ME40064: Systems Modelling & Simulation
ME50344: Engineering Systems Simulation
Lecture 4

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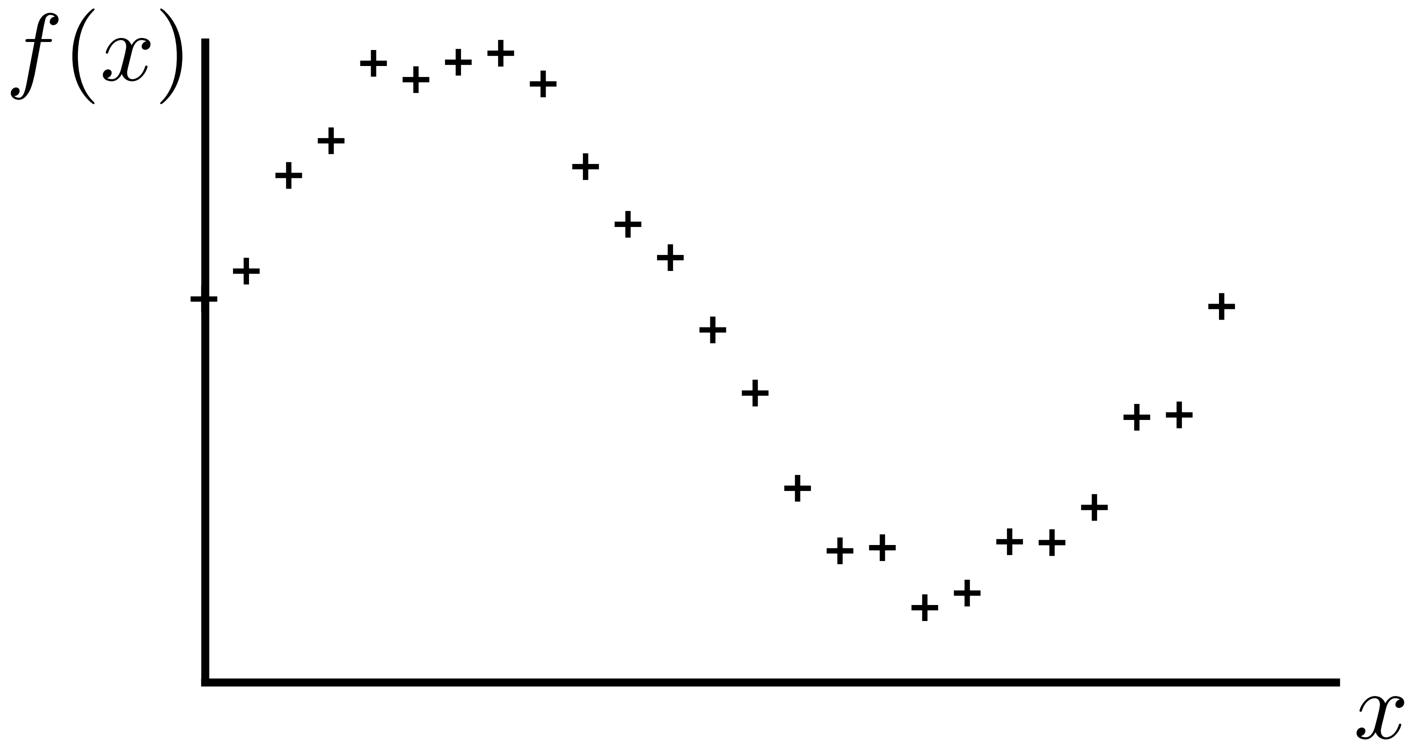
University of Bath, October 2019-20

LECTURE 4
FEM: Basis Functions

- Understand how fields of data might be represented in space
- Understand the different components of a finite element mesh
- Ability to represent a continuous field using finite elements

HOW TO REPRESENT SOME DATA?
What Function Can Join The Dots?



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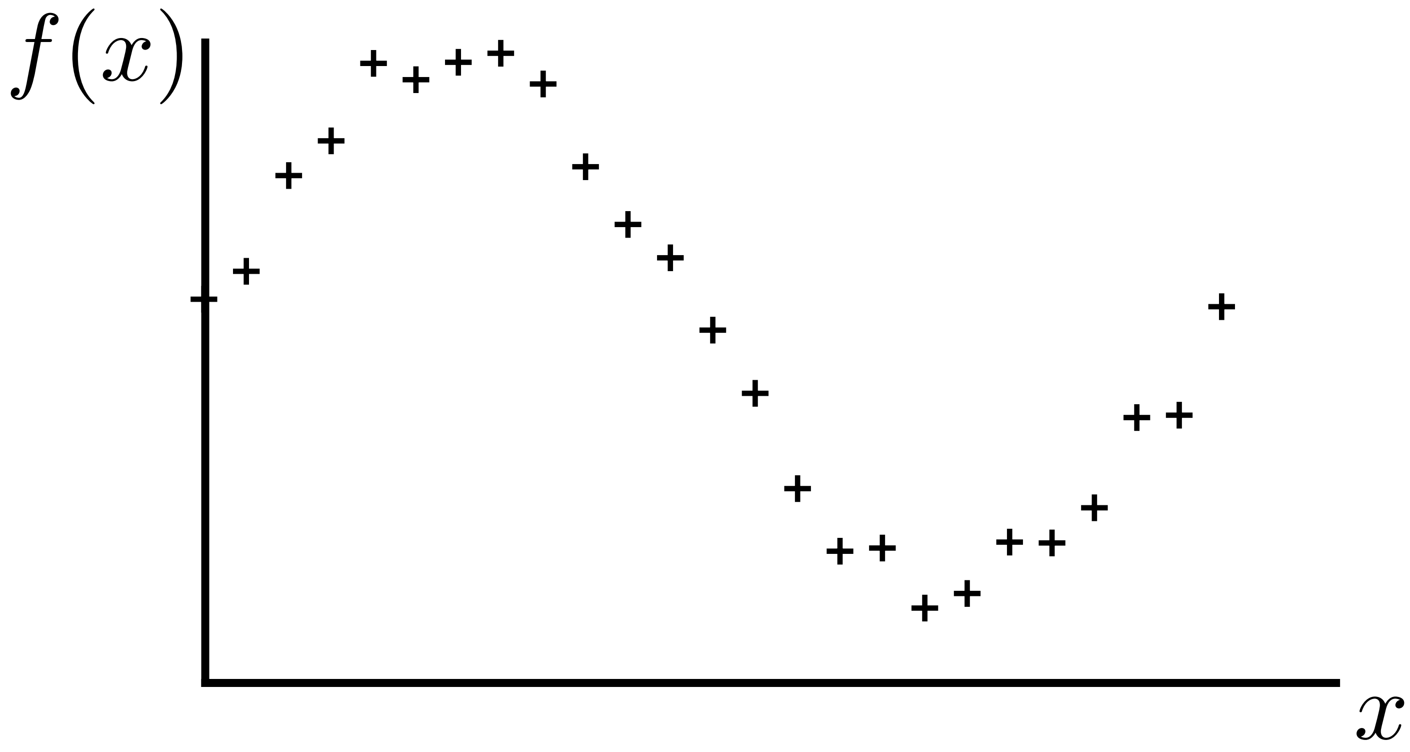

Fourier series - but can have oscillations & need periodicity

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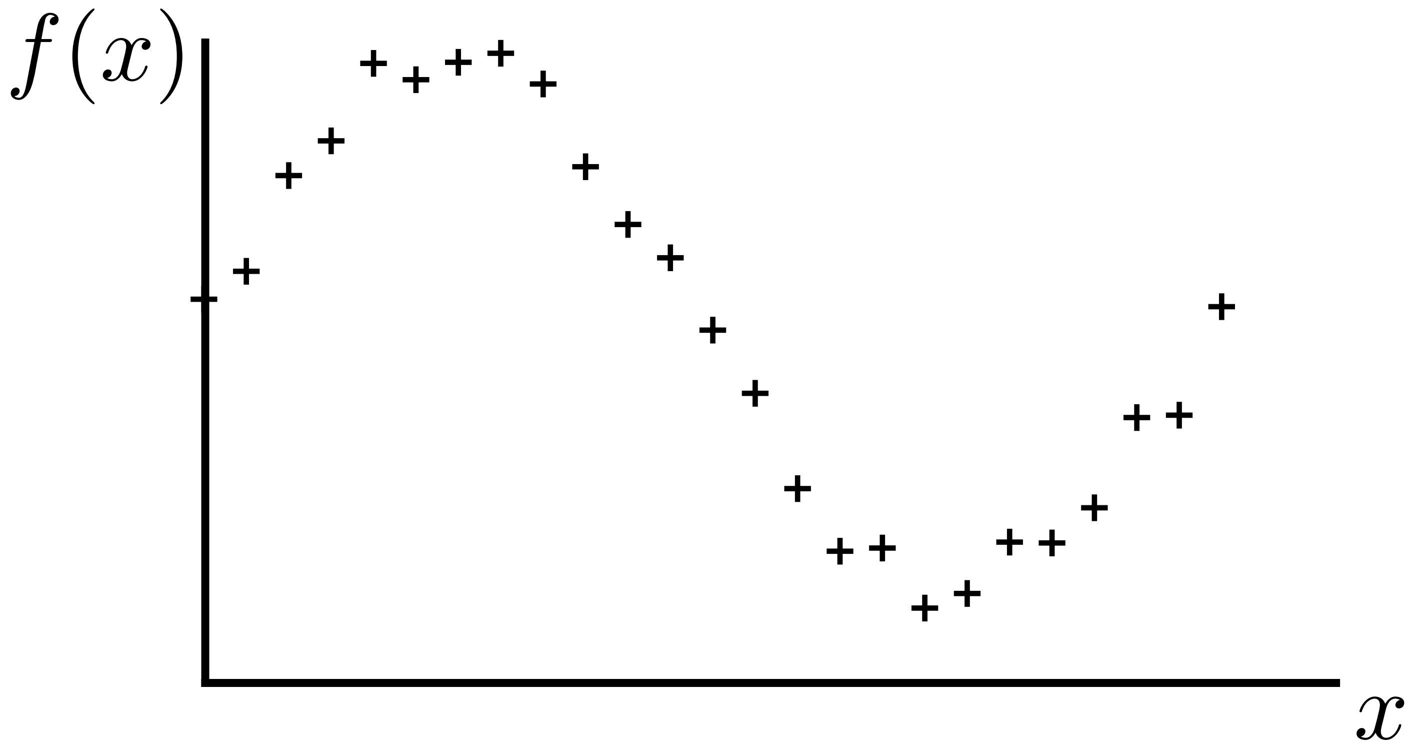

Polynomial fit - but can generate oscillations & need appropriate choice of functions

