Modeling and Simulating Dice

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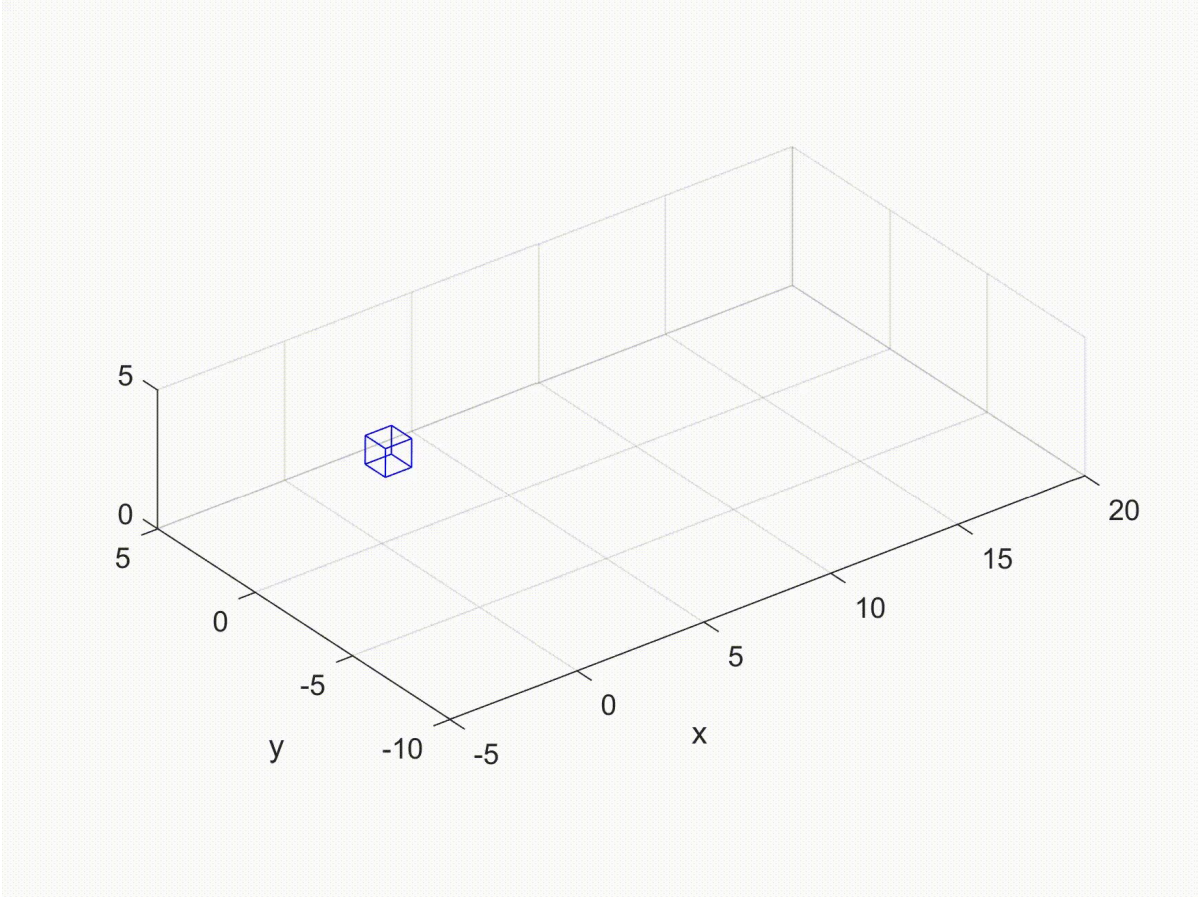
Consider effects of:

1. gravity
2. spinning
3. friction
4. bouncing


5
0
5
0
-5
-10
y
-5
0
5
10
15
20
x

Configuring the Cube & the Table

The Cube

$$m = \text{total mass}$$
$$s = \text{side length}$$

Note: we distribute the total mass uniformly throughout the cube.

The Cube: moment of inertia tensor

$$I = (1/6)ms^2$$

I is a **scalar** and stays **constant** throughout the simulation

The Table

We represent the table as a plane through the origin.

$$n = \text{unit normal of table}$$

$$S = \text{stiffness}$$
$$D = \text{damping}$$
$$\mu = \text{coefficient of sliding friction}$$

Governing Equations

Rigid Body Motion

Central assumption: The motion of the body is only affected by external forces.

