Advanced Web Security

Secure Electronic Payments

BLOCKCHAIN

Paul Stankovski Wagner - EITN41 LP2 2022 - introduction
Slides by Elena Pagnin

Quick History of (Digital) Money

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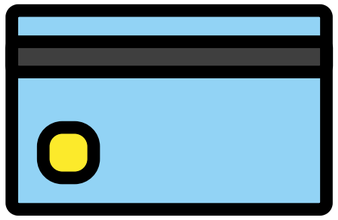

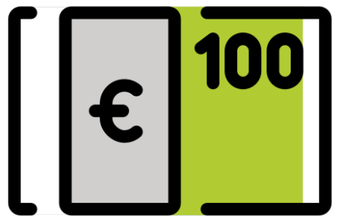

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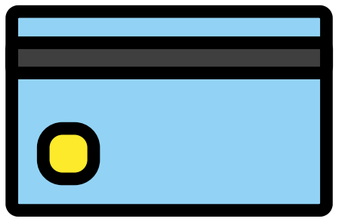

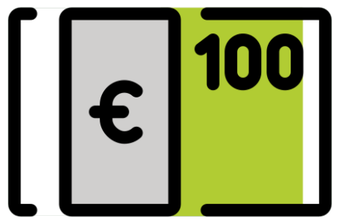

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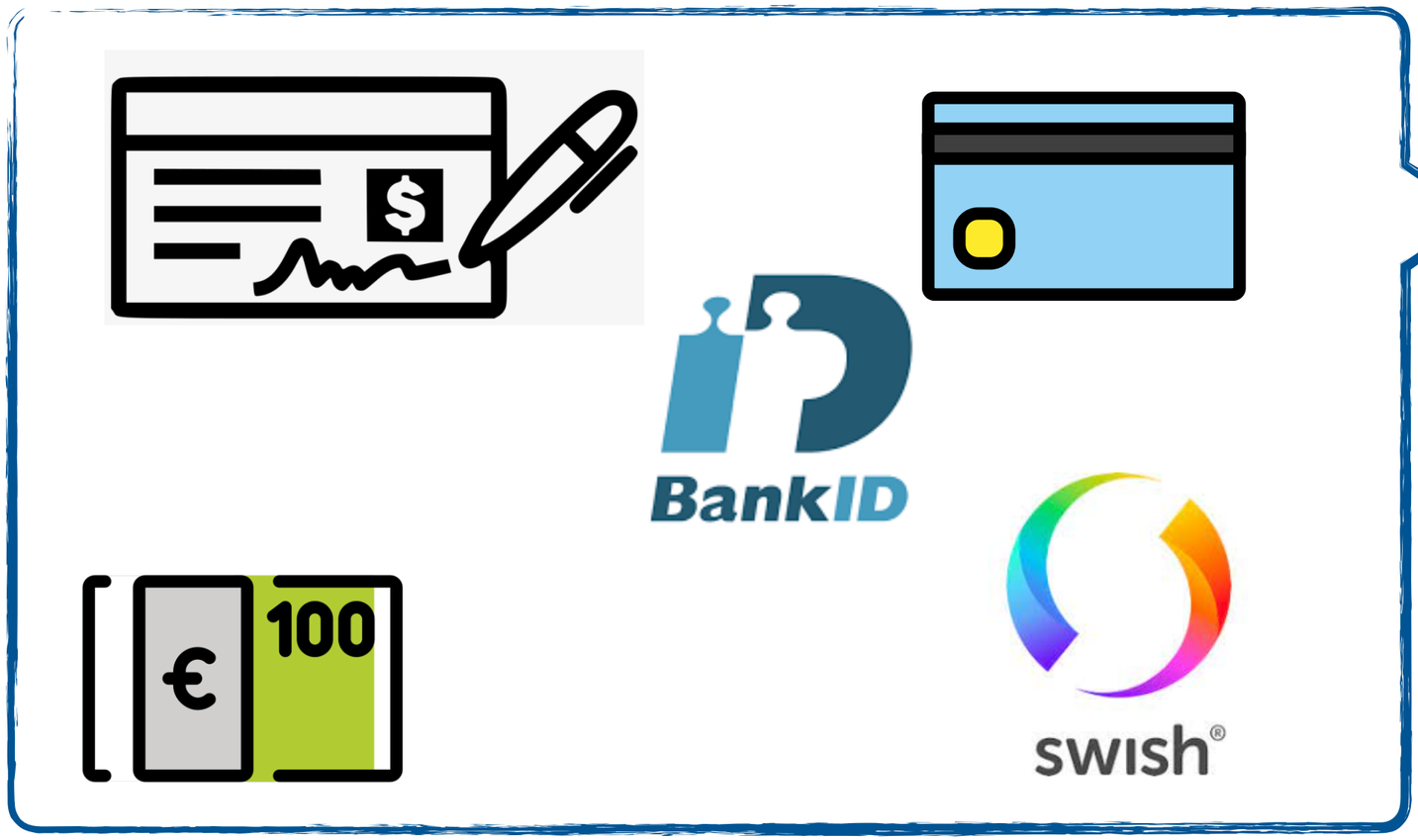
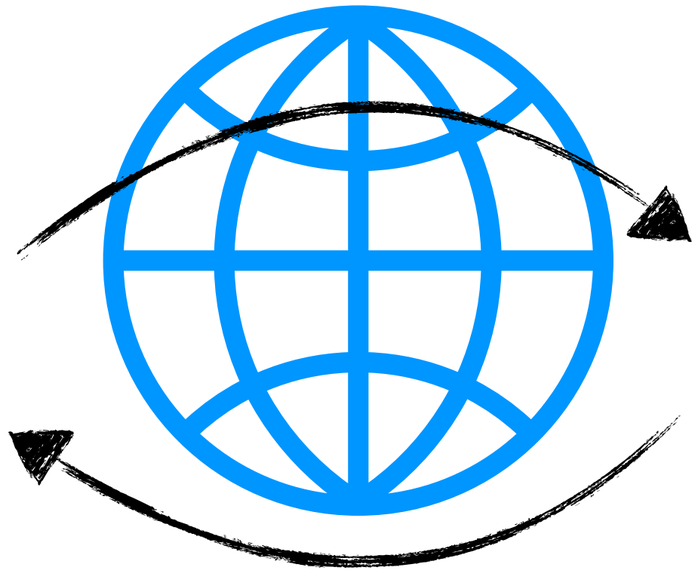


More details on card-based transactions in Lecture 2.

