Aufgabe 25

a) $x = a z + b \quad \rightarrow \quad x_1 = a z_1 + b, \quad x_2 = a z_2 + b \quad \Rightarrow \quad a = \frac{x_2 - x_1}{z_2 - z_1}$

$$b = x_2 \cdot \frac{z_1}{z_1 - z_2} - x_1 \frac{z_2}{z_1 - z_2}$$

als Matrix: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} z_1 & 1 \\ z_2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{z_1 - z_2} \begin{pmatrix} 1 & -1 \\ -z_2 & z_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Kovarianzmatrix (mit BVB Formel):

$$V_{a,b} = \frac{1}{(z_1 - z_2)^2} \begin{pmatrix} 1 & -1 \\ -z_2 & z_1 \end{pmatrix} \begin{pmatrix} \sigma_{x_1}^2 & 0 \\ 0 & \sigma_{x_2}^2 \end{pmatrix} \begin{pmatrix} 1 & -z_2 \\ -1 & z_1 \end{pmatrix}$$

$$= \frac{1}{(z_1 - z_2)^2} \begin{pmatrix} \sigma_{x_1}^2 & -\sigma_{x_2}^2 \\ -\sigma_{x_1}^2 z_2 & \sigma_{x_2}^2 z_1 \end{pmatrix} \begin{pmatrix} 1 & -z_2 \\ -1 & z_1 \end{pmatrix}$$

$$= \frac{1}{(z_1 - z_2)^2} \begin{pmatrix} \sigma_{x_1}^2 + \sigma_{x_2}^2 & -(\sigma_{x_1}^2 z_2 + \sigma_{x_2}^2 z_1) \\ -(\sigma_{x_1}^2 z_2 + \sigma_{x_2}^2 z_1) & \sigma_{x_1}^2 z_2^2 + \sigma_{x_2}^2 z_1^2 \end{pmatrix}$$

Fehler:

$$\sigma_a = \frac{1}{(z_1 - z_2)} \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2} \qquad \sigma_b = \frac{1}{(z_1 - z_2)} \sqrt{\sigma_{x_1}^2 z_2^2 + \sigma_{x_2}^2 z_1^2}$$

Korrelationskoeff:

$$\rho(a,b) = \frac{1}{\sigma_a \sigma_b} \operatorname{cov}(a,b) = - \frac{(z_2 \sigma_{x_1}^2 + z_1 \sigma_{x_2}^2)}{\sqrt{(\sigma_{x_1}^2 + \sigma_{x_2}^2)(\sigma_{x_1}^2 z_2^2 + \sigma_{x_2}^2 z_1^2)}}$$

b) $x_3 = a \cdot z_3 + b = \frac{x_2 - x_1}{z_2 - z_1} \cdot z_3 - x_1 \frac{z_2}{z_1 - z_2} + x_2 \frac{z_1}{z_1 - z_2} = \frac{1}{z_1 - z_2} \left(x_2 z_1 - x_1 z_2 + (x_1 - x_2) z_3 \right)$

$$V_{x_3} = \begin{pmatrix} \frac{\partial x_3}{\partial a} & \frac{\partial x_3}{\partial b} \end{pmatrix} \cdot \frac{1}{(z_1 - z_2)^2} \begin{pmatrix} \sigma_{x_1}^2 + \sigma_{x_2}^2 & -(\sigma_{x_1}^2 z_2 + \sigma_{x_2}^2 z_1) \\ -(\sigma_{x_1}^2 z_2 + \sigma_{x_2}^2 z_1) & \sigma_{x_1}^2 z_2^2 + \sigma_{x_2}^2 z_1^2 \end{pmatrix} \begin{pmatrix} \frac{\partial x_3}{\partial a} \\ \frac{\partial x_3}{\partial b} \end{pmatrix}$$

$$= \begin{pmatrix} z_3 & 0 \end{pmatrix} \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} z_3 \\ 0 \end{pmatrix}$$

$$= \frac{1}{(z_1 - z_2)^2} \begin{pmatrix} z_3 & 0 \end{pmatrix} \begin{pmatrix} z_3 (\sigma_{x_1}^2 + \sigma_{x_2}^2) \\ z_3 (\sigma_{x_1}^2 z_2 + \sigma_{x_2}^2 z_1) \end{pmatrix} = \frac{1}{(z_1 - z_2)^2} \cdot z_3^2 (\sigma_{x_1}^2 + \sigma_{x_2}^2)$$

$$= \frac{\cancel{(x_1 - x_2)}^2}{z_3} \cdot (\sigma_{x_1}^2 + \sigma_{x_2}^2)$$

$$\sigma_{x_3} = \sqrt{V_{x_3}} = \frac{\cancel{a}}{z_3} \cancel{(x_1 - x)} \cdot (\sigma_{x_1}^2 + \sigma_{x_2}^2)$$

c) für diesen Vektor $\begin{pmatrix} \frac{\partial x_3}{\partial a} \\ \frac{\partial x_3}{\partial b} \end{pmatrix} = \begin{pmatrix} z_3 \\ 0 \end{pmatrix}$ macht es keinen Unterschied,

da die Korrelation eh weg fällt $\rightarrow$ stimmt vermutlich nicht.