ME40064: System Modelling & Simulation
ME50344: Engineering Systems Simulation
Lecture 14

Dr Andrew Cookson
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LECTURE 14
Coursework Assignment 2

- Introduce the coursework problem
- Introduce the modelling concepts relevant to the coursework
- Highlight relevant numerical implementation strategies

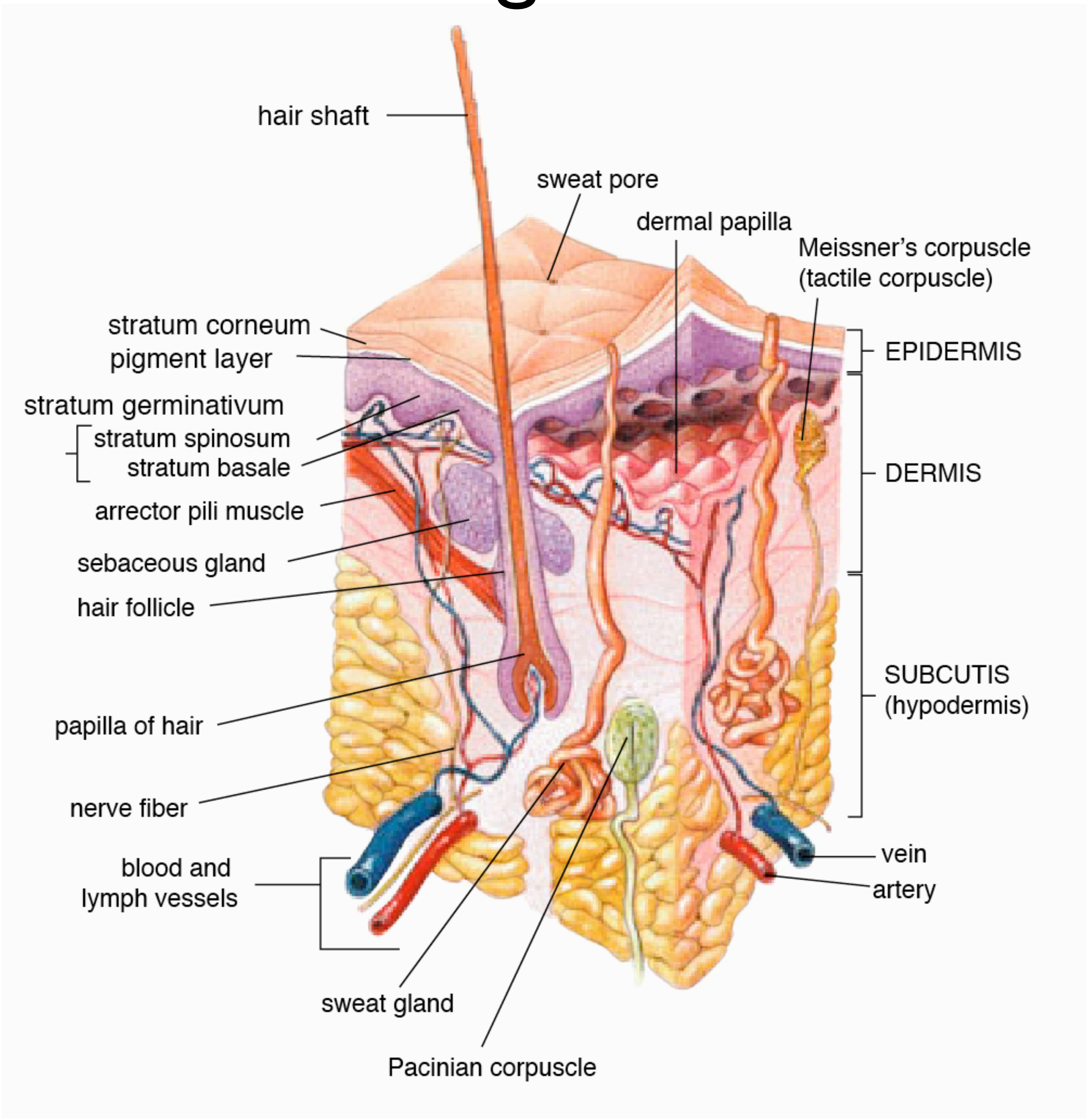
LECTURE 14
Coursework Assignment 2

Need a model to evaluate performance of heat protective clothing for preventing tissue burns:

- what is temperature distribution in the skin?
- how do we evaluate the damage that this temperature will cause?

