

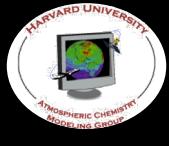


Reduced-Cost Construction of Jacobian Matrices for High-Resolution Inverse Modeling

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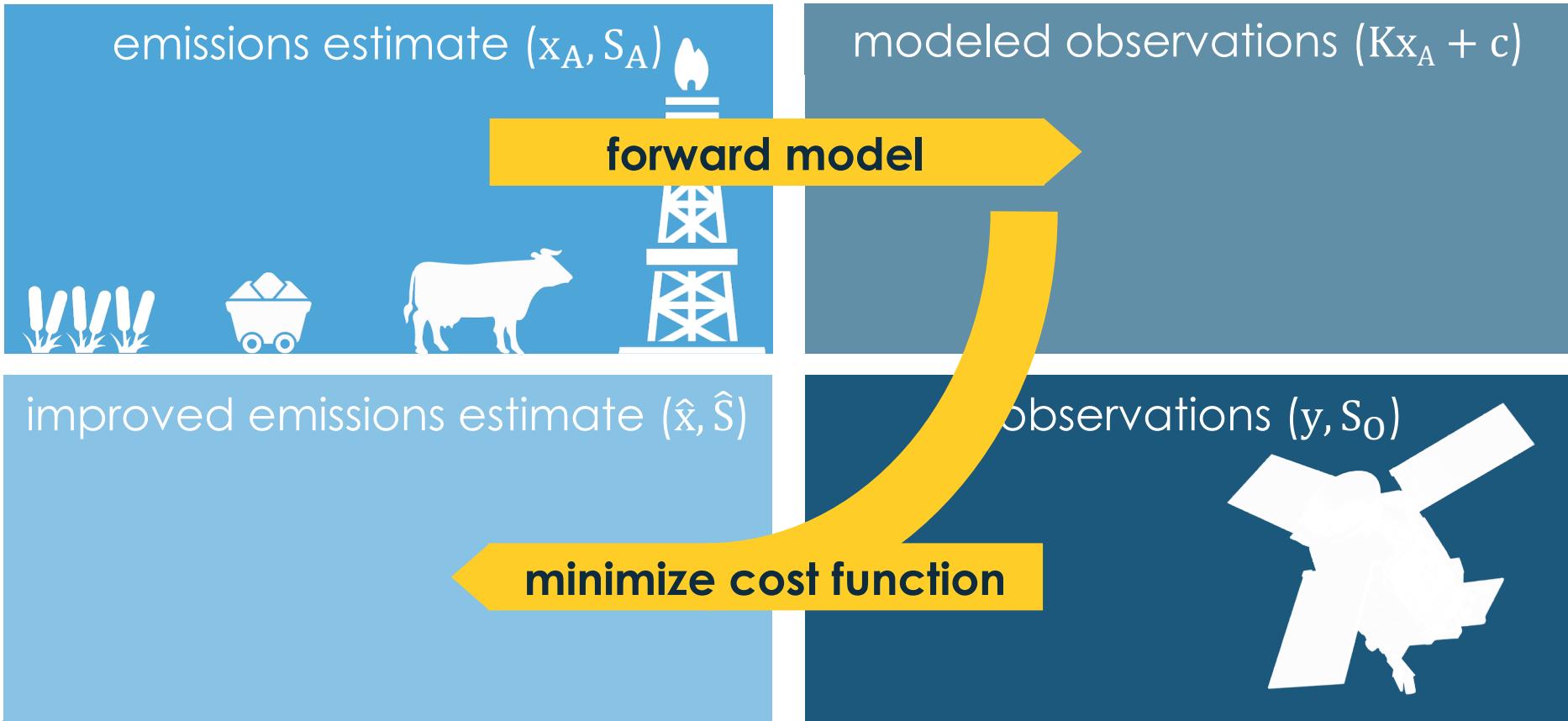
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TROPOMI provides daily, global retrievals of atmospheric methane columns

Satellite retrievals can be used in inverse frameworks to improve constraints on emissions estimates



Minimization method:

Variational

Analytic

Computational cost is not limited by resolution



Finds true minimum of shallow cost function



Characterizes posterior error and information content

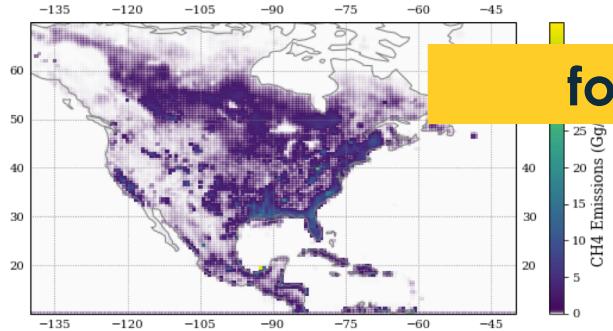


Sensitivity tests require no significant additional computational cost



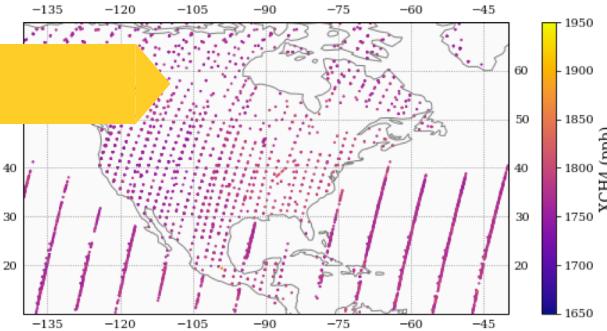
The computational cost of an analytic inversion is limited by constructing the Jacobian \mathbf{K}

