Exercise 2a: Define RE

Definition: A RE with sequential trading is a set of a_0, $A(s)$, $Q(s'|s)$, $v(a, s)$, $c = h(a, s)$, $a'(s') = g(a, s, s')$, such that

i. The state-by-state borrowing constraints satisfy the recursion

$$A(s) = d(s) + \sum_{s'} Q(s'|s)A(s'|s)$$

ii. Given a_0, $A(s)$, and $Q(s'|s)$, the value function $v(a, s)$ and the decision rules $h(a, s)$, $g(a, s, s')$ solve

$$v(a, s) = \max_{c, a'(s')} \left\{ u(c) + \beta \sum_{s'} v[a'(s'), s']\pi(s'|s) \right\}$$

$$\text{s.t.} \quad c(s) + \sum_{s'} a'(s')Q(s|s') \leq d(s) + a(s)$$

$$c(s) \geq 0, \quad -a'(s') \leq A(s') \quad \forall s'.$$

iii. For all realisations of $\{s_t\}_{t=0}^{\infty}$, $\{c_t\}_{t=0}^{\infty}$ and $\{a_{t+1}(s_{t+1}\}_{t=0}^{\infty}$ satisfy:

$$c_t = h(a_t, s_t) = d(s_t), \quad a_{t+1}(s_{t+1}) = g(a_t, s_t, s_{t+1}) = 0.$$