## Notes on Pset 1 #2

## S&DS 361

## 2024-01-25

There was some confusion, and a typo by me, for #2, so here are some notes. First let's load the data.

```
d = readRDS('data/water.usage.rds')
head(d)
```

```
## # A tibble: 6 x 5
##
     GEOID crop
                      src
                             year value
##
     <int> <chr>
                      <chr> <chr> <dbl>
## 1 1001 barley
                      gwd
                             2008
## 2 1001 corn
                             2008
                      gwd
## 3 1001 cotton
                             2008
                                       0
                       gwd
## 4 1001 millet
                       gwd
                             2008
                                       0
## 5 1001 oats
                             2008
                                       0
                       gwd
## 6 1001 other_sctg2 gwd
                             2008
                                       0
```

We want average water usage by year for each crop and source. The first incorrect thing we did is this

```
dd1 = d %>%
  group_by(crop, src, year) %>%
  summarise(value = mean(value))

## 'summarise()' has grouped output by 'crop', 'src'. You can override using the
## '.groups' argument.
```

```
## # A tibble: 780 x 4
## # Groups:
              crop, src [60]
##
                           value
     crop
           src
                  year
##
     <chr> <chr> <chr>
                           <dbl>
##
   1 barley gwa
                  2008 0.000376
  2 barley gwa
                  2009
                        0.000368
                  2010
                        0.000357
  3 barley gwa
   4 barley gwa
                  2011
                        0.000476
                  2012 0.000455
  5 barley gwa
   6 barley gwa
                  2013
                        0.000395
  7 barley gwa
                  2014 0.000266
##
   8 barley gwa
                  2015 0.000428
## 9 barley gwa
                  2016 0.000347
## 10 barley gwa
                  2017 0.000359
## # i 770 more rows
```

```
dd2 = dd1 \%
  group_by(crop, src) %>%
 summarise(value = mean(value))
## 'summarise()' has grouped output by 'crop'. You can override using the
## '.groups' argument.
dd2
## # A tibble: 60 x 3
## # Groups: crop [20]
     crop src
     <chr> <chr>
##
                     <dbl>
## 1 barley gwa 0.000372
## 2 barley gwd 0.000222
                0.000688
## 3 barley sw
## 4 corn gwa 0.00176
## 5 corn gwd 0.00110
## 6 corn sw
                0.00171
## 7 cotton gwa 0.000623
## 8 cotton gwd
                0.000442
## 9 cotton sw
                 0.000517
## 10 millet gwa
                0.0000231
## # i 50 more rows
I meant to say sum here. What I meant was
dd3 = d \%
 group_by(crop, src, year) %>%
summarise(value = sum(value)) ## sum not mean
## 'summarise()' has grouped output by 'crop', 'src'. You can override using the
## '.groups' argument.
dd3
## # A tibble: 780 x 4
## # Groups: crop, src [60]
##
     crop src
                year value
##
     <chr> <chr> <chr> <dbl>
## 1 barley gwa 2008 1.21
## 2 barley gwa
                2009 1.19
                2010 1.11
## 3 barley gwa
## 4 barley gwa
                2011 1.53
## 5 barley gwa 2012 1.46
## 6 barley gwa
                2013 1.27
## 7 barley gwa
                2014 0.857
## 8 barley gwa
                2015 1.33
## 9 barley gwa
                 2016 1.12
                 2017 1.16
## 10 barley gwa
## # i 770 more rows
```

```
dd4 = dd3 %>%
  group_by(crop, src) %>%
  mutate(mean = mean(value))

dd4
```

```
## # A tibble: 780 x 5
## # Groups:
               crop, src [60]
                   year
##
      crop
             src
                        value
                                mean
##
      <chr> <chr> <chr> <dbl> <dbl>
   1 barley gwa
##
                   2008
                         1.21
                                1.19
##
   2 barley gwa
                   2009
                         1.19
                                1.19
##
                   2010 1.11
   3 barley gwa
                                1.19
##
   4 barley gwa
                   2011
                        1.53
                                1.19
##
   5 barley gwa
                   2012 1.46
                                1.19
##
   6 barley gwa
                   2013
                         1.27
                                1.19
##
                   2014
                         0.857 1.19
   7 barley gwa
   8 barley gwa
                   2015
                        1.33
                                1.19
                   2016 1.12
                                1.19
   9 barley gwa
## 10 barley gwa
                   2017
                        1.16
                                1.19
## # i 770 more rows
```

1 barley gwa

0.000372

The data frame dd3 has the total water usage for each crop, source, and year, for all census tracts in the US, and dd4 is the average annual water usage for each crop and source. For example, for barley and gwa, the average annual water usage in the US is 1.19 km<sup>3</sup>. These numbers match the table in the article, except that the surface water numbers are slightly off for some reason.

I'll leave it to you to compute the change and percent change. Hint: check out pivot\_wider, the inverse of our friend pivot\_longer.