emissions estimate

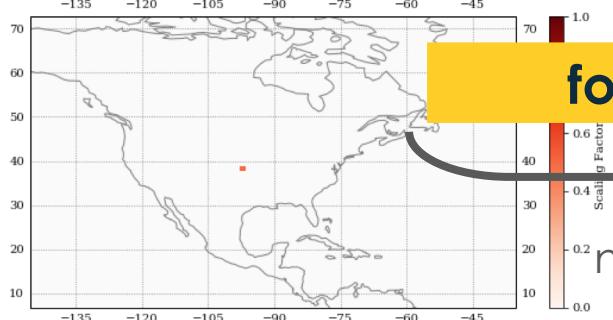


forward model

modeled observations

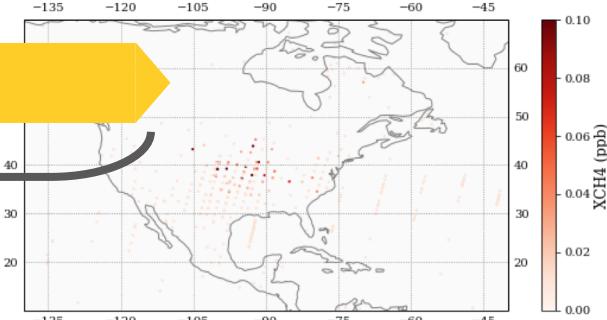


Δx



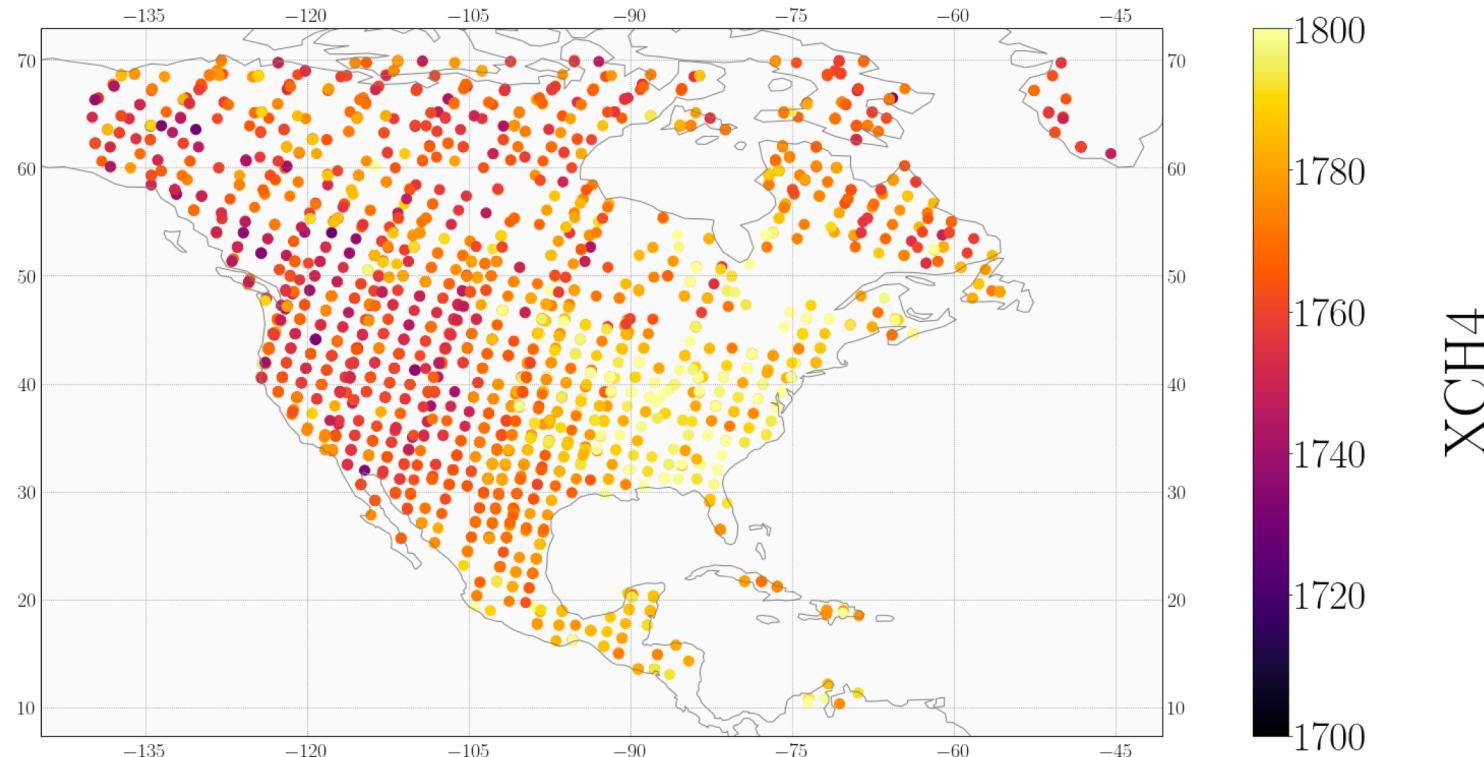
forward model

Δy



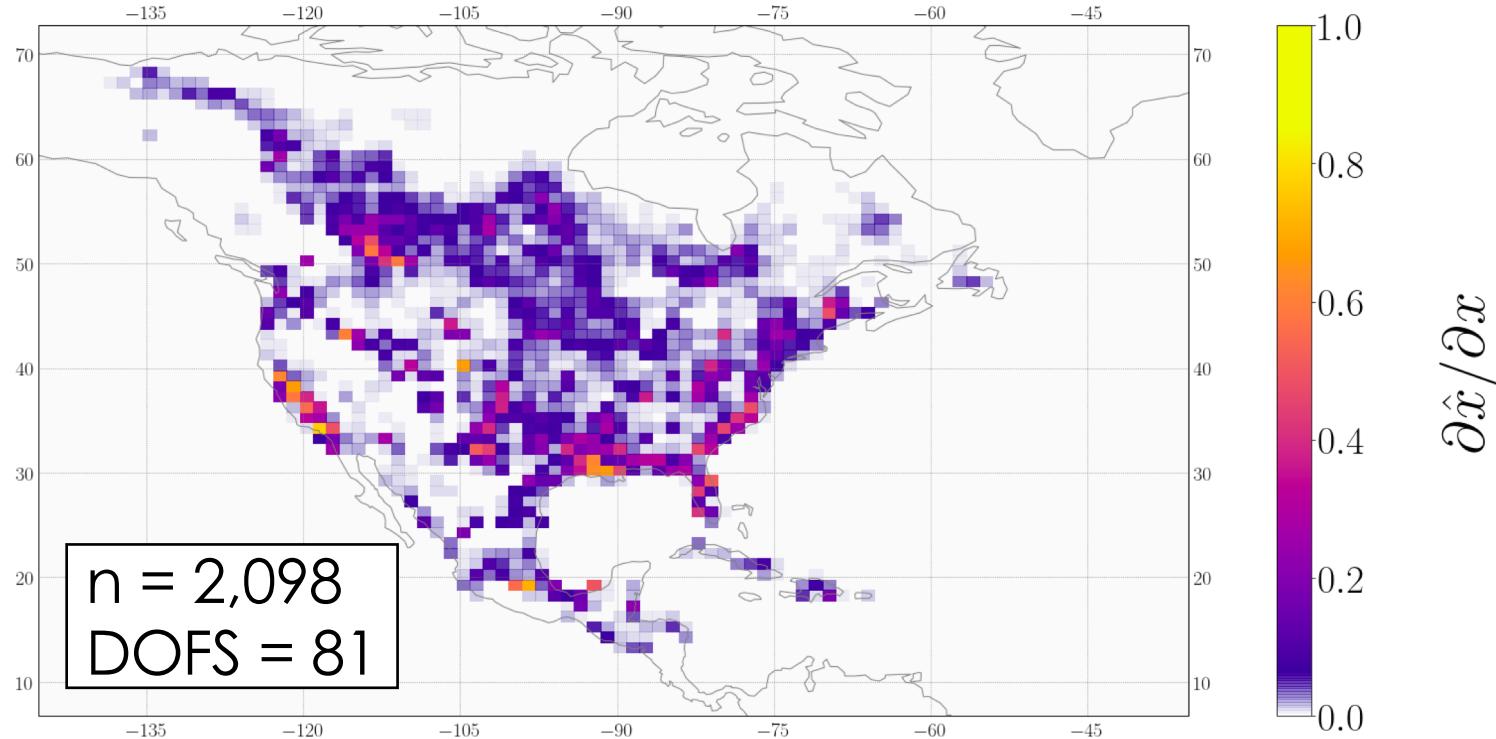
n perturbations,
n model runs

Analytic inversion of July 2009 GOSAT observations over North America