# Assignment 1

**AI1110**: Probability and Random Variables

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**A** person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\left(\frac{1}{100}\right)$. What is the probability that he will win a prize (a) at least once (b) at least twice ?

## Answer:

a. $1-\left(\frac{99}{100}\right)^{50}$

b. $1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$

**Solution**: X$\in \{0, 1, 2, 3..., ., 50\}$ .
We will solve this using CDF.
The trials are Bernoulli trials.
X has a binomial distribution with n=50 and p=$\frac{1}{100}$

$$\therefore q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$$
$$\therefore \Pr(X = x) = {}^{n}C_x q^{n-x} p^x = {}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \left(\frac{1}{100}\right)^x \tag{1}$$

(a) $\Pr(X \geq 1) = 1 - F_X(0) = 1 - \Pr(X \leq 0)$

$$\begin{aligned} &= 1 - \Pr(X = 0) \\ &= 1 - {}^{50}C_x \left(\frac{99}{100}\right)^{50} \\ &= 1 - \left(\frac{99}{100}\right)^{50} \end{aligned} \tag{2}$$

(b) $\Pr(X \geq 2) = 1 - F_X(1) = 1 - \Pr(X \leq 1)$

$$\begin{aligned} &= 1 - \Pr(X \leq 1) \\ &= 1 - [\Pr(X = 0) + \Pr(X = 1)] \\ &= 1 - \Pr(X = 0) - \Pr(X = 1) \\ &= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2}\left(\frac{99}{100}\right)^{49} \\ &= 1 - \left(\frac{99}{100}\right)^{49}\left[\frac{99}{100} + \frac{1}{2}\right] \\ &= 1 - \frac{149}{100}\left(\frac{99}{100}\right)^{49} \end{aligned} \tag{3}$$