A close up of a map

Description automatically generated

**Figure 1**. Dimension and rank reductions of a gridded emissions field. The linear transformation matrix reduces the dimension of the original state space (upper left) either discretely by aggregating grid cells to generate a multiscale grid (upper right) or non-discretely by projecting along the patterns given by the rows of (lower right, with positive values in red and negative in blue). The reverse transformation restores the dimension but not the rank, producing a low-rank subspace of the original state space (lower right). The projection reduces rank but not dimension.

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**Figure 2**. Averaging kernel sensitivities for the demonstration inversion of GOSAT observations for July 2009 and their dependence on prior error (lower left) and observational density (lower right). The top left panel shows the averaging kernel sensitivities for the native resolution averaging kernel matrix **A** and the top right panel shows the same for the initial estimate **A**(0).

A screenshot of a cell phone

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**Figure 3.** [INCOMPLETE] Multiscale grid construction scheme. We show the DOFS per cluster for each state vector

**A close up of a map

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**Figure 4.** The multiscale grid that preserves native resolution in areas with highest information content and aggregates grid boxes elsewhere. The native resolution grid is clustered into ~200 grid boxes with cluster size 1, ~100 with cluster size ~3, ~100 with cluster size ~5, and ~150 with cluster size ~8 using a K-means clustering algorithm. The resulting grid has dimension ~550 and requires ~550 forward model simulations to construct the associated Jacobian matrix.

A picture containing room

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**Figure 4**. Results from the demonstration inversion using the reduced-dimension and reduced-rank methods, as compared to the native-resolution inversion. The Figure shows the averaging kernel sensitivities and posterior scaling factors. The legend gives the degrees of freedom for signal (DOFS) representing the information content of the inversion.

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**Figure 6.** Ability of the reduced-rank method to capture the structure of the native-resolution inversion with a factor of 4 decrease in computational cost. Scatterplots, 1:1 lines, and correlation coefficients are shown for the mxn Jacobian matrix elements [units], the posterior estimate (scaling factors, dimensionless), the posterior error standard deviation [units], and the averaging kernel sensitivities (dimensionless). Individual points are summed (counts) over finite ranges to better visualize the density of results.

A screenshot of a cell phone

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**Figure 7.** The sensitivity of the reduced-rank inversion DOFS to the number of model runs conducted in the first (x axis) and second (y axis) update. The star represents the inversion solved here. The correlation improves as the total number of model runs increases (diagonal contours) but there is a stronger dependence on the number of model runs conducted in the second update than in the first update.