We track (1) translational motion X_{cm}

(2) angular momentum L (rotational motion)

both in three dimensions.

Governing Equations

$$(1) \quad \frac{d}{dt}u_{cm} = F/m = \sum_k F_k/m \qquad \text{Change in } u_{cm}$$

$$(2) \quad \boldsymbol{\frac{d}{dt}x_{cm} = u_{cm}} \qquad \text{Change in } x_{cm}$$

$$(3) \quad \frac{d}{dt}L = \tau = \sum_k \tilde{X}_k \times F_k \qquad \text{Change in } L$$

$$(4) \quad \boldsymbol{\omega = (I^{-1})L} \qquad \text{Value of } \omega$$

Where

$\tilde{X}_k$ denotes the position of corner k relative to the center of mass

F_k denotes the force acted on corner k

Corner Positions and Velocities

We need these to animate our simulation and calculate the interaction with the table later.

$$X_k = x_{cm} + \tilde{X}_k \qquad \text{Updating X}$$

$$U_k = u_{cm} + \omega \times \tilde{X}_k \qquad \text{Updating U}$$

Updating Relative Positions $\tilde{X}_k$

Updating Relative Positions

$$\tilde{X}_k = R\tilde{X}_k$$

$$\text{where} \quad R = P + cos(||\omega||\Delta t)(E - P) + sin(||\omega||\Delta t)\left(\frac{\omega_{cross}}{||\omega||}\right)$$

$$P = \frac{\omega}{||\omega||}\left(\frac{\omega}{||\omega||}\right)^{\mathsf{T}} \qquad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \omega_{cross} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

Updating Forces F_k

Forces on the body

$$F_k = F_{gravity} + F_{table}$$

$$F_{gravity} = mg$$

$$F_{table} = \ ?$$

Force of the Table

$$F_{table} = F_n + F_f$$

where $F_n =$ normal force, $F_f =$ sliding friction

Force of the Table (Normal)

$$F_n = ||F_n|| \, n$$

$$\text{where} \quad ||F_n|| = S\left(-n^\top X_k\right) - D\left(n^\top U_k\right)$$

$-n^\top X_k =$ distance into the table

$n^\top U_k =$ component of velocity normal to the table

Force of the Table (Friction)

$$F_f = -\mu||F_n||\left(\frac{U_{tan}}{||U_{tan}||}\right)$$

$$\text{where} \quad U_{\tan} = U_k - \left(n^\top U_k\right)n$$

Finally, we can evaluate the total force by the table:

$$F_{table} = F_n + F_f$$

Results

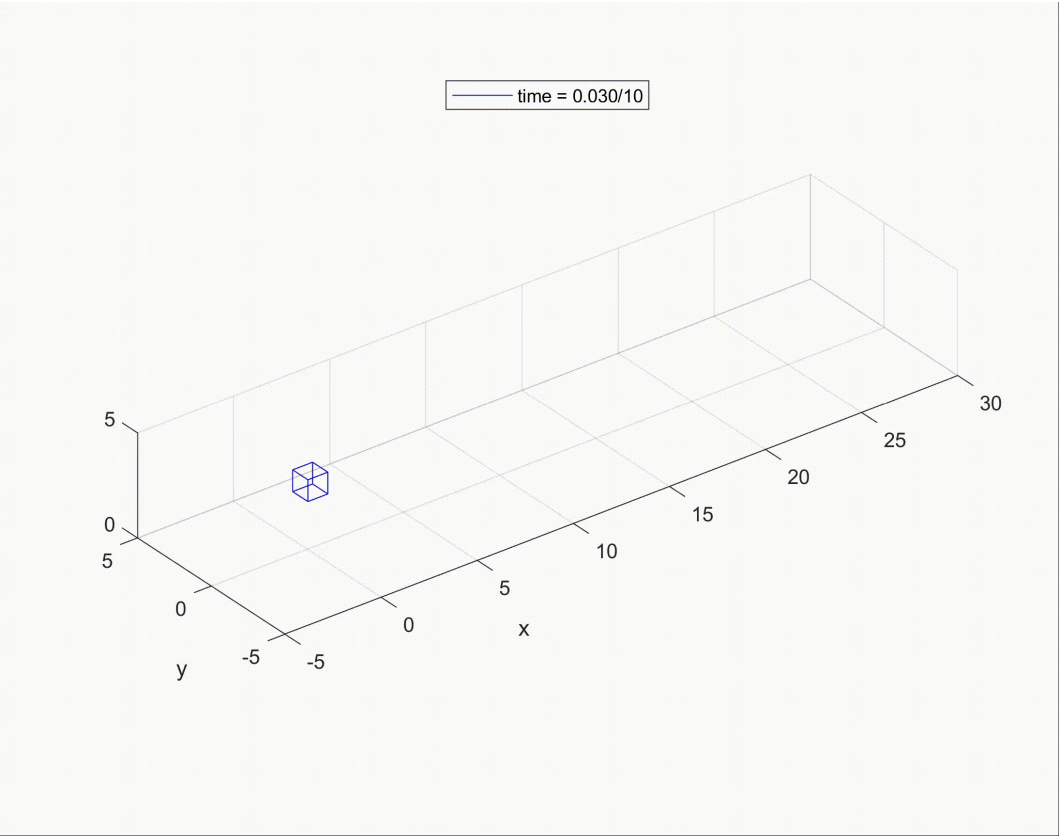
Changing Table Parameters (No Rotation Version)

