Minimax-Bayes Reinforcement Learning

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Problem set up

- MDP $\mu = (S, A, P, \rho, T) \in \mathcal{M}$
- Utility $\mathscr{U} = \sum_{t=1}^{T} r_t$

For a fixed MDP $\mu \in \mathcal{M}$, we define the

- Utility $\mathscr{U}(\pi, \mu) = E^{\pi}_{\mu}[\mathscr{U}]$
- Optimal Utility $\mathscr{U}^*(\mu) = \max_\pi \mathscr{U}(\pi, \mu)$

For a distribution β over MDPs, we define the

- Utility $\mathscr{U}(\pi, \beta) = E^{\pi}_{\beta}[\mathscr{U}] = \int_{\mathcal{M}} \mathscr{U}(\pi, \mu) d\beta(\mu)$
- Bayes-optimal utility $\mathscr{U}^*(\beta) = \sup_\pi \mathscr{U}(\pi, \beta)$

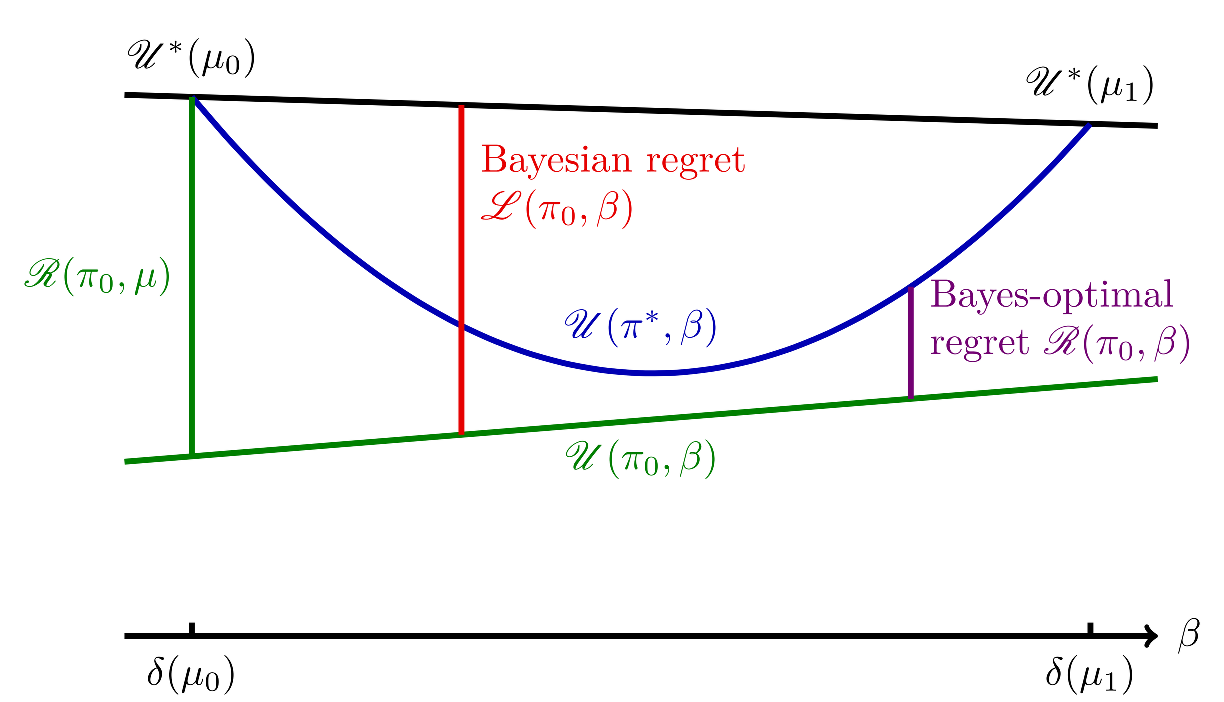
Interpretation of β:

1. The agent's subjective belief about which MDP is the most likely a priori.
2. The MDP is actually drawn randomly from distribution.

Suppose Nature choses β arbitrarily or adversarially, then we are interested in finding

$$\max_\pi \min_\beta \mathscr{U}(\pi, \beta)$$

However, for an unrestricted set of priors, Nature could pick a prior such that all rewards are zero, thus trivially achieving minimal utility. Instead we consider regret.



