Information Representation

(a) A seven-bit cell.

Figure 3.1
(Continued)

0	1	1	0	1	0	1

1	1	0	1	1	0	0

0	0	0	0	0	0	0

(b) Some possible values in a
seven-bit cell.

Figure 3.1
(Continued)

| 6 | 8 | 0 | 7 | 2 | 5 | 1 |

| J | A | N | U | A | R | Y |

| | | 1 | 1 | 0 | 1 | 0 |

(c) Some impossible values in a seven-bit cell.

Counting in decimal

0	7	14	21	28	35
1	8	15	22	29	36
2	9	16	23	30	37
3	10	17	24	31	38
4	11	18	25	32	.
5	12	19	26	33	.
6	13	20	27	34	.

Counting in octal

0	7	16	25	34	43
1	10	17	26	35	44
2	11	20	27	36	45
3	12	21	30	37	46
4	13	22	31	40	.
5	14	23	32	41	.
6	15	24	33	42	.

Counting in base 3

0	21	112	210	1001	1022
1	22	120	211	1002	1100
2	100	121	212	1010	1101
10	101	122	220	1011	1102
11	102	200	221	1012	.
12	110	201	222	1020	.
20	111	202	1000	1021	.

Counting in binary

0	111	1110	10101	11100	100011
1	1000	1111	10110	11101	100100
10	1001	10000	10111	11110	100101
11	1010	10001	11000	11111	100110
100	1011	10010	11001	100000	.
101	1100	10011	11010	100001	.
110	1101	10100	11011	100010	.

Figure 3.2

1 0 1 1 0

1's place

2's place

4's place
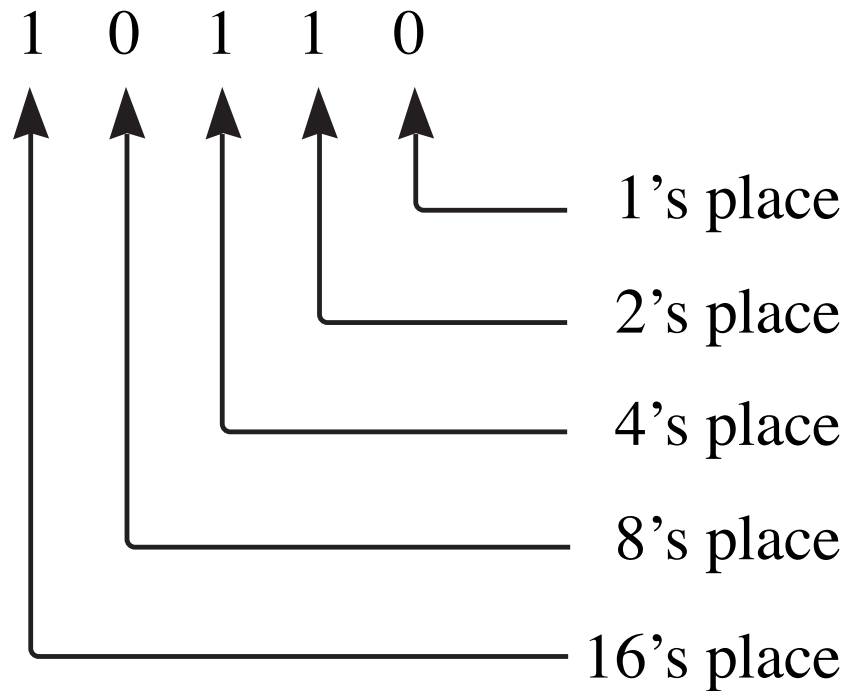
8's place

16's place

(a) The place values for 10110 (bin).

0 1's place = 0

1 2's place = 2

1 4's place = 4

0 8's place = 0

1 16's place = 16
_____
22 (dec)

(b) Converting 10110 (bin) to decimal.

Figure 3.3

Figure 3.4
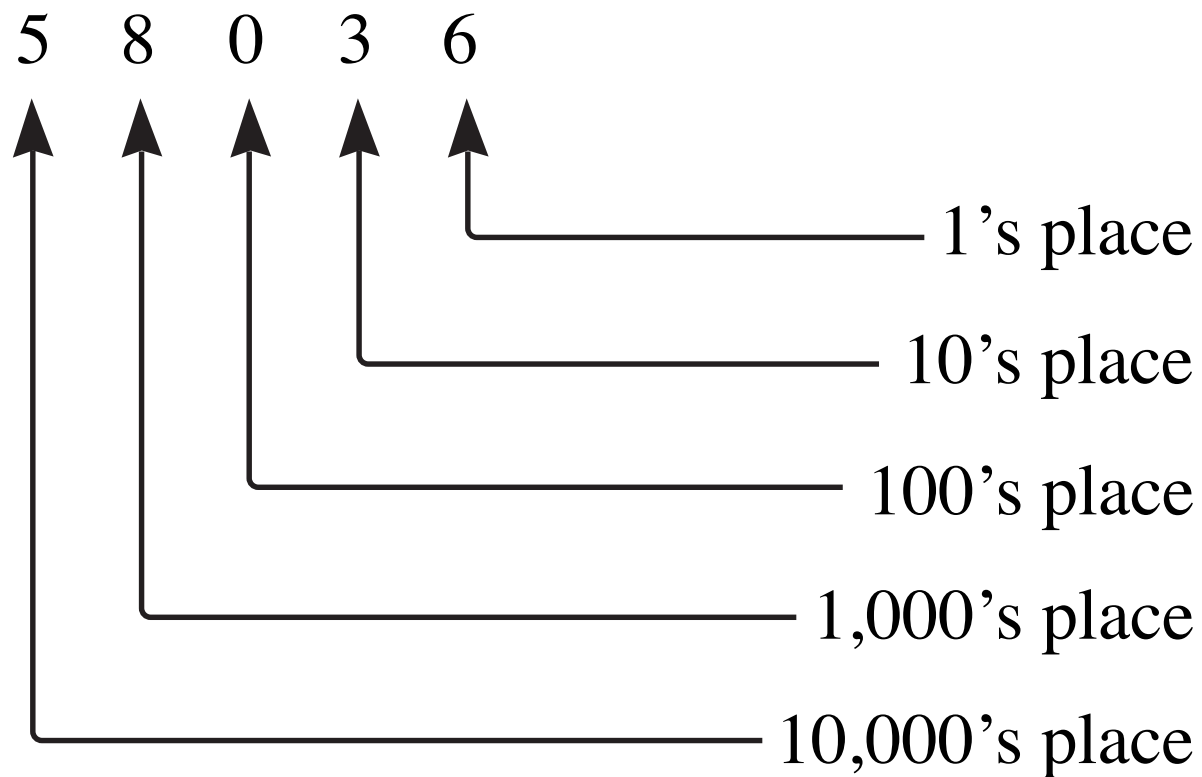
$$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

(a) The binary number 10110.

$$5 \times 10^4 + 8 \times 10^3 + 0 \times 10^2 + 3 \times 10^1 + 6 \times 10^0$$

(b) The decimal number 58,036.

