Random walk task

walks either way by 50% chance.

$V(s) = \sum_{s'} p(s'|a,s)\,V(s')$

So we have:

$V(A) = 0.5V(B)$

$V(B) = 0.5(V(A) + V(C))$

$V(C) = 0.5(V(B) + V(D))$

$V(D) = 0.5(V(C) + V(E))$

$V(E) = 0.5 + 0.5V(D)$

$\Downarrow$

$V(E)_1 = 0.5$

$V(D)_1 = 0.5(0.5)$

$V(C)_1 = 0.5(0.5(0.5))$

$V(B)_1 = 0.5(0.5(0.5(0.5)))$

$V(A)_1 = 0.5(0.5(0.5(0.5(0.5))))$

$V(E)_2 = 0.5 + 0.5(0.5(0.5))$

$V(D)_2 = 0.5(0.5 + 0.5(0.5(0.5)) + 0.5(0.5(0.5)))$

$V(C)_2 = 0.5(0.5(0.5 + 0.5(0.5(0.5)) + 0.5(0.5(0.5)))$
$\quad + 0.5(0.5(0.5(0.5))))$

$V(B)_2 = \ldots$

$$
\begin{aligned}
V(E) &= 0.5 + 0.5V(D) = 0.5 + 0.5^2(V(C) + V(E)) \\
&= 0.5 + 0.5^2(V(C) + 0.5 + 0.5V(D)) \\
&= 0.5 + 0.5^2(V(C) + 0.5 + 0.5^2(V(C) + V(E)) \\
&= 0.5 + 0.5^2 V(C) + 0.5^3 + 0.5^4 V(C) + 0.5^5 + 0.5^5 V(D) \\
&= \sum_{n=1} 0.5^{2n-1} + 0.5^{2n} V(C)
\end{aligned}
$$

$$
\begin{aligned}
V(A) &= 0.5V(B) = 0.5^2(V(A) + V(C)) \\
&= 0.5^2 V(C) + 0.5^3(V(B)) \\
&= 0.5^2 V(C) + 0.5^4(V(C) + V(B)) \\
&= 0.5^2 V(C) + 0.5^4(V(C) + 0.5^2(V(A) + V(C)) \\
&= \sum_{n=1} 0.5^{2n} V(C)
\end{aligned}
$$

$$V(E) = \sum_{n=1} 0.5^{2n-1} + 0.5^{2n} V(C)$$

$$V(A) = \sum_{n=1} 0.5^{2n} V(C)$$

$$V(B) = 0.5(V(A) + V(C))$$
$$= 0.5V(C)(1 + \sum_{n=1} 0.5^{2n})$$

$$V(D) = 0.5(V(C) + V(E))$$
$$= 0.5V(C)(1 + \sum_{n=1} 0.5^{2n}) + 0.5\sum_{n=1} 0.5^{2n-1}$$

$$V(C) = 0.5(V(B) + V(D))$$
$$= 0.5V(C) + 0.5V(C)\sum_{n=1} 0.5^{2n} + 0.25\sum_{n=1} 0.5^{2n-1}$$

$V(C_0) \approx 0.1\overline{6}$

$V(C_1) \approx 0.08333 + 0.1944 = 0.27773$

$V(C_2) \approx 0.3509$ $\qquad V(C_5) \approx 0.4509$

$V(C_3) \approx 0.3990$

$V(C_4) \approx 0.4290$

Trying $V(C) = 0.5$, it seems like the solution converges

$V(C) = 1/2 \Rightarrow V(A) = 1/6$, $V(E) = 5/6$

$V(B) = 0.5(1/6 + 1/2) = 1/3$

$V(D) = 0.5(1/2 + 5/6) = 2/3$

$V(A) = 1/6$

$V(B) = 1/3$

$V(C) = 1/2$

$V(D) = 2/3$

$\underline{V(E) = 5/6}$

This could probably be done simpler