Notions of regret

For a fixed MDP $\mu \in \mathcal{M}$, we define

$$\mathscr{R}(\pi, \mu) = \mathscr{U}^*(\mu) - \mathscr{U}(\pi, \mu)$$

For a prior β, we define the

- Bayes-optimal regret

$$\mathscr{R}(\pi, \beta) = \mathscr{U}^*(\beta) - \mathscr{U}(\pi, \beta)$$

- Bayesian regret (comparing against an oracle)

$$\mathscr{L}(\pi, \beta) = \mathrm{E}_{\mu \sim \beta}[\mathscr{R}(\pi, \mu)] = \int_{\mathcal{M}} \mathscr{U}^*(\mu) - \mathscr{U}(\pi, \mu) d\beta(\mu)$$

Of course, we always have $\mathscr{R}(\pi, \beta) \le \mathscr{L}(\pi, \beta)$.

Minimax game against Nature

We define minimax games with respect to the Bayes-optimal regret $\min_\pi \max_\beta \mathscr{R}(\pi, \beta)$ and Bayesian regret $\min_\pi \max_\beta \mathscr{L}(\pi, \beta)$.

Corollary (value of the game)

The minimax game with resepct to the utility and the Bayesian regret have a value, i.e. it holds that

$$\max_\pi \min_\beta \mathscr{U}(\pi, \beta) = \min_\beta \max_\pi \mathscr{U}(\pi, \beta), \quad \min_\pi \max_\beta \mathscr{L}(\pi, \beta) = \max_\beta \min_\pi \mathscr{L}(\pi, \beta).$$

Lemma

The minimax game with respect to the Bayes-optimal regret may not have a value, i.e.,

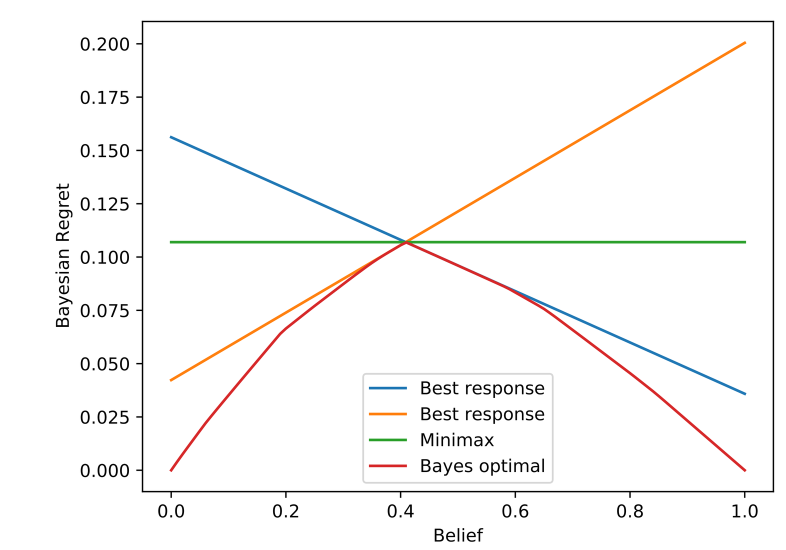
$$\min_\pi \max_\beta \mathscr{R}(\pi, \beta) < \max_\beta \min_\pi \mathscr{R}(\pi, \beta).$$

Lemma (Bayesian regret of the Bayes-optimal policy)

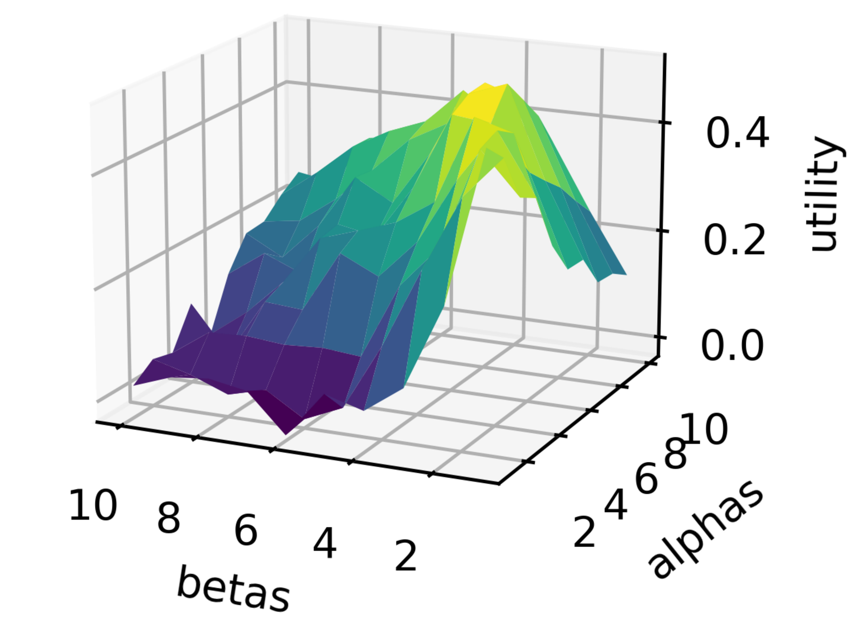
The worst-case Bayesian regret of the Bayes-optimal policy equals the minimax Bayesian regret, i.e.