Figure 3.5

$$\begin{array}{c|c}
22 & \\
\hline
11 & 0 \\
5 & 1 \\
2 & 1 \\
1 & 0 \\
0 & 1
\end{array}$$

Remainders

Dividends

Figure 3.6

0	0	1	0	1	1	0

Binary addition rules

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

Figure 3.7

Figure 3.8

Magnitude

Sign bit
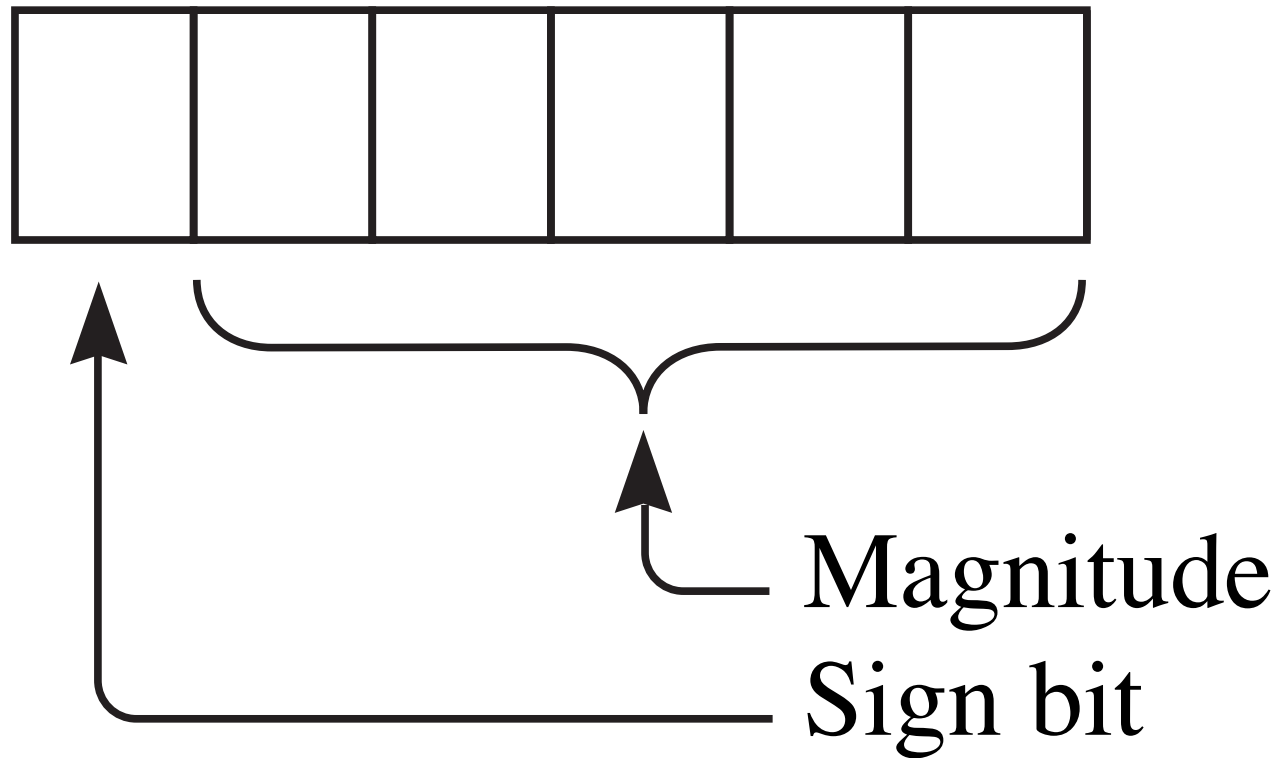
- The NEG operation

  ▸ Taking the two's complement

- The NOT operation
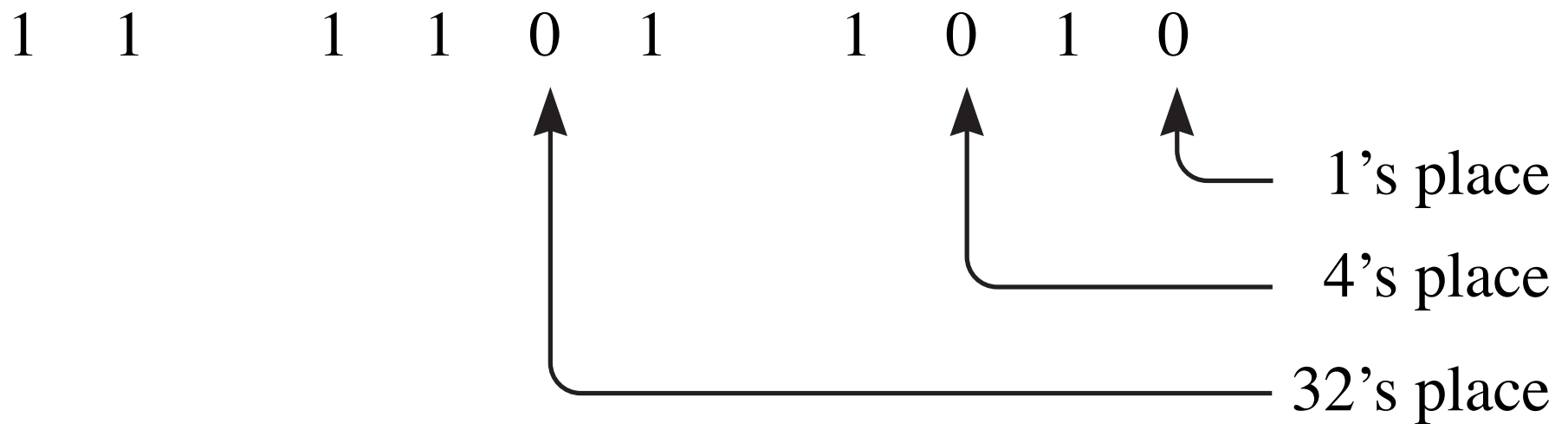
  ▸ Change the 1's to 0's and the 0's to 1's

The two's complement rule
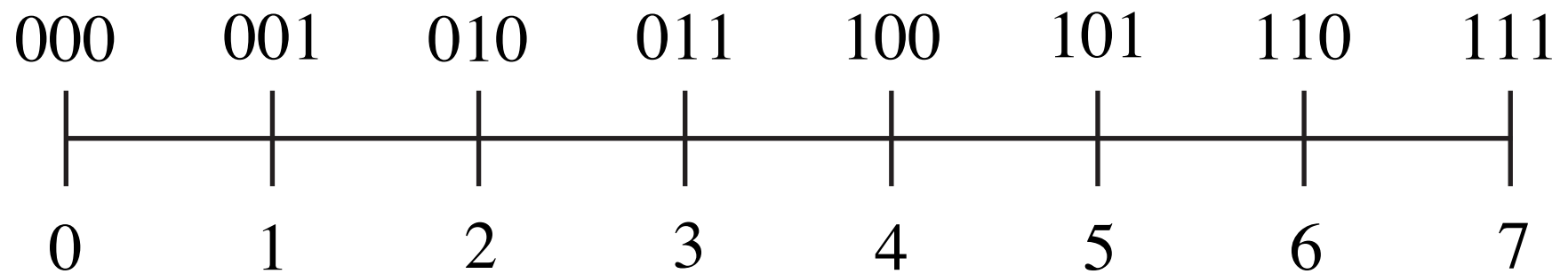
- The two's complement of a number is 1 plus its one's complement

