In[∘]:=
```
ClearAll["Global`*"]
```

analytical solution for v_2(T)

note that in the manuscript, a = α, b = β, d = μ, v0 = v*, s0 = s^*

solution for v_1(t) for t < t < T

In[∘]:=
```
v11t := {v1'[t] == -δ v1[t], v1[0] == v0}
FullSimplify[DSolve[v11t, {v1[t]}, t]]
```

Out[∘]= $\{\{v1[t] \to e^{-t\,\delta}\, v0\}\}$

$$v1[t] = e^{-t\,\delta}\, v0$$

solution for s(t) for 0 < t < t_l

In[∘]:= $vs1t := \{s'[t] == s0 / tl - d\, s[t] - a\, s[t]\, e^{-t\,\delta}\, v0,\ s[0] == 0\}$
```
FullSimplify[DSolve[vs1t, {s[t]}, t]]
```

Out[∘]= $\left\{\left\{s[t] \to e^{-d\,t + \frac{a\,e^{-t\,\delta}\,v0}{\delta}} \int_0^t \frac{e^{-\frac{a\,e^{-\delta K[1]}\,v0}{\delta} + d\,K[1]}\, s0}{tl}\, dK[1]\right\}\right\}$

$$s[t] = \frac{s0}{t_l}\, e^{-d\,t + \frac{a\,e^{-t\,\delta}\,v0}{\delta}} \int_0^t e^{-\frac{a\,e^{-\delta\,u}\,v0}{\delta} + d\,u}\, du$$

solution for s(t) for t > t_l

In[∘]:= $vs2t := \{s'[t] == -d\, s[t] - a\ s[t] * e^{-(t+tl)\,\delta}\, v0,\ s[0] == stl\}$
```
FullSimplify[DSolve[vs2t, {s[t]}, t]]
```

Out[∘]= $\left\{\left\{s[t] \to e^{-d\,t - \frac{a\,e^{-((t+tl)\,\delta)}\,(-1+e^{t\,\delta})\,v0}{\delta}}\, stl\right\}\right\}$

t needs to be shifted by t_l in $e^{-d\,t}$ term

$$s[t] = s\,(t_l)\, e^{-d\,(t-t_l) - \frac{a\,e^{-(t+t_l)\,\delta}\,(-1+e^{t\,\delta})\,v0}{\delta}}$$

solution for $v_2(t)$ when $\tau < (T - t_l)$ for τ < t < (t_l + τ)

In[∘]:= $v2t := \left\{v2'[t] == -\delta\, v2[t] + a\, b\, e^{-\tau\,d}\, e^{-d\,t + \frac{a\,e^{-t\,\delta}\,v0}{\delta}} \int_0^t \frac{e^{-\frac{a\,e^{-\delta\,x}\,v0}{\delta} + d\,x}\, s0}{tl}\, dx\ e^{-t\,\delta}\, v0,\ v2[0] == 0\right\}$
```
FullSimplify[DSolve[v2t, {v2[t]}, t]]
```

Out[∘]= $\left\{\left\{v2[t] \to e^{-t\,\delta} \int_0^t a\, b\, e^{\frac{a\,e^{-\delta K[1]}\,v0}{\delta} - d\,(\tau + K[1])}\, v0 \int_0^{K[1]} \frac{e^{d\,x - \frac{a\,e^{-x\,\delta}\,v0}{\delta}}\, s0}{tl}\, dx\, dK[1]\right\}\right\}$

t needs to be shifted by τ in $e^{-t\,\delta}$ term and integration limit

$$v2[t] = \frac{a\, b\, e^{-\tau\,d}\, v0\, s0}{t_l}\, e^{-(t-\tau)\,\delta} \int_0^{t-\tau} e^{\frac{a\,e^{-\delta\,u}\,v0}{\delta} - d\,u} \int_0^u e^{d\,x - \frac{a\,e^{-x\,\delta}\,v0}{\delta}}\, dx\, du$$

solution for $v_2(t)$ when $\tau < (T - t_l)$ for t > (t_l + τ)

In[ ]:= $$\texttt{v2tt} := \left\{\texttt{v2}'[t] == -\delta\,\texttt{v2}[t] + a\,b\,e^{-\tau d}\,e^{-d\,t-\frac{a\,e^{-((t+tl)\,\delta)}\,\left(-1+e^{t\,\delta}\right)\,v0}{\delta}}\,stl\,e^{-(t+tl)\,\delta}\,v0,\ \texttt{v2}[0] == v2x\right\}$$

```
FullSimplify[DSolve[v2tt, {v2[t]}, t]]
```