$$\max_\beta \mathscr{L}(\pi^*(\beta), \beta) = \min_\pi \max_\beta \mathscr{L}(\pi, \beta)$$

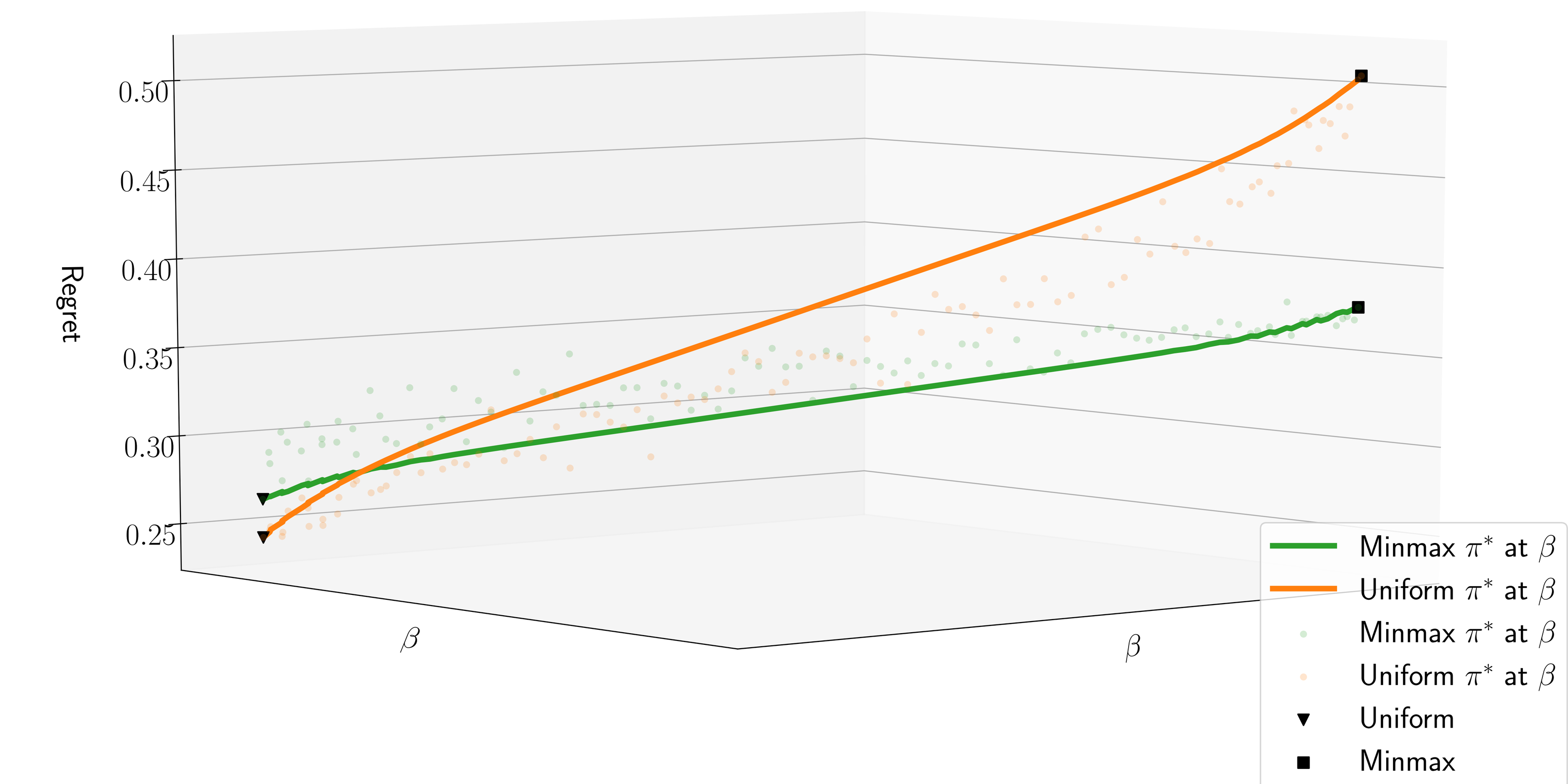
Experiments



(a) Regret for a two MDP task for a variety of policies.



(b) Bayesian regret of the Bayes-optimal policy in two armed Bernoulli bandit task. The first arms prior is fixed to $\mathcal{B}eta(4, 2)$ while the other is given by the values on the x- and y-axis.



(c) t-SNE embeddings for (approximately) minimax and uniform beliefs with their corresponding Bayesian regret $\mathscr{L}$.