- NEG x = 1 + NOT x

Figure 3.9

Decimal	Binary
−7	1001
−6	1010
−5	1011
−4	1100
−3	1101
−2	1110
−1	1111

Figure 3.10

Decimal	Binary
−8	1000
−7	1001
−6	1010
−5	1011
−4	1100
−3	1101
−2	1110
−1	1111
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

Figure 3.11

1 1   1 1 0 1   1 0 1 0

1's place

4's place

32's place

Figure 3.12

Figure 3.13

000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7
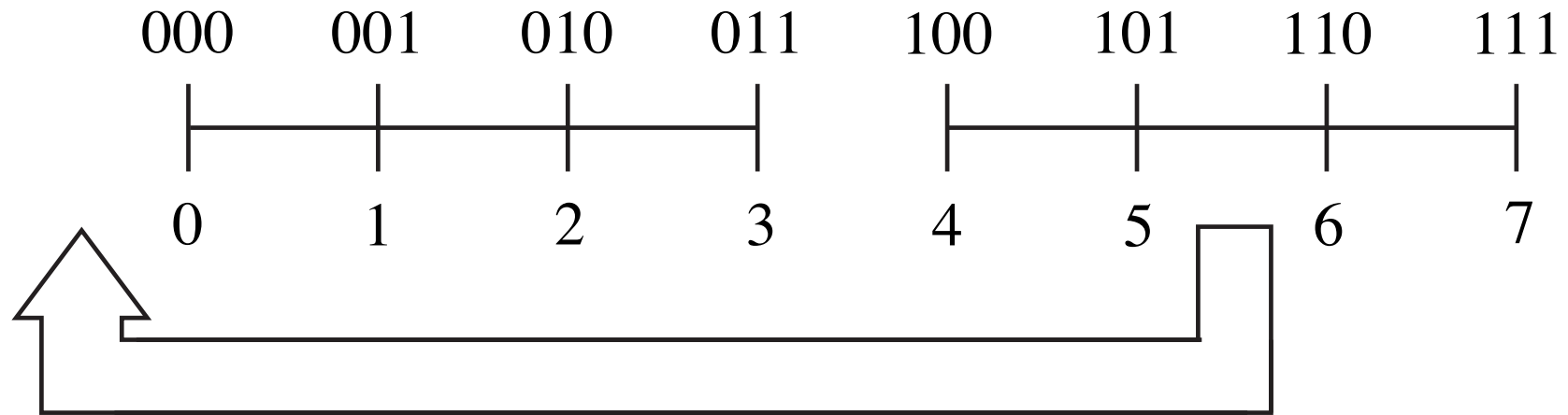
(a) Breaking the number line in the middle.

100	101	110	111	000	001	010	011
−4	−3	−2	−1	0	1	2	3
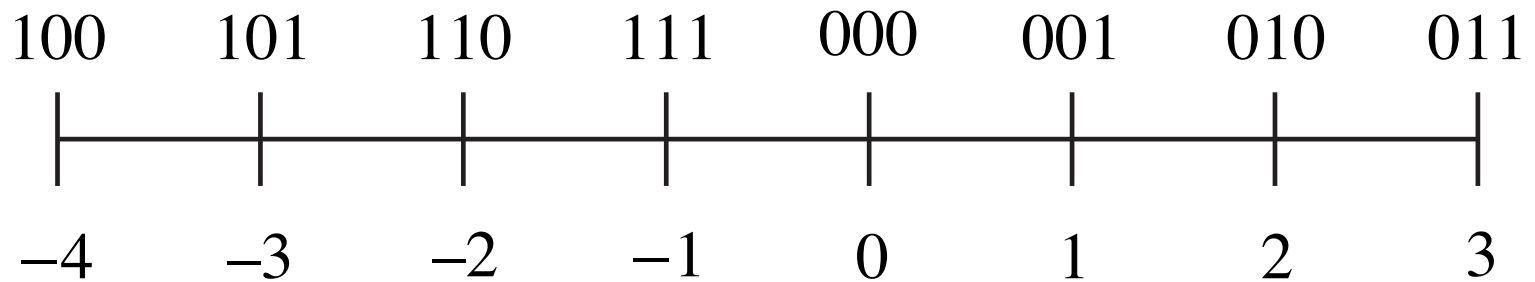
(b) Shifting the right part to the left side.

Figure 3.14

4	5	6	7	0	1	2	3
−4	−3	−2	−1	0	1	2	3
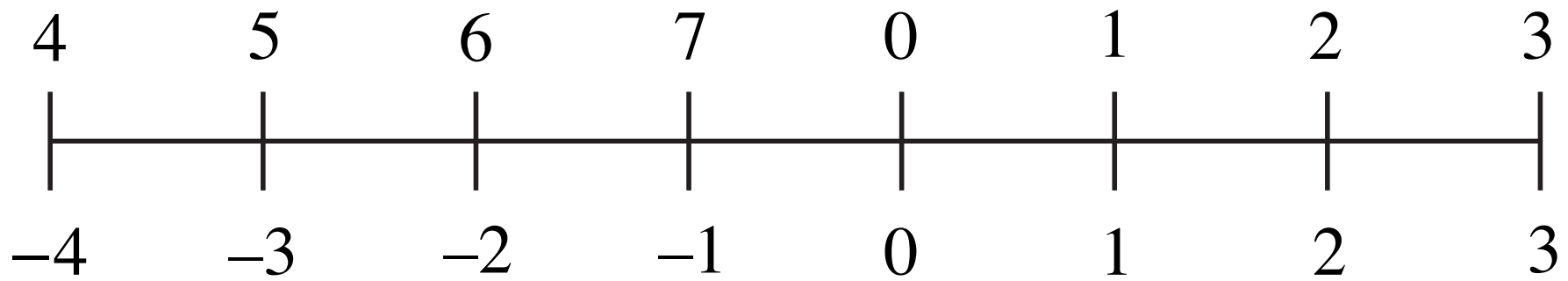
The status bits

- N = 1 if the result is negative

  N = 0 otherwise

- Z = 1 if the result is all zeros

  Z = 0 otherwise

- V = 1 if a signed integer overflow occurred

  V = 0 otherwise

- C = 1 if an unsigned integer overflow occurred

  C = 0 otherwise

Figure 3.15

p	q	p AND q
0	0	0
0	1	0
1	0	0
1	1	1

p	q	p OR q
0	0	0
0	1	1
1	0	1
1	1	1

p	q	p XOR q
0	0	0
0	1	1
1	0	1
1	1	0

Figure 3.16

p	q	p AND q
true	true	true
true	false	false
false	true	false
false	false	false

p	q	p OR q
true	true	true
true	false	true
false	true	true
false	false	false

p	q	p XOR q
true	true	false
true	false	true
false	true	true
false	false	false

Figure 3.17

Operation	RTL Symbol
AND	$\wedge$
OR	$\vee$
XOR	$\oplus$
NOT	$\neg$
Implies	$\Rightarrow$
Transfer	$\leftarrow$
Bit index	$\langle\rangle$
Informal description	$\{\ \}$
Sequential separator	;
Concurrent separator	,