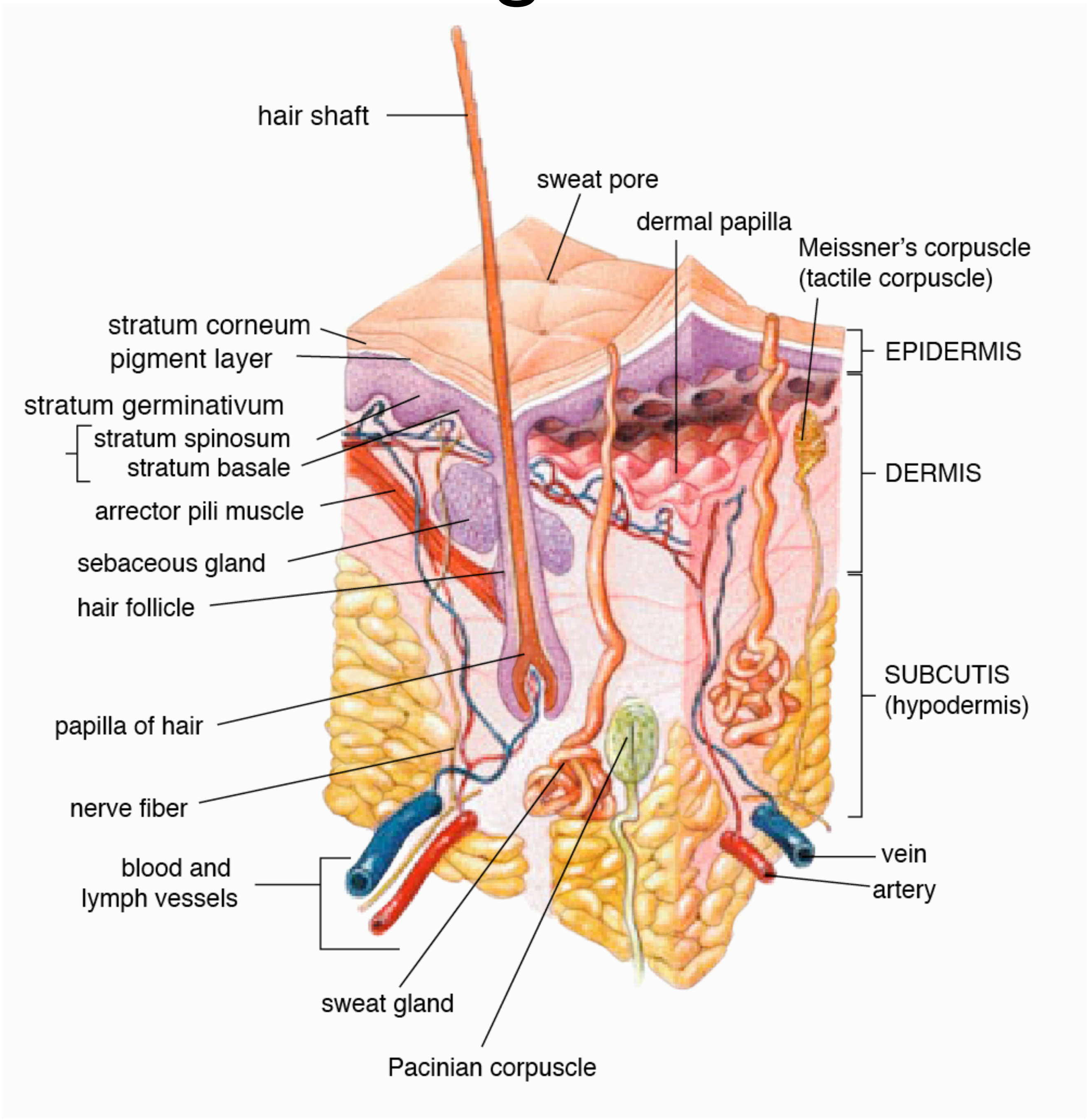
MODELLING TISSUE DAMAGE
The Geometry & Material Parameters

