If a strict partial order also has the property of

- *totality:* $\forall x, y : x \rhd y \vee y \rhd x \vee x = y$,

then it is a *total order.*

Definition 2 (preference formula (pf) [Chomicki, 2003])**.** *A preference formula (pf) $C(r, s)$ is a first-order formula defining a preference relation $\rhd$ in the standard sense, namely*

$$r \rhd s \quad \text{iff} \quad C(r, s).$$

Because of this close relationship between the first-order formula C and the binary relation $\rhd$ we will later use $\text{skyline}_{\rhd}$ and skyline_C interchangeably.

In our case we only focus on *intrinsic* preference formulas, i.e. the value of $C(r, s)$ only depends on the attributes of r and s and not on other tuples in the same or other relations. If the preference formula depends on other tuples, like in *"we prefer tuples that are better than the average"*, which involves aggregation, it is called *extrinsic* preference formula. We now define a special form of preference formula.

Definition 3 (Skyline Preference Formula)**.** *Given the two tuples*

$$r = (\underbrace{r_1, \ldots, r_k}_{\texttt{MIN}}, \underbrace{r_{k+1}, \ldots, r_l}_{\texttt{MAX}}, \underbrace{r_{l+1}, \ldots, r_m}_{\texttt{DIFF}}, \underbrace{r_{m+1}, \ldots, r_n}_{extra})$$

and

$$s = (\underbrace{s_1, \ldots, s_k}_{\texttt{MIN}}, \underbrace{s_{k+1}, \ldots, s_l}_{\texttt{MAX}}, \underbrace{s_{l+1}, \ldots, s_m}_{\texttt{DIFF}}, \underbrace{s_{m+1}, \ldots, s_n}_{extra})$$

both elements of an n-ary relation $\mathbf{R}$, then the skyline preference formula *is of the form:*

$$\begin{aligned} C(r, s) = {} & \overbrace{\left(\bigwedge_{1 \le i \le k} r_i \le s_i\right)}^{\text{“}\texttt{MIN} \textit{ at least as good”}} \wedge \overbrace{\left(\bigwedge_{k+1 \le i \le l} r_i \ge s_i\right)}^{\text{“}\texttt{MAX} \textit{ at least as good”}} \wedge \overbrace{\left(\bigwedge_{l+1 \le i \le m} r_i = s_i\right)}^{\text{“}\texttt{DIFF} \textit{ equal”}} \wedge \\ & \wedge \left(\underbrace{\left(\bigvee_{1 \le i \le k} r_i < s_i\right)}_{\text{“}\texttt{MIN} \textit{ better than”}} \vee \underbrace{\left(\bigvee_{k+1 \le i \le l} r_i > s_i\right)}_{\text{“}\texttt{MAX} \textit{ better than”}} \right). \end{aligned} \tag{2.1}$$

To state (2.1) in an informal way, a tuple r dominates a tuple s if it is equal in all `DIFF` dimensions, at least as good in all `MIN`/`MAX` dimensions and better than in at least one of the `MIN`/`MAX` dimensions. The above formula corresponds to a skyline query with the following skyline clause:

$$\texttt{SKYLINE OF } a_1\,\texttt{MIN}, \ldots, a_k\,\texttt{MIN}, a_{k+1}\,\texttt{MAX}, \ldots, a_l\,\texttt{MAX}, a_{l+1}\,\texttt{DIFF}, \ldots, a_m\,\texttt{DIFF}$$

To simplify the definition of formula (2.1) and combine the "`MIN`- and `MAX`-at least as good" condition to a just "at least as good" condition we define:

$$\succeq_i := \begin{cases} \le & \text{if } 1 \le i \le k, \\ \ge & \text{if } k+1 \le i \le l, \\ \text{undefined} & \text{otherwise} \end{cases} \tag{2.2}$$