

Assignment 1

AI1110: Probability and Random Variables

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A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\left(\frac{1}{100}\right)$. What is the probability that he will win a prize (a) at least once (b) at least twice ?

Answer:

a. $1 - \left(\frac{99}{100}\right)^{50}$

b. $1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$

Solution: $X \in \{0, 1, 2, 3, \dots, 50\}$.

We will solve this using CDF.

The trials are Bernoulli trials.

X has a binomial distribution with $n=50$ and $p=\frac{1}{100}$

$$\begin{aligned}
 &= 1 - \Pr(X \leq 1) \\
 &= 1 - [\Pr(X = 0) + \Pr(X = 1)] \\
 &= 1 - \Pr(X = 0) - \Pr(X = 1) \\
 &= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2}\left(\frac{99}{100}\right)^{49} \\
 &= 1 - \left(\frac{99}{100}\right)^{49} \left[\frac{99}{100} + \frac{1}{2} \right] \\
 &= 1 - \frac{149}{100}\left(\frac{99}{100}\right)^{49}
 \end{aligned} \tag{3}$$

Parameter	Value	Description
n	50	no of lotteries
x	0,1	random variable
p	$\frac{1}{100}$	Probability of winning a prize
q	$\frac{99}{100}$	Probability of losing a prize

$$\therefore q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$$

$$\therefore \Pr(X = x) = {}^n C_x q^{n-x} p^x = {}^{50} C_x \left(\frac{99}{100}\right)^{50-x} \left(\frac{1}{100}\right)^x \tag{1}$$

$$(a) \Pr(X \geq 1) = 1 - F_X(0) = 1 - \Pr(X \leq 0)$$

$$\begin{aligned}
 &= 1 - \Pr(X = 0) \\
 &= 1 - {}^{50} C_0 \left(\frac{99}{100}\right)^{50} \\
 &= 1 - \left(\frac{99}{100}\right)^{50}
 \end{aligned} \tag{2}$$

$$(b) \Pr(X \geq 2) = 1 - F_X(1) = 1 - \Pr(X \leq 1)$$