Why Cryptocurrencies?



Bitcoin: A Peer-to-Peer Electronic Cash System by Satoshi Nakamoto (2008)

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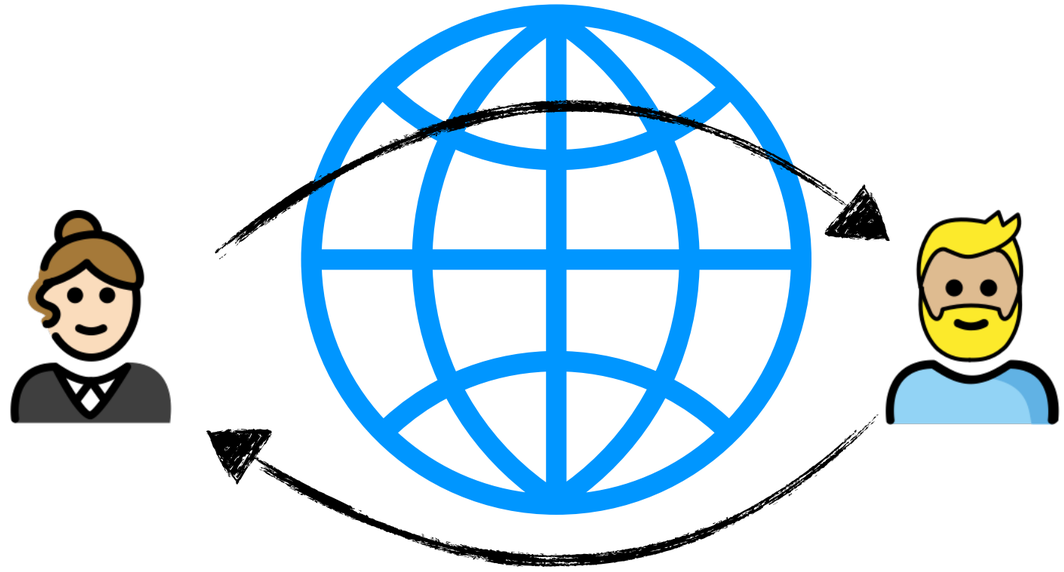

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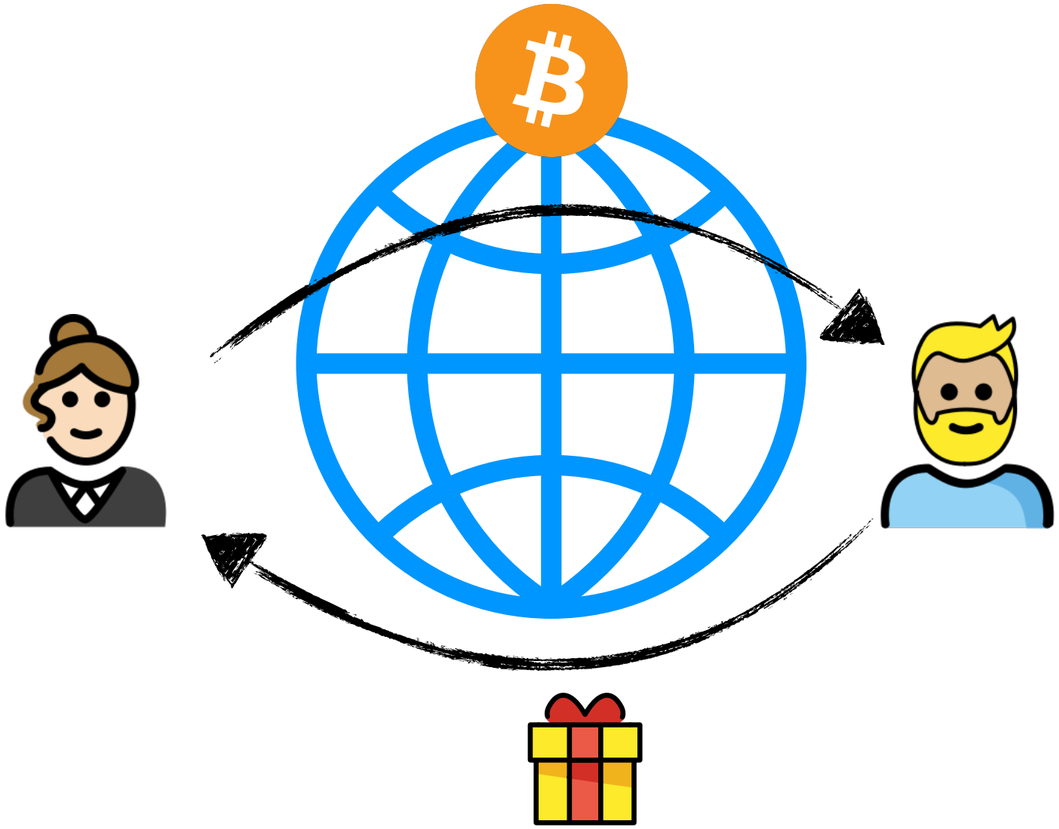