at $1^{\circ} \times 1.25^{\circ}$ requires 2,099 model runs



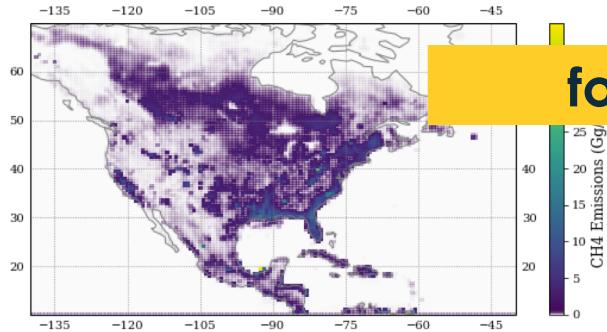
Analytic solution of the inversion shows that not all grid cells are equally well constrained

Averaging Kernel



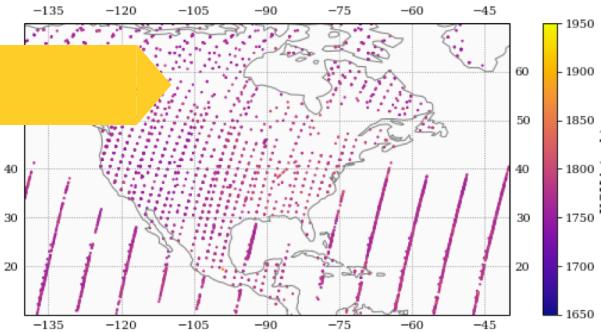
Perturbing the dominant patterns of information content would require $k < n$ model runs