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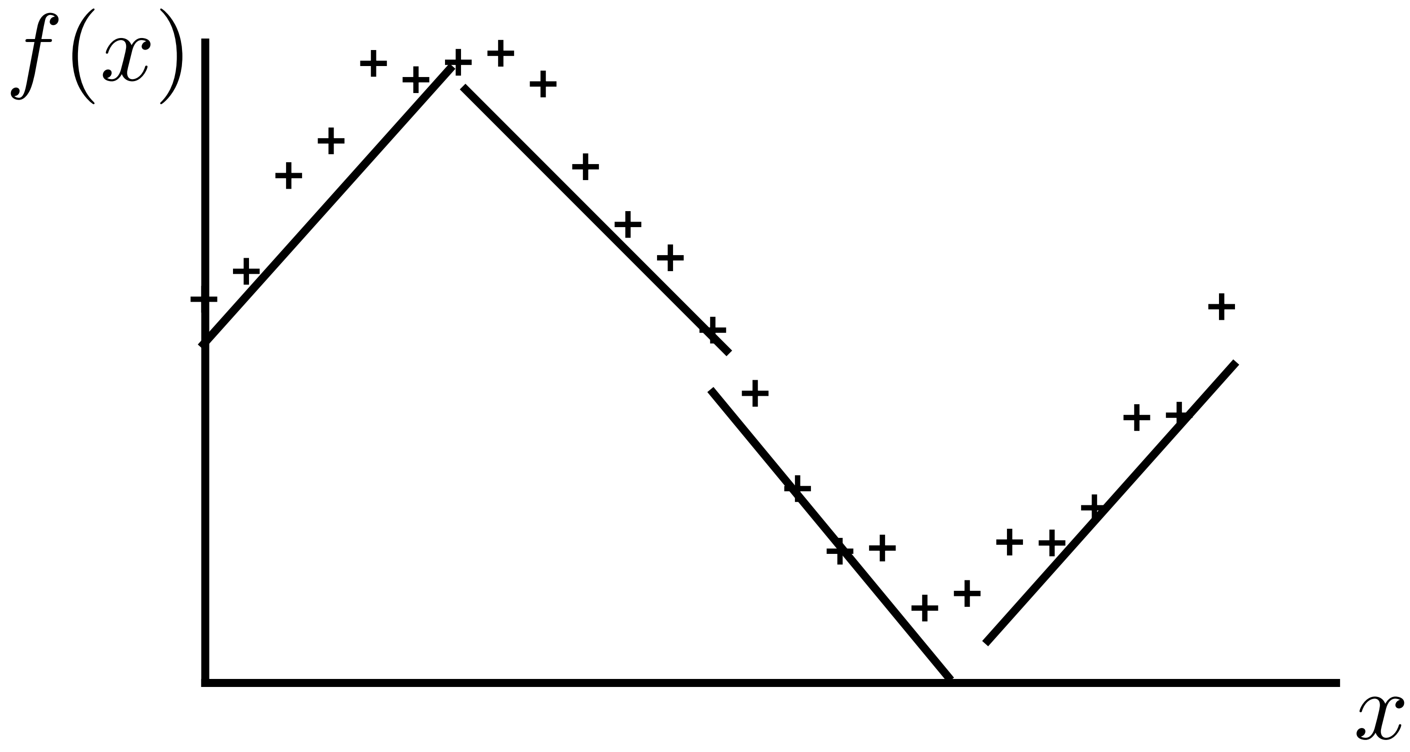
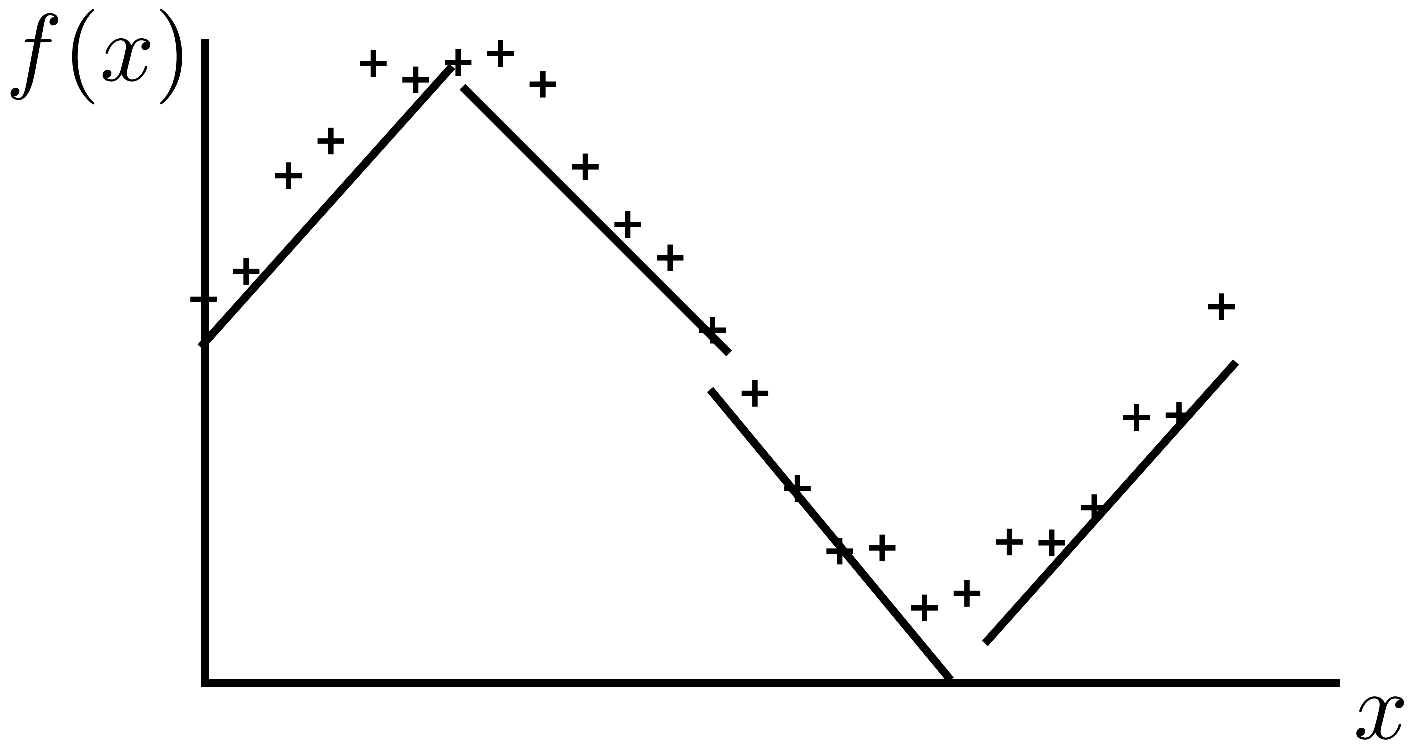


Piecewise linear functions - a simple & robust solution

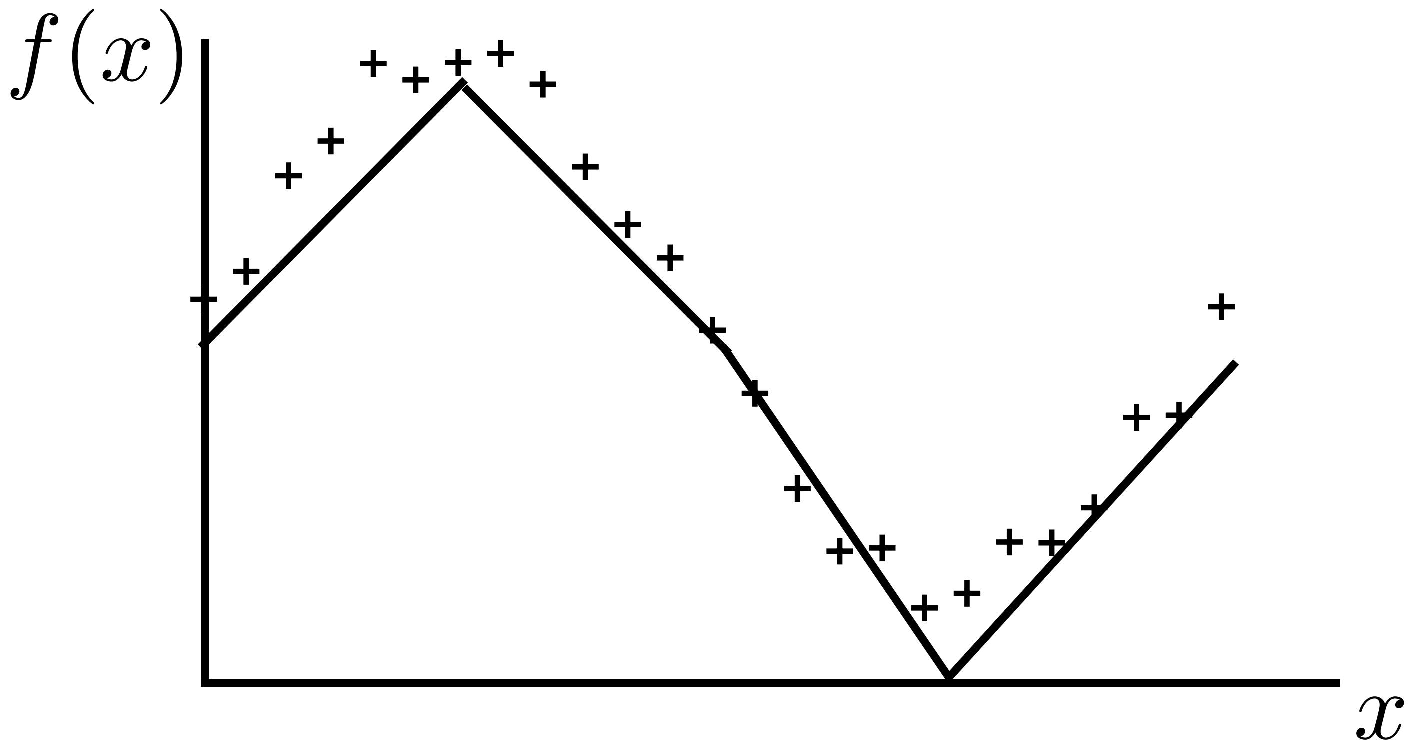
HOW TO REPRESENT SOME DATA?
What Function Can Join The Dots?



Piecewise linear functions - a simple & robust solution

Actually all are used in the finite element method, but we will focus on the piecewise discretisation in this course

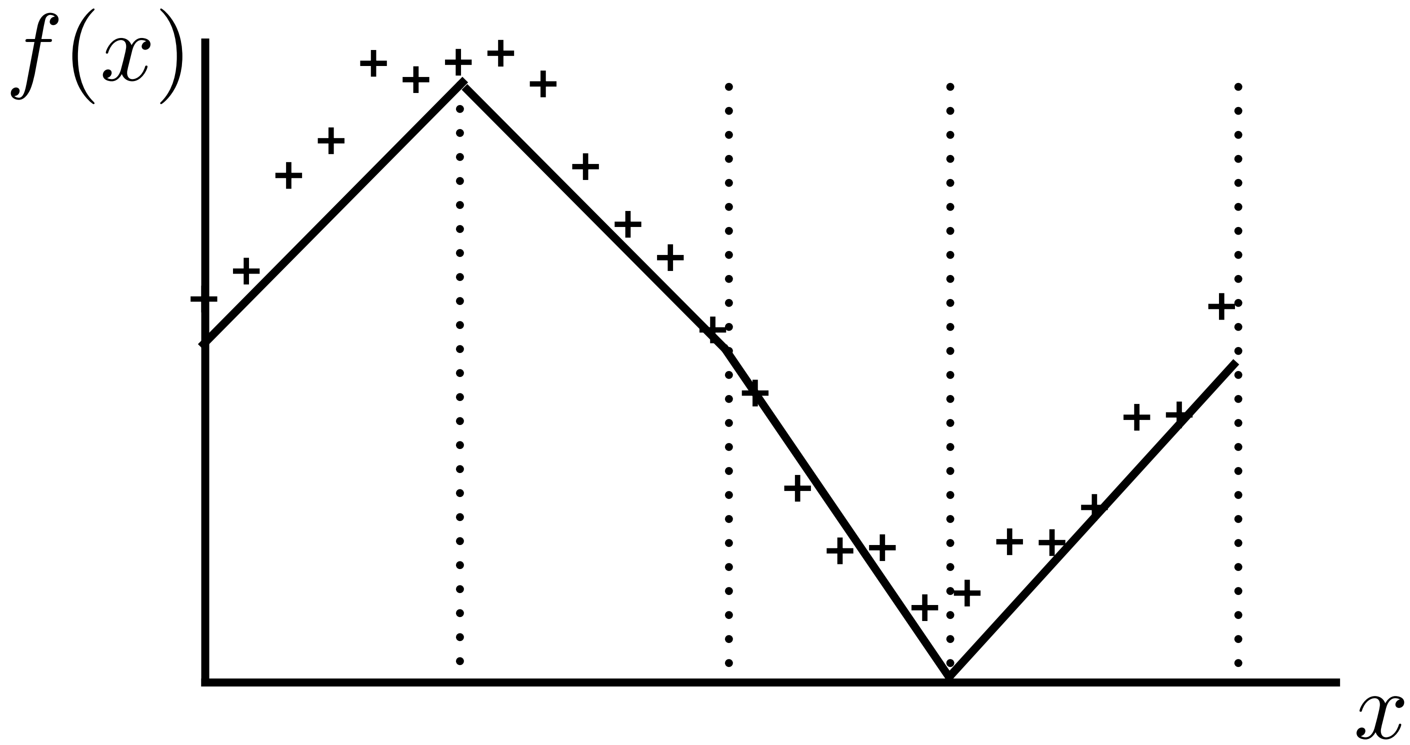
DISCRETISATIONS
A C0 Continuous Approach



