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a)

$$E(\hat{\mu}) = E(\bar{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n} \mu = \frac{1}{n} n\mu = \mu$$

b)

$$Var(\bar{X}) = Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i,j=1}^{n} Cov(X_i, X_j) \underset{\text{unkorreliert}}{=} \frac{1}{n}\sum_{i=1}^{n} Var(X_i) = \frac{1}{n}\sigma^2$$

c)

$$E(S_0^2) = E\left(\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2\right) = \frac{1}{n}\sum_{i=1}^{n} E((X_i-\mu)^2) = \frac{1}{n}\sum_{i=1}^{n}\sigma^2 = \sigma^2$$

d)

$$E(S_n'^2) = E\left(\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2\right) = \frac{1}{n}E\left(\sum(X_i-\mu+\mu-\bar{X})^2\right)$$

$$\underset{\text{bin. Formel}}{=} \frac{1}{n}E\left(\sum (X_i-\mu)^2 - 2(X_i-\mu)(\bar{X}-\mu) + (\bar{X}-\mu)^2\right)$$

$$= \frac{1}{n}E\left(\sum(X_i-\mu)^2 - 2\sum(X_i-\mu)(\bar{X}-\mu) + \sum(\bar{X}-\mu)^2\right)$$

$$= \frac{1}{n}E\left(\sum(X_i-\mu)^2 - 2n(\bar{X}-\mu)(\bar{X}-\mu) + n(\bar{X}-\mu)^2\right)$$

$$= \frac{1}{n}E\left(\sum(X_i-\mu)^2 - n(\bar{X}-\mu)^2\right)$$

$$= \frac{1}{n}\sum E(X_i-\mu)^2 - nE(\bar{X}-\mu)^2)$$

$$= \frac{1}{n}(Var(X_i) - n Var(\bar{X}))$$

$$= \sigma^2 - \frac{\sigma^2}{n} = \frac{n-1}{n}\sigma^2 \quad \rightarrow \text{Verzerrt}$$

=> Korrektur

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$$