emissions estimate

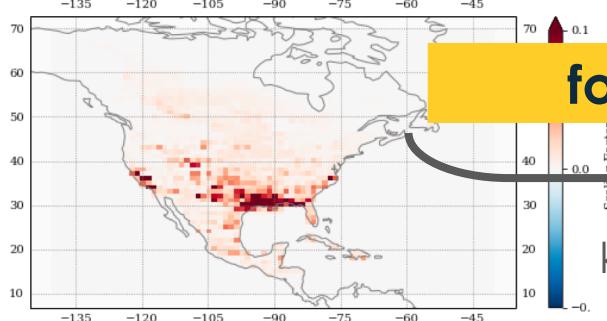


forward model

modeled observations



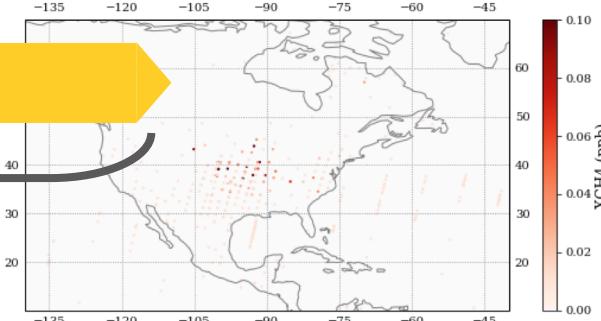
Δx



forward model

Δy

k perturbations,
 k model runs



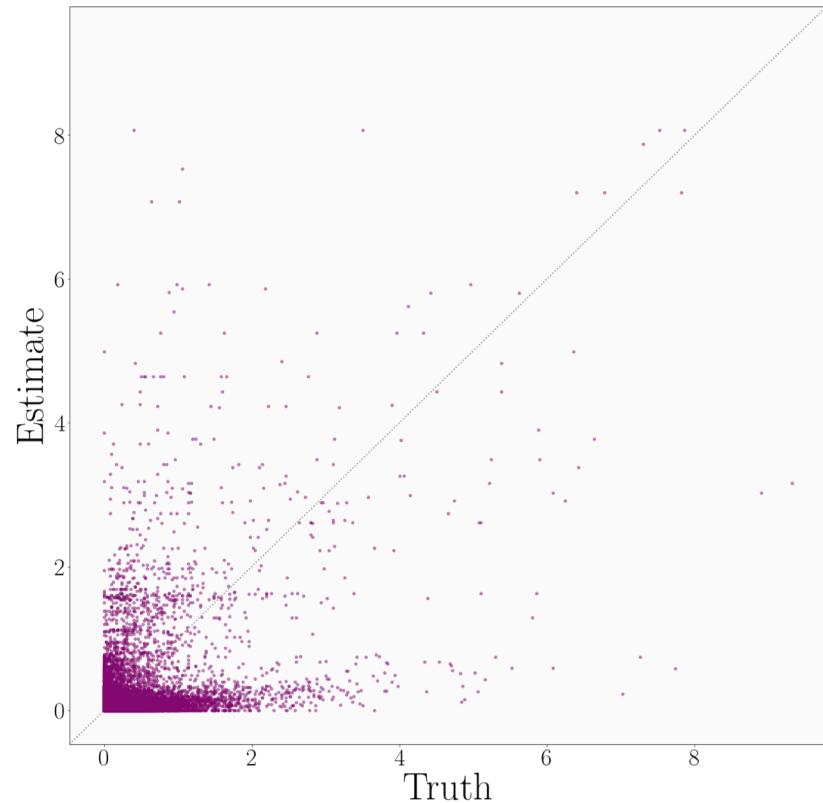
Initialize the Jacobian

Find the dominant patterns of information content

Perturb those patterns in the forward model

Transform the resulting Jacobian to the original resolution

Mass-Balance Jacobian

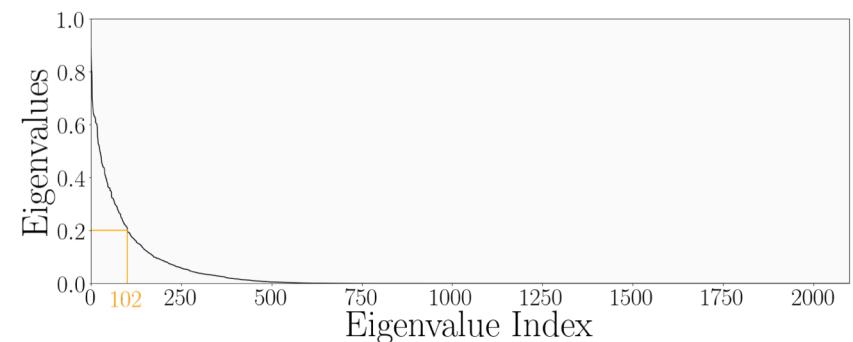
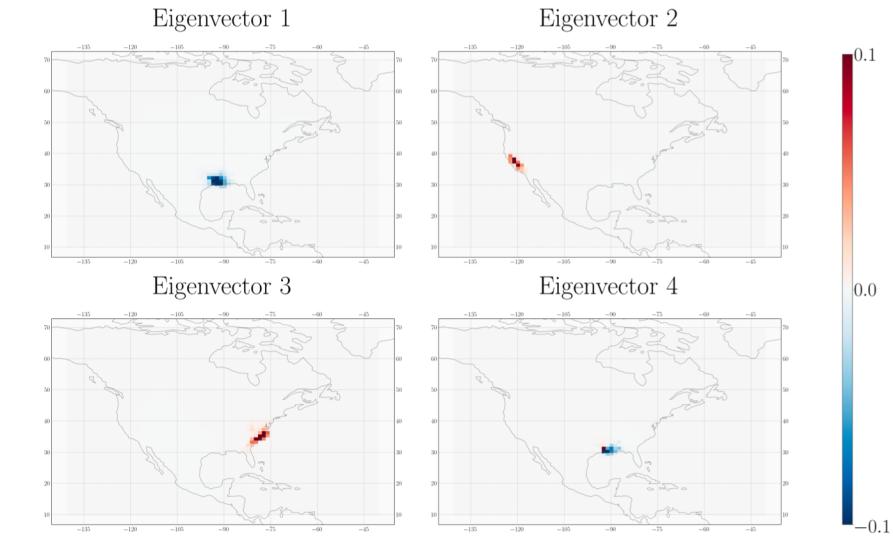