RTL specification of OR operation

$$c \leftarrow a \vee b \; ; \; \mathrm{N} \leftarrow c < 0 \, , \, \mathrm{Z} \leftarrow c = 0$$

Figure 3.18

Arithmetic shift left (ASL)
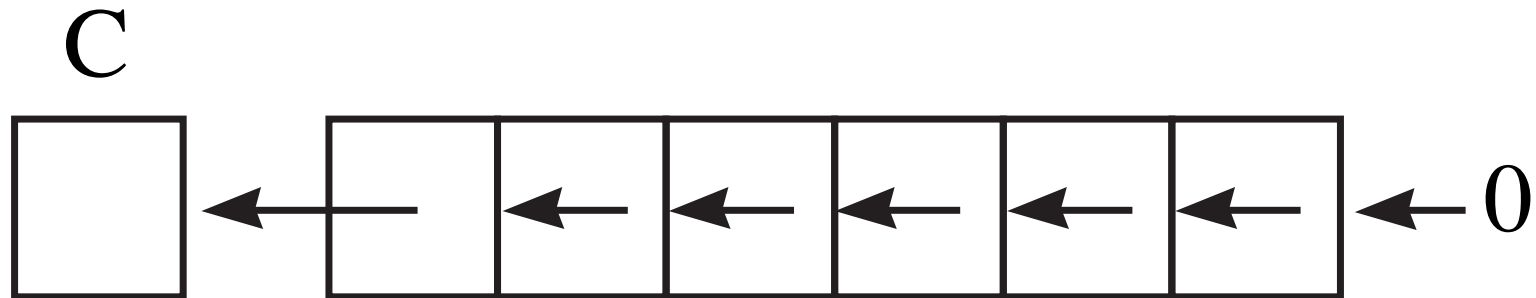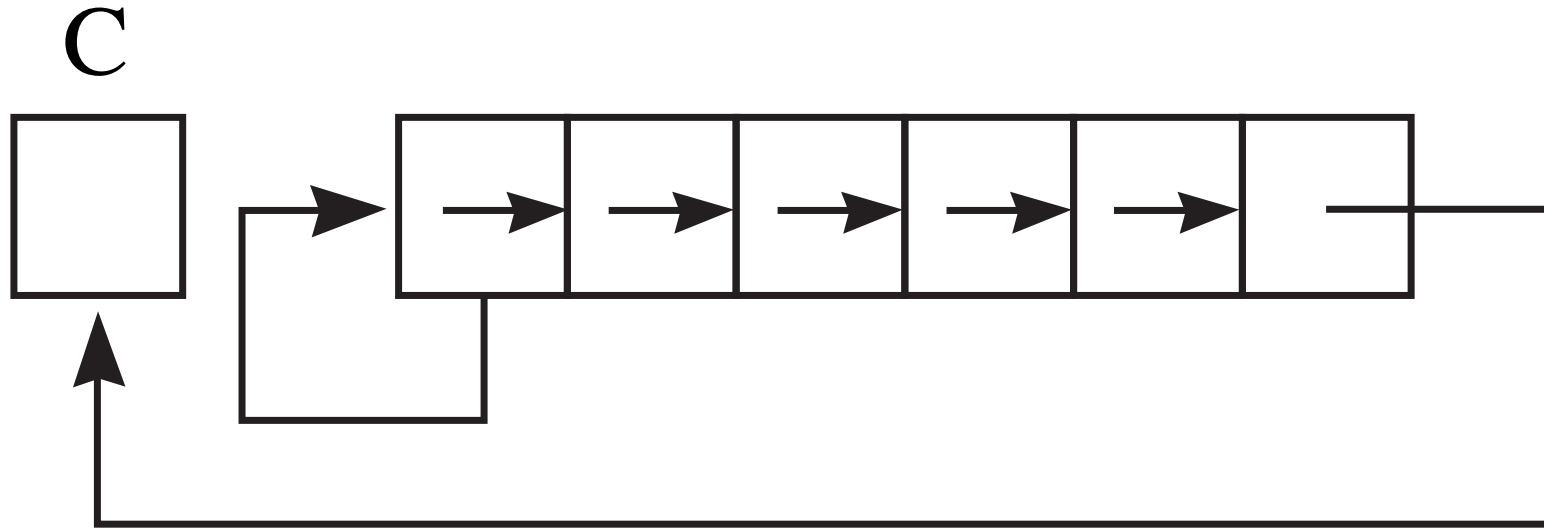

$$C \leftarrow r\langle 0 \rangle \,,\; r\langle 0..4 \rangle \leftarrow r\langle 1..5 \rangle \,,\; r\langle 5 \rangle \leftarrow 0 \,;$$

$$N \leftarrow r < 0 \,,\; Z \leftarrow r = 0 \,,\; V \leftarrow \{overflow\}$$

Figure 3.19

Arithmetic shift right (ASR)

RTL specification is a problem for the student

Figure 3.20

Rotate left (ROL)

Figure 3.20
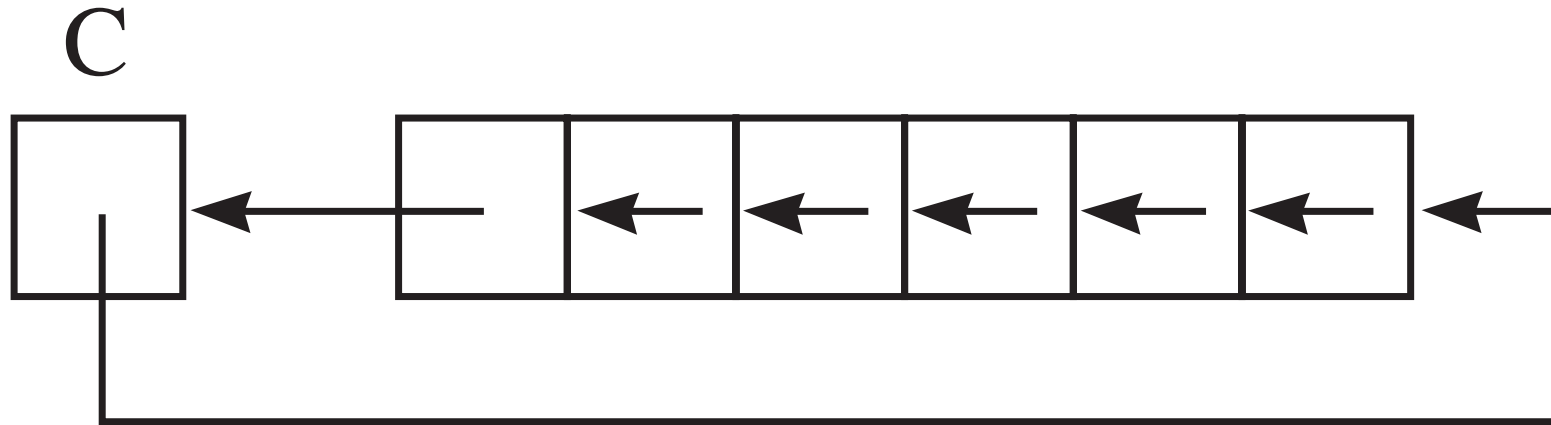
Rotate right (ROR)
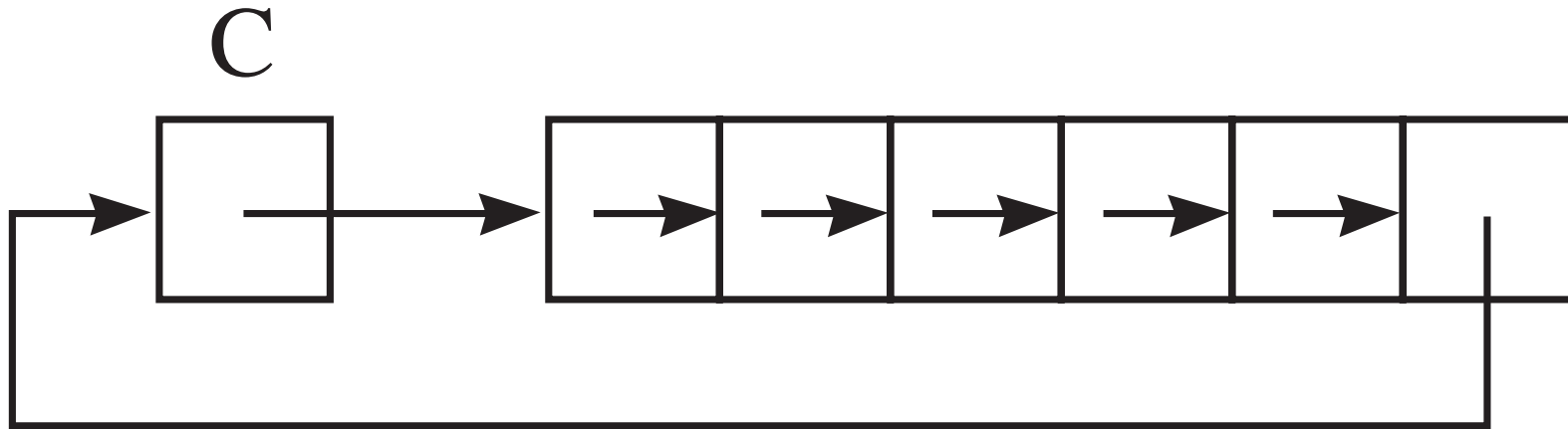
Counting in hexadecimal

