Lecture 15

Consider the forced mass-spring-damper system written as the following 2nd order differential equation

$$m\ddot{x} + c\dot{x} + kx = f$$

where :

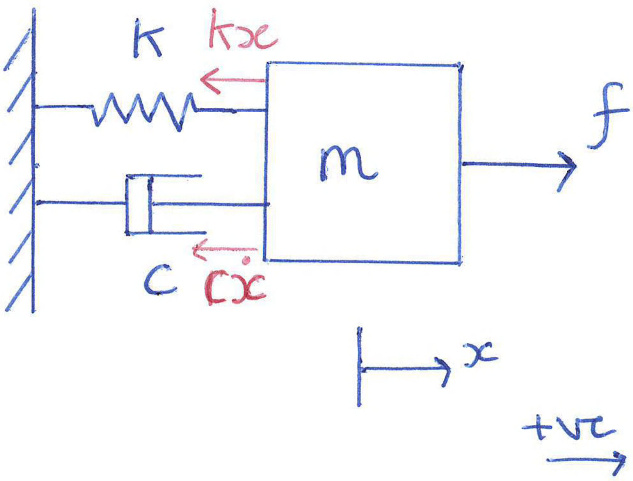
m = mass of the body

c = damping rate

k = linear spring stiffness

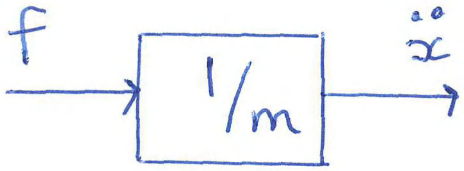
f = external force

x = position of body



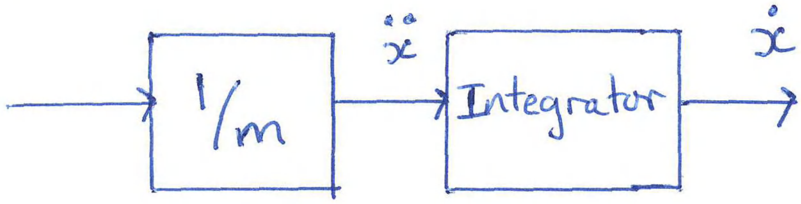
Converting this to a block-diagram representation proceeds as follows.

Start with the mass-acceleration term:

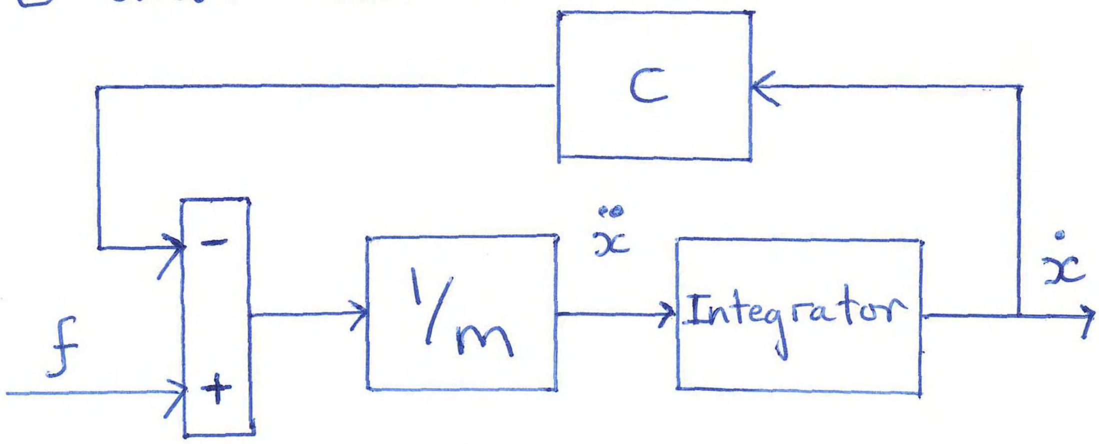


i.e $\ddot{x} = f/m$

Now connect an integrator block. i.e $\int dt$ to output $\dot{x}$:



Want to multiply $\dot{x}$ by damping coefficient c and feed it back:

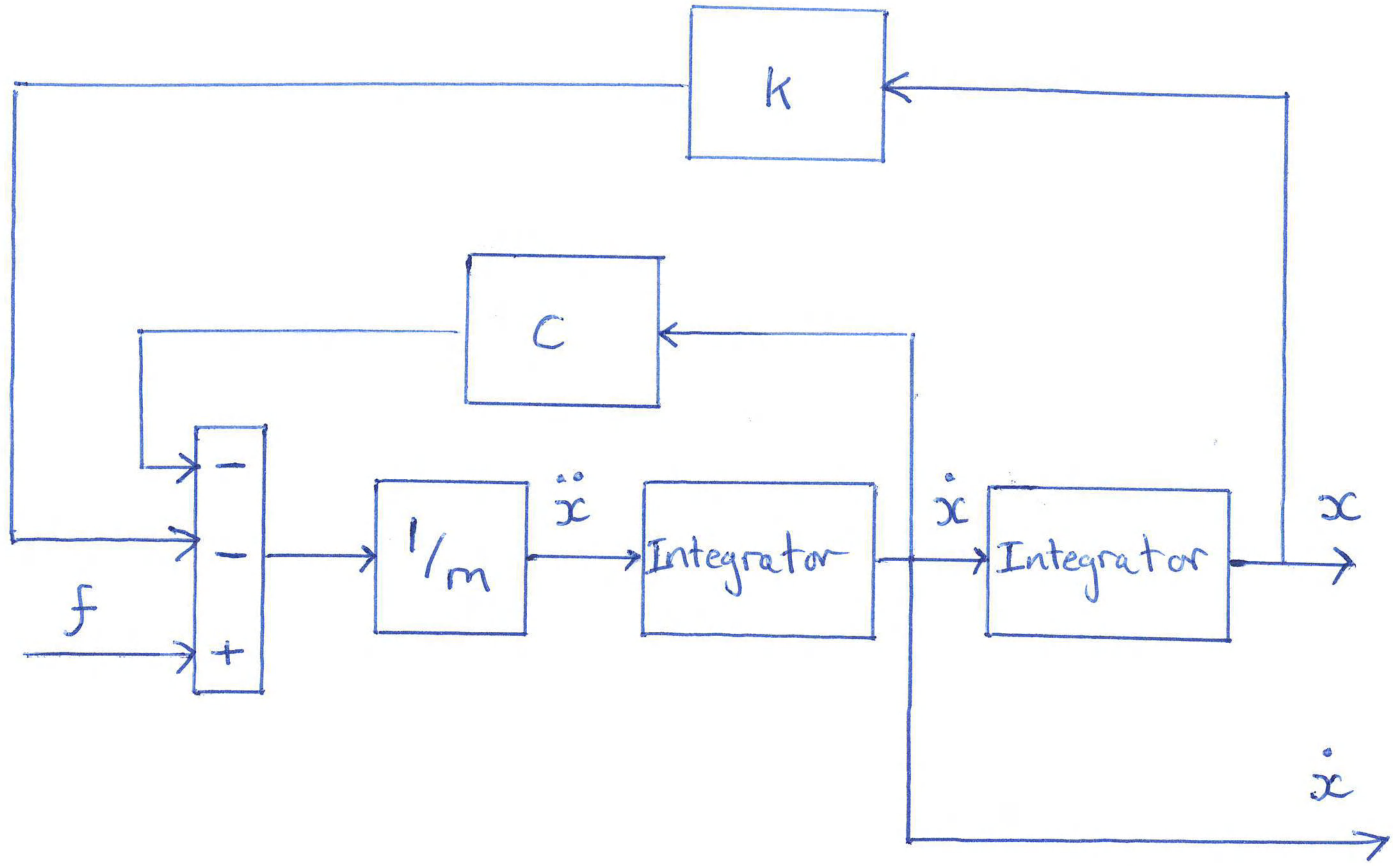


Summation Block

i.e $\frac{1}{m}\left(f - c\dot{x}\right) = \ddot{x}$

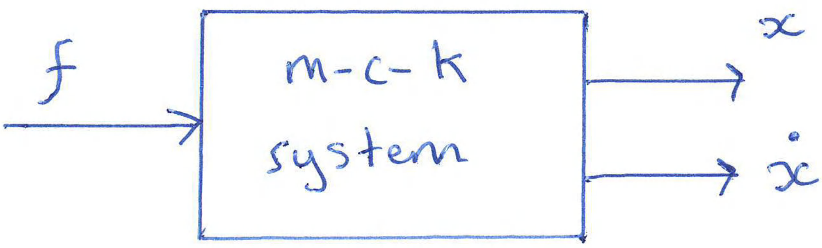
In Laplace domain, s, an integrator block is $\frac{1}{s}$. This notation used in Simulink.

Finally integrate $\dot{x}$ to produce x, multiply by spring stiffness k, and feed it back to the summation block:



Can view x, $\dot{x}$ as outputs

For ease of viewing can represent as a single block:



Can also represent this using state space.

State space representations link inputs and outputs of a system by a set of first-order differential equations.

State space vector for mass-spring-damper system is:

$$\underline{z} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

The block form for this:

$$\underline{u} \longrightarrow \boxed{\begin{aligned} \dot{\underline{z}} &= A\underline{z} + B\underline{u} \\ \underline{y} &= C\underline{z} + D\underline{u} \end{aligned}} \longrightarrow \underline{y}$$

$$\underline{u} = \begin{bmatrix} f \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}, \quad \underline{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$