$S = \text{stiffness}$

$D = \text{damping}$

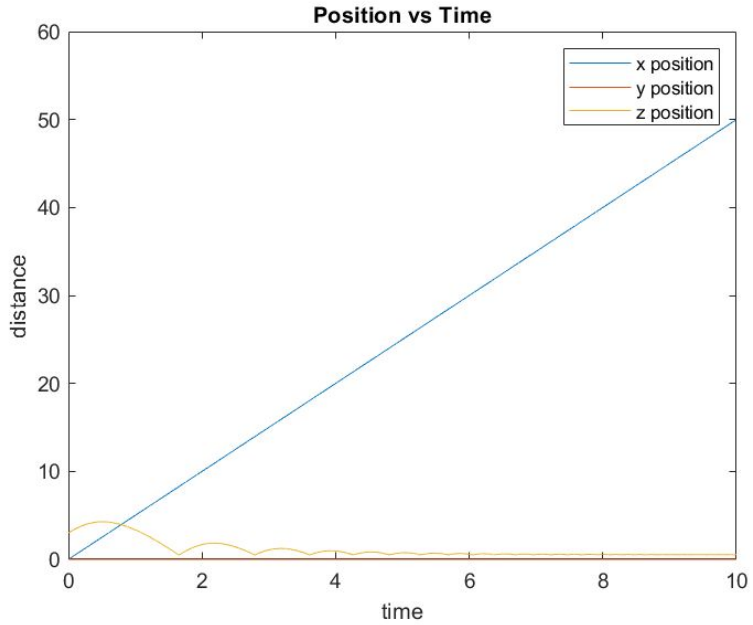
$\mu = \text{coefficient of sliding friction}$

NO FRICTION:

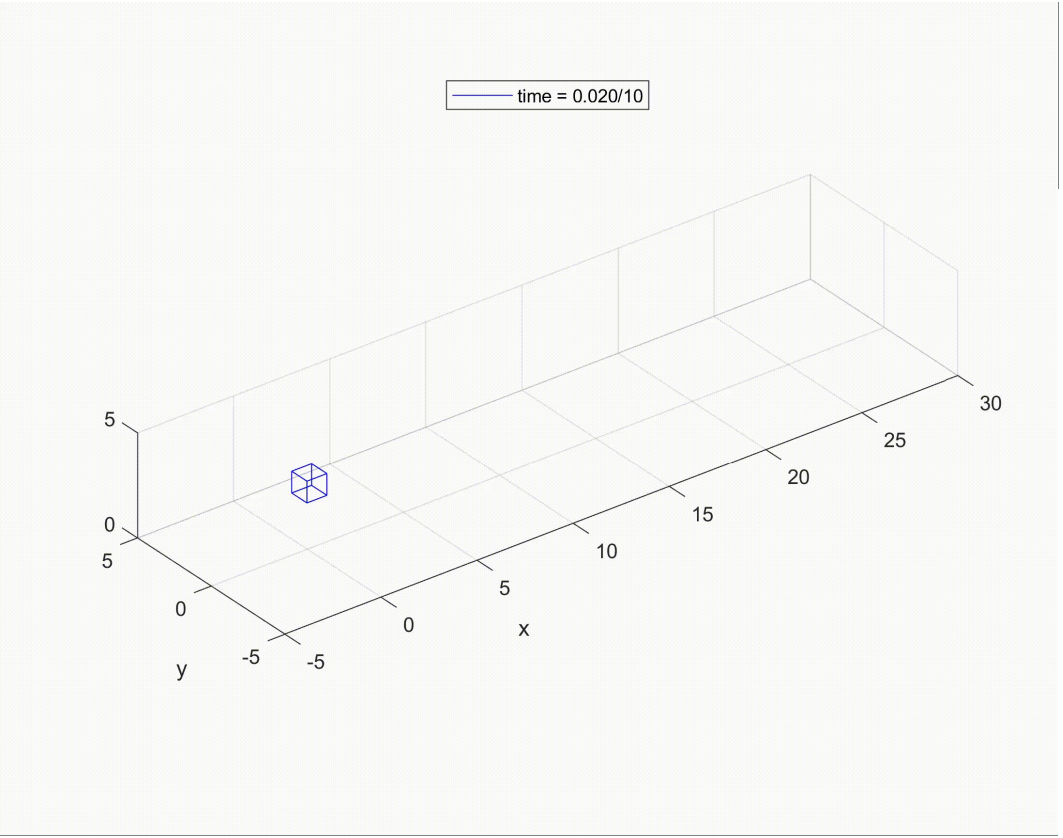


S = 10^4
D = 0.2
mu = 0

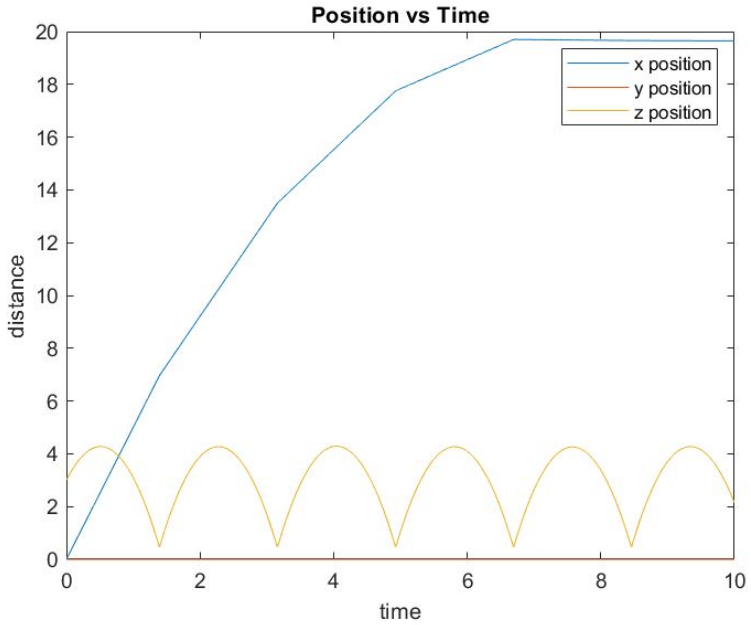
tmax = 10
clockmax = 10000
dt = tmax/clockmax