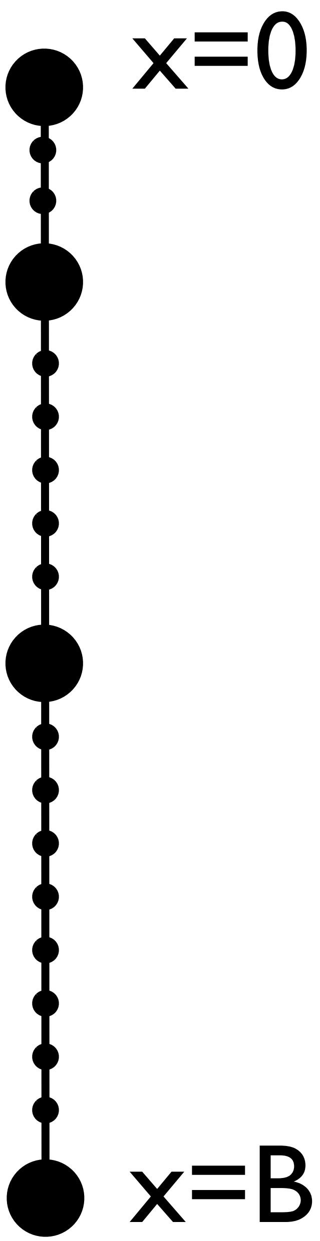
Consider the following block of tissue:



MODELLING TISSUE DAMAGE
Defining The Material Parameters

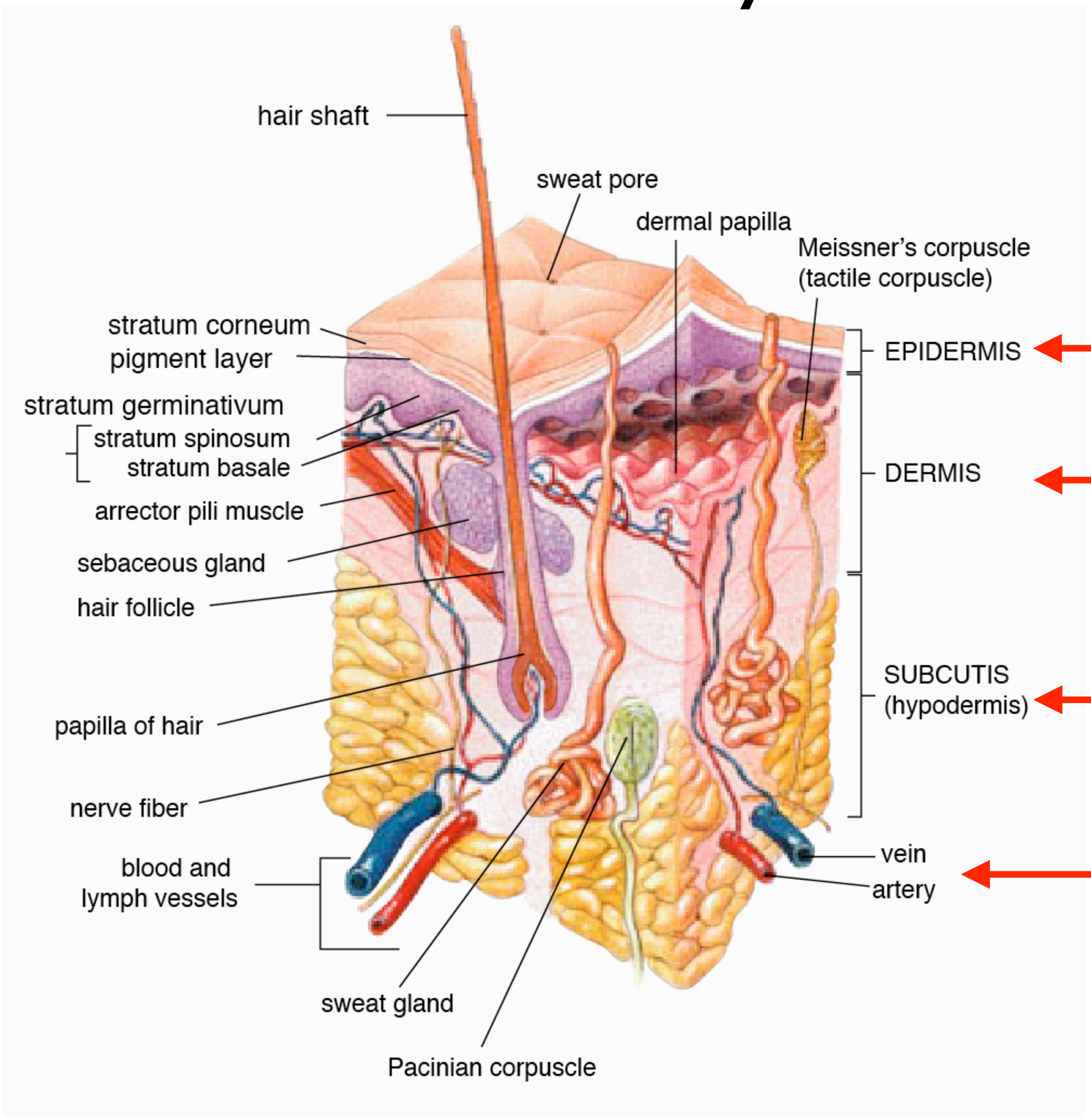
Consider the following block of tissue:





MODELLING TISSUE DAMAGE
Defining The Material Parameters

There are several discrete layers of tissue types:



EPIDERMIS

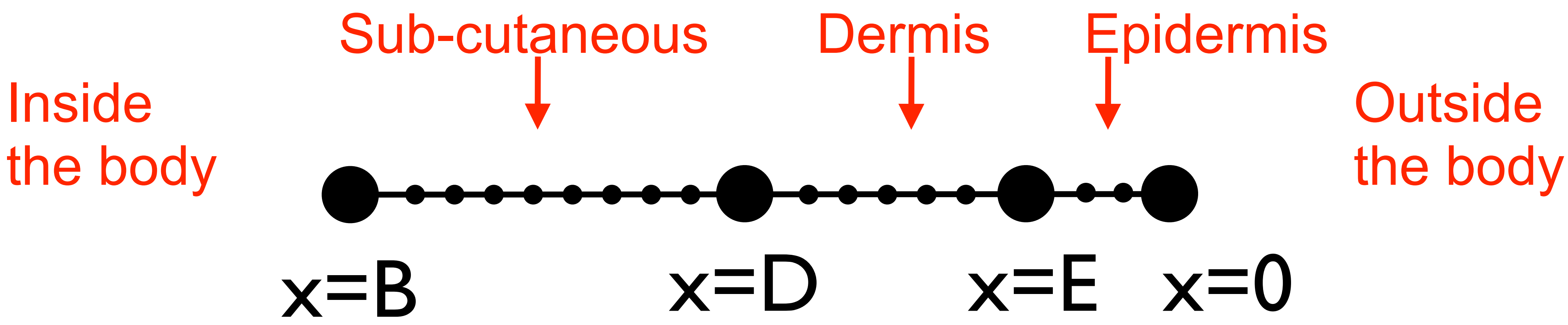
DERMIS

SUB-
CUTANEOUS

BLOOD
VESSELS

MODELLING TISSUE DAMAGE
The Geometry & Material Parameters

Define a mesh that represents the those layers:



These layers of tissue will have different thicknesses and transport properties

MODELLING TISSUE DAMAGE
Other Effects To Be Modelled?

Is there any other physiology to take account of?

MODELLING TISSUE DAMAGE
Other Effects To Be Modelled?

Is there any other physiology to take account of?

Transient heat equation models heat conduction through a tissue layer:

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}$$

where T is the temperature, and k the thermal conductivity

MODELLING TISSUE DAMAGE
Other Effects To Be Modelled?

Is there any other physiology to take account of?

Transient heat equation models heat conduction through a tissue layer

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}$$

where T is the temperature, and k the thermal conductivity

Where does the heat energy go?

MODELLING TISSUE DAMAGE
Other Effects To Be Modelled?

Flow in blood vessels (arteries, capillaries, veins) removes heat energy from particular location in tissue

Physiologically, capillaries dilate or widen to increase blood flow, and hence cooling when we become hot

How do we describe this mathematically?

MODELLING TISSUE DAMAGE
Other Effects To Be Modelled?

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How do we describe this mathematically?

$$Heat\ Sink \propto T - T_{blood}$$

This is analogous to Fick's First Law of Diffusion but consider only a difference rather than a gradient

MODELLING TISSUE DAMAGE
Other Effects To Be Modelled?

What is the constant of proportionality?

$$Heat\ Sink = G\rho_b c_b \left(T - T_b\right)$$

- blood flow rate: G
- specific heat capacity of blood: c_b
- blood density: ρ_b

Specific heat capacity = amount of heat energy per unit mass required to raise temperature of a substance by 1 degree Kelvin

MODELLING TISSUE DAMAGE
Other Effects To Be Modelled?

What is the constant of proportionality?

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- blood flow rate: G
- specific heat capacity of blood: c_b
- blood density: ρ_b

Governing equation becomes:

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - G\rho_b c_b (T - T_b)$$

Tissue properties

MODELLING TISSUE DAMAGE
Other Effects To Be Modelled?