0	7	E	15	1C	23
1	8	F	16	1D	24
2	9	10	17	1E	25
3	A	11	18	1F	26
4	B	12	19	20	.
5	C	13	1A	21	.
6	D	14	1B	22	.

Figure 3.20

8 B E 7

1's place

16's place

256's place

4096's place
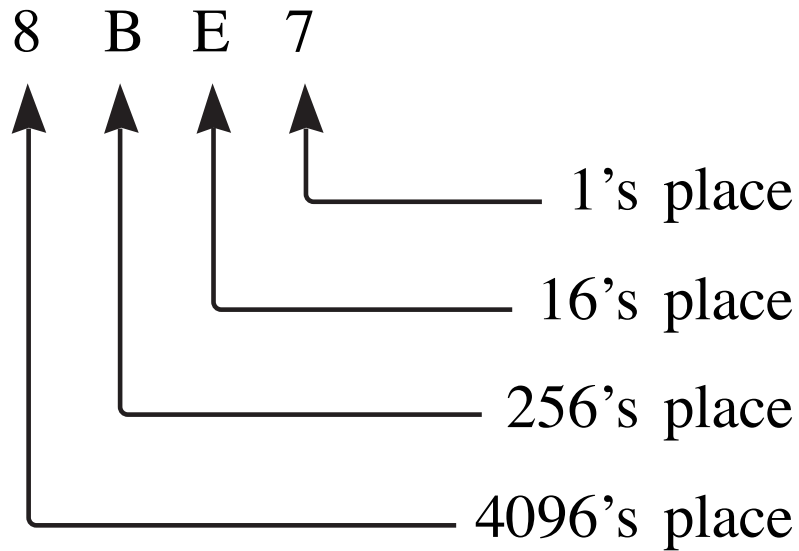
(a) The place values for 8BE7.

$$7 \times 1 = 7$$
$$14 \times 16 = 224$$
$$11 \times 256 = 2,816$$
$$8 \times 4096 = 32,768$$
$$\overline{ 35,815}$$

(b) Converting 8BE7 to decimal.

Figure 3.21

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0_	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1_	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
2_	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
3_	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
4_	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
5_	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
6_	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
7_	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127
8_	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
9_	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159
A_	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175
B_	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191
C_	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207
D_	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223
E_	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
F_	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255

Figure 3.22

Hexadecimal	Binary		Hexadecimal	Binary
0	0000		8	1000
1	0001		9	1001
2	0010		A	1010
3	0011		B	1011
4	0100		C	1100
5	0101		D	1101
6	0110		E	1110
7	0111		F	1111

Char	Bin	Hex	Char	Bin	Hex	Char	Bin	Hex	Char	Bin	Hex
NUL	000 0000	00	SP	010 0000	20	@	100 0000	40	`	110 0000	60
SOH	000 0001	01	!	010 0001	21	A	100 0001	41	a	110 0001	61
STX	000 0010	02	"	010 0010	22	B	100 0010	42	b	110 0010	62
ETX	000 0011	03	#	010 0011	23	C	100 0011	43	c	110 0011	63
EOT	000 0100	04	$	010 0100	24	D	100 0100	44	d	110 0100	64
ENQ	000 0101	05	%	010 0101	25	E	100 0101	45	e	110 0101	65
ACK	000 0110	06	&	010 0110	26	F	100 0110	46	f	110 0110	66
BEL	000 0111	07	'	010 0111	27	G	100 0111	47	g	110 0111	67
BS	000 1000	08	(	010 1000	28	H	100 1000	48	h	110 1000	68
HT	000 1001	09	)	010 1001	29	I	100 1001	49	i	110 1001	69
LF	000 1010	0A	*	010 1010	2A	J	100 1010	4A	j	110 1010	6A
VT	000 1011	0B	+	010 1011	2B	K	100 1011	4B	k	110 1011	6B
FF	000 1100	0C	,	010 1100	2C	L	100 1100	4C	l	110 1100	6C
CR	000 1101	0D	-	010 1101	2D	M	100 1101	4D	m	110 1101	6D
SO	000 1110	0E	.	010 1110	2E	N	100 1110	4E	n	110 1110	6E
SI	000 1111	0F	/	010 1111	2F	O	100 1111	4F	o	110 1111	6F
DLE	001 0000	10	0	011 0000	30	P	101 0000	50	p	111 0000	70
DC1	001 0001	11	1	011 0001	31	Q	101 0001	51	q	111 0001	71
DC2	001 0010	12	2	011 0010	32	R	101 0010	52	r	111 0010	72
DC3	001 0011	13	3	011 0011	33	S	101 0011	53	s	111 0011	73
DC4	001 0100	14	4	011 0100	34	T	101 0100	54	t	111 0100	74
NAK	001 0101	15	5	011 0101	35	U	101 0101	55	u	111 0101	75
SYN	001 0110	16	6	011 0110	36	V	101 0110	56	v	111 0110	76
ETB	001 0111	17	7	011 0111	37	W	101 0111	57	w	111 0111	77
CAN	001 1000	18	8	011 1000	38	X	101 1000	58	x	111 1000	78
EM	001 1001	19	9	011 1001	39	Y	101 1001	59	y	111 1001	79
SUB	001 1010	1A	:	011 1010	3A	Z	101 1010	5A	z	111 1010	7A
ESC	001 1011	1B	;	011 1011	3B	[	101 1011	5B	{	111 1011	7B
FS	001 1100	1C	<	011 1100	3C	\	101 1100	5C	|	111 1100	7C
GS	001 1101	1D	=	011 1101	3D	]	101 1101	5D	}	111 1101	7D
RS	001 1110	1E	>	011 1110	3E	^	101 1110	5E	~	111 1110	7E
US	001 1111	1F	?	011 1111	3F	_	101 1111	5F	DEL	111 1111	7F

