Selective Laser Sintering 3D Printer

ELEC 341 Project 2017 - Part 1

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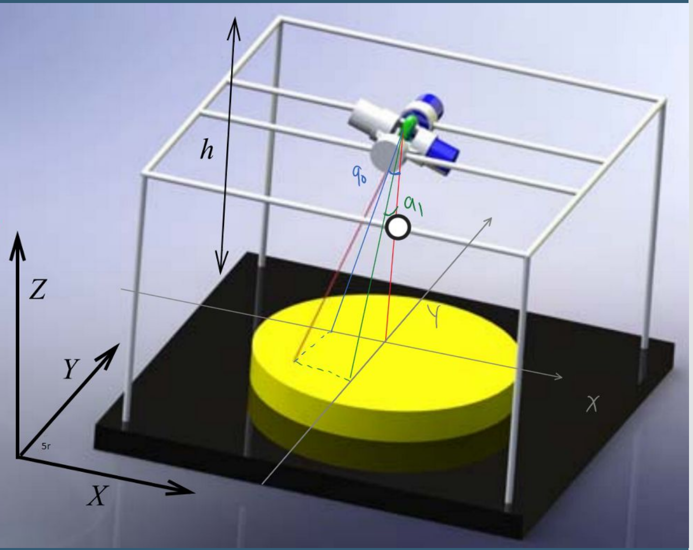
Inverse and Direct Kinematics

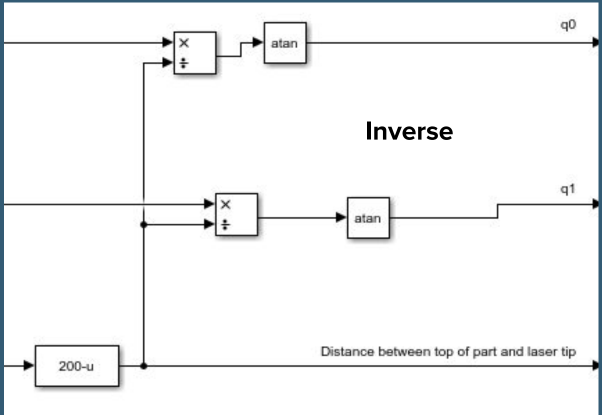


Z0: the current height of the object, which increases in 5mm increments.

Z: the distance between the between the laser and the point on the part that is being sintered.

Since the laser is located at [0,0,200mm]:

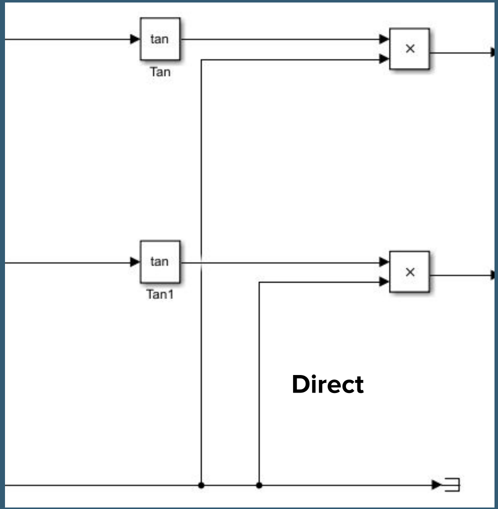
$$z = 200\text{mm} - z_0$$



Q0 rotates about the the y-axis and sweeps the x-axis, and Q1 rotates about the x-axis and sweeps the y-axis. So the angles are given by

$$\theta_0 = tan^{-1}\left(\frac{x}{z}\right)$$

$$\theta_1 = tan^{-1}\left(\frac{y}{z}\right)$$



For direct kinematics, the actual angles are converted into cartesian parameters (x and y coordinates).