Initialize the Jacobian

Find the dominant
patterns of information
content

Perturb those patterns in
the forward model

Transform the resulting
Jacobian to the original
resolution



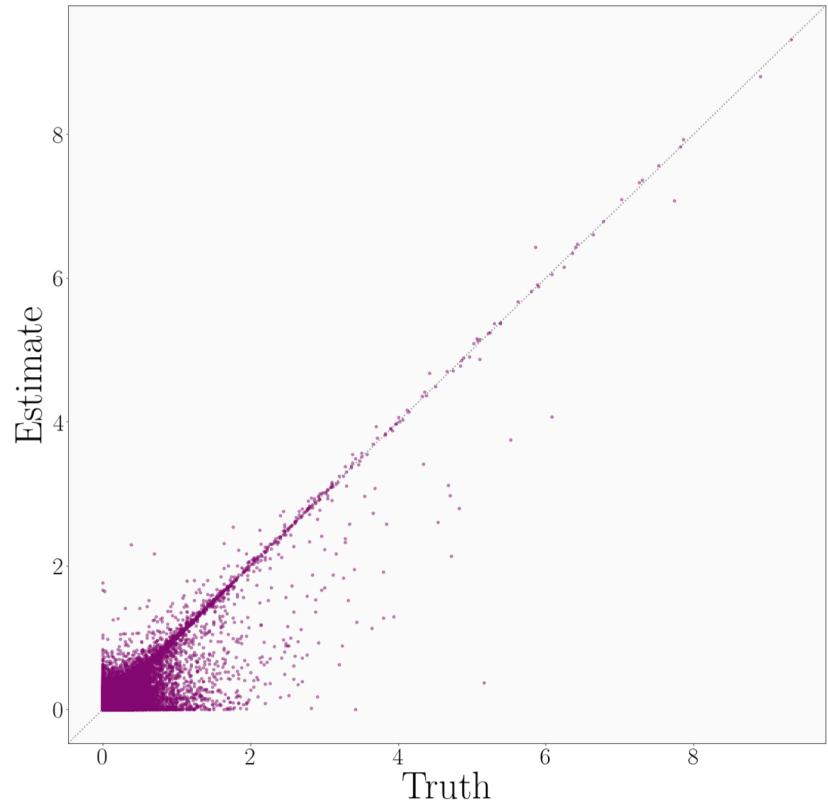
Initialize the Jacobian

Find the dominant
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Perturb those patterns in
the forward model

Transform the resulting
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Second Jacobian Update



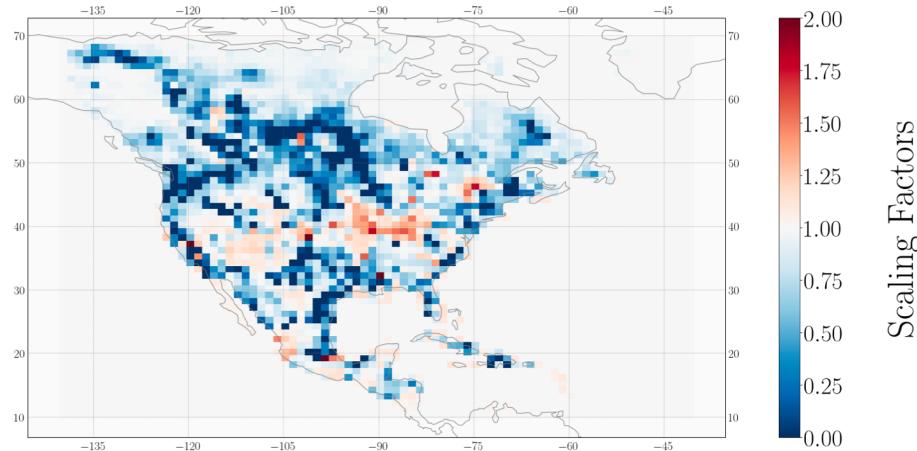
Can the low-rank Jacobian reproduce the true posterior solution?