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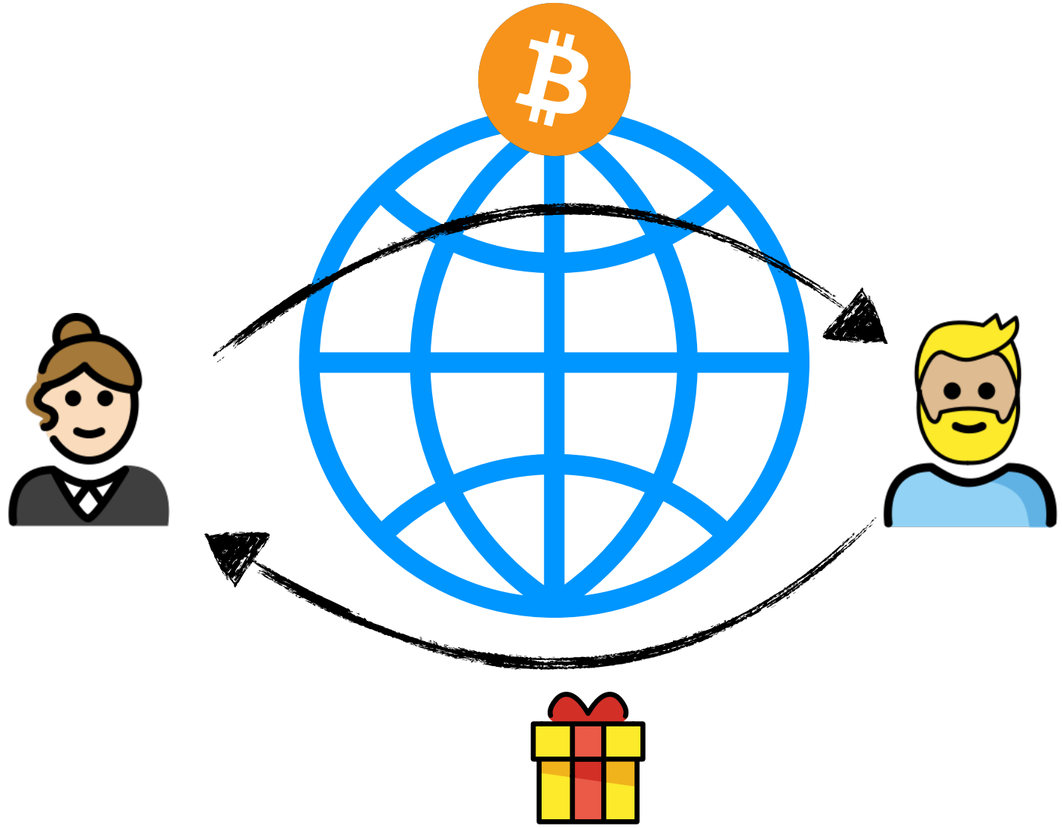

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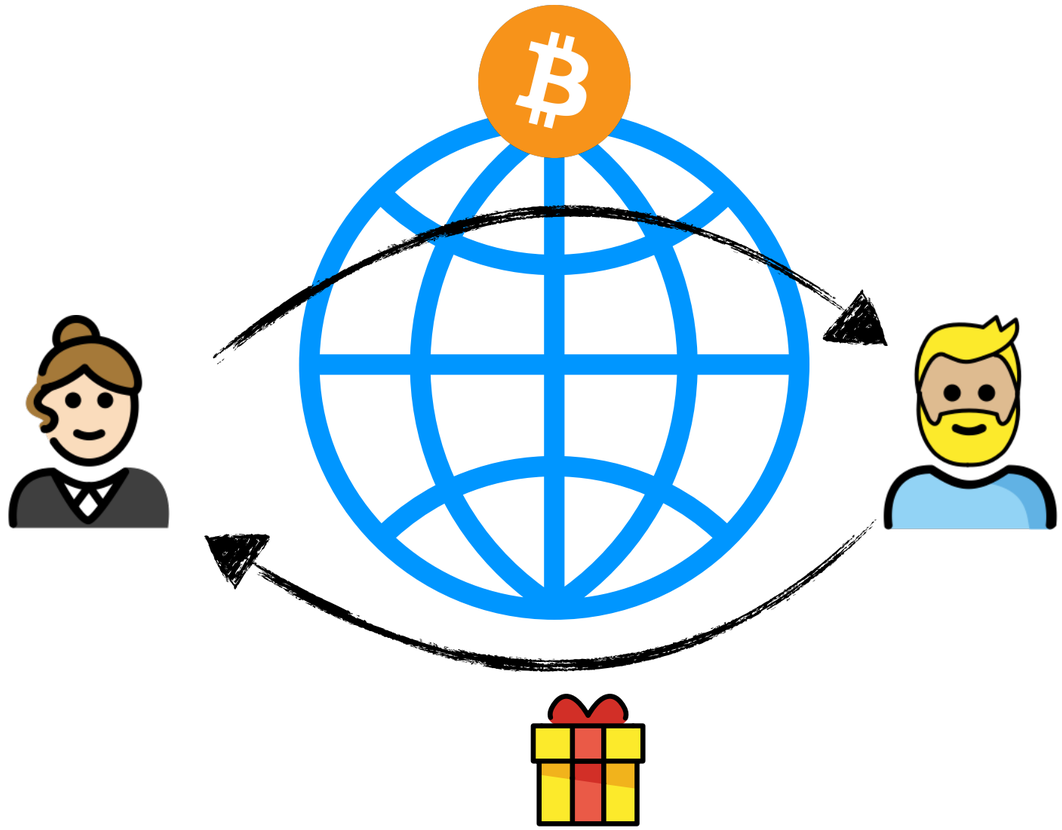