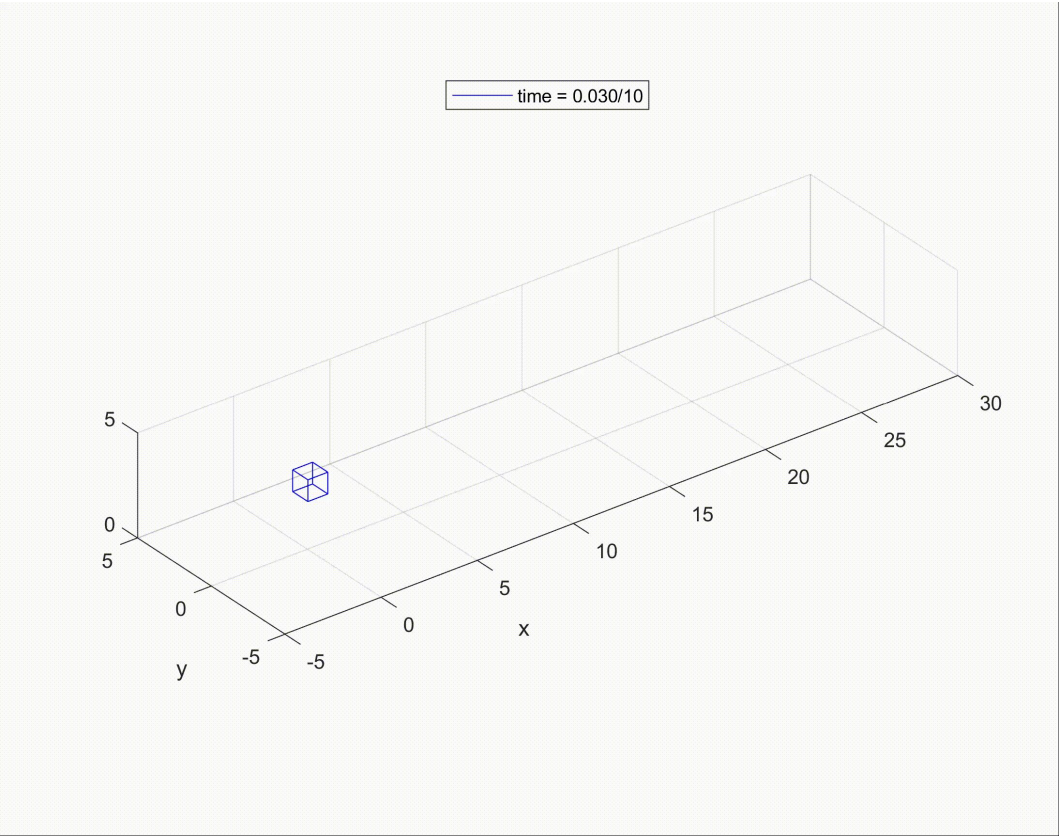
NO DAMPING:



S = 10^4 tmax = 10
D = 0 clockmax = 10000
mu = 0.75 dt = tmax/clockmax

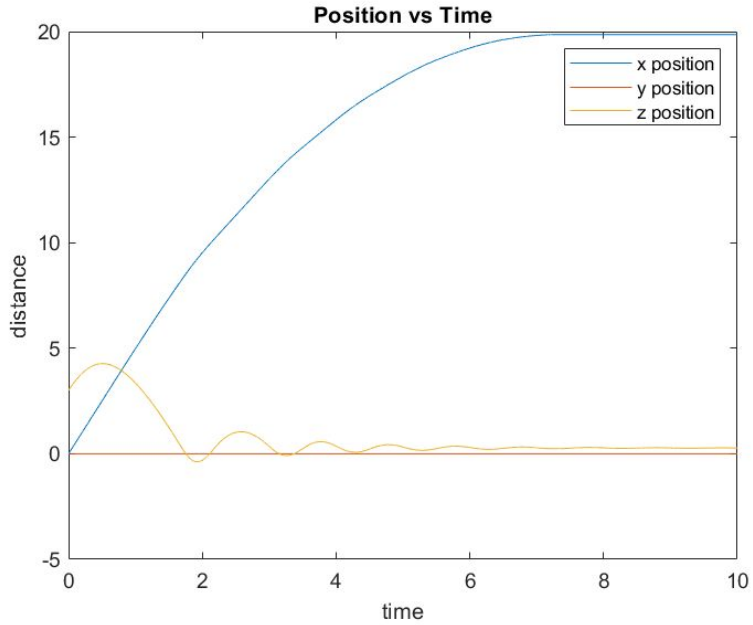


LOW STIFFNESS:

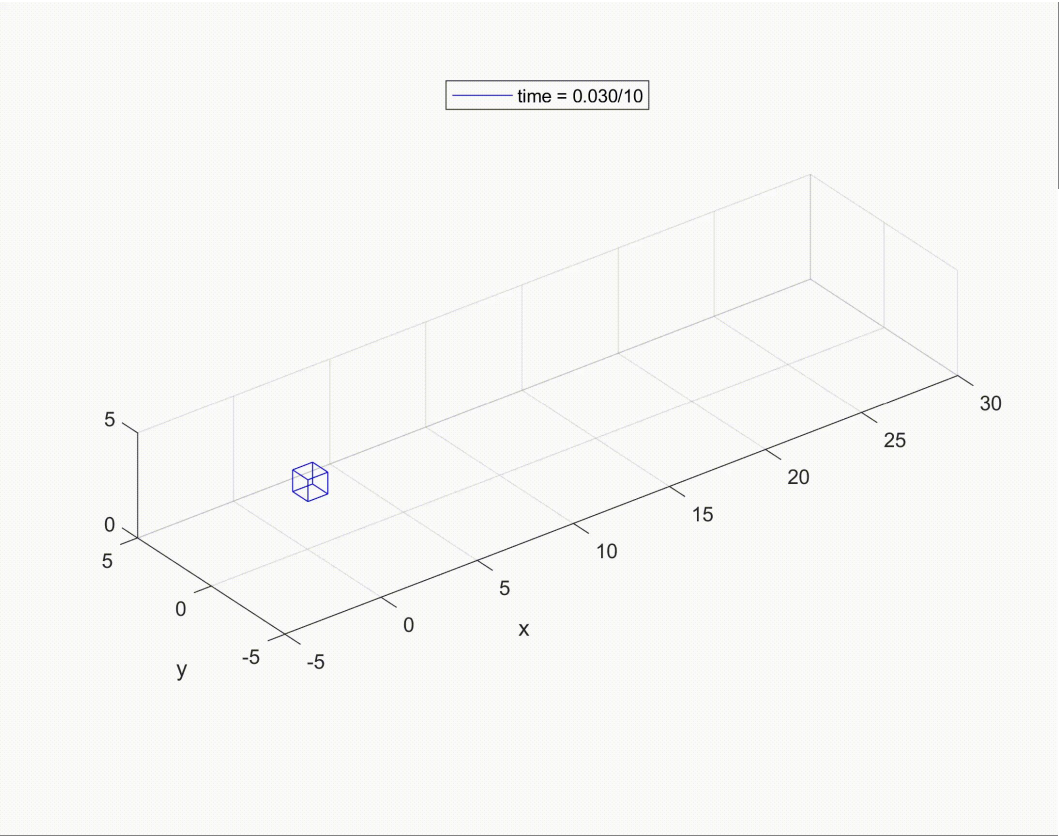


S = 10^1
D = 0.2
mu = 0.75

tmax = 10
clockmax = 10000
dt = tmax/clockmax

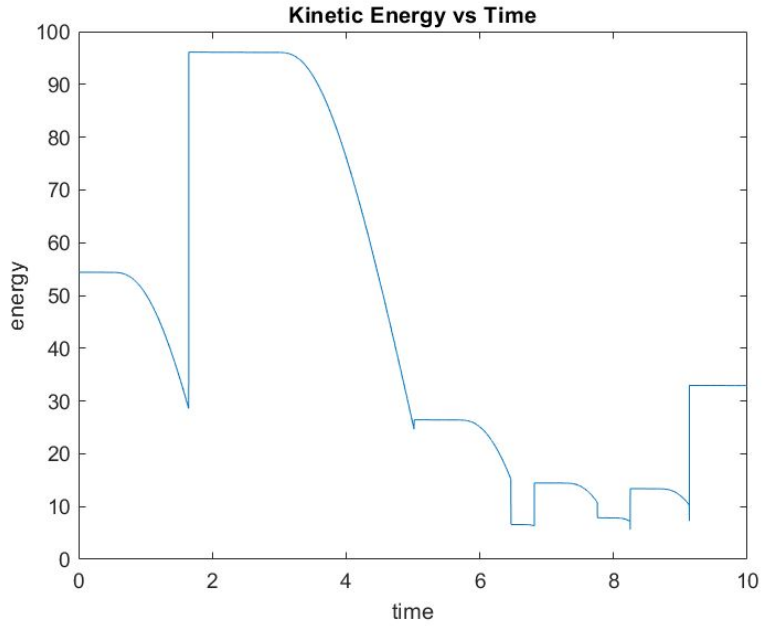


HIGH STIFFNESS:

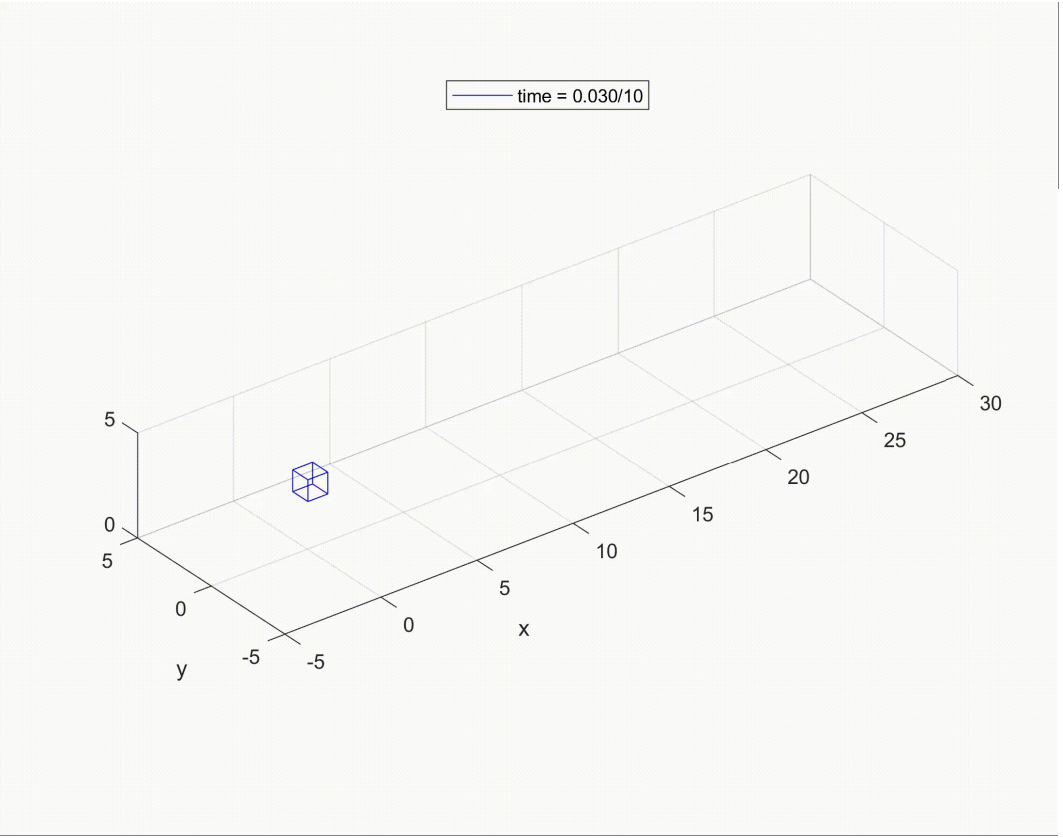


S = 10^6
D = 0.2
mu = 0.75

tmax = 10
clockmax = 10000
dt = tmax/clockmax

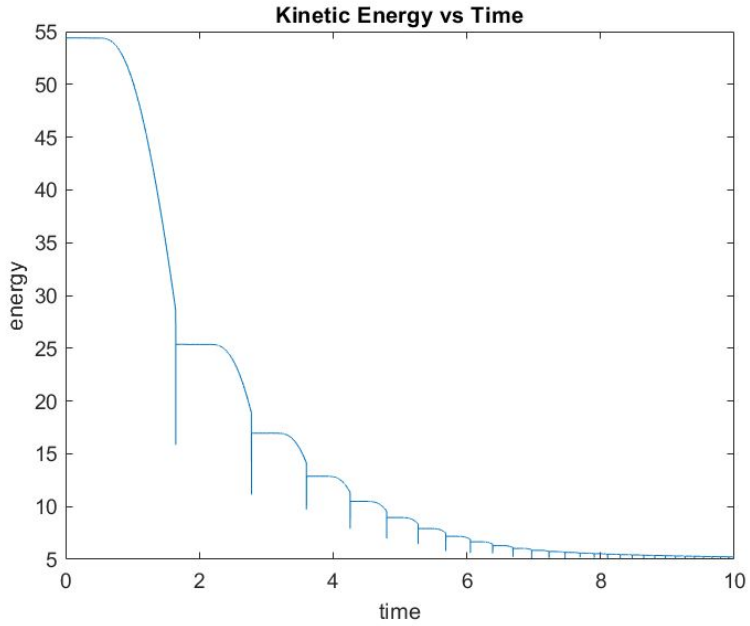


HIGH STIFFNESS (NUMERICALLY STABLE):