Function value is continuous across elements, but gradients are not

- C0 continuous discretisation

DISCRETISATIONS
A C0 Continuous Approach



Function value is continuous across elements, but gradients are not

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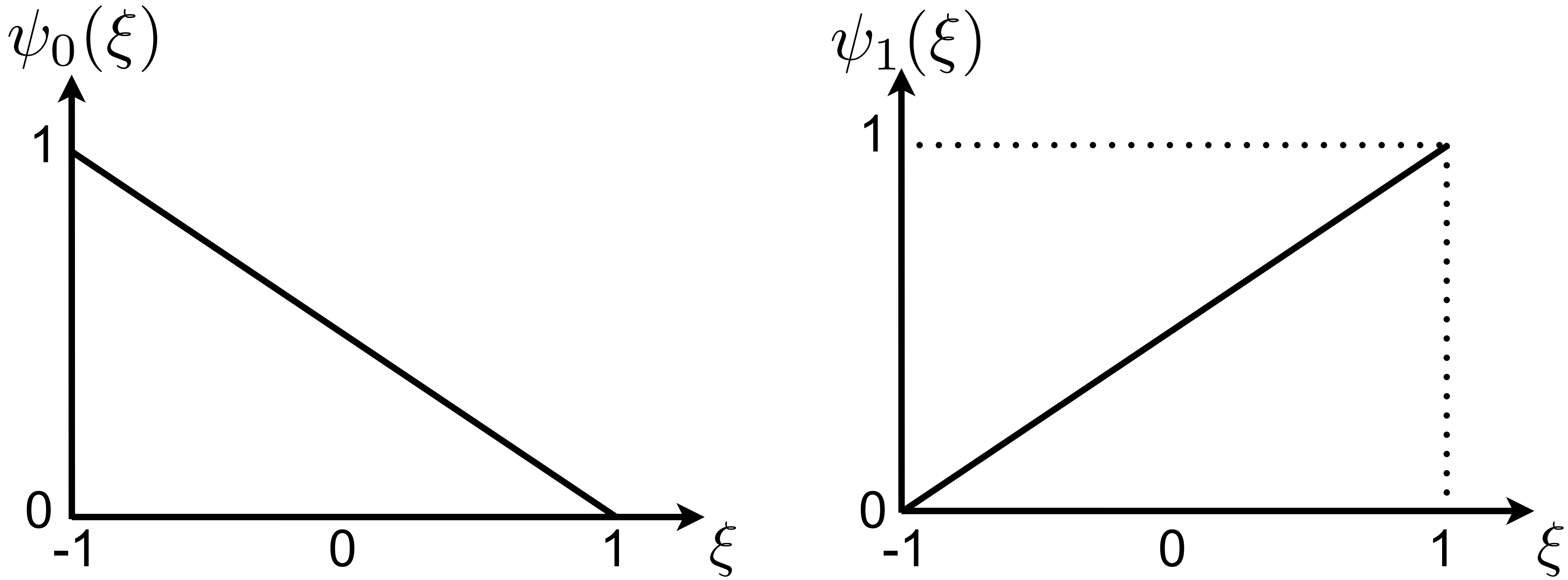
BASIS FUNCTIONS
Linear Nodal Lagrange

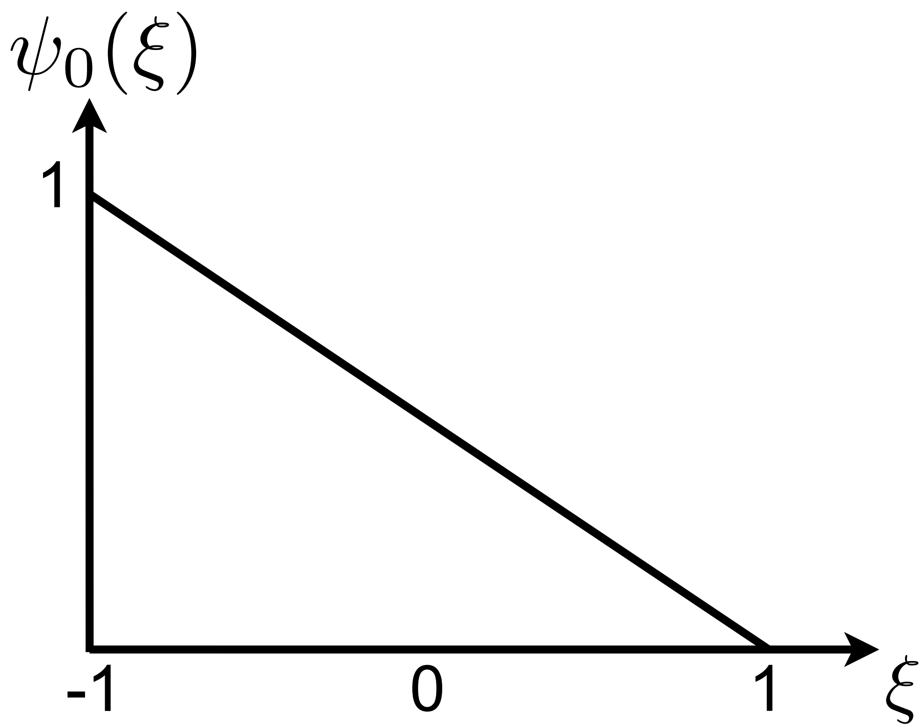
Define linear functions in the standard element $\Omega_{st} = [-1, 1]$

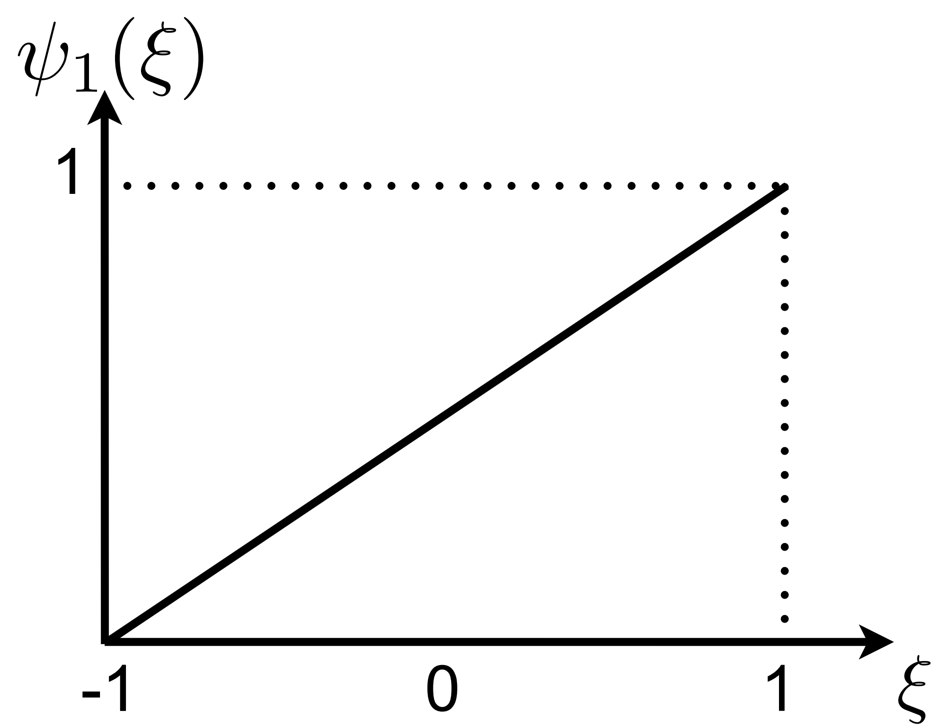


$$\psi_0(\xi) = \begin{cases} \frac{1-\xi}{2} & \xi \in \Omega_{st} \\ 0 & \xi \notin \Omega_{st} \end{cases} \qquad \psi_1(\xi) = \begin{cases} \frac{1+\xi}{2} & \xi \in \Omega_{st} \\ 0 & \xi \notin \Omega_{st} \end{cases}$$

Use sum of two linear functions to represent each linear segment in the line fitted to the data

BASIS FUNCTIONS
Linear Nodal Lagrange

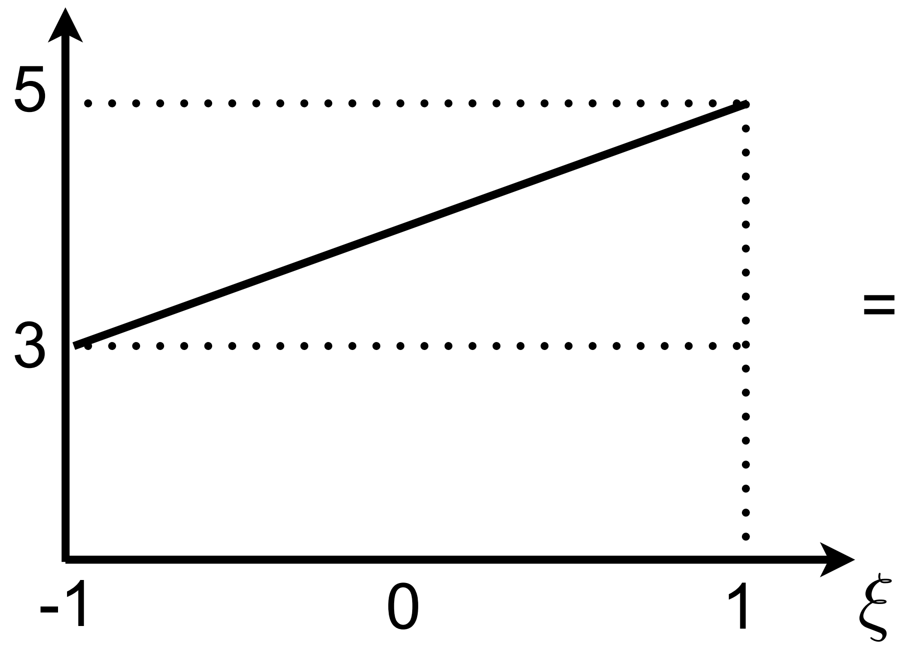




<u>Note</u> The following definitions generates the nodal property of these basis functions

$$\psi_0(-1) = 1, \quad \psi_0(1) = 0$$
$$\psi_1(-1) = 0, \quad \psi_1(1) = 1$$

WORKED EXAMPLE
How To Represent This Line?