Figure 3.23
(Continued)

Abbreviations for Control Characters

NUL	null, or all zeros	**FF**	form feed	**CAN**	cancel
SOH	start of heading	**CR**	carriage return	**EM**	end of medium
STX	start of text	**SO**	shift out	**SUB**	substitute
ETX	end of text	**SI**	shift in	**ESC**	escape
EOT	end of transmission	**DLE**	data link escape	**FS**	file separator
ENQ	enquiry	**DC1**	device control 1	**GS**	group separator
ACK	acknowledge	**DC2**	device control 2	**RS**	record separator
BEL	bell	**DC3**	device control 3	**US**	unit separator
BS	backspace	**DC4**	device control 4	**SP**	space
HT	horizontal tabulation	**NAK**	negative acknowledge	**DEL**	delete
LF	line feed	**SYN**	synchronous idle		
VT	vertical tabulation	**ETB**	end of transmission block		

Figure 3.24

EBCDIC	Binary
A	1100 0001
B	1100 0010
C	1100 0011
.	.
.	.
.	.
0	1111 0000
1	1111 0001
2	1111 0010
.	.
.	.
.	.

Figure 3.25

1   0   1 . 0   1   1

¼'s place

¼'s place

½'s place

1's place

2's place

4's place

1   ⅛'s place = 0.125

1   ¼'s place = 0.25

0   ½'s place = 0.0

1   1's place = 1.0

0   2's place = 0.0

1   4's place = 4.0
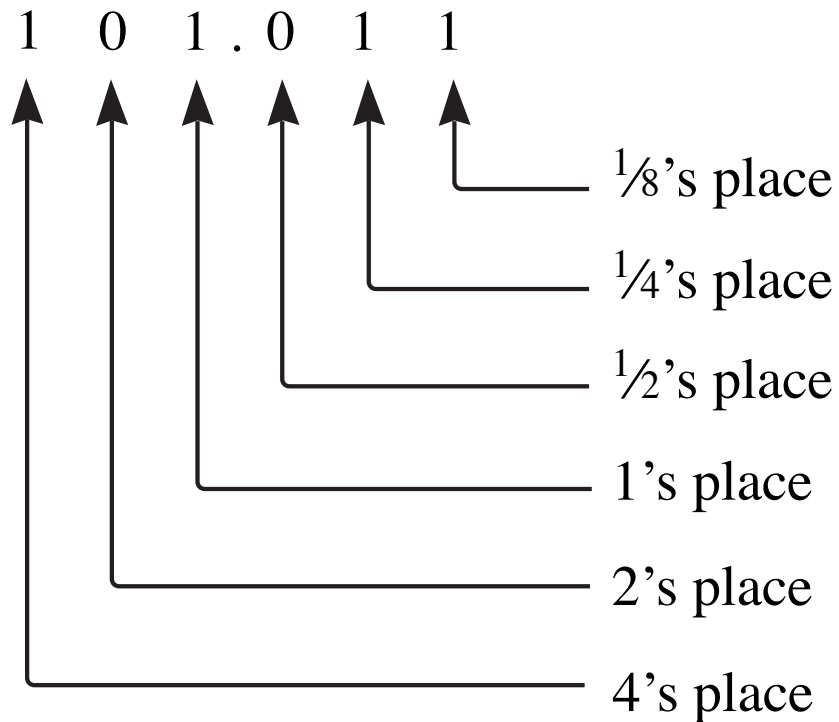
5.375 (dec)

(a) The place values for 101.011 (bin).

(b) Converting 101.011 (bin) to decimal.

Figure 3.26

$$1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$$

(a) The binary number 101.011.

$$5 \times 10^2 + 0 \times 10^1 + 6 \times 10^0 + 7 \times 10^{-1} + 2 \times 10^{-2} + 1 \times 10^{-3}$$
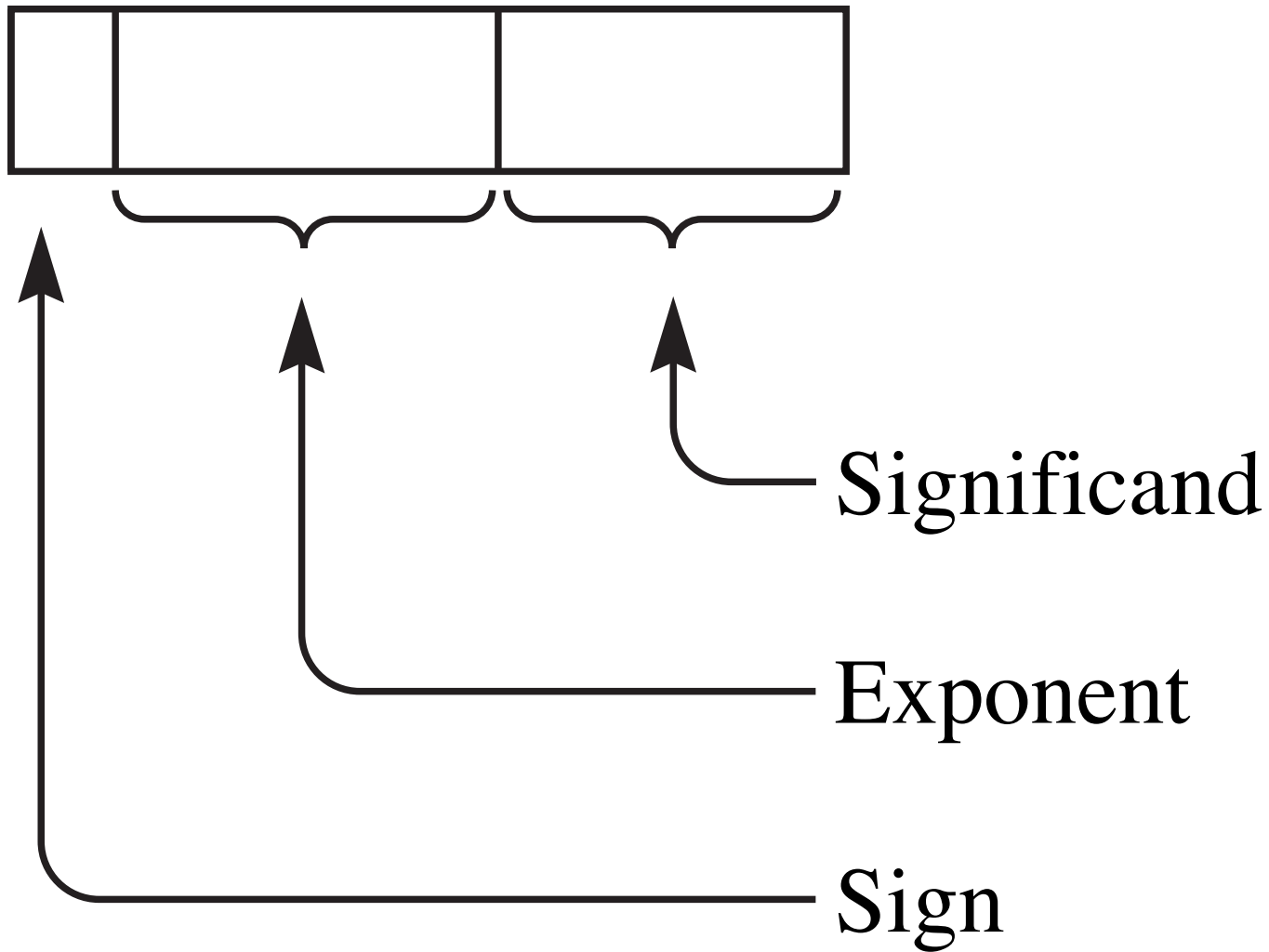
(b) The decimal number 506.721.