Out[ ]= $$\left\{\left\{\texttt{v2}[t] \to e^{-t\,\delta}\left(v2x + \int_0^t a\,b\,e^{-\frac{a\,e^{-\delta\,(tl+K[1])}\,\left(-1+e^{\delta\,K[1]}\right)\,v0}{\delta}-tl\,\delta-d\,(\tau+K[1])}\,stl\,v0\,dK[1]\right)\right\}\right\}$$

t needs to be shifted by $\tau + t_l$ in $e^{-t\,\delta}$ term and integration limit

$$\texttt{v2}[t] = e^{-(t-\tau-t_l)\,\delta}\left(v2\,(t_l) + a\,b\,e^{-\tau\,d}\,v0\,s\,(t_l)\int_0^{t-\tau-t_l} e^{-\frac{a\,v0\,e^{-\delta\,(t_l+u)}\,\left(-1+e^{\delta\,u}\right)}{\delta}-t_l\,\delta-d\,u}\,du\right)$$

solution for $v_2(t)$ when $\tau > (T - t_l)$ for $\tau < t < (t_l + \tau)$

In[ ]:= $$\texttt{v2t2} := \left\{\texttt{v2}'[t] == -\delta\,\texttt{v2}[t] + a\,b\,e^{-\tau\,d}\,e^{-d\,t+\frac{a\,e^{-t\,\delta}\,v0}{\delta}}\int_0^t \frac{e^{-\frac{a\,e^{-\delta\,x}\,v0}{\delta}+d\,x}\,s0}{tl}\,dx\,e^{-t\,\delta}\,v0,\ \texttt{v2}[0] == 0\right\}$$

```
FullSimplify[DSolve[v2t2, {v2[t]}, t]]
```

Out[ ]= $$\left\{\left\{\texttt{v2}[t] \to e^{-t\,\delta}\int_0^t a\,b\,e^{\frac{a\,e^{-\delta\,K[1]}\,v0}{\delta}-d\,(\tau+K[1])}\,v0\int_0^{K[1]}\frac{e^{d\,x-\frac{a\,e^{-x\,\delta}\,v0}{\delta}}\,s0}{tl}\,dx\,dK[1]\right\}\right\}$$

t needs to be shifted by τ in $e^{-t\,\delta}$ term and integration limit

$$\texttt{v2}[t] = \frac{a\,b\,e^{-\tau\,d}\,v0\,s0}{t_l}\,e^{-(t-\tau)\,\delta}\int_0^{t-\tau} e^{\frac{a\,e^{-\delta\,u}\,v0}{\delta}-d\,u}\int_0^u e^{d\,x-\frac{a\,e^{-x\,\delta}\,v0}{\delta}}\,dx\,du$$

analytical solution for $v_{2\,m}$(T)

note that the solution is for the specific case when $\tau_m$and τ and v0 and v0m are different from one another but all other parasite parameters are identical between the resident and mutant strains

In[ ]:=
```
ClearAll["Global`*"]
```

solution for $v_{1\,m}$(t) for t < t < T

In[ ]:=
```
v1mt := {v1m'[t] == -δ v1m[t], v1m[0] == vm0}
FullSimplify[DSolve[v1mt, {v1m[t]}, t]]
```

Out[ ]= $$\left\{\left\{\texttt{v1m}[t] \to e^{-t\,\delta}\,vm0\right\}\right\}$$

$$\texttt{v1m}[t] = e^{-t\,\delta}\,vm0$$

solution for s(t) for 0 < t < t_l

$$\texttt{vs1tm} := \left\{s'[t] == s0\,/\,tl - d\,s[t] - a\,s[t]\,e^{-t\,\delta}\,(v0 + v0m),\ s[0] == 0\right\}$$

```
FullSimplify[DSolve[vs1tm, {s[t]}, t]]
```

Out[ ]= $$\left\{\left\{s[t] \to e^{-d\,t+\frac{a\,e^{-t\,\delta}\,(v0+v0m)}{\delta}}\int_0^t \frac{e^{-\frac{a\,e^{-\delta\,K[1]}\,(v0+v0m)}{\delta}+d\,K[1]}\,s0}{tl}\,dK[1]\right\}\right\}$$

$$s[t] = \frac{s0}{t_l}\,e^{-d\,t+\frac{a\,e^{-t\,\delta}\,(v0+v0m)}{\delta}}\int_0^t e^{-\frac{a\,e^{-\delta\,u}\,(v0+v0m)}{\delta}+d\,u}\,du$$

solution for s(t) for t > t_l

In[∘]:=
$$\text{vs2tm} := \left\{s'[t] == -d\, s[t] - a\, s[t] * e^{-(t+tl)\,\delta}\,(v0 + v0m),\ s[0] == stl\right\}$$

```
FullSimplify[DSolve[vs2tm, {s[t]}, t]]
```

Out[∘]=
$$\left\{\left\{s[t] \to e^{-d\,t - \frac{a\,e^{-((t+tl)\,\delta)}\,\left(-1+e^{t\,\delta}\right)\,(v0+v0m)}{\delta}}\ stl\right\}\right\}$$

need to shift t by $t_l$in $e^{-t\,\delta}$ term

$$s[t] = s\,(t_l)\ e^{-d\,(t-t_l) - \frac{a\,e^{-(t+t_l)\,\delta}\,\left(-1+e^{t\,\delta}\right)\,(v0+v0m)}{\delta}}$$

solution for $v_{2\,m}(t)$ when $\tau < (T - t_l)$ for $\tau < t < (t_l + \tau)$

In[∘]:= v2tm :=

$$\left\{v2m'[t] == -\delta\, v2m[t] + a\,b\,e^{-\tau\,d}\,e^{-d\,t + \frac{a\,e^{-t\,\delta}\,(v0+v0m)}{\delta}} \int_0^t \frac{e^{-\frac{a\,e^{-\delta\,x}\,(v0+v0m)}{\delta} + d\,x}\ s0}{tl}\,dx\ e^{-t\,\delta}\,v0m,\ v2m[0] == 0\right\}$$

```
FullSimplify[DSolve[v2tm, {v2m[t]}, t]]
```