$$x = ztan(\theta_0)$$

$$y = ztan(\theta_1)$$

Amplifier Dynamics

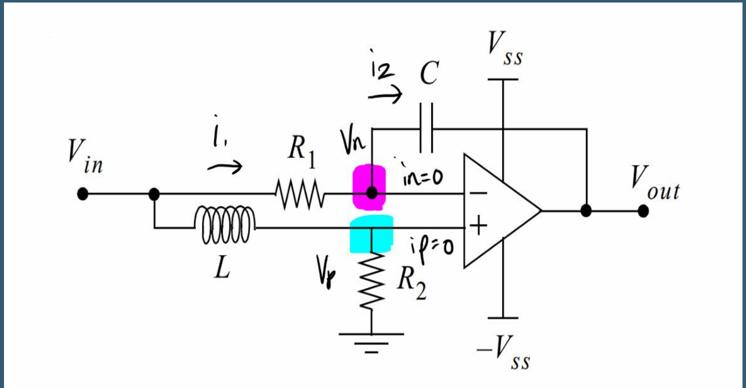
$$V_n = V_p \qquad i_n = i_p = 0 \qquad i_1 = i_2$$

$$i_1 = \frac{V_{in} - V_n}{R_1} \qquad i_2 = (V_n - V_{out})sC$$

$$V_n = V_{in}\frac{R_2}{R_2 + sL}$$

Using these equations, the transfer function of the amplifier can be determined.

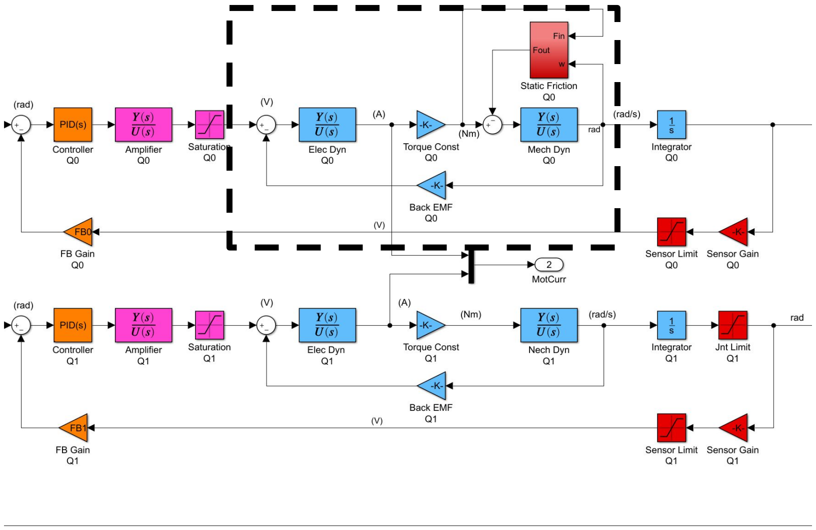
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{C_1R_1R_2 - L}{sLCR_1 + CR_1R_2}$$

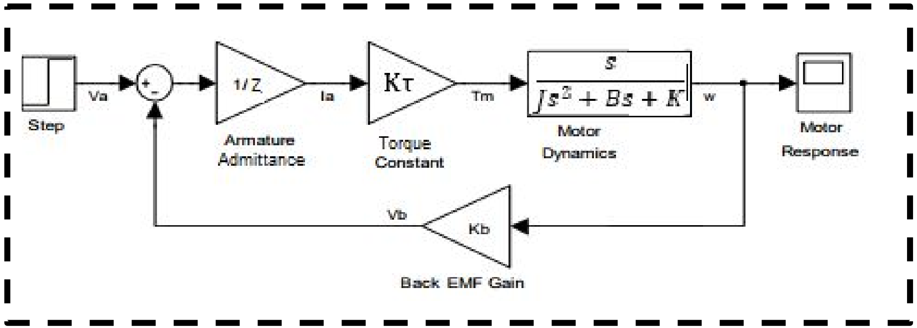


In an ideal operational amplifier, the currents going into the amplifier will be 0 and the voltages at both the non-inverting and inverting inputs will be the same.

