TDT4171 Assignment 1

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1. A_i = person has i siblings.

$P(A_0) = 0.15$

$P(A_1) = 0.49$

$P(A_2) = 0.27$

$P(A_3) = 0.06$

$P(A_4) = 0.02$

$P(A_5) = 0.01$

a) $P(A_{\leq 2}) = P(A_0 \vee A_1 \vee A_2) = P(A_0) + P(A_1) + P(A_2) = \underline{0.91}$

b) $P(A_{>2} \mid A_{\geq 1}) = \dfrac{P(A_{>2} \cap A_{\geq 1})}{P(A_{\geq 1})}$

$$= \frac{P(A_{>2})}{P(A_{\geq 1})} = \frac{1 - P(A_{\leq 2})}{P(A_{\geq 1})} = \frac{1 - 0.91}{0.85} = \underline{\underline{0.11}}$$

c)

1	2	3	Σ
1	1	1	3
0	2	1	3
2	0	1	3
2	1	0	3
0	1	2	3
1	2	0	3
1	0	2	3
0	0	3	3
0	3	0	3
3	0	0	3

$$\Rightarrow P(3 \text{ combined}) = P(A_1)^3 + 6 \cdot P(A_0 \wedge A_1 \wedge A_2) + 3P(A_0 \wedge A_0 \wedge A_3)$$

$$= P(A_1)^3 + 6 \cdot P(A_0) \cdot P(A_1) \cdot P(A_2) + 3 \cdot P(A_0)^2 \cdot P(A_3)$$

$$= \underline{\underline{0.24}}$$

d) Let E = Emmas # of siblings
and J = Jacobs # of siblings

$P(E+J=3)$:

E	J	Σ
1	2	3
2	1	3
0	3	3
3	0	3

$$\Rightarrow P(E+J=3) = 2 \cdot P(A_1) \cdot P(A_2) + 2 \cdot P(A_0) \cdot P(A_3)$$

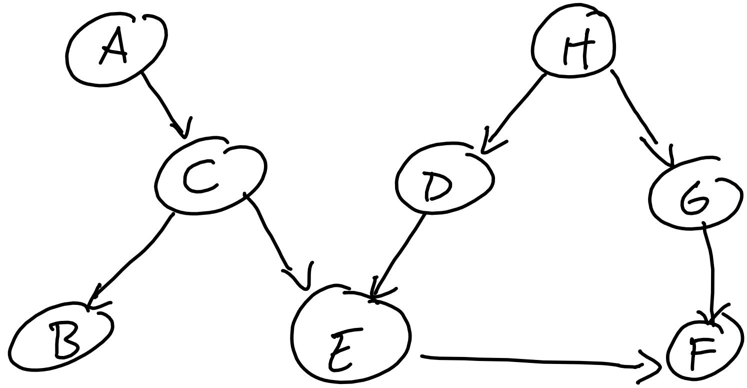
$$= 2 \cdot 0.49 \cdot 0.27 + 2 \cdot 0.15 \cdot 0.06$$

$$= 0.28$$

$$P(E=0 \mid J+E=3) = \frac{P(J=3 \wedge E=0)}{P(E+J=3)}$$

$$= \frac{0.06 \cdot 0.15}{0.28} = \underline{\underline{0.03}}$$

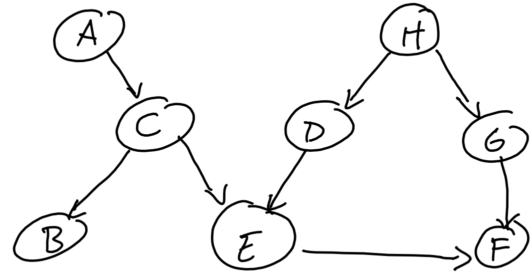
2.



a)

Variable	# needed to represent
A	1
B	2
C	2
D	2
E	4
F	4
G	2
H	1
Σ	18

(Need 2^k numbers for each node, where k is the number of parents). Checks out!



b) $G \perp\!\!\!\perp A$ is true due to neither being descendants of one another.

c) $E \perp\!\!\!\perp H \mid \{D, G\}$?

Yes, this is true. As long as we know D, E becomes indep of H.

d) $E \perp\!\!\!\perp H \mid \{C, D, F\}$?

No, this is not true. Knowing F makes E dependent on H again, via it being the grandparent of E's child F.