Out[∘]=
$$\left\{\left\{v2m[t] \to e^{-t\,\delta} \int_0^t a\,b\,e^{\frac{a\,e^{-\delta\,K[1]}\,(v0+v0m)}{\delta} - d\,(\tau+K[1])}\ v0m \int_0^{K[1]} \frac{e^{d\,x - \frac{a\,e^{-x\,\delta}\,(v0+v0m)}{\delta}}\ s0}{tl}\,dx\,dK[1]\right\}\right\}$$

need to shift t by τ_m in $e^{-t\,\delta}$ term and integration limit

$$v2m[t] = \frac{a\,b\,e^{-\tau_m\,d}\ v0m\ s0}{t_l}\ e^{-(t-\tau_m)\,\delta} \int_0^{t-\tau_m} e^{\frac{a\,e^{-\delta u}\,(v0+v0m)}{\delta} - d\,u} \int_0^u e^{d\,x - \frac{a\,e^{-x\,\delta}\,(v0+v0m)}{\delta}}\,dx\,du$$

solution for $v_{2\,m}(t)$ when $\tau < (T - t_l)$ for $t > (t_l + \tau)$

In[∘]:= v2ttm :=

$$\left\{v2m'[t] == -\delta\, v2m[t] + a\,b\,e^{-\tau\,d}\,e^{-d\,t - \frac{a\,e^{-((t+tl)\,\delta)}\,\left(-1+e^{t\,\delta}\right)\,(v0+v0m)}{\delta}}\ stl\ e^{-(t+tl)\,\delta}\,v0m,\ v2m[0] == v2mx\right\}$$

```
FullSimplify[DSolve[v2ttm, {v2m[t]}, t]]
```

Out[∘]=
$$\left\{\left\{v2m[t] \to e^{-t\,\delta}\left(v2mx + \int_0^t a\,b\,e^{-\frac{a\,e^{-\delta\,(tl+K[1])}\,\left(-1+e^{\delta\,K[1]}\right)\,(v0+v0m)}{\delta} - tl\,\delta - d\,(\tau+K[1])}\ stl\ v0m\,dK[1]\right)\right\}\right\}$$

need to shift t by $\tau_m + t_l$ in $e^{-t\,\delta}$ term and integration limit

$$v2m[t] = e^{-(t-\tau_m-t_l)\,\delta}\left(v2\,(t_l) + a\,b\,e^{-\tau_m\,d}\ v0m\ s\,(t_l) \int_0^{t-\tau_m-t_l} e^{-\frac{a\,e^{-\delta\,(tl+u)}\,\left(-1+e^{\delta\,u}\right)\,(v0+v0m)}{\delta} - t_l\,\delta - d\,u}\,du\right)$$

solution for $v_{2\,m}(t)$ when $\tau > (T - t_l)$ for $\tau < t < T$

In[∘]:= v2tm2 :=

$$\left\{v2m'[t] == -\delta\, v2m[t] + a\,b\,e^{-\tau\,d}\,e^{-d\,t + \frac{a\,e^{-t\,\delta}\,(v0+v0m)}{\delta}} \int_0^t \frac{e^{-\frac{a\,e^{-\delta\,x}\,(v0+v0m)}{\delta} + d\,x}\ s0}{tl}\,dx\ e^{-t\,\delta}\,v0m,\ v2m[0] == 0\right\}$$

```
FullSimplify[DSolve[v2tm2, {v2m[t]}, t]]
```

Out[∘]=
$$\left\{\left\{v2m[t] \to e^{-t\,\delta} \int_0^t a\,b\,e^{\frac{a\,e^{-\delta\,K[1]}\,(v0+v0m)}{\delta} - d\,(\tau+K[1])}\ v0m \int_0^{K[1]} \frac{e^{d\,x - \frac{a\,e^{-x\,\delta}\,(v0+v0m)}{\delta}}\ s0}{tl}\,dx\,dK[1]\right\}\right\}$$

need to shift t by $\tau_m$in $e^{-t\,\delta}$ term and integration limit

$$\mathrm{v2m}[t] = \frac{a\, b\, e^{-\tau_m d}\, \mathrm{v0m}\, \mathrm{s0}}{t_l}\, e^{-(t-\tau_m)\,\delta} \int_0^{t-\tau_m} e^{\frac{a\, e^{-\delta u}\,(\mathrm{v0}+\mathrm{v0m})}{\delta} - d\, u} \int_0^u e^{d\, x - \frac{a\, e^{-x\,\delta}\,(\mathrm{v0}+\mathrm{v0m})}{\delta}}\, dx\, du$$