Same for the "better than" condition:

$$\succ_i := \begin{cases} < & \text{if } 1 \le i \le k, \\ > & \text{if } k+1 \le i \le l, \\ \text{undefined} & \text{otherwise} \end{cases} \tag{2.3}$$

Using the above two definitions formula (2.1) simplifies to:

$$r \rhd s := \underbrace{\left(\bigwedge_{l+1 \le i \le m} r_i = s_i \right)}_{\text{“\texttt{DIFF} \textit{equal}”}} \wedge \underbrace{\left(\bigwedge_{1 \le i \le l} r_i \succeq_i s_i \right)}_{\text{“\textit{at least as good}”}} \wedge \underbrace{\left(\bigvee_{1 \le i \le l} r_i \succ_i s_i \right)}_{\text{“\textit{better than}”}} \tag{2.4}$$

We now define the *skyline operator* which is a special case of the *winnow operator* [Chomicki, 2003], where the preference formula has exactly the form of formula (2.1).

Definition 4 (Skyline Operator)**.** *Let $\mathcal{R}$ be a relation schema and C a skyline preference formula defining a preference relation $\rhd$ over $\mathcal{R}$, then the* skyline operator *is denoted by* $\text{skyline}_{\rhd}(\mathcal{R})$*, and for every instance $\mathbf{R} \in \mathcal{R}$:*

$$\text{skyline}_{\rhd}(\mathbf{R}) := \{r \in \mathbf{R} | \nexists s \in \mathbf{R} \colon s \rhd r\}$$

To give the intuition, the skyline operator returns all tuples which are *not dominated*, in other words all tuples that do *not* have a *witness*.

We now like to introduce some further concepts:

Definition 5 (Weak Preference)**.** *There is a weak preference between r and s if r is equal in all* `DIFF` *dimensions and at least as good in all other skyline dimensions compared to s, formally:*

$$r \trianglerighteq s := \underbrace{\left(\bigwedge_{l+1 \le i \le m} r_i = s_i \right)}_{\text{“\texttt{DIFF} \textit{equal}”}} \wedge \underbrace{\left(\bigwedge_{1 \le i \le l} r_i \succeq_i s_i \right)}_{\text{“\textit{at least as good}”}} \tag{2.5}$$

Definition 6 (Non Distinct)**.** *If two tuples r and s are equal on all skyline dimensions, we say r and s are* non distinct*, denoted by:*

$$r \stackrel{\rhd}{=} s := \left(\bigwedge_{1 \le i \le m} r_i = s_i \right) \tag{2.6}$$

If $r \stackrel{\rhd}{=} s$ and in case of `SKYLINE OF DISTINCT`, then it is left up to the implementation to include either r or s in the skyline, otherwise r *and* s are included. Please note that in general $r = s$ is not equivalent to $r \stackrel{\rhd}{=} s$, only when $m = n$, i.e. all attributes are subject to skyline computation.

For convenience we define the following two *commutator* relations:

$$s \lhd r :\Leftrightarrow r \rhd s.$$

$$s \trianglelefteq r :\Leftrightarrow r \trianglerighteq s.$$