The saturation voltage is the maximum voltage of the motor (the nominal voltage), so that the maximum is not exceeded.

$$V_{SS} = 12V$$



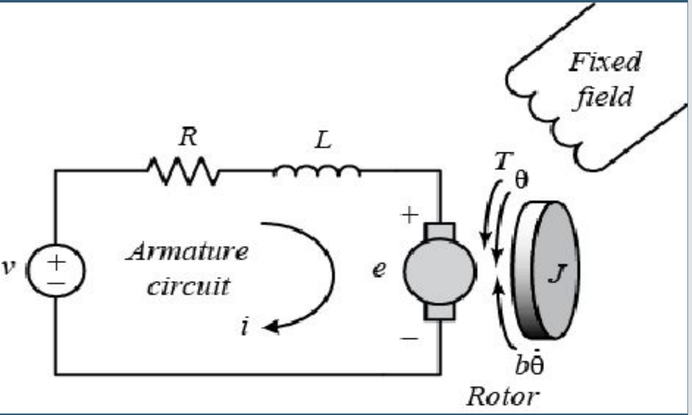


$$V \times \frac{1}{Z} \times K_\tau \times T(s) \times K_b = V$$

$$V \times \frac{1}{\Omega} \times \frac{\text{Nm}}{\text{A}} \times \frac{\text{rad/sec}}{\text{Nm}} \times \frac{\text{Vsec}}{\text{rad}} = V$$

Unit conversion validation

Motor Model



Electrical Motor Dynamics

$$Z_a = sL_a + R_a \longrightarrow Y_a = \frac{1}{sL_a + R_a}$$

The armature admittance is the electrical component of motor response. The motor's electrical characteristics can be modeled with an armature circuit, where the terminal impedance is represented as a resistor and inductor in series.

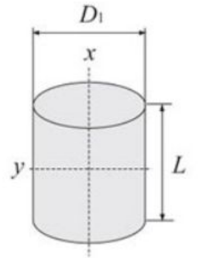
Back-EMF Constant

$$\text{Back-EMF constant}: \ K_b = \frac{1}{K_v}$$

Kv is the speed constant of a motor. It describes the relationship between RPM and Volts for a motor, and has units of rpm/V (or rad/Vs). Kb is the back-EMF constant, and is the inverse of Kv.

$$Jx = \frac{1}{8} m D_1^2 = \frac{\pi}{32} \rho L D_1^4$$

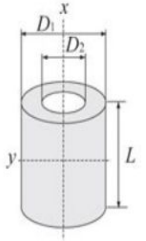
$$Jy = \frac{1}{4} m \left(\frac{D_1^2}{4} + \frac{L^2}{3} \right)$$



Inertia of cylinder

$$Jx = \frac{1}{8} m (D_1^2 + D_2^2) = \frac{\pi}{32} \rho L (D_1^4 - D_2^4)$$

$$Jy = \frac{1}{4} m \left(\frac{D_1^2 + D_2^2}{4} + \frac{L^2}{3} \right)$$



Inertia of hollow cylinder (ring)

Moment of Inertia (J)

Motor Q0

$$J_{Motor_0} = J_{Motor'_1} + J_{Ring} + J_{rot_0}$$
$$= 1.58 \times 10^{-7} + 7.67 \times 10^{-5} + 4.36 \times 10^{-7}$$
$$= 8 \times 10^{-5} Nms^2$$

Note: rotational inertia is given

$$J_{Motor'_1} = \sqrt{J_x^2 + J_y^2} = J_y = J_{2Length+2Link_{off}} - J_{2Link_{off}}$$
$$= 2(m_1 \frac{(Length + Link_{off})^2}{3} - m_2 \frac{Link_{off}^2}{3})$$
$$= 2(J_{Length+Link_{off}} - J_{Link_{off}})$$

Inertia of motor Q1 due to motor Q0

$$m_2 = m_{Link_{off}} = \frac{Link_{Off}}{Length} \times Weight = 1.52 \times 10^{-2} kg$$
$$m_1 = m_{Length+Link_{off}} = \frac{Length + Link_{off}}{Length} \times Weight = 6.92 \times 10^{-2} kg$$
$$J_{Link_{off}} = \frac{Link_{off}^2}{3} \times m_2$$
$$J_{Length+Link_{off}} = \frac{(Length + Link_{off})^2}{3} \times m_1$$
$$J_{Motor'_1} = 2(J_{Length+Link_{off}} - J_{Link_{off}}) = 7.67 \times 10^{-5} kgm^2 = 7.67 \times 10^{-5} Nms^2$$