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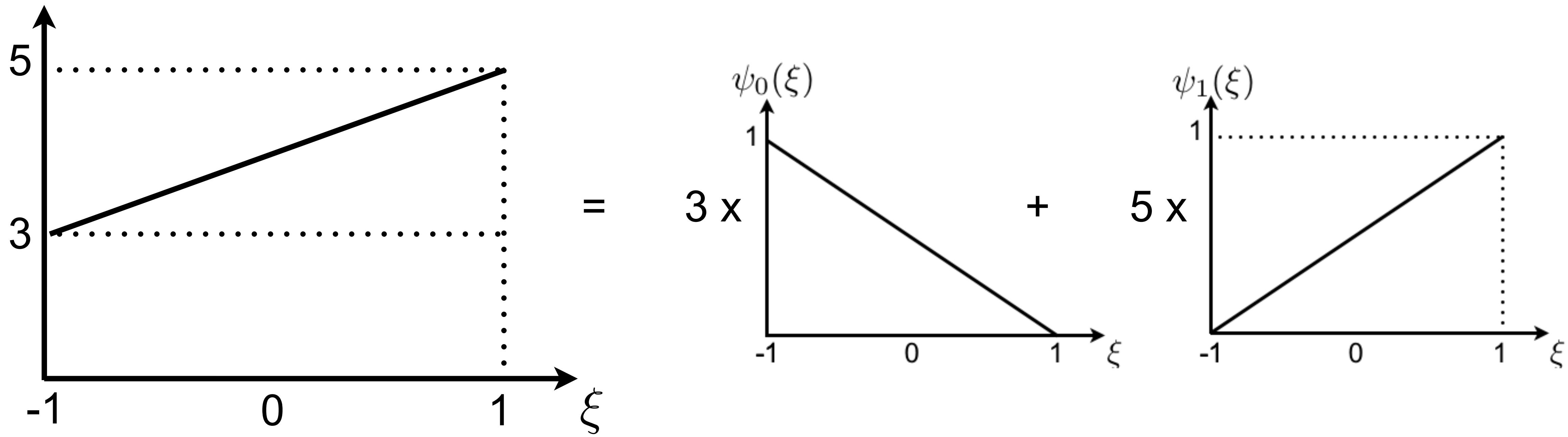
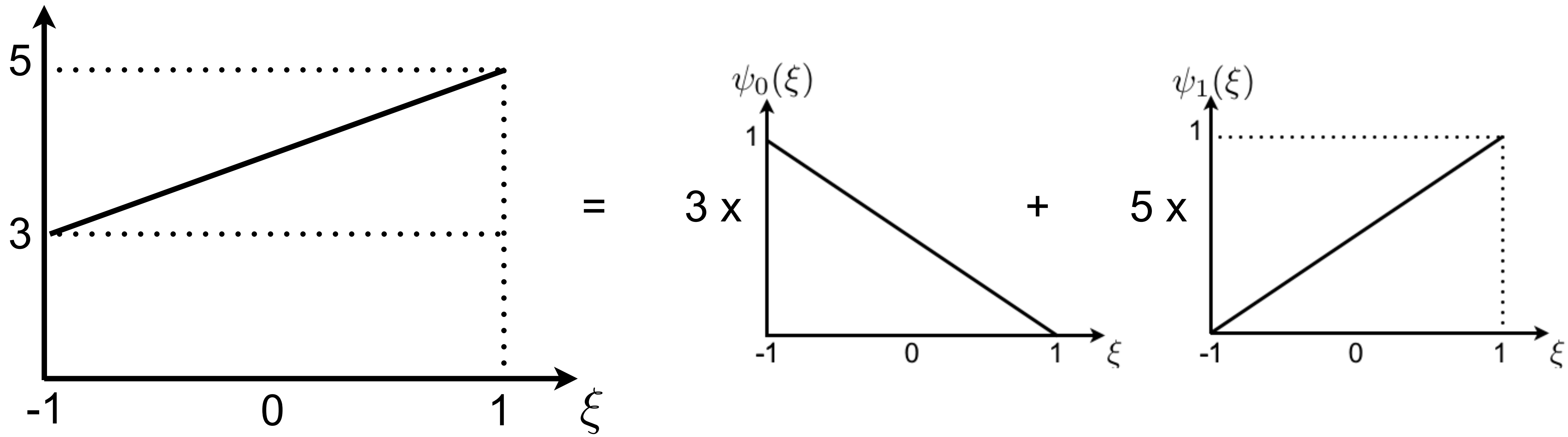
WORKED EXAMPLE
Sum Of Two Linear Basis Functions



WORKED EXAMPLE
It Works!



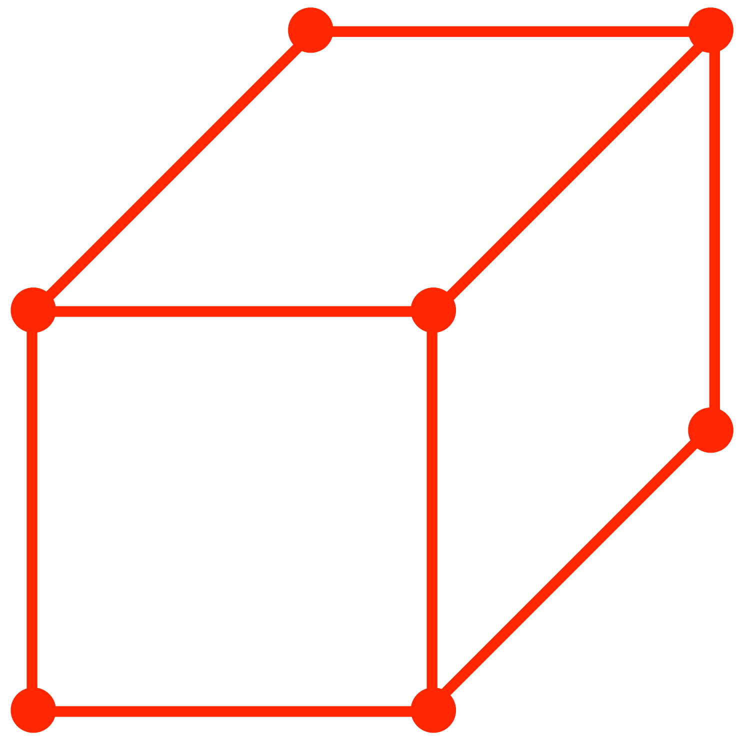
$$\xi = -1 \ : \ x(-1) = 3\psi_0(-1) + 5\psi_1(-1) = 3.1 + 5.0 = 3$$

$$\xi = 1 \ : \ x(1) = 3\psi_0(1) + 5\psi_1(1) = 3.0 + 5.1 = 5$$

$$\xi = 0 \ : \ x(0) = 3\psi_0(0) + 5\psi_1(0) = 3.\left(\frac{1-0}{2}\right) + 5.\left(\frac{1+0}{2}\right) = 4$$

FINITE ELEMENT MESH TOPOLOGY
What Does A Mesh Consist Of?

- In general, for a 3D element, such as a hexahedral or tetrahedral, there is:
- element
- faces
- lines
- nodes



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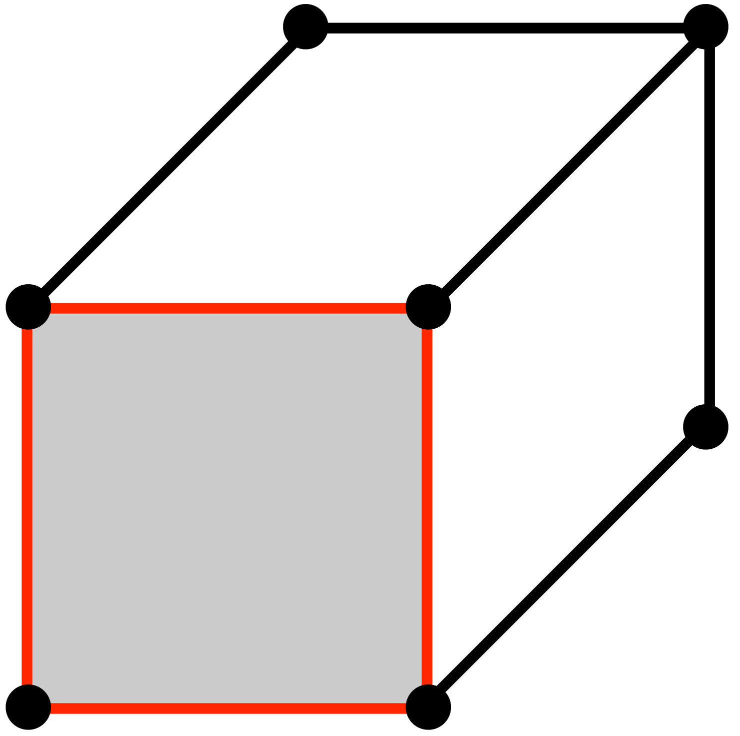

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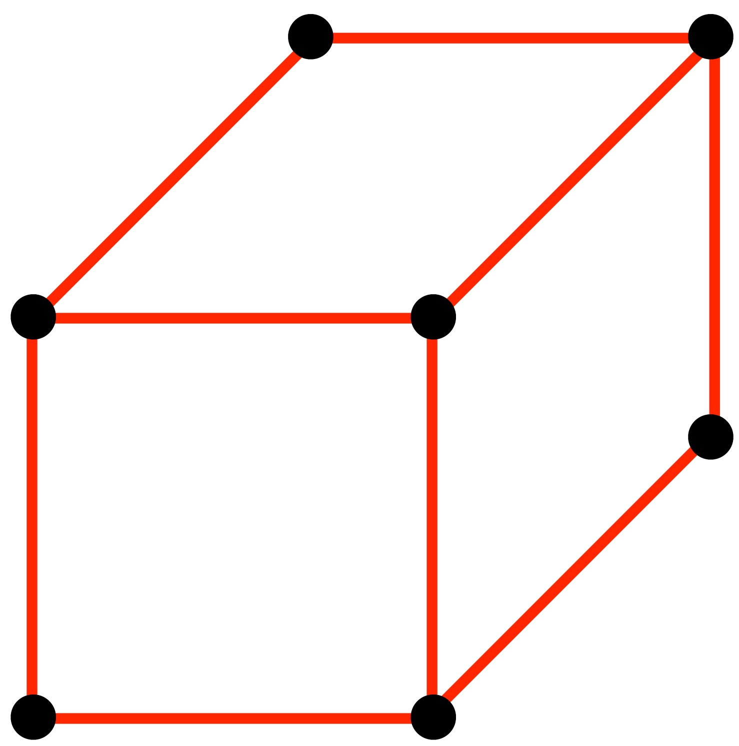