2,099 model runs

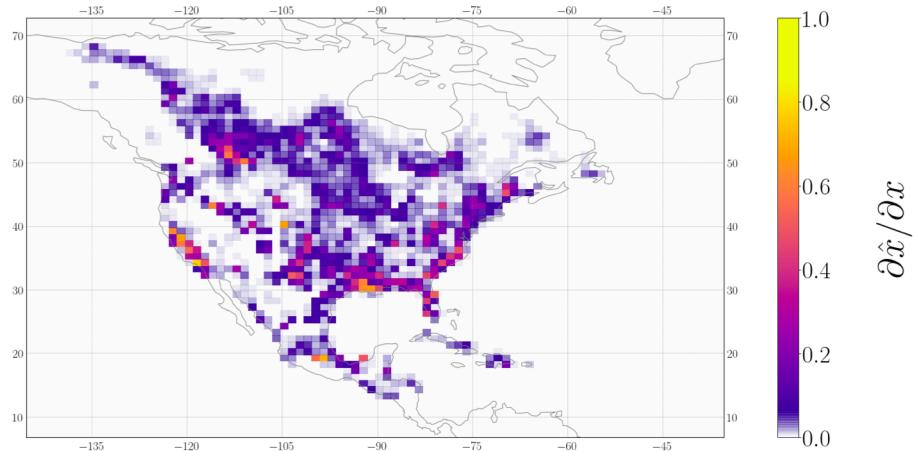
2,098 grid boxes

81 DOFs

True Posterior Mean



True Averaging Kernel



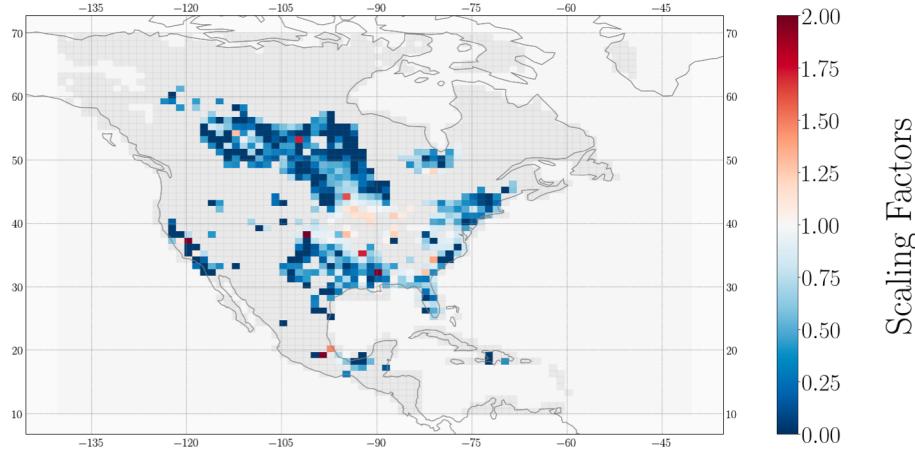
The low-rank Jacobian solves the inversion accurately in the grid cells with highest information content