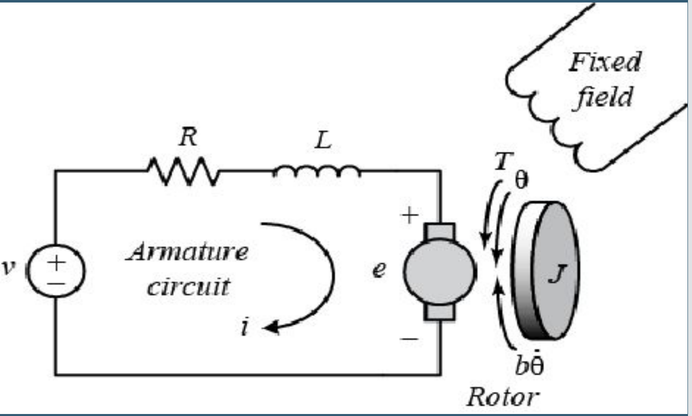
$$m_{Ring} = \rho \times V_{Ring} = RhoAl \times \pi \times (LinkR2^2 - LinkR1^2) \times LinkD = 9.665 \times 10^{-3} kg$$
$$J_{Ring} = \sqrt{J_x^2 + J_y^2} = J_y$$
$$= \frac{1}{4} \times m_{Ring} (\frac{(LinkR2^2 - LinkR1^2)}{4} + \frac{(LinkD^2)}{3}) = 1.58 \times 10^{-7} kgm^2 = 1.58 \times 10^{-7} Nms^2$$

Inertia of the ring

Motor Q1

$$J_{Motor_1} = J_{rot_1} = 4.36 \times 10^{-7} Nms^2$$

Inertia of motor 1 is its rotational inertia (given)



Viscous Damping Coefficient (B)

Viscous damping is the mechanical rotational equivalent of conductance (B=1/R) as it impedes rotation. The higher the damping, the higher the applied torque must be to produce the same rotation, and therefore speed.

The damping describes the relationship between the change in speed and torque, which is the speed-torque gradient.

$$\text{Speed-torque gradient} = \frac{\Delta\text{speed}}{\Delta\text{torque}}$$

$$B = \frac{1}{\text{Speed-torque gradient}}$$

Dynamic Spring Constant (K)

$$K = 7\text{mNm/rev}$$

The spring constant of the motor is dependent on the material of the motor. The spring characteristics of the motor contribute minimally to the dynamic friction.

Transfer Functions

Mechanical Motor Dynamics

Q0

$$\frac{12644s}{s^2 + 0.2799s + 14.11}$$

Q1

$$\frac{2.396 \times 10^6}{s + 50.7s}$$

Amplifier

$$\frac{49.138}{s + 49.17}$$

The amplifier transfer function is the same for both motors.

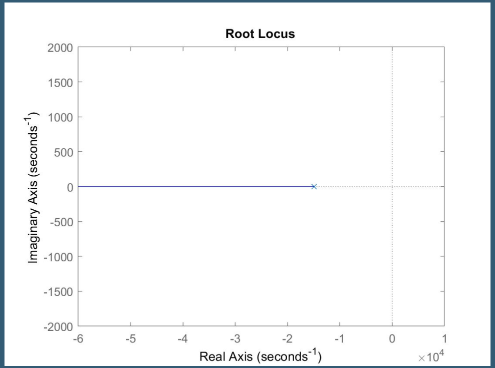
Electrical Motor Dynamics

$$\frac{2762.4}{s + 1.489 \times 10^4}$$

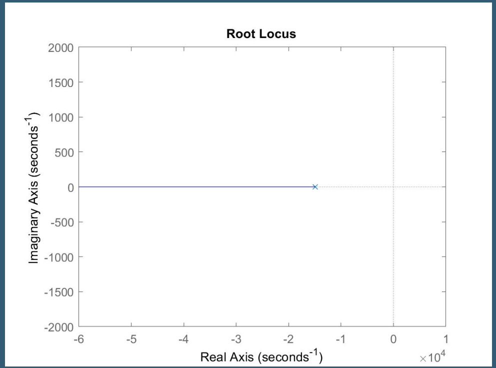
The same motor model is used for both Q0 and Q1.

