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ME40064: System Modelling & Simulation
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Tutorial 3: Constructing Element Matrices for FEM

1. For the local element matrix for the diffusion term, shown in the lecture, evaluate the matrix for the second element in the 3-element mesh, that is, for the element defined by $x_0 = 1/3$ and $x_1=2/3$. What do you notice about this element matrix compared to the one shown in the notes?

2. In this question you will derive the element matrix operator for the **linear reaction term** from the advection-diffusion-reaction equation. First, you will derive this for a general local element, before substituting in the appropriate values for specific elements within a mesh.

    a. The weighted residual integral for a linear reaction term in an element given by $x=[x_0, x_1]$ is:

    $$\int_{x_0}^{x_1} \lambda c v dx$$

    Moving this integral to the standard element, with coordinate system $\boldsymbol{\xi}$, the integral becomes:

    $$\int_{-1}^{1} \lambda c v J d\xi$$

    Calculate this integral in the standard element by following the same approach that was used for a diffusion operator in the lectures. As in that example, you should arrive at a 2-by-2 local element matrix, which is of the form:

    $$\begin{bmatrix} Int_{00} & Int_{01} \\ Int_{10} & Int_{11} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

    For this question, just calculate the integral for a general local element – in the next two parts of Question 2 you will evaluate the values of the integrals for specific elements.

    b. Evaluate this element matrix for the first element in the 3-element mesh, which is defined between the points $x_0 = 0$ and $x_1 = 1/3$.

c. Now evaluate this element matrix for the second element in a 6-element equally spaced mesh, defined between the points $x_0 = 1/6$ & $x_1 = 1/3$. Again, what do you notice, and why?

As in the lectures, the solution, c, space, x, and test function, v, are all represented using linear Lagrange basis functions as follows:

$$c = c_0\psi_0(\xi) + c_1\psi_1(\xi)$$

$$x = x_0\psi_0(\xi) + x_1\psi_1(\xi)$$

$$v = \psi_0, \psi_1$$

3. Write a MATLAB function with the name, `TestMatrixCreate`, that will take in the following scalar inputs, `a, b, c`, and return a symmetric 2-by-2 matrix, given by the following expressions:

$$\begin{bmatrix} c^3 + 2bc + a & b^2 + a \\ b^2 + a & 2c^3 + 4bc + 5a \end{bmatrix}$$

Test your code for the particular case of `a=1, b=3, c=6`, which has a resultant matrix of:

$$\begin{bmatrix} 253 & 10 \\ 10 & 509 \end{bmatrix}$$

Note: the Matlab function, `zeros(n,m)`, will create an n-by-m matrix, with each entry initialised to zero.