306 model runs

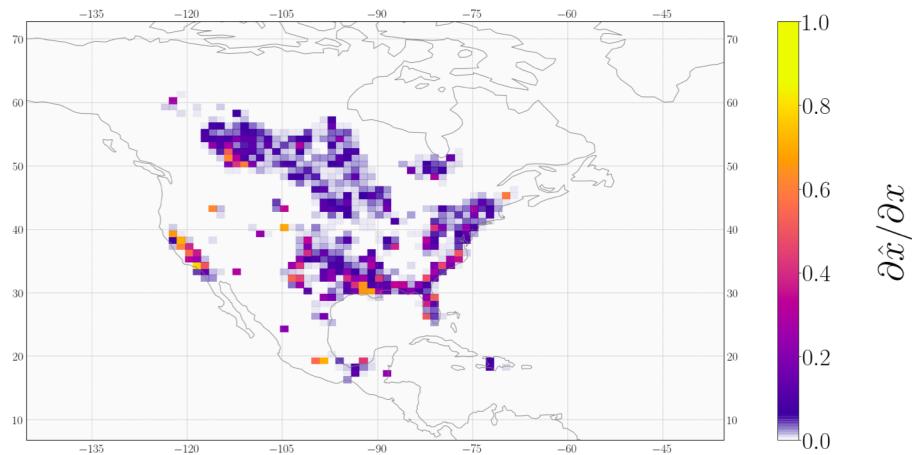
622 grid boxes

49 DOFs

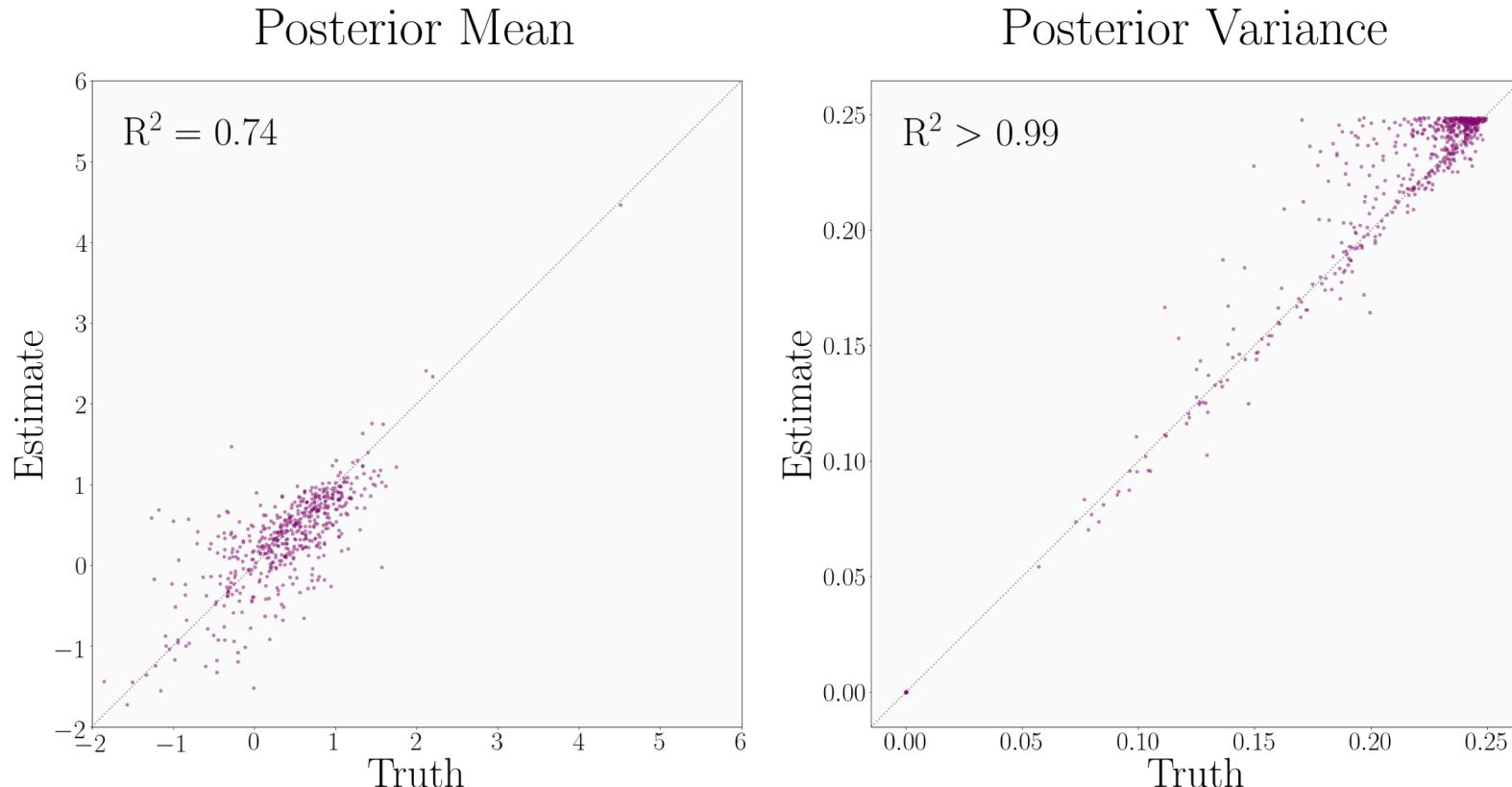
Approximate Posterior Mean



Approximate Averaging Kernel



The low-rank Jacobian solves the inversion accurately in the grid cells with highest information content



Low-rank Jacobian approximations significantly reduce the computational cost of analytic Bayesian inversions

- Perturbing the primary directions of information content can decrease the computational cost of constructing the Jacobian by an order of magnitude.
- The resulting Jacobian produces a posterior solution that is accurate in areas with sufficient information content.