Figure 3.27

	.5859375
1	.171875
0	.34375
0	.6875
1	.375
0	.75
1	.5
1	.0

6.5859375

↑

6 (dec) = 110 (bin)

(a) Convert the whole part   (b) Convert the fractional part

Figure 3.28

$$
\begin{array}{c|c}
 & .2 \\
\hline
0 & .4 \\
0 & .8 \\
1 & .6 \\
1 & .2 \\
0 & .4 \\
0 & .8 \\
1 & .6 \\
\vdots & \vdots \\
\end{array}
$$

Figure 3.29

Significand

Exponent

Sign

Figure 3.30

Decimal	Excess 3	Two's Complement
−4		100
−3	000	101
−2	001	110
−1	010	111
0	011	000
1	100	001
2	101	010
3	110	011
4	111	

Special value

- Zero

  ▸ Exponent field all 0's

  ▸ Significand all 0's

  ▸ There is a +0 and a –0

Figure 3.31
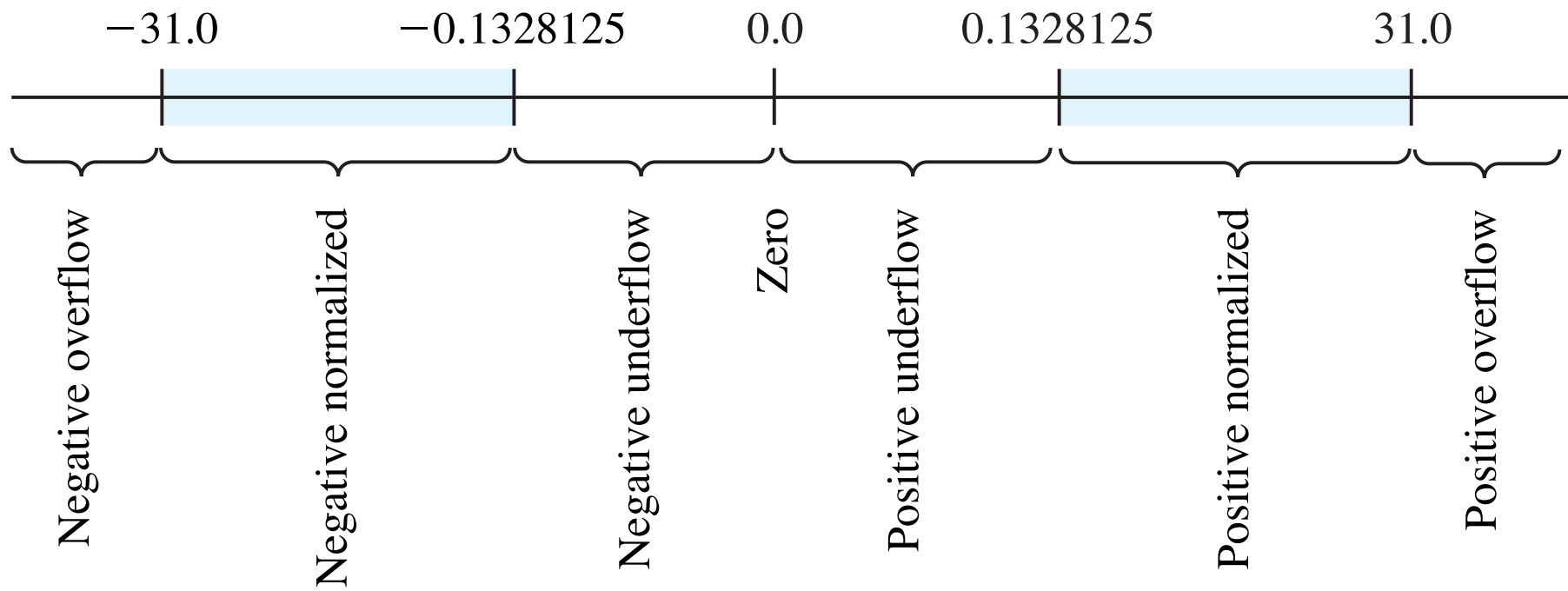
Special value

- Infinity

  ‣ Exponent field all 1's

  ‣ Significand all 0's

  ‣ There is a $+\infty$ and a $-\infty$

  ‣ Produced by operation that gives result in overflow region

Special value

- Not a Number (NaN)

  ‣ Exponent field all 1's

  ‣ Significand nonzero

  ‣ Produced by illegal math operations

Special value

- Denormalized number

  ‣ Exponent field all 0's

  ‣ Significand nonzero

  ‣ Hidden bit is assumed to be 0 instead of 1

  ‣ If the exponent is stored in excess n for normalized numbers, it is stored in excess $n - 1$ for denormalized numbers

Figure 3.32

	Binary	Scientific notation	Decimal
Not a Number	1 111 nonzero		
Negative infinity	1 111 0000		$-\infty$
Negative normalized	1 110 1111	-1.1111×2^3	-15.5
	1 110 1110	-1.1110×2^3	-15.0
	...	...	...
	1 011 0001	-1.0001×2^0	-1.0625
	1 011 0000	-1.0000×2^0	-1.0
	1 010 1111	-1.1111×2^{-1}	-0.96875
	...	...	...
	1 001 0001	-1.0001×2^{-2}	-0.265625
	1 001 0000	-1.0000×2^{-2}	-0.25
Negative denormalized	1 000 1111	-0.1111×2^{-2}	-0.234375
	1 000 1110	-0.1110×2^{-2}	-0.21875
	...	...	...
	1 000 0010	-0.0010×2^{-2}	-0.03125
	1 000 0001	-0.0001×2^{-2}	-0.015625
Negative zero	1 000 0000		-0.0
Positive zero	0 000 0000		$+0.0$

Figure 3.32
(Continued)