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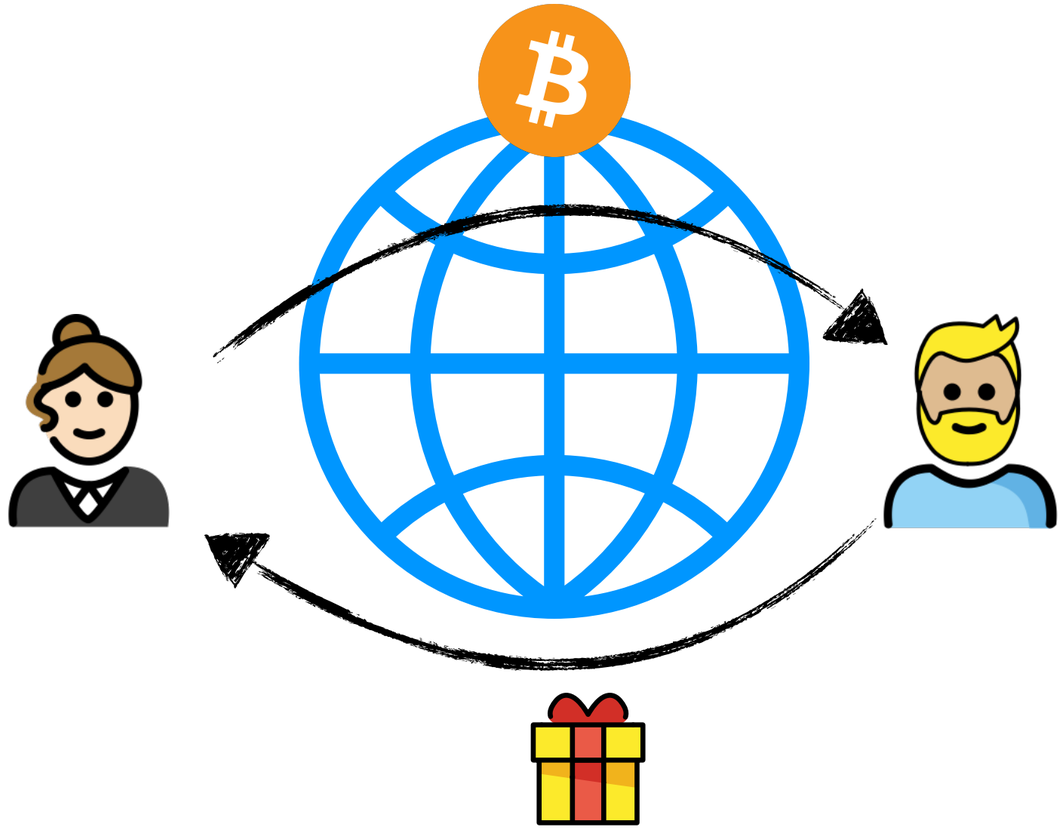

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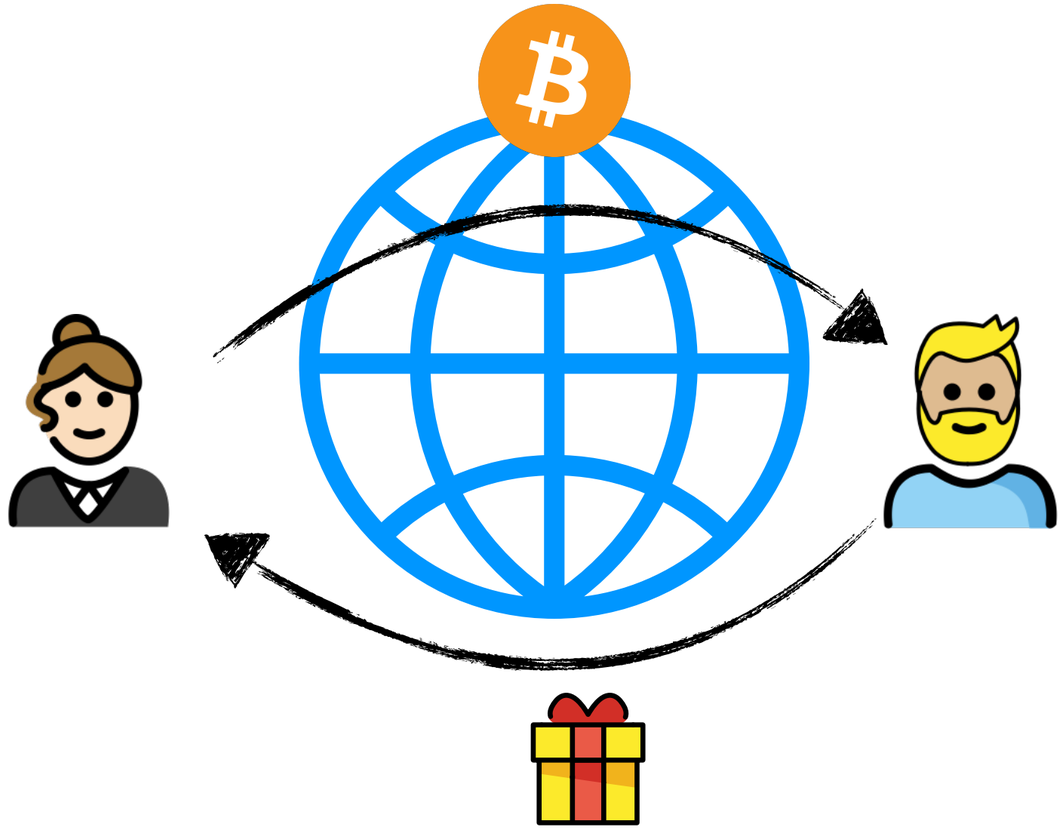

