Assignment 1

AI1110: Probability and Random Variables

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12.13.5.10: Question.
A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\left(\frac{1}{100}\right)$. What is the probability that he will win a prize (a) at least once (b) at least twice ?

Answer:

a. $1-\left(\frac{99}{100}\right)^{50}$

b. $1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$

Solution: Let X represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials. Clearly, X has a binomial distribution with n=50 and p=$\frac{1}{100}$

$$\begin{aligned} &\therefore q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100} \\ &\therefore \Pr(X = x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{50}C_{x} \left(\frac{99}{100}\right)^{50-x} \left(\frac{1}{100}\right)^{x} \end{aligned} \tag{1}$$

(a) $\Pr(winingatleastonce) = \Pr(X \geq 1)$

$$\begin{aligned} &= 1 - \Pr(X < 1) \\ &= 1 - \Pr(X = 1) \\ &= 1 - {}^{50}C_{x} \left(\frac{99}{100}\right)^{50} \\ &= 1 - \left(\frac{99}{100}\right)^{50} \end{aligned} \tag{2}$$

(b) $\Pr(atleasttwice) = \Pr(X \geq 2)$

$$\begin{aligned} &= 1 - \Pr(X < 2) \\ &= 1 - \Pr(X \leq 1) \\ &= 1 - [\Pr(X = 0) + \Pr(X = 1)] \\ &= 1 - \Pr(X = 0) - \Pr(X = 1) \\ &= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2}\left(\frac{99}{100}\right)^{49} \\ &= 1 - \left(\frac{99}{100}\right)^{49} \left[\frac{99}{100} + \frac{1}{2}\right] \\ &= 1 - \frac{149}{100}\left(\frac{99}{100}\right)^{49} \end{aligned} \tag{3}$$