Negative zero	1 000 0000		-0.0
Positive zero	0 000 0000		$+0.0$
Positive denormalized	0 000 0001	0.0001×2^{-2}	0.015625
	0 000 0010	0.0010×2^{-2}	0.03125
	...	...	...
	0 000 1110	0.1110×2^{-2}	0.21875
	0 000 1111	0.1111×2^{-2}	0.234375
Positive normalized	0 001 0000	1.0000×2^{-2}	0.25
	0 001 0001	1.0001×2^{-2}	0.265625
	...	...	...
	0 010 1111	1.1111×2^{-1}	0.96875
	0 011 0000	1.0000×2^{0}	1.0
	0 011 0001	1.0001×2^{0}	1.0625
	...	...	...
	0 110 1110	1.1110×2^{3}	15.0
	0 110 1111	1.1111×2^{3}	15.5
Positive infinity	0 111 0000		$+\infty$
Not a Number	0 111 nonzero		

Figure 3.33

Bits   1   8   23

(a) Single precision

Bits   1   11   52
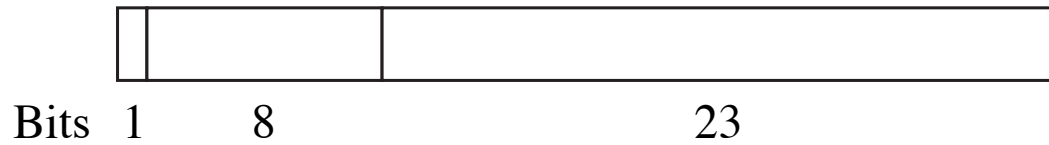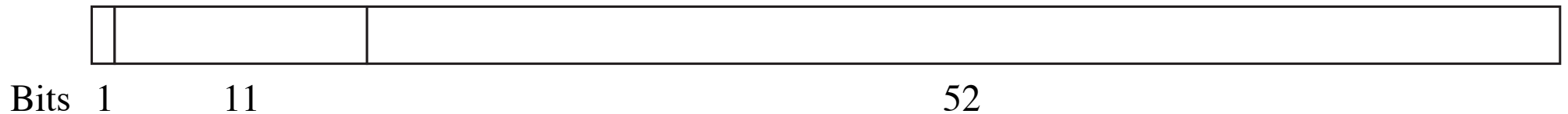
(b) Double precision

Figure 3.34

Figure 3.35
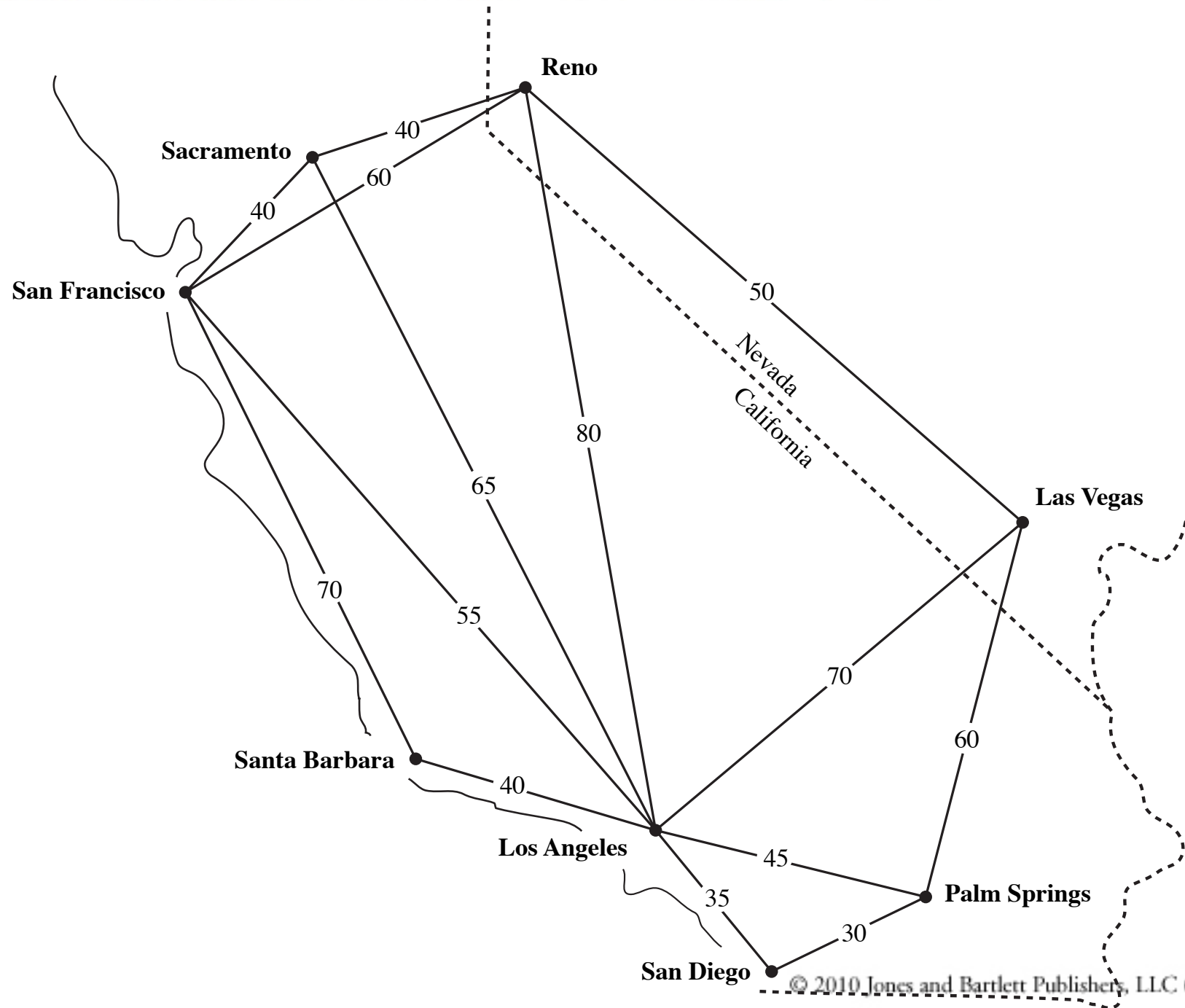
	0	1	2	3	4	5	6	7
0	1e30	35	45	40	55	65	80	70
1	35	1e30	30	1e30	1e30	1e30	1e30	1e30
2	45	30	1e30	1e30	1e30	1e30	1e30	60
3	40	1e30	1e30	1e30	70	1e30	1e30	1e30
4	55	1e30	1e30	70	1e30	40	60	1e30
5	65	1e30	1e30	1e30	40	1e30	40	1e30
6	80	1e30	1e30	1e30	60	40	1e30	50
7	70	1e30	60	1e30	1e30	1e30	50	1e30

City Numbers

0 Los Angeles   **4** San Francisco

1 San Diego   **5** Sacramento

2 Palm Springs   **6** Reno

3 Santa Barbara   **7** Las Vegas

Figure 3.36

Route number	From	To	Cost
0	0	1	35
1	0	2	45
2	0	3	40
3	0	4	55
4	0	5	65
5	0	6	80
6	0	7	70
7	1	2	30
8	2	7	60
9	3	4	70
10	4	5	40
11	4	6	60
12	5	6	40
13	6	7	50