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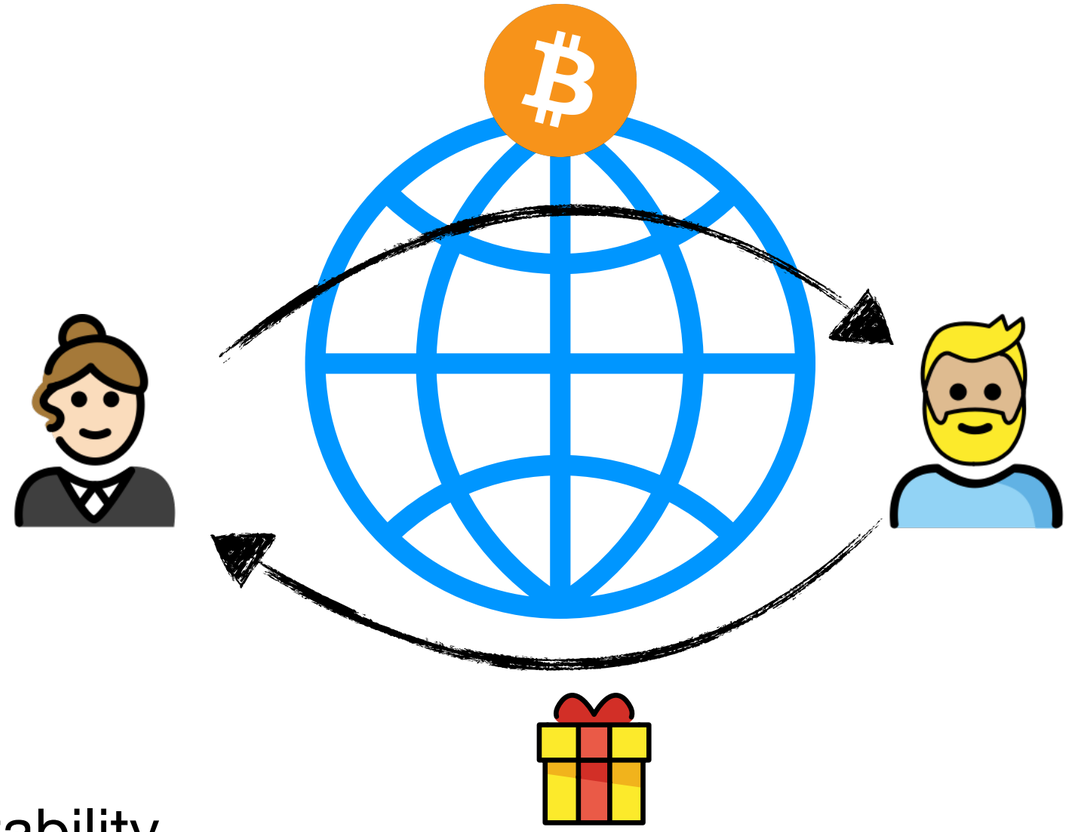

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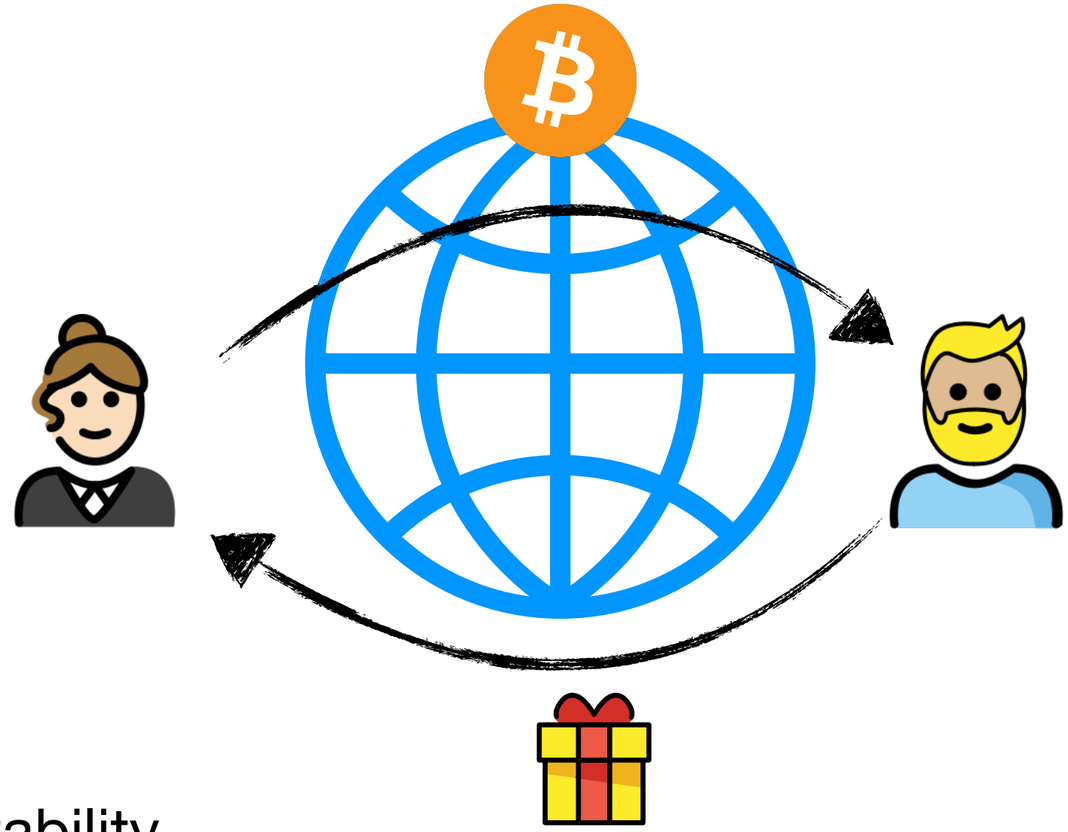

