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ME40064: System Modelling & Simulation
ME50344: Engineering Systems Simulation

Tutorial 6: Error Analysis & Matrices for Time Integration

Part A: Matrices for Time Integration

1. Hand derivations for the mass element matrix integrated using by Gaussian quadrature are as follows:

Mass element matrix.

$$Int_{mn} = \int_{-1}^{1} \psi_n \psi_m J \, d\xi$$

Will calculate for Int_{00} and Int_{01}, using Gaussian quadrature.

Function to be integrated is quadratic, there need GQ for $N=2$, where $w_1 = w_2 = 1$ and $\xi_1 = -\sqrt{\frac{1}{3}}$ and $\xi_2 = \sqrt{\frac{1}{3}}$

Int_{00}

$$\int_{-1}^{1} \psi_0 \psi_0 J \, d\xi = \int_{-1}^{1} \left(\frac{1-\xi}{2}\right)\left(\frac{1-\xi}{2}\right) J \, d\xi$$

$$\therefore Int_{00} = \int_{-1}^{1} \frac{J}{4}(1 - 2\xi + \xi^2) \, d\xi$$

Using G.Q:

$$\Rightarrow 1 \cdot \frac{J}{4}\left[1 - \cancel{2\left(-\sqrt{\tfrac{1}{3}}\right)} + \frac{1}{3}\right] + 1 \cdot \frac{J}{4}\left[1 - \cancel{2\sqrt{\tfrac{1}{3}}} + \frac{1}{3}\right]$$

$$= \frac{J}{4}\left[2 + \frac{2}{3}\right] = \underline{\underline{\frac{2J}{3}}}$$

Similarly for Int_{01}:

$$Int_{01} = \frac{J}{4}\int_{-1}^{1} (1-\xi)(1+\xi) \, d\xi = \frac{J}{4}\int_{-1}^{1} (1-\xi^2) \, d\xi$$

$$\Rightarrow 1 \cdot \frac{J}{4}\left[1 - \frac{1}{3}\right] + 1 \cdot \frac{J}{4}\left[1 - \frac{1}{3}\right] = \underline{\underline{\frac{J}{3}}}$$

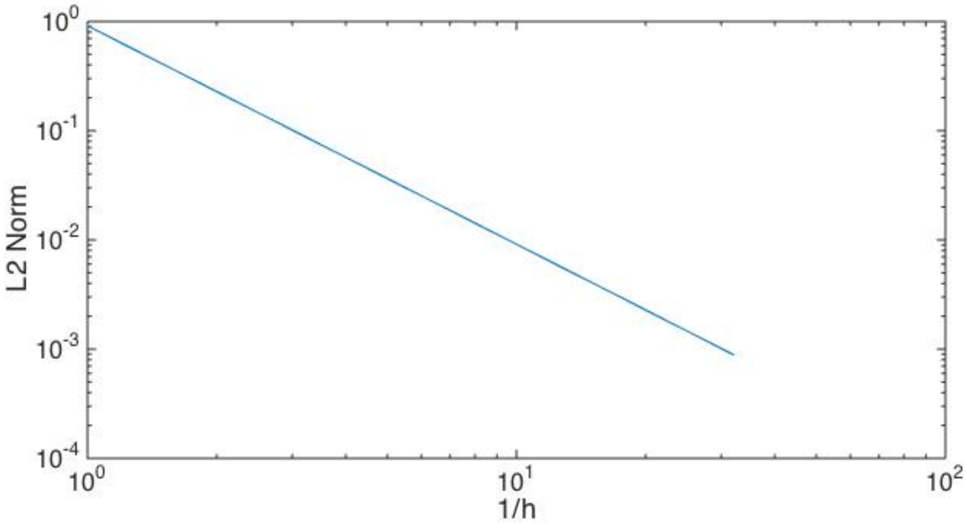
Therefore the mass element matrix for linear basis functions is:

$$\mathrm{Int}_{mn} = \begin{bmatrix} 2J/3 & J/3 \\ J/3 & 2J/3 \end{bmatrix}$$

Part B: Error Analysis

1. The L_2 norm for the 4-element mesh representation is: 0.05705.

2. The convergence plot on log-log axes is:



The gradient of this line is – 2.000.