Linking this back to familiar operators:

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - G \rho_b c_b T + G \rho_b c_b T_b$$

Transient | Diffusion | Linear reaction | Source term

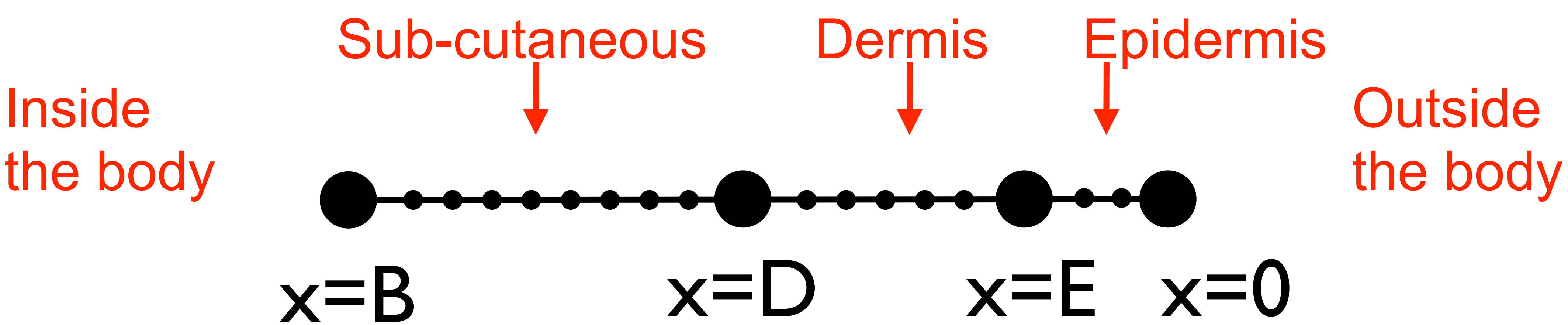
MODELLING TISSUE DAMAGE
Putting It All Together

Linking this back to familiar operators:

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - G \rho_b c_b T + G \rho_b c_b T_b$$

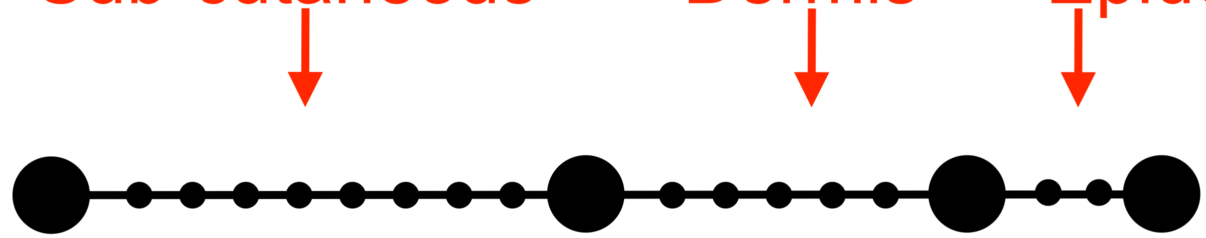
Transient | Diffusion | Linear reaction | Source term

Solve this using 1D mesh, with different values of k and G depending on the tissue layer i.e discontinuous material fields



MODELLING TISSUE DAMAGE
Implementation Suggestion

Can use discontinuous, constant basis function to represent these distinct regions



A simpler, but less general method is a conditional statement based on element ID or local node position:

```
if(eID >=1 && eID <= 3)
  Dc = 1.0;
elseif(eID >= 4 && eID <= 25)
  Dc = 0.5;
else
  Dc = 0.1;
end
```

Place inside diffusion local element matrix function

CALCULATING TISSUE DAMAGE
An Integrated Damage Equation

Once we have spatial-temporal temperature distribution, convert to tissue damage using an Arrhenius rate equation for the damage, Γ

$$\frac{d\Gamma}{dt} = P \exp\left(-\frac{E_a}{RT}\right)$$

Integrating:

$$\Gamma = \int_{t_{burn}}^{t} P \exp\left(-\frac{E_a}{RT}\right) dt$$

Damage only begins when T reaches a threshold temperature at the tissue layer interfaces, at time: t_{burn}

CALCULATING TISSUE DAMAGE
An Integrated Damage Equation

$$\Gamma = \int_{t_{burn}}^{t} P \exp\left(-\frac{E_a}{RT}\right) dt$$

Different values of Γ correspond to either second or third degree burns, depending on which tissue layer is considered

Parameter values for all these equations will be provided in the Coursework Handout

CALCULATING TISSUE DAMAGE
Implementation Suggestion

Evaluate the integral numerically using the trapezium rule, for the same time step used to solve the model

Define two variables and store the nodal solution value at each time step for the nodes at the interfaces between the skin layers

Evaluation of exponential function can be done on these entire vectors in Matlab