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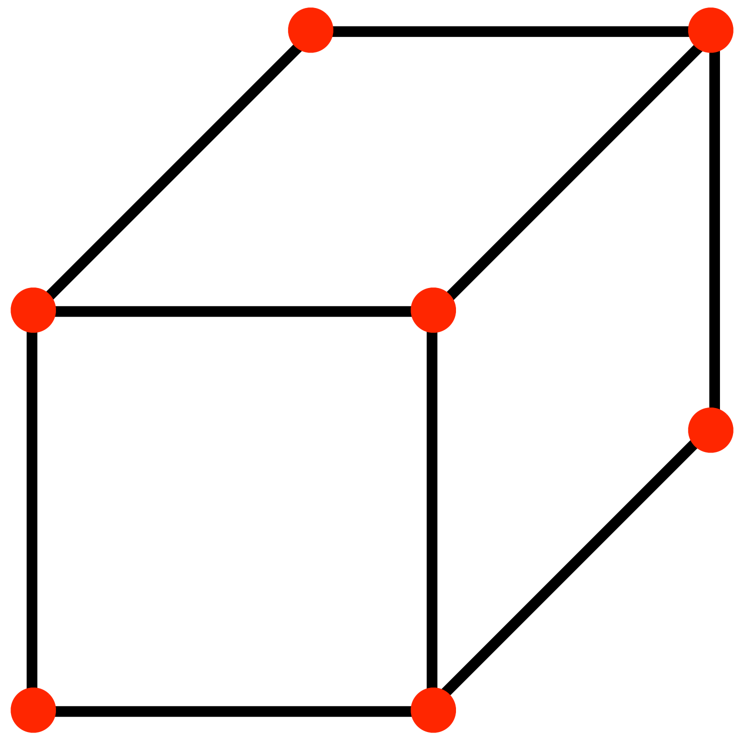
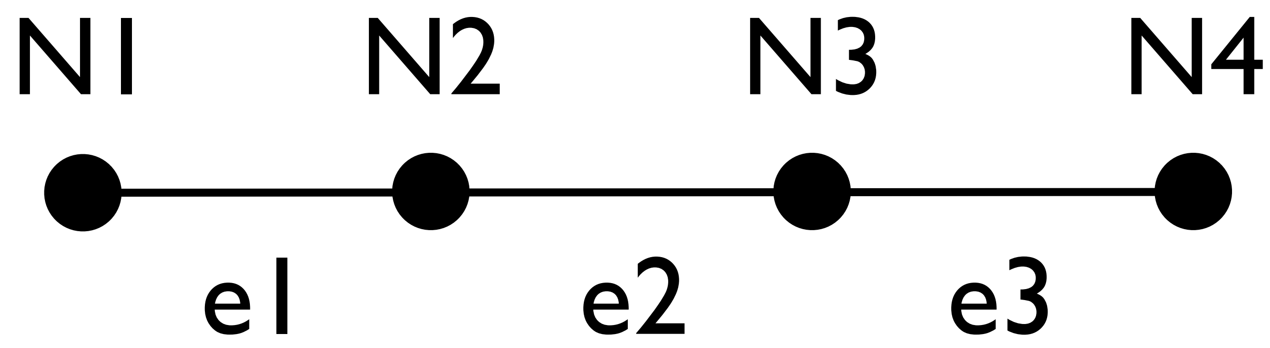


FINITE ELEMENT MESH
What Do We Use It For?

Represents

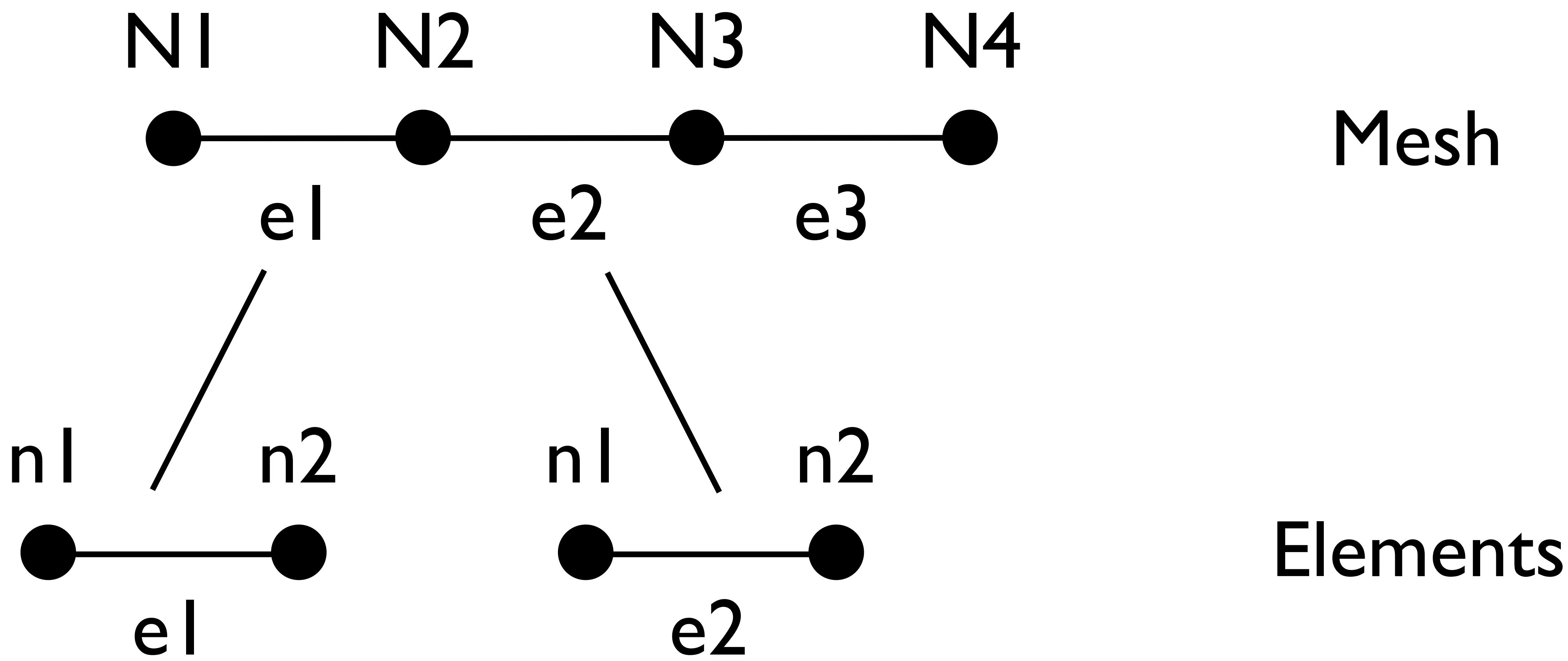
- geometry - volume & boundaries
- material parameters
- the model solution

A 1-D FINITE ELEMENT MESH



Mesh

A 1-D FINITE ELEMENT MESH



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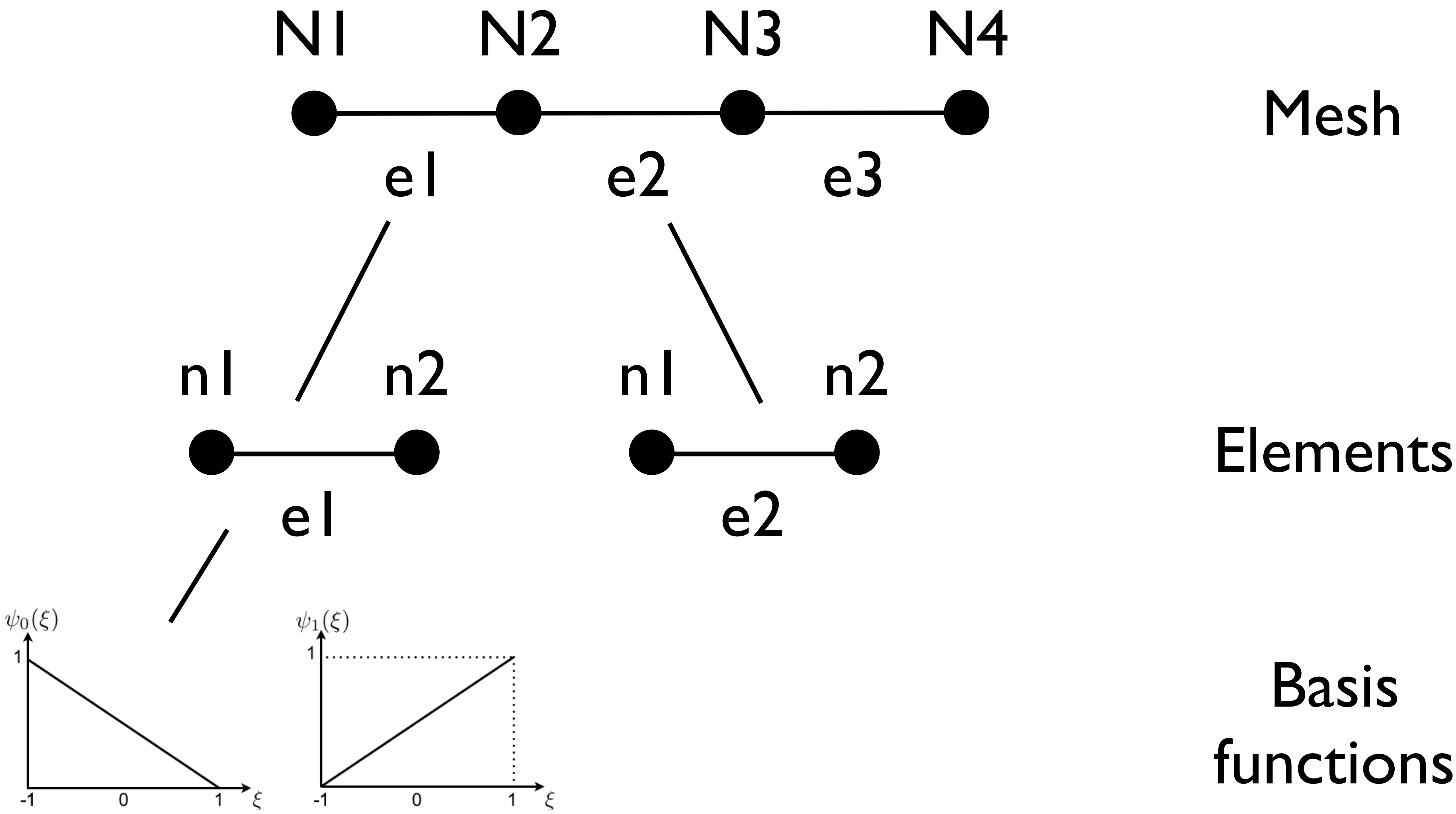