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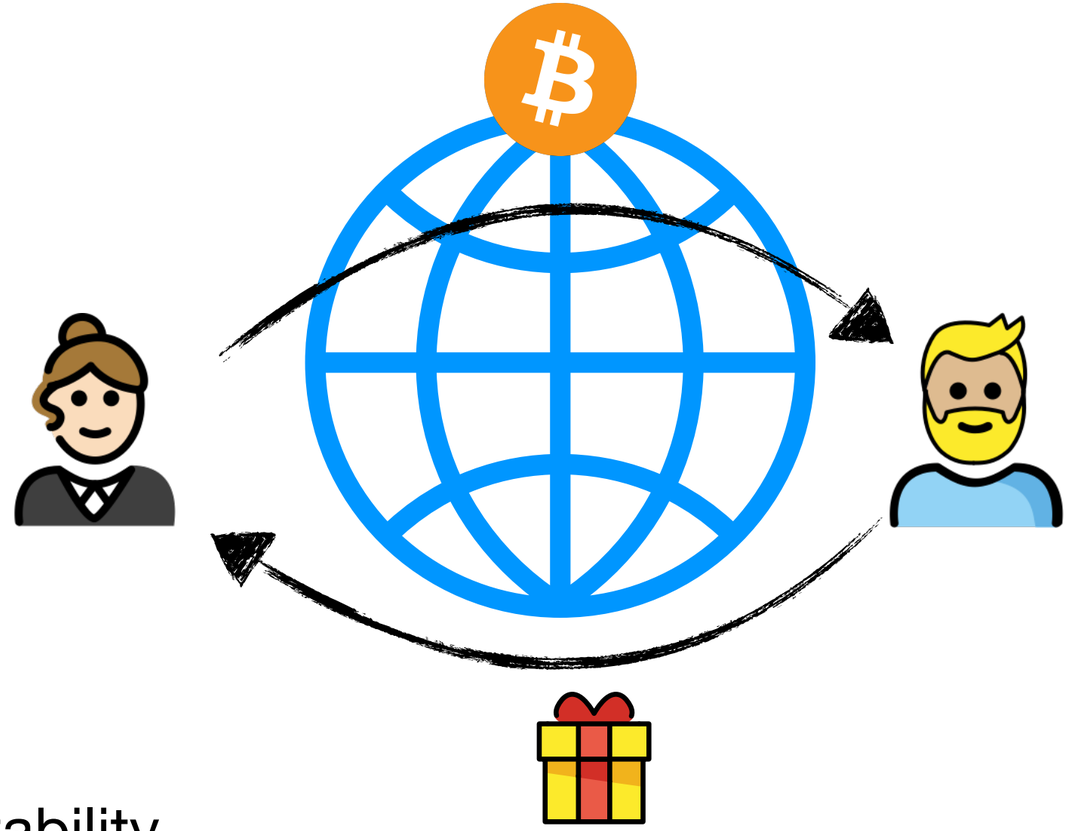

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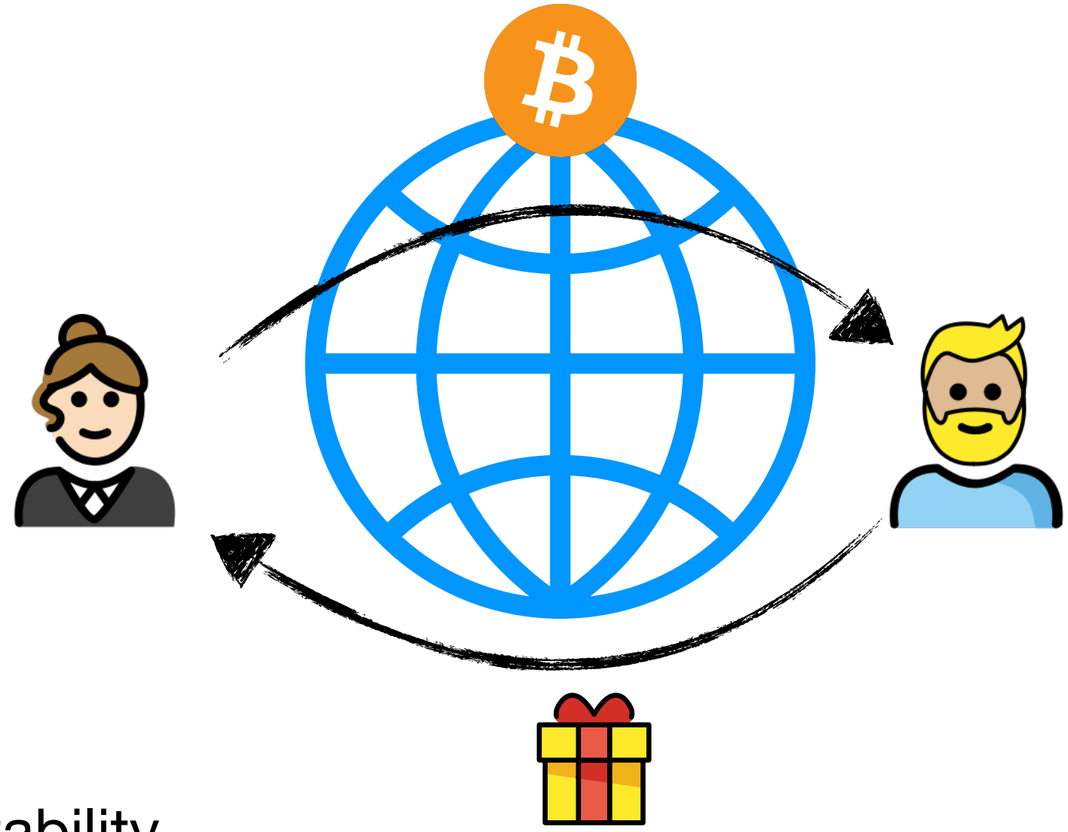