## The two means we computed in class are not equal

I wanted to return to the (incorrect) code we were working on in class, where many people thought that two different ways of computing the mean that we tried should have been equal. Note that dd2, the mean across years of the values in dd1, is **not** exactly the same as simply finding the mean of the original data frame with all location-year combinations:

```
dd5 = d %>%
  group_by(crop, src) %>%
  summarise(value = mean(value)) %>%
  ungroup()

## 'summarise()' has grouped output by 'crop'. You can override using the
## '.groups' argument.

dd5

## # A tibble: 60 x 3
## crop src value
## <chr> <chr> <dbl>
```

```
## 4 corn
            gwa
                  0.00176
## 5 corn
                  0.00110
            gwd
##
   6 corn
            sw
                  0.00172
  7 cotton gwa
                  0.000623
##
                  0.000442
  8 cotton gwd
## 9 cotton sw
                  0.000517
## 10 millet gwa
                  0.0000231
## # i 50 more rows
dd5 %>%
  left_join(dd2,
            by = c('crop', 'src'),
            suffix = c('.dd5', '.dd2')) %>%
 mutate(diff = value.dd5 - value.dd2) ## difference is not 0
## # A tibble: 60 x 5
##
                  value.dd5 value.dd2
                                               diff
      crop
            src
##
                       <dbl>
                                 <dbl>
                                              <dbl>
      <chr> <chr>
##
                  0.000372 0.000372 -0.000000118
   1 barley gwa
##
   2 barley gwd
                  0.000222 0.000222 -0.000000293
                  0.000688 0.000688 -0.000000603
  3 barley sw
## 4 corn
                  0.00176
                            0.00176
                                       0.00000175
            gwa
                            0.00110
                                       0.0000133
## 5 corn
                  0.00110
            gwd
## 6 corn
                  0.00172
                            0.00171
                                       0.00000117
            SW
                  ## 7 cotton gwa
                  0.000442 0.000442
                                       0.000000299
## 8 cotton gwd
## 9 cotton sw
                  0.000517 0.000517
                                       0.000000906
                  0.0000231 0.0000231 0.0000000173
## 10 millet gwa
## # i 50 more rows
The differences are small but not zero. To see why this is, let's simplify let's look at just barley and gwa
and find the yearly averages. Let's also include a column showing the number of locations.
dd6 = d \%
  filter(crop == 'barley',
         src == 'gwa') %>%
  group_by(crop, src, year) %>%
  summarise(mean = mean(value),
            count = n(),
            sum = sum(value))
## 'summarise()' has grouped output by 'crop', 'src'. You can override using the
## '.groups' argument.
dd6
## # A tibble: 13 x 6
## # Groups:
               crop, src [1]
##
           src
      crop
                 year
                            mean count
      <chr> <chr> <chr>
                           <dbl> <int> <dbl>
##
```

2 barley gwd

## 3 barley sw

##

0.000222

0.000688

```
1 barley gwa
                  2008
                        0.000376 3222 1.21
##
                        0.000368 3222 1.19
##
   2 barley gwa
                  2009
  3 barley gwa
                  2010
                        0.000357 3108 1.11
                  2011
                        0.000476 3222 1.53
##
  4 barley gwa
##
  5 barley gwa
                  2012
                        0.000455
                                 3222 1.46
                  2013
                        0.000395 3222 1.27
##
  6 barley gwa
                        0.000266 3222 0.857
##
   7 barley gwa
                  2014
##
   8 barley gwa
                  2015
                        0.000428 3108 1.33
## 9 barley gwa
                  2016
                        0.000347
                                  3223 1.12
## 10 barley gwa
                  2017
                        0.000359 3223 1.16
## 11 barley gwa
                  2018
                        0.000334 3223 1.08
                        0.000277
                                  3223 0.892
## 12 barley gwa
                  2019
## 13 barley gwa
                  2020
                        0.000396 3223 1.28
```

Note that the count column is not the same for every year. Uh oh. So averaging over locations first, and then averaging over years, is not the same as the average over all location-year combinations:

```
mean(dd6$mean) ## this weights each year the same
```

## [1] 0.0003718526

## [1] 0.0003717348

We would have to use a weighted mean to get the same result

```
weighted.mean(dd6$mean, w = dd6$count) ## this weights each location-year the same
```

## [1] 0.0003717348

```
sum(dd6$sum)/sum(dd6$count) ## same as above
```

```
## [1] 0.0003717348
```

Let's use mathematical notation. We'll again limit ourselves to barley and gwa for simplicity. Let  $x_{jk}$  be the amount of groundwater gwa used by barley at location j in year k. Let  $n_k$  be the number of locations for which there is data in year k. Note that there are 13 years.

```
length(unique(d$year ))
```

## [1] 13

Then the yearly average water usage in year k is

$$\frac{1}{n_k} \sum_{j=1}^{n_k} x_{jk}$$

The average of that across years is

$$\frac{1}{13} \sum_{k=2008}^{2020} \frac{1}{n_k} \sum_{j=1}^{n_k} x_{jk}$$

Since  $n_k$  depends on k, we can't pull it out of the sum.