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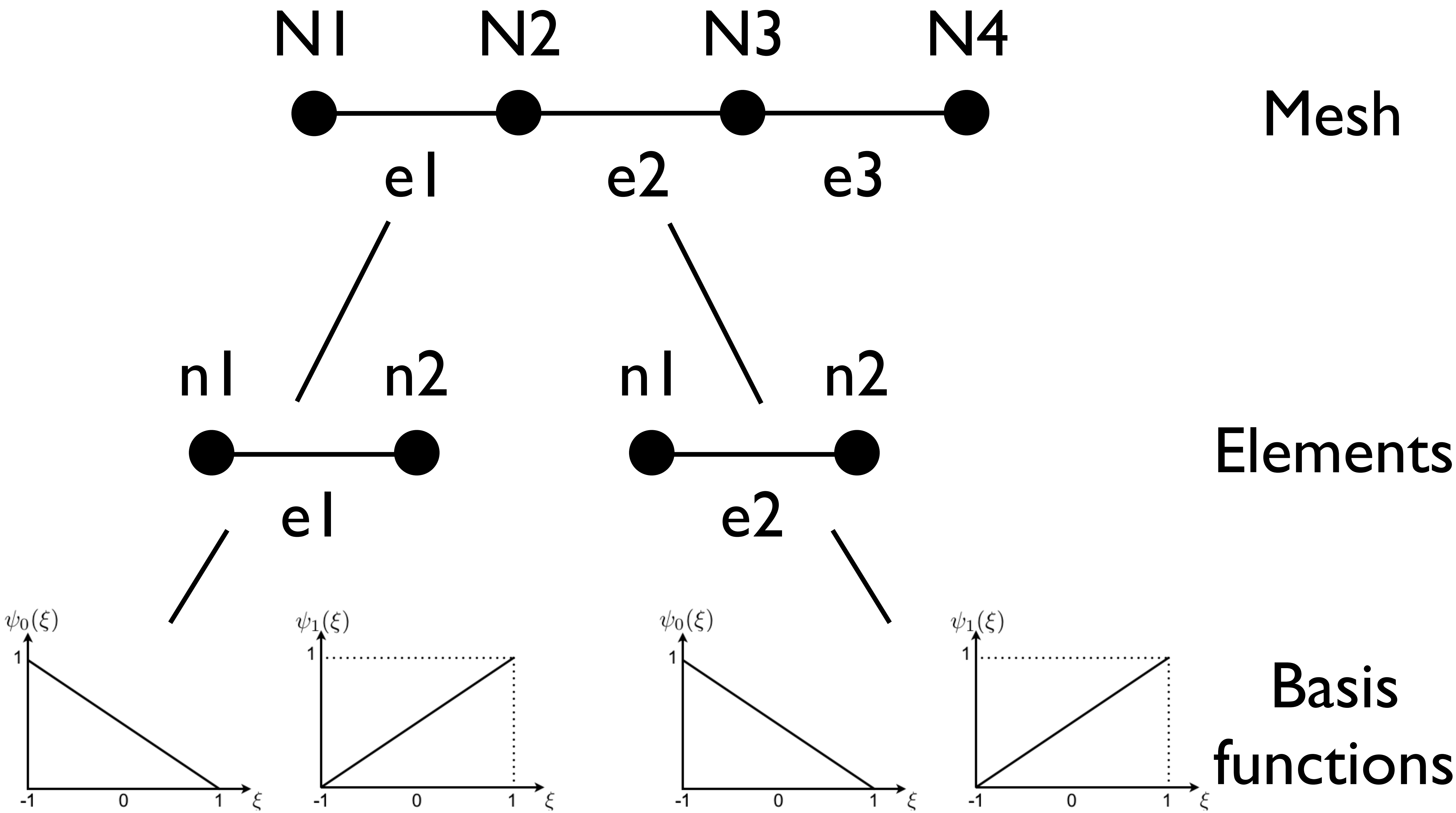
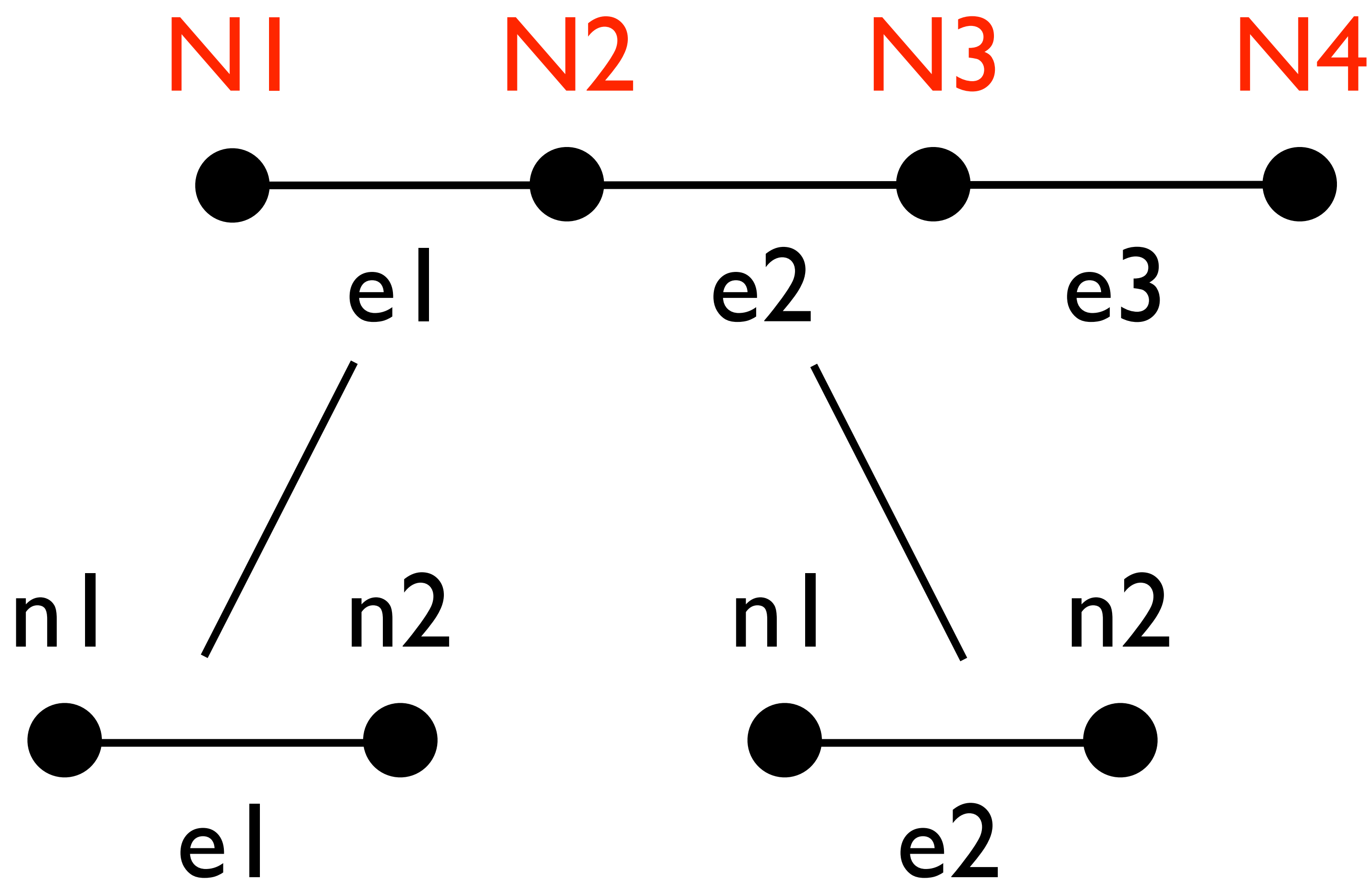


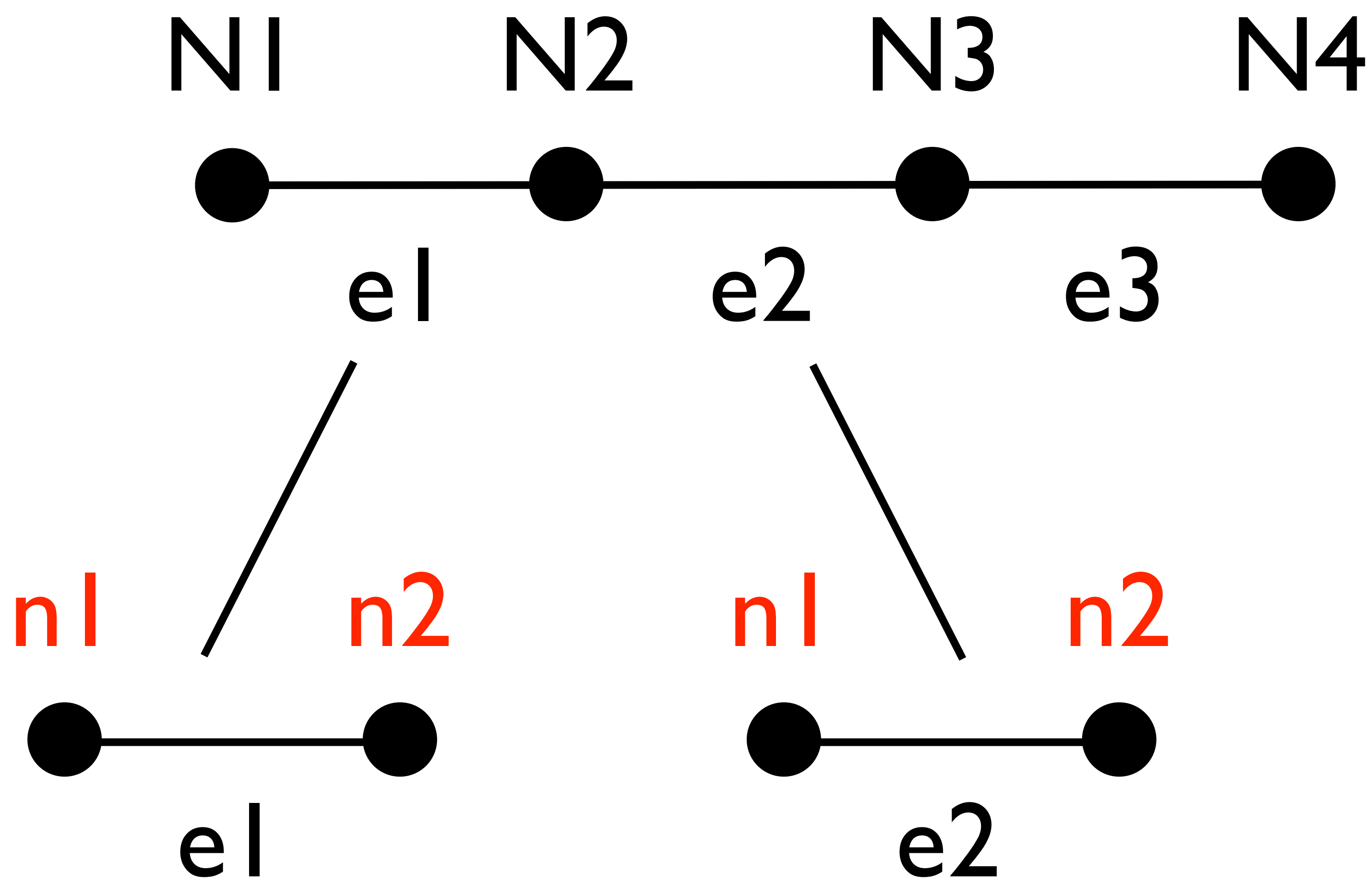
A 1-D FINITE ELEMENT MESH
Global Nodes



In 1D mesh there are only elements and nodes to consider **but** two kinds of nodes