Root Locus

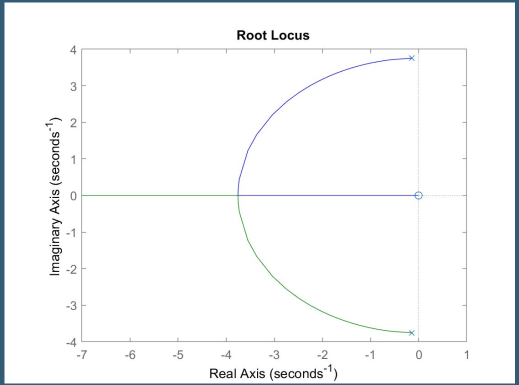
Amplifier



Electrical Motor Dynamics



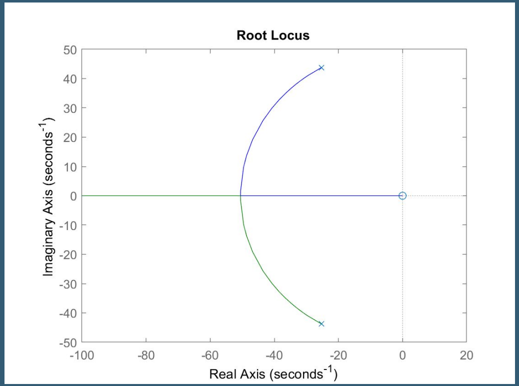
Q0



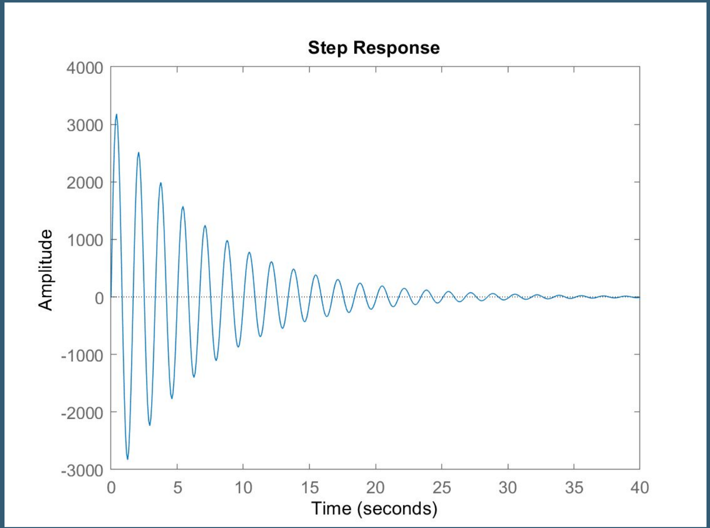
The differences in the Q0 and Q1 root locus are due to the difference in the inertia of the motors.

Since the poles of Q0 are further towards the right, the damping constant will be smaller than that of Q1. It is expected that Q0 will have a much longer settle time than Q1.

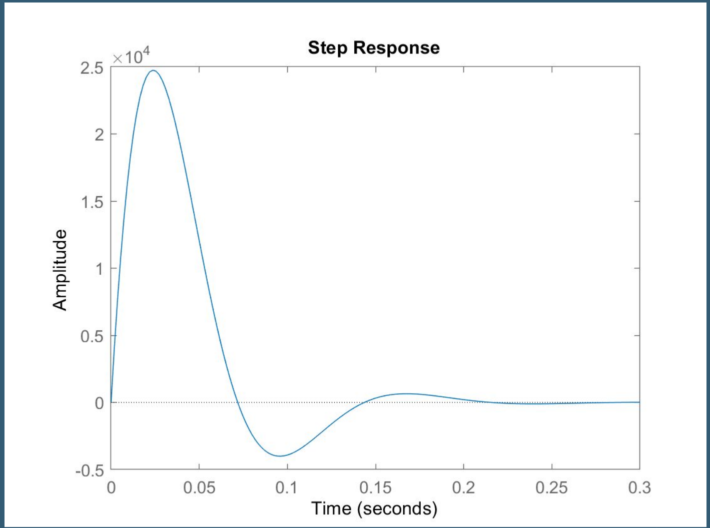
Q1



Step Response



Q0



Q1

As indicated by the root locus on the previous slide, Q0 has a longer settle time than Q1 as well as a higher peak amplitude.