3.

a	$\neg a$
0.8	0.2

$$\begin{tikzcd} & & D \\ A \arrow[r] & B \arrow[ur] \arrow[dr] & \\ & & C \end{tikzcd}$$

	d	$\neg d$
b	0.6	0.4
$\neg b$	0.8	0.2

	b	$\neg b$
a	0.5	0.5
$\neg a$	0.2	0.8

	c	$\neg c$
b	0.1	0.9
$\neg b$	0.3	0.7

a) $p(b) = p(a) \cdot p(b|a) + p(\neg a) \cdot p(b|\neg a)$

$= 0.8 \cdot 0.5 + 0.2 \cdot 0.2 = \underline{\underline{0.44}}$

b) $p(d) = p(b) \cdot p(d|b) + p(\neg b) \cdot p(d|\neg b)$

$p(\neg b) = 1 - p(b) = \underline{0.56}$

$\Rightarrow p(d) = 0.44 \cdot 0.6 + 0.56 \cdot 0.8 \approx \underline{\underline{0.71}}$

c) $p(c|\neg d)$: we get "new probabilities" for b:

$$p(b|\neg d) = \frac{p(\neg d|b)\cdot p(b)}{p(\neg d)} = \frac{0.4\cdot 0.44}{1-0.71} \approx \underline{0.61}$$

$$p(\neg b|\neg d) = \frac{p(\neg d|\neg b)\cdot p(\neg b)}{p(\neg d)} = \frac{0.2\cdot 0.56}{1-0.71} \approx \underline{0.39}$$

$$p(c|\neg d) = p(b|\neg d)\cdot p(c|b) + p(\neg b|\neg d)\cdot p(c|\neg b)$$

$$= 0.61\cdot 0.1 + 0.39\cdot 0.3$$

$$= \underline{\underline{0.18}}$$

$\Rightarrow$ knowing $\neg d$ makes the probability of c a bit smaller

(originally it is

$$p(c) = p(b)\cdot p(c|b) + p(\neg b)\, p(c|\neg b)$$

$$= 0.44\cdot 0.1 + 0.56\cdot 0.3 = 0.21\ )$$

d) $P(a \mid \neg c, d) = \dfrac{P(a, \neg c, d)}{P(\neg c, d)}$

$$P(\neg c, d) = P(b) \cdot P(\neg c \mid b) \cdot P(d \mid b)$$
$$+ P(\neg b) \cdot P(\neg c \mid \neg b) \cdot P(d \mid \neg b)$$

$$= 0.44 \cdot 0.9 \cdot 0.6$$
$$+ 0.56 \cdot 0.7 \cdot 0.8 \approx \underline{0.55}$$

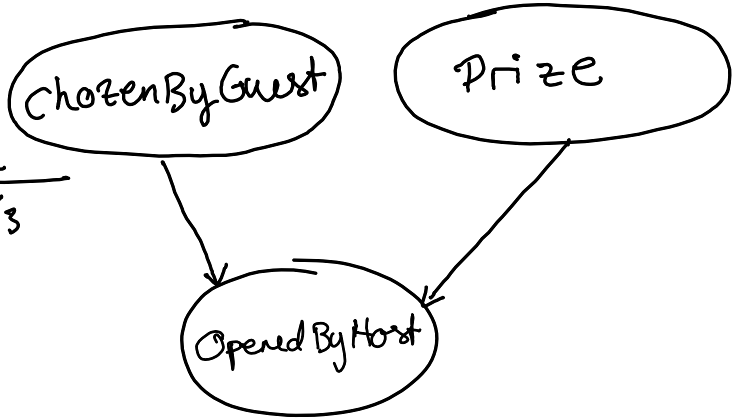
$$P(a, \neg c, d) = P(a) \cdot P(b \mid a) \cdot P(\neg c \mid b) \cdot P(d \mid b)$$
$$+ P(a) \cdot P(\neg b \mid a) \cdot P(\neg c \mid \neg b) \cdot P(d \mid \neg b)$$

$$= 0.8 \cdot 0.5 \cdot 0.9 \cdot 0.6$$
$$+ 0.8 \cdot 0.5 \cdot 0.7 \cdot 0.8 = 0.44$$

$$\Rightarrow P(a \mid \neg c, d) = \frac{0.44}{0.55} \approx \underline{\underline{0.8}}$$

4.c)

A	B	C
1/3	1/3	1/3



A	B	C
1/3	1/3	1/3

ChozenByGuest		A	A	A	B	B	B	C	C	C
Prize		A	B	C	A	B	C	A	B	C
OpenedByHost	A	0	0	0	0	0.5	1	0	1	0.5
	B	0.5	0	1	0	0	0	1	0	0.5
	C	0.5	1	0	1	0.5	0	0	0	0

From the code, we get

P(Prize | ChosenByGuest = 1, OpenedByHost = 3)

=

A	0
B	0,333
C	0,667