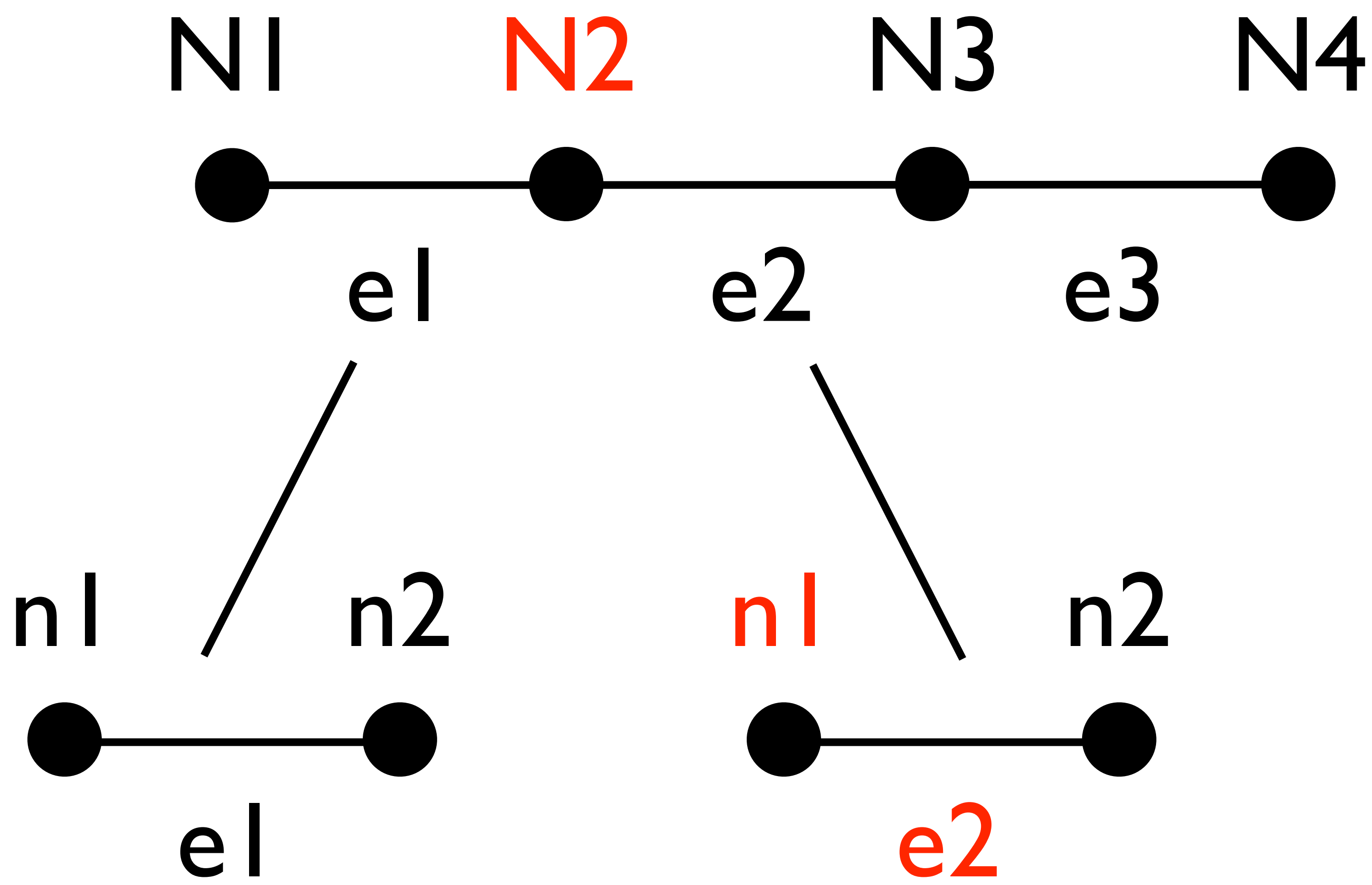
Global nodes = N1, N2, ...

A 1-D FINITE ELEMENT MESH
Local Nodes



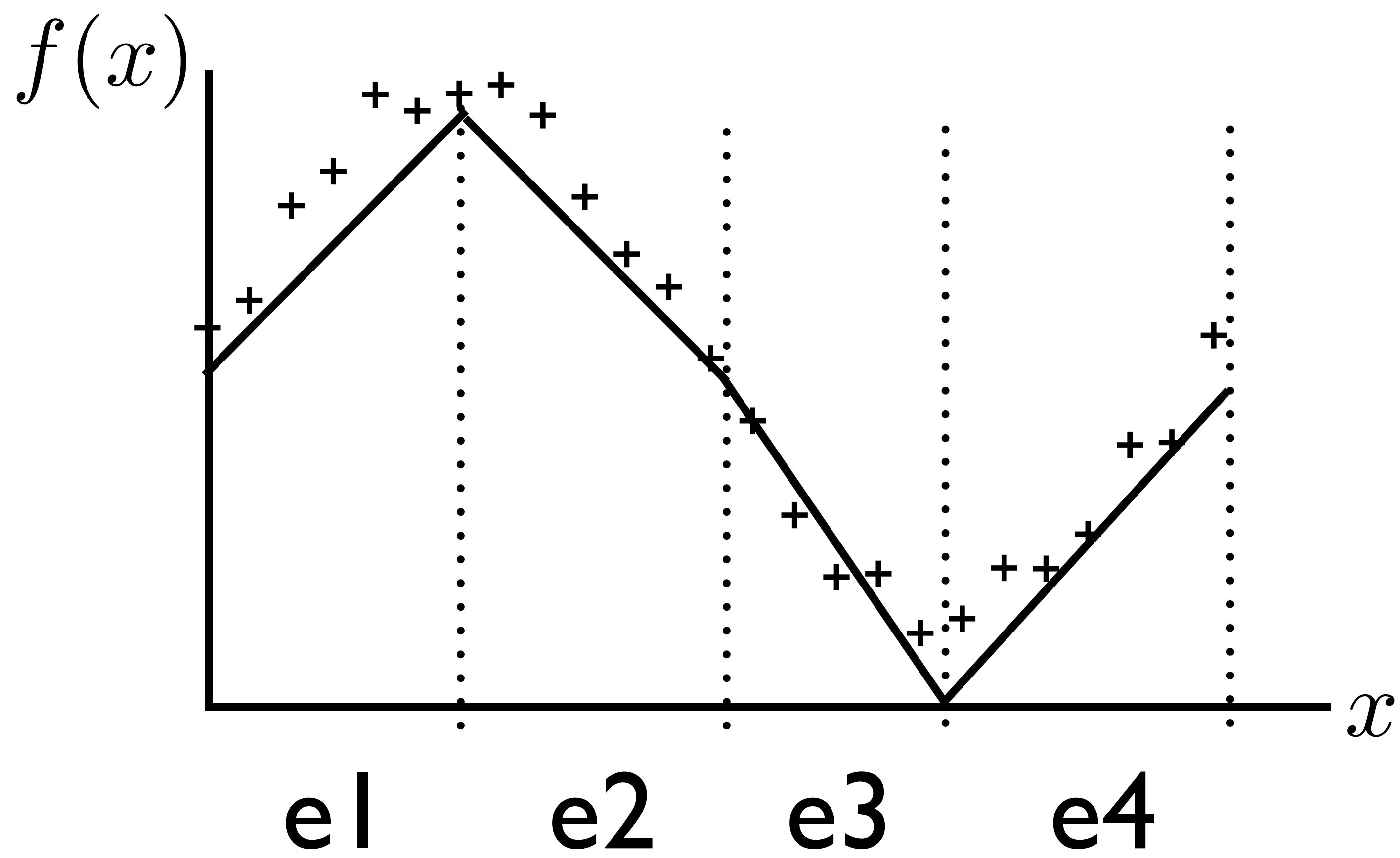
Local nodes = n1, n2, for each element e

