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ME40064: System Modelling & Simulation
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Tutorial 5: The Use of Numerical Integration Techniques within the Finite Element Method

Part A: Gaussian Quadrature Schemes

1. In Lecture 9 a data structure for Gaussian quadrature schemes, and a partially complete GQ scheme creation function, were introduced. Download the file, `CreateGQScheme.m`, from Moodle and complete this function, so that it will generate GQ schemes for N=2 and N=3.

2. For the local element integrals for the linear reaction term (assuming here that λ is spatially constant) manually use the Gaussian quadrature method to evaluate this integral for the two cases m=n=0 and m=0, n=1, i.e. the following two integrals:

$$Int_{00} = \int_{-1}^{1} \lambda \psi_0 \psi_0 J d\xi$$

$$Int_{01} = \int_{-1}^{1} \lambda \psi_0 \psi_1 J d\xi$$

As these require quadratic polynomials to be integrated, a GQ scheme with N=2 is required.

However, evaluate the integrals using N=1, 2, and 3 and check that for N=1 the answer is incorrect and that N=2 and 3, produce the same, correct answer. These integrals were previously evaluated analytically in Tutorial 3.

3. Write a short MATLAB script that uses Gaussian integration to numerically evaluate the following 5th order polynomial. This will require a Gauss scheme with N=3:

$$f(x) = x^5 - 3x^4 + 2x^3 + x^2 + 4x + 8$$

Check your numerical solution against the analytical one (evaluated with limits of x = -1 to 1, as is required by the Gaussian quadrature method).

Part B: Basis Functions Revisited

1. Write a function that will evaluate the linear Lagrange basis functions at a given Gauss point, ξ. An empty function definition, `EvalBasis.m`, is available on Moodle to provide a starting point. A similar function, `EvalBasisGrad.m`, has also been provided that evaluates the gradients of the basis functions (as are required in the diffusion operator local element matrix), which may point the way.

2. Write a function to evaluate a material parameter field that is defined on the mesh, for example the diffusion coefficient, D, or linear reaction coefficient, lambda, at a given Gauss point. Your function will need to call the function written in answer to Part B.Q1. A script, `MaterialFieldEvalScript.m`, to help you write & run your function is again provided on Moodle.

3. In this question, we are now at the point of using numerical integration within the finite element method context and making our code general.

   Therefore, write a function that evaluates the 2-by-2 local element matrix for the linear reaction term, using Gaussian quadrature and assuming that the coefficient may now be spatially varying and hence represented on the mesh.

   This requires calling the function written for Part B.Q2 to evaluate the reaction coefficient, $\boldsymbol{\lambda}$, at the Gauss points and the function written in Part B.Q1 to evaluate the basis functions at the Gauss points. Note again that, as this element matrix involves quadratic expressions, a Gauss scheme of at least N=2 is required for accurate evaluation.

Part C: Optional Challenge Questions

1. Following Part B.Q1, write functions that will evaluate the quadratic Lagrange basis functions and basis gradients. One neat solution may be to use the same function to compute both the linear & quadratic basis functions, and have the required order specified as an input to the function.

2. Repeat Part B.Q3 but for a reaction coefficient, $\boldsymbol{\lambda}$, that is represented using the quadratic basis functions, on the linearly spaced mesh. To test your code, generate a vector of nodal $\boldsymbol{\lambda}$ coefficients, which represent a $\boldsymbol{\lambda}$ that varies quadratically with x. Use the script, `MaterialFieldEvalScript.m`, as a guide.