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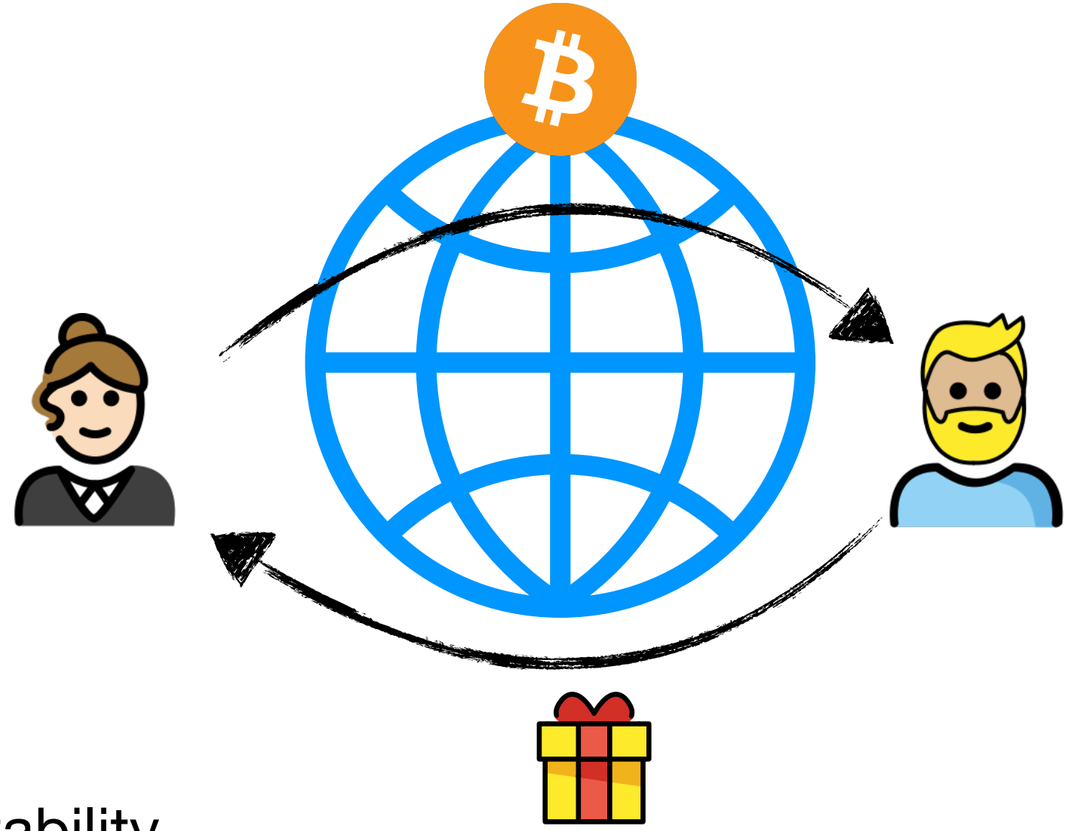

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+ (Pseudo) Anonymity & no double-spending

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replace the bank with
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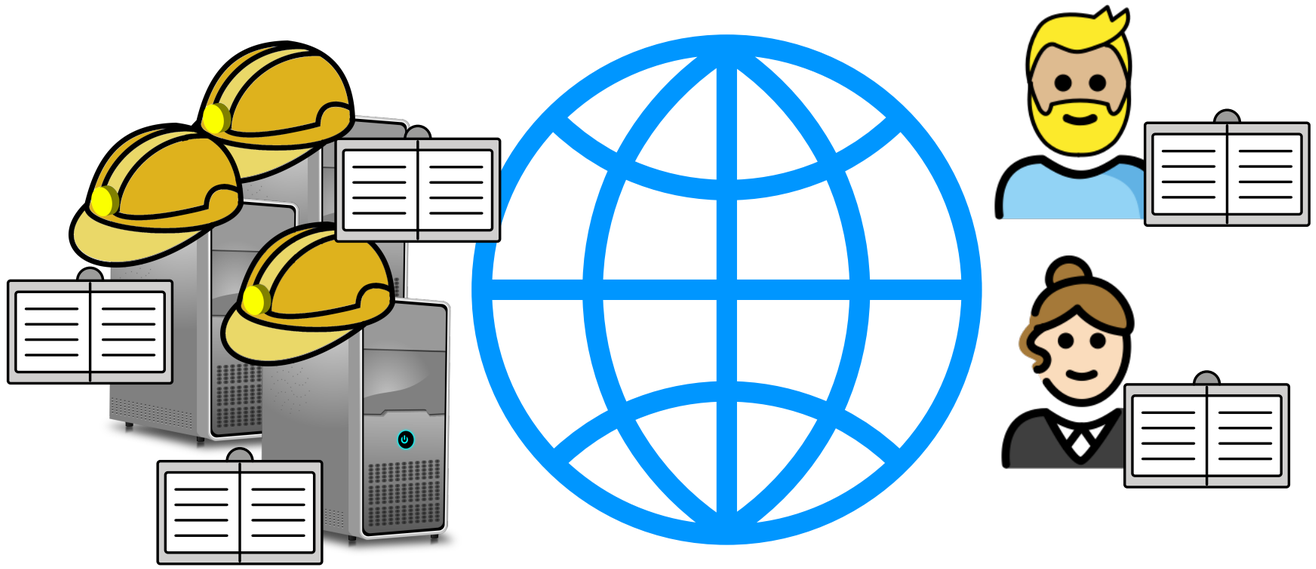