A 1-D FINITE ELEMENT MESH
Link Between Local & Global Nodes

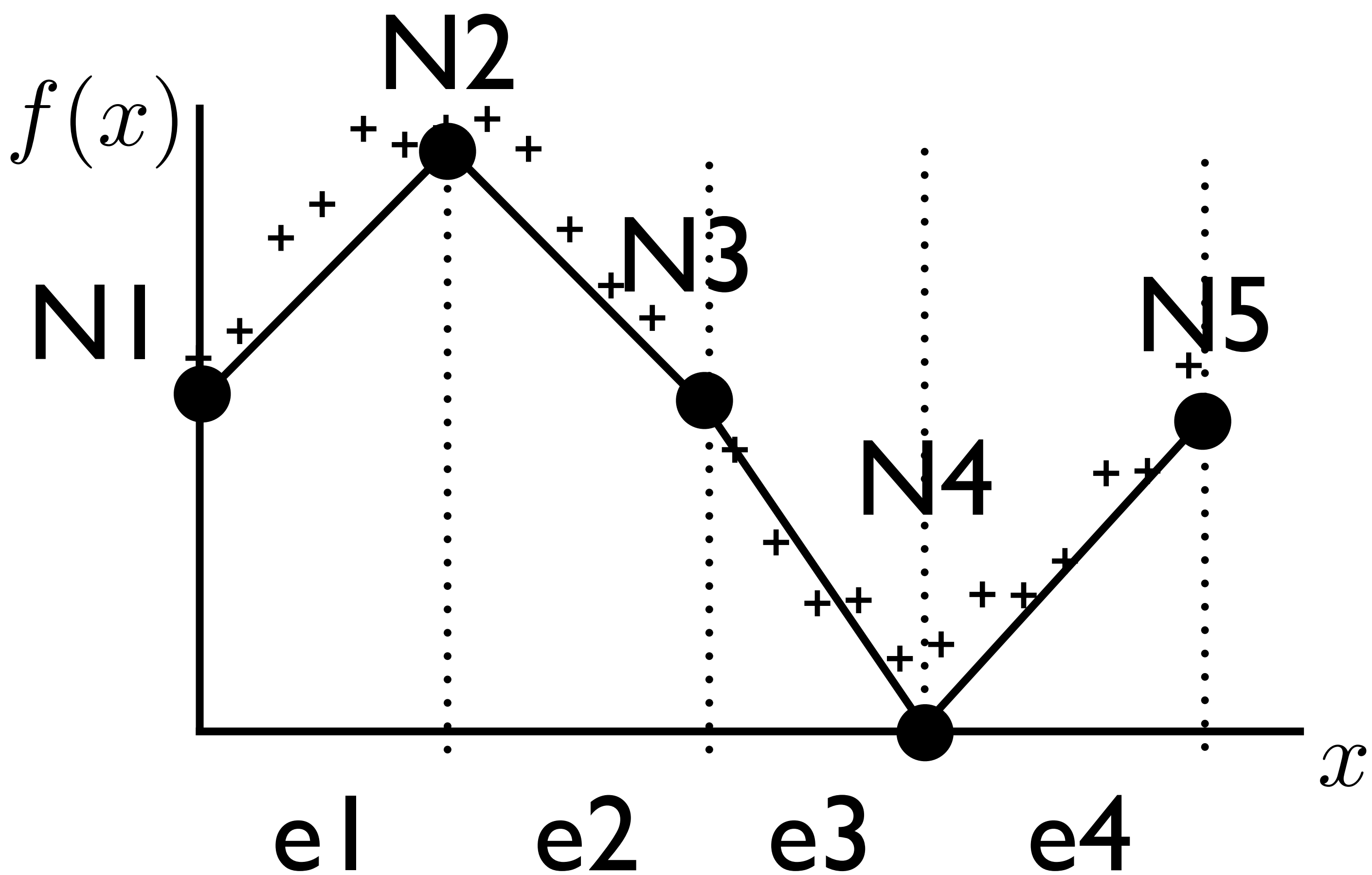


e.g. local node, n1 in element, e2, is global node N2

DISCRETISATIONS
Revisiting Curve Fitting



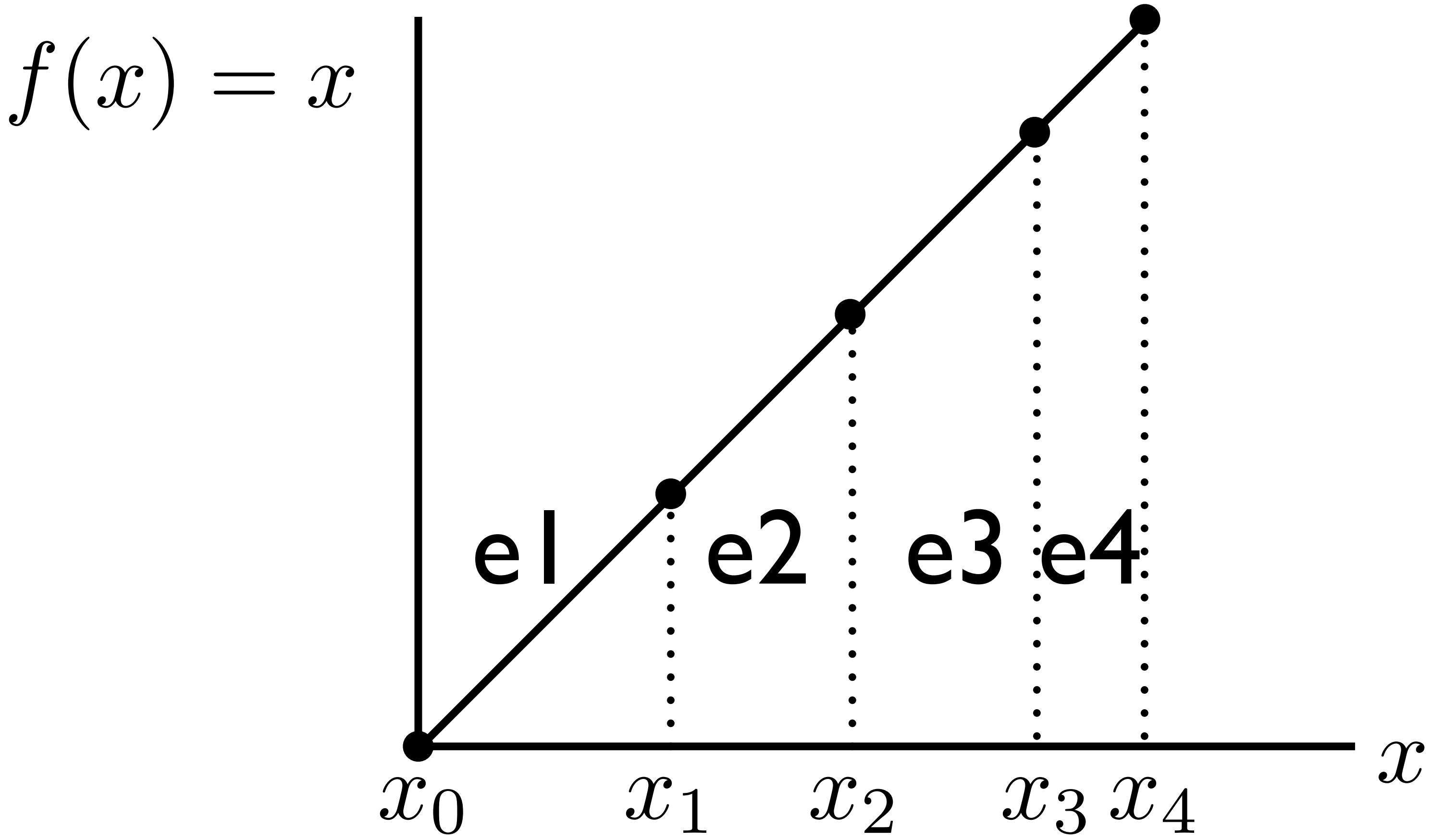
DISCRETISATIONS
Revisiting Curve Fitting



The original series of points now represented on a 4 element mesh, containing 5 global node values

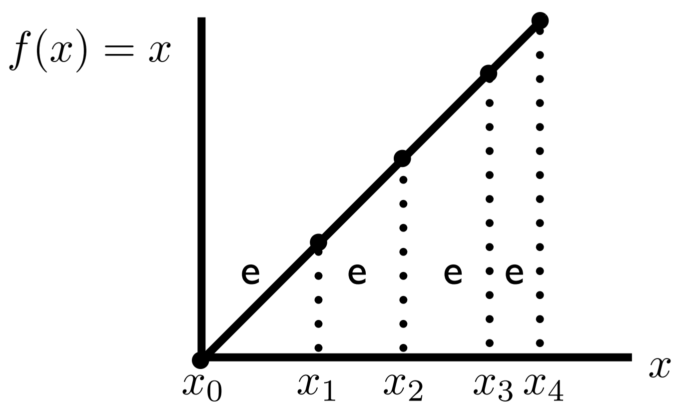
Each mesh node has a spatial position

DISCRETISATIONS
Representing Space Using The Mesh



The space variable, x, can be represented just like any other variable by the finite element mesh

DISCRETISATIONS
Representing Space Using The Mesh



Interpolate x between the elemental nodes using the sum of basis functions:

Element 1

$$x(\xi) = x_0\psi_0(\xi) + x_1\psi_1(\xi)$$

Element 2

$$x(\xi) = x_1\psi_0(\xi) + x_2\psi_1(\xi)$$

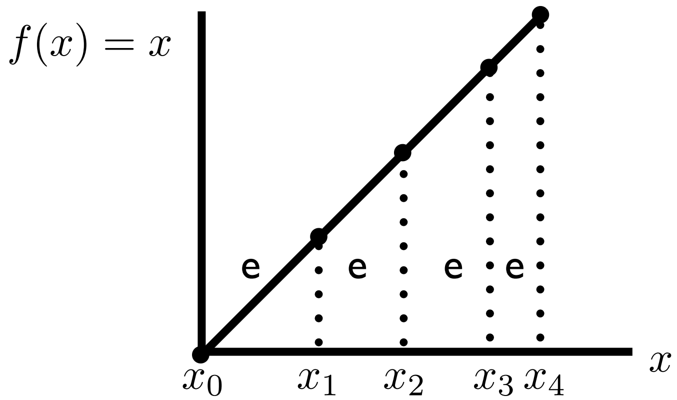
Element 3

$$x(\xi) = x_2\psi_0(\xi) + x_3\psi_1(\xi)$$

Element 4

$$x(\xi) = x_3\psi_0(\xi) + x_4\psi_1(\xi)$$

DISCRETISATIONS
Representing Space Using The Mesh



Interpolate x between the elemental nodes using the sum of basis functions:

Element 1

$$x(\xi) = x_0\psi_0(\xi) + x_1\psi_1(\xi) \quad x(\xi) = x_0\left(\frac{1-\xi}{2}\right) + x_1\left(\frac{1+\xi}{2}\right)$$

Element 2

$$x(\xi) = x_1\psi_0(\xi) + x_2\psi_1(\xi) \quad x(\xi) = x_1\left(\frac{1-\xi}{2}\right) + x_2\left(\frac{1+\xi}{2}\right)$$

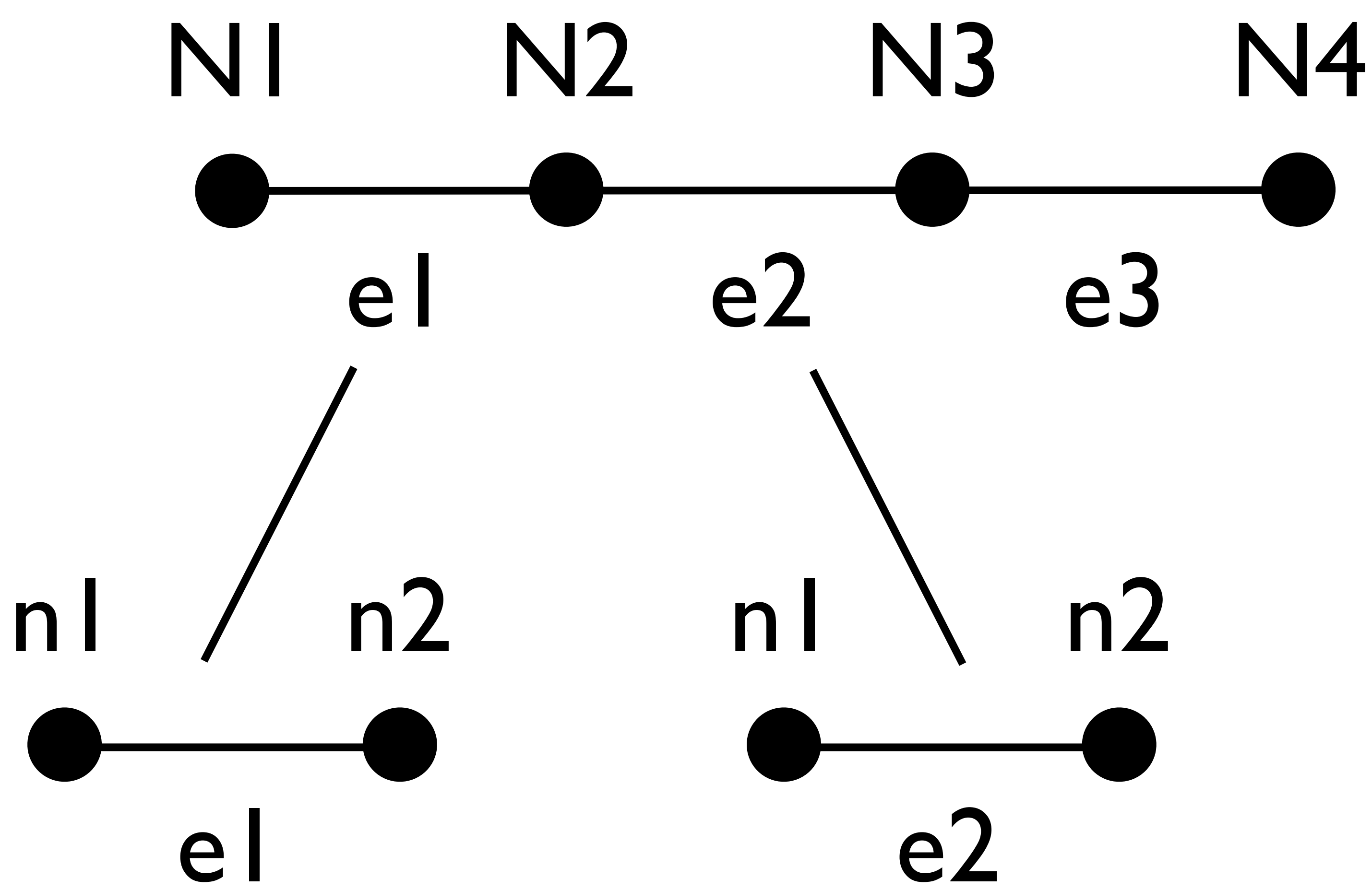
General Form for Element e

$$x(\xi) = x_{e-1}\psi_0(\xi) + x_e\psi_1(\xi)$$

$$x(\xi) = x_{e-1}\left(\frac{1-\xi}{2}\right) + x_e\left(\frac{1+\xi}{2}\right)$$

A 1-D FINITE ELEMENT MESH

Making A Mesh



To generate a 1D mesh with 10 elements, between x=0 and x=1, call following function:

- `mesh = OneDimLinearMeshGen(0,1,10);`

A 1-D FINITE ELEMENT MESH

A Mesh Data Structure

```
mesh.ne = Ne; %set number of elements
mesh.ngn = Ne+1; %set number of global nodes

%set spatial positions of nodes
mesh.elem(i).x(1)
mesh.elem(i).x(2)

%set global IDs of the nodes
mesh.elem(i).n(1)
mesh.elem(i).n(2)
```