A map with text

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**Figure 1.** Dimension and rank reductions of a gridded emissions field. The linear transformation matrix reduces the dimension of the original state space (upper left) either discretely by aggregating grid cells to generate a multiscale grid (upper right) or non-discretely by projecting along the patterns given by the rows of (lower right, with positive values in red and negative in blue). The reverse transformation restores the dimension but not the rank, producing a low-rank subspace of the original state space (lower right). The projection reduces rank but not dimension.

A close up of a map

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**Figure 2**. Averaging kernel sensitivities for the demonstration inversion of GOSAT observations for July 2009 and their dependence on prior error (lower left) and observational density (lower right). The top left panel shows the averaging kernel sensitivities for the native resolution averaging kernel matrix **A** and the top right panel shows the same for the initial estimate **A**(0).

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**Figure 3.** Initial multiscale grid construction scheme for the demonstration inversion. We show the DOFS per cluster as a function of the number of native-resolution grid cells added to the state vector. The color-coding corresponds to cluster size. The solid line shows the DOFS per cluster for the final state vector while the dashed lines show state vectors where the additional grid cells were not included. We summarize the final configuration in the table.

**A close up of a map

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**Figure 4.** The multiscale grid generated by the reduced-dimension method. The grid preserves native resolution in areas with highest information content and aggregates grid boxes elsewhere. The clusters are generated using a K-means algorithm following the configuration shown in Figure 3. The resulting grid has dimension 423 and the corresponding reduced-dimension Jacobian required 534 model runs.

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**Figure 5**. Results from the demonstration inversion at native-resolution compared to the reduced-dimension and reduced-rank methods. The figure shows the averaging kernel sensitivities and posterior scaling factors for each inversion. The subtitle gives the degrees of freedom for signal (DOFS), the number of pieces of information each inversion can independently constrain.

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**Figure 6.** The sensitivity of the reduced-rank inversion DOFS to the number of model runs conducted in the first (x axis) and second (y axis) update. The star represents the inversion solved here. The correlation improves as the total number of model runs increases (diagonal contours) but there is a stronger dependence on the number of model runs conducted in the second update than in the first update.

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**Figure 7.** Correlation of the Jacobian matrix and posterior solution of the reduced-rank method compared to the native-resolution system given a 75% reduction in computational cost. Scatterplots, 1:1 lines, and correlation coefficient are shown for the Jacobian matrix elements [ppb], the posterior scaling factors [dimensionless], the posterior error standard deviation [dimensionless], and the averaging kernel sensitivities [dimensionless]. Individual points are summed over finite ranges to yield the count and better visualize the density of results.