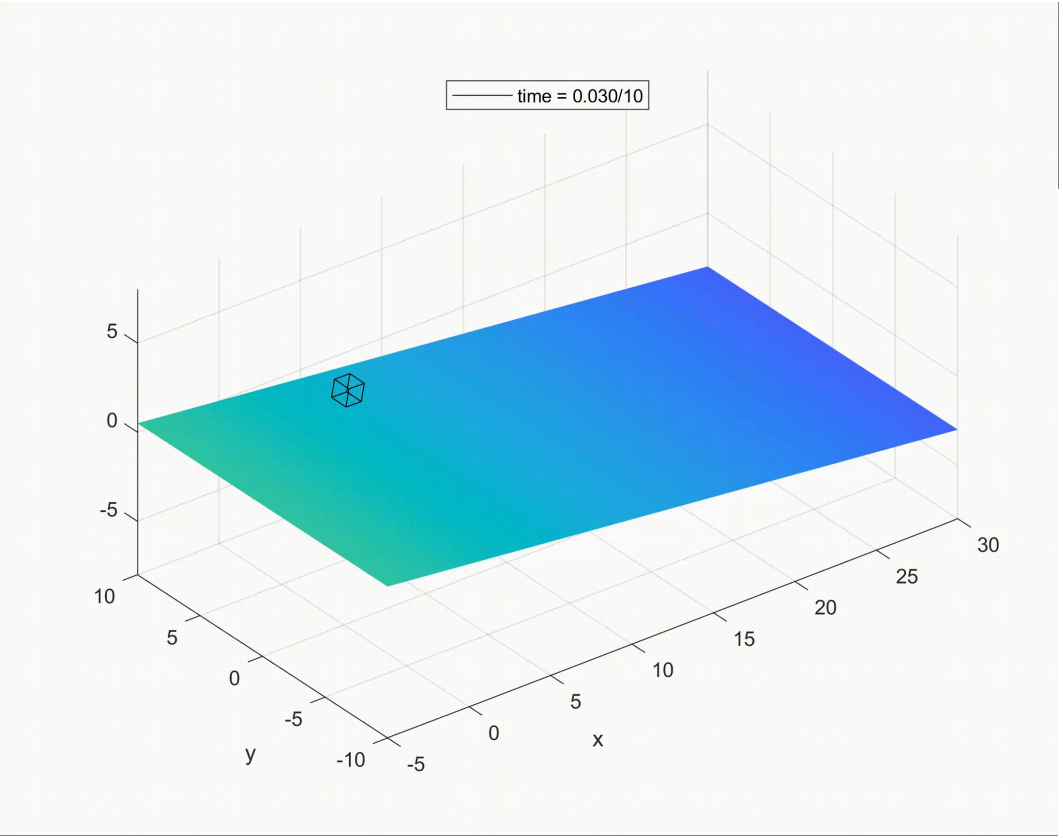
S = 10^6
D = 0.2
mu = 0.75

tmax = 10
clockmax = 20000
dt = tmax/clockmax

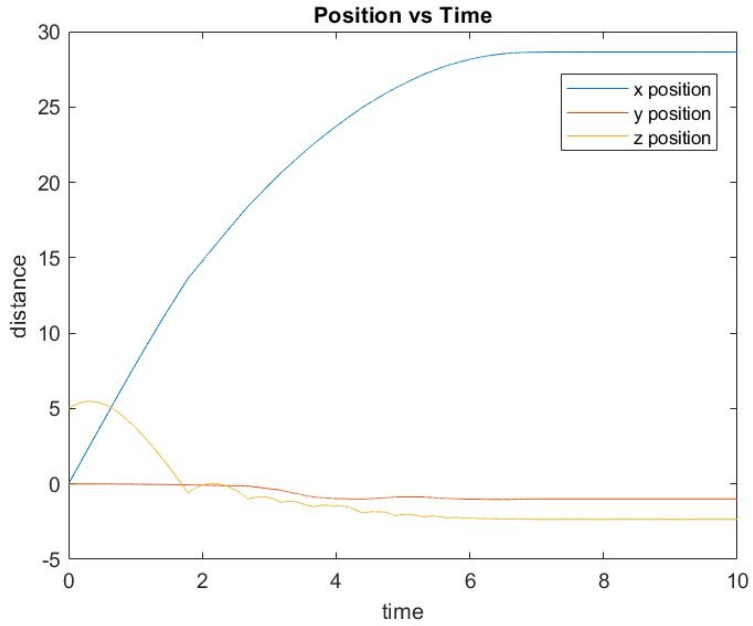


Tilting the Table

TILTED TABLE:




```
S = 10^4    ag = 1/101   x_cm = [0,0,5]
D = 0.2     bg = 0       u_cm = [8,0,3]
mu = 0.25   cg = 10/101  L = [0.6; 0.6; 0.6]
```



Acknowledgements

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