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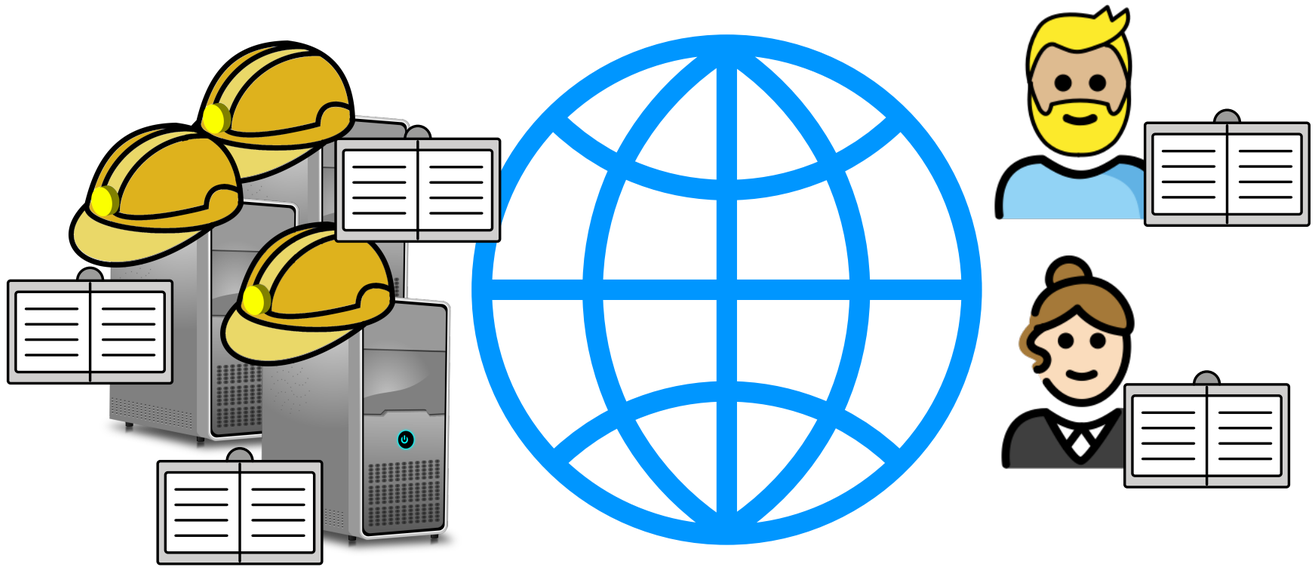

Main challenges

1) How to create a digital bulletin board (distributed ledger)?
2) How to agree on ONE ledger view (finality layer)?
3) How to create money?
4) How to prevent double-spending?

Bitcoin.org hack nets giveaway scammers $17,000 overnight

Adam Bannister 24 September 2021 at 13:28 UTC
Updated: 05 October 2021 at 08:36 UTC

Social Engineering | Cryptocurrency | Cybercrime

Open source project back online after fraudsters dangled double-your-money lure

Cryptocurrency funds removed from 6,000 Coinbase accounts due to flaw in SMS authentication

Jessica Haworth 04 October 2021 at 14:09 UTC
Updated: 06 October 2021 at 13:21 UTC

2FA | Cryptocurrency | Cybercrime

Lecture Agenda

How To Set Up a Bulletin Board

- Hash Functions
- Proof of Work (Cryptographic Puzzles)
- Example: Bitcoin

How To Generate Money

- Mining Blocks
- Merkle Trees
- Block Reward System

How To Circulate Money

- Transactions
- ECDSA

Some Problems With Blockchains

- Freshness of the Genesis Block
- 51% Attack
- Proof of Work VS Proof of Stake

Advanced Tools

- Proofs of Knowledge
- zkSNARKs
- MPC

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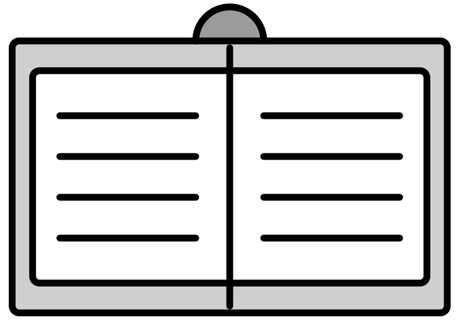

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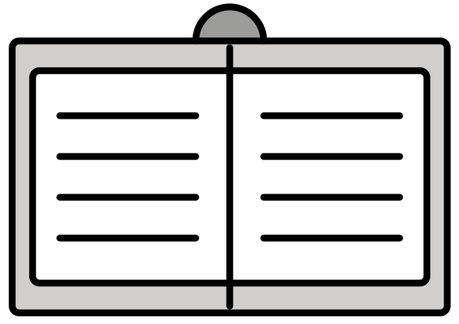

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1) the past is immutable
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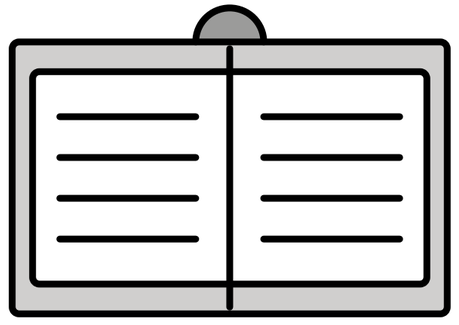

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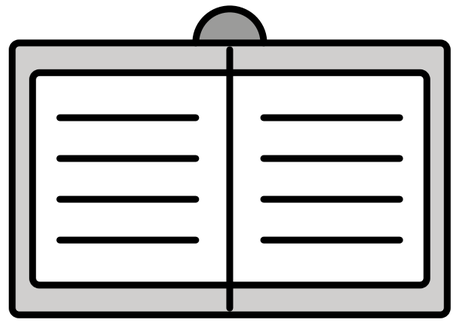

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