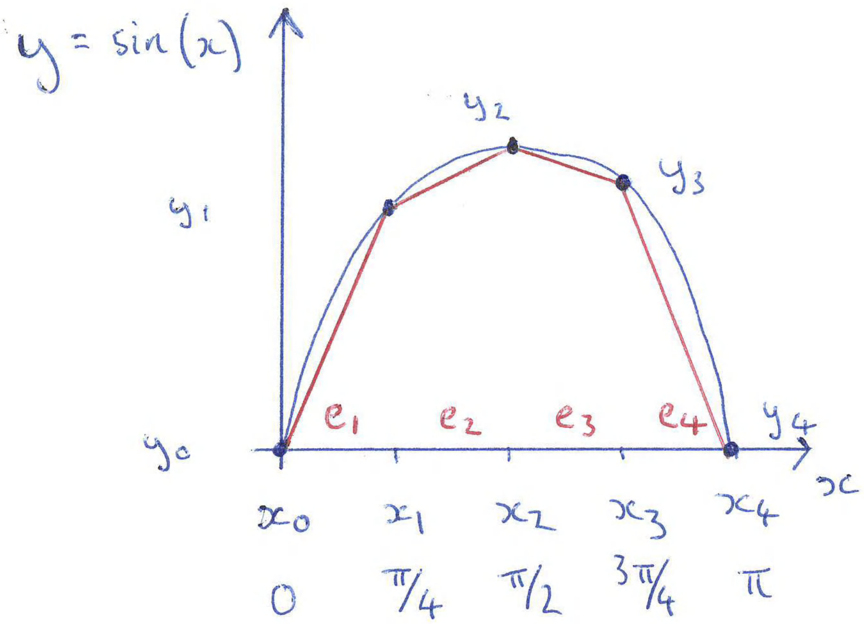
Example : approximate the function $\sin(x)$ using linear basis functions. ①



At $x_0, \dots, x_4$ evaluate $\sin(x)$. That is:

$$y_0 = \sin(x_0) = \sin(0)$$
$$y_1 = \sin(x_1) = \sin(\pi/4)$$
$$y_2 = \sin(x_2) = \sin(\pi/2)$$
$$y_3 = \sin(x_3) = \sin(3\pi/4)$$
$$y_4 = \sin(x_4) = \sin(\pi)$$

$\therefore$ At the values of $x_0, \dots, x_4$ y will be exactly equal to $\sin(x)$.

Inside the element there will be an error due to the linear approximation. We will address how to compute this error later in the course.

Following lecture 4 on basis functions, the value of y can be linearly interpolated within a given element using the weighted sum of basis functions.

E.g. in element 1:

$$y(\xi) = y_0 \psi_0(\xi) + y_1 \psi_1(\xi)$$

$$\Rightarrow y_0\left(\frac{1-\xi}{2}\right) + y_1\left(\frac{1+\xi}{2}\right).$$

In element 2:

$$y(\xi) = y_1\left(\frac{1-\xi}{2}\right) + y_2\left(\frac{1+\xi}{2}\right).$$

In general element e:

$$y(\xi) = y_{e-1}\left(\frac{1-\xi}{2}\right) + y_e\left(\frac{1+\xi}{2}\right)$$

<u>Example</u>

Evaluate y at $\xi = 0$ in element 1:

$$y(0) = 0 \cdot \left(\frac{1-0}{2}\right) + \sin(\pi/4) \cdot \left(\frac{1+0}{2}\right)$$

$$= \frac{1}{2\sqrt{2}} = 0.3536 \quad (4\text{d.p.})$$

This is the linear approximation

The coordinate $\xi = 0$ in element 1 corresponds to an x coordinate of $\pi/8$ i.e. it is halfway in this element.

$$y\left(\frac{\pi}{8}\right) = \sin\left(\pi/8\right) = 0.3827 \quad (4 \text{ d.p.})$$

Using only four elements to represent half a sine wave clearly produces a non-trivial error, which at $x = \pi/8$ is roughly 7.5%.
Increasing the number of elements will therfore reduce this error.