1. Your source attribution approach appears to go from a larger space (full rank) to a reduced space (Lets say M to L where M > L).  That’s fine, but how does that work for the sectoral attribution when that goes from M to N (where N > M), i.e. do you assume a relative weighting approach for sectoral attribution such that the prior distribution defines the relative weighting?    Some discussion on this would be good as we are starting to take into account the prior uncertainties in the sectoral attribution (e.g. Cusworth et al. Nature Com: Earth and Envirornment 2021, Worden et al. ACP 2022).

Thanks for bringing this up. I added a sentence on the new methods to section 2.8 (see attached document with tracked changes)

The only case in which we go from a lower dimensional space to a higher dimensional space is for our landfill results. I added a sentence clarifying this, also in section 2.8. As you note, this is a bit of a risky endeavor, which is why we spend so much time validating the landfill results (and we make special note of the Puente Hills Landfill, which demonstrates the limit of this approach).

1. Also, do you find any instability in W when you calculate the pseudo-inverse? (that is you can go back and forth between the larger to the smaller state space).   Assuming I interpreted your attribution correctly, W is a summation matrix  such that if M is the number of elements in the “red” state vector and N is the number of elements in the “FR” state vector, with M < N.  In this scenario, W can be unstable such that you cant go back and forth between the reduced and larger space. On the other hand if you put W on the other side of the equation than the pseudo-inverse is always stable but then its harder to use in this context.

Thanks for catching this! I doubt anyone else would have noticed. You’re correct that I should have written instead of since I’ve defined so that the rows (not the columns) are linearly independent. Indeed, this is how I calculate it in practice. In this case, I encountered no instability (I think—though I don’t remember—that I may have encountered some instability using the previous definition). (It’s also the case that we don’t go back and forth between the state space and the reduced space since it’s so hard (impossible) to recover lost information after dimension reduction.) I fixed this in the draft!

1. You could also compare to our 2019 emissions in my 2022 paper (Worden et al. 2022 ACP) which is based on GOSAT data and uses the sectoral attribution approach from Cusworth et al.. In general there is good agreement for the total USA, livestock, and oil and gas, but worse agreement for waste and coal. However, its quite possible that the sensitivity of the GOSAT data to coal and waste is small such that our results are strongly affected by the prior.   I found this was the case when using GOSAT data for 2009 to 2018 based on Zhangs results. One way to show this is to look at the relative decline in uncertainty (a proxy for DOFS)…

Thanks for the suggestion! I’m happy to compare our results to your results for total emissions, livestock, oil and gas, and coal. Because you don’t separate landfills and wastewater, I can’t do a direct comparison for your waste result. Please see the updated figure below, and the associated edits in the attached document (in tracked changes). How I should interpret the error bounds on your total for the United States? Your supplemental table has the value 27.6 ± (1.5–3.3). Is that equivalent to a range of (27.6 – 1.5 to 27.6 + 3.3)? This was my best guess.

Graphical user interface

Description automatically generated