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ME40064: System Modelling & Simulation
ME50344: Engineering Systems Simulation

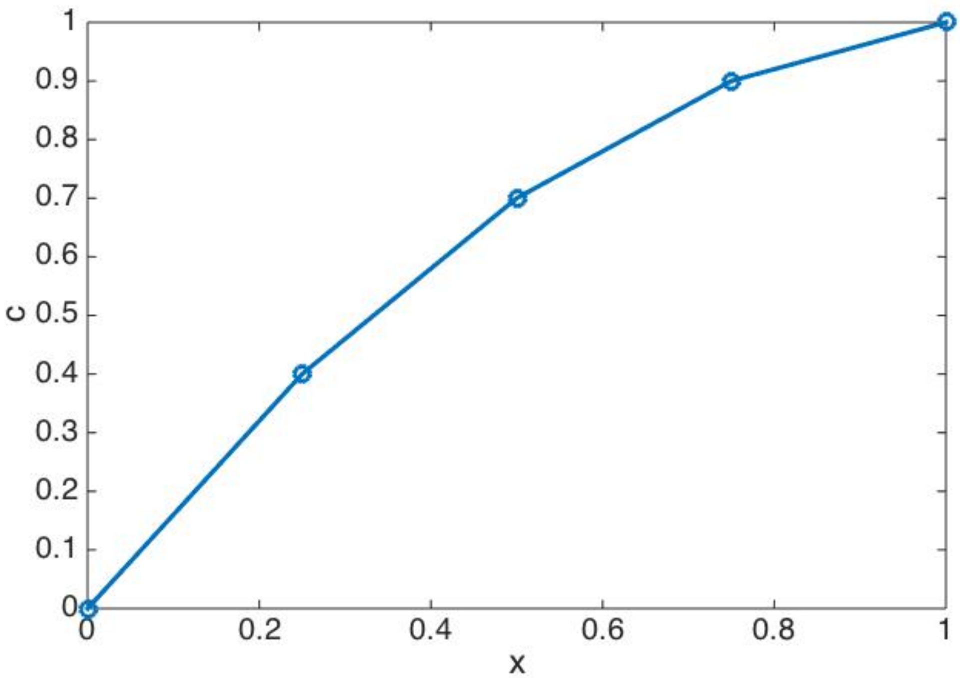
Tutorial 4: Applying Boundary Conditions and Solving the Finite Element Matrix System

Example Solutions

1. The modified linear system for the two Dirichlet boundary conditions is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 \\ 0 & -2 & 4 & -2 & 0 \\ 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.2000 \\ 0.2000 \\ 0.2000 \\ 1.000 \end{bmatrix}$$

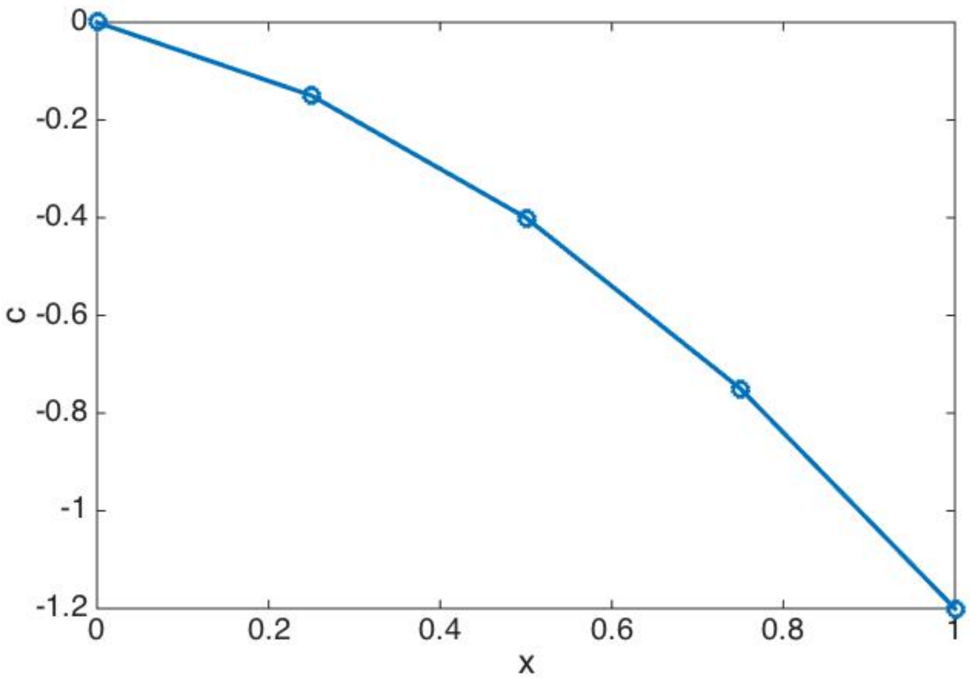
Solving this linear system produces the following:



2. The modified linear system for one Dirichlet and one Neumann is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 \\ 0 & -2 & 4 & -2 & 0 \\ 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0.0000 \\ 0.2000 \\ 0.2000 \\ 0.2000 \\ -0.9000 \end{bmatrix}$$

Solving this linear system produces the following:



In this solution, the Dirichlet condition at x=0 means the solution starts at the same value. However, due to the negative value of the Neumann BC, the gradient of the solution is now negative there. Physically this indicates that concentration is leaving the system at x=1 and hence reducing the value inside the rest of the domain.

3. If a Dirichlet condition is set at only one end of the domain, then at the other end a Neumann boundary condition of zero has been implicitly specified at the other, i.e. a zero flux condition. This is because we not modified the corresponding row in the matrix and we have effectively added zero onto the right hand side vector. In the weak form weighted residual method, a zero Neumann boundary condition therefore arises naturally.

   Physically a zero flux condition implies a perfect insulating boundary.

Additional points about boundary conditions

- Dirichlet boundary conditions are enforced exactly at the nodes, due to the way the matrix & vector are modified to alter the matrix algebra.
- Neumann boundary conditions are specified as an integral, and therefore the accuracy of the boundary condition enforcement depends on the mesh resolution. In the tutorial example, there are only four elements in the mesh, which means that specifying a zero Neumann BC does not give a zero gradient in the solution at that boundary. Increasing the number of elements would help the boundary condition converge to its true value.
- To help check whether your boundary condition implementation is correct, you can solve your problem with the desired BCs. If your solution matches the boundary conditions then the chances are things are correct. The other thing to do is to look at the Matlab variables for your global matrix and global vector to check that your code has modified them in the way you intended.
- Another way to use a zero Neumann boundary condition is to model a symmetry condition in a system, thereby reducing computational effort. This is because at the point of symmetry in the system, we know that the value of the solution a small distance either side of the symmetry line is the same, and therefore the gradient there is zero, which we can then represent with a zero Neumann condition.