

Assignment 1

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AI1110: Probability and Random Variables

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A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\left(\frac{1}{100}\right)$. What is the probability that he will win a prize (a) at least once (b) at least twice?

Answer:

a. $1 - \left(\frac{99}{100}\right)^{50}$

b. $1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$

Solution: $X \in \{0, 1, 2, 3, \dots, 50\}$.

We will solve this using CDF.

The trials are Bernoulli trials.

X has a binomial distribution with $n=50$ and $p=\frac{1}{100}$

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Parameter	Value	Description
n	50	no of lotteries
X	0,1	random variable
p	1/100	Probability of winning a prize
q	99/100	Probability of losing a prize

TABLE 0

$$\begin{aligned}\therefore q &= 1 - p = 1 - \frac{1}{100} = \frac{99}{100} \\ \therefore \Pr(X = x) &= {}^n C_x q^{n-x} p^x = {}^{50} C_x \left(\frac{99}{100}\right)^{50-x} \left(\frac{1}{100}\right)^x\end{aligned}\tag{1}$$

$$\begin{aligned}\text{(a) } \Pr(X \geq 1) &= 1 - F_X(0) = 1 - \Pr(X \leq 0) \\ &= 1 - \Pr(X = 0) \\ &= 1 - {}^{50} C_0 \left(\frac{99}{100}\right)^{50} \\ &= 1 - \left(\frac{99}{100}\right)^{50}\end{aligned}\tag{2}$$

$$(b) \Pr(X \geq 2) = 1 - F_X(1) = 1 - \Pr(X \leq 1)$$

$$= 1 - \Pr(X \leq 1)$$

$$= 1 - [\Pr(X = 0) + \Pr(X = 1)]$$

$$= 1 - \Pr(X = 0) - \Pr(X = 1)$$

$$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2}\left(\frac{99}{100}\right)^{49} \tag{3}$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \left[\frac{99}{100} + \frac{1}{2} \right]$$

$$= 1 - \frac{149}{100} \left(\frac{99}{100}\right)^{49}$$