bp' + aq'$

2) To apply twice, substitute a & b back into the equation

$T_{pq}(T_{pq})$: $a \leftarrow (bp + aq)q + (bq + aq + ap)q + (bq + aq + ap)p$
$\qquad bpq + aq^2 + bq^2 + aq^2 + apq + bpq + apq + ap^2$
$\qquad (bpq + bq^2 + bpq) + (aq^2 + aq^2 + apq + apq + ap^2)$
$\qquad b(2pq + q^2) + a(2q^2 + 2pq + p^2)$

according to step 1: $bq' + aq' + ap' = b(2pq + q^2) + a(2q^2 + 2pq + p^2)$
$\quad - q' = 2pq + q^2$
$\quad - q' + p' = 2q^2 + 2pq + p^2 \rightarrow p' = q^2 + p^2$

$T_{pq}(T_{pq})$: $b \leftarrow (bp + aq)p + (bq + aq + ap)q$
$\qquad bp^2 + apq + bq^2 + aq^2 + apq$
$\qquad (bp^2 + bq^2) + (aq^2 + apq + apq)$
$\qquad b(p^2 + q^2) + a(q^2 + 2pq)$

according to step 1: $bp' + aq' = b(p^2 + q^2) + a(q^2 + 2pq)$
$\quad - p' = p^2 + q^2$
$\quad - q' = q^2 + 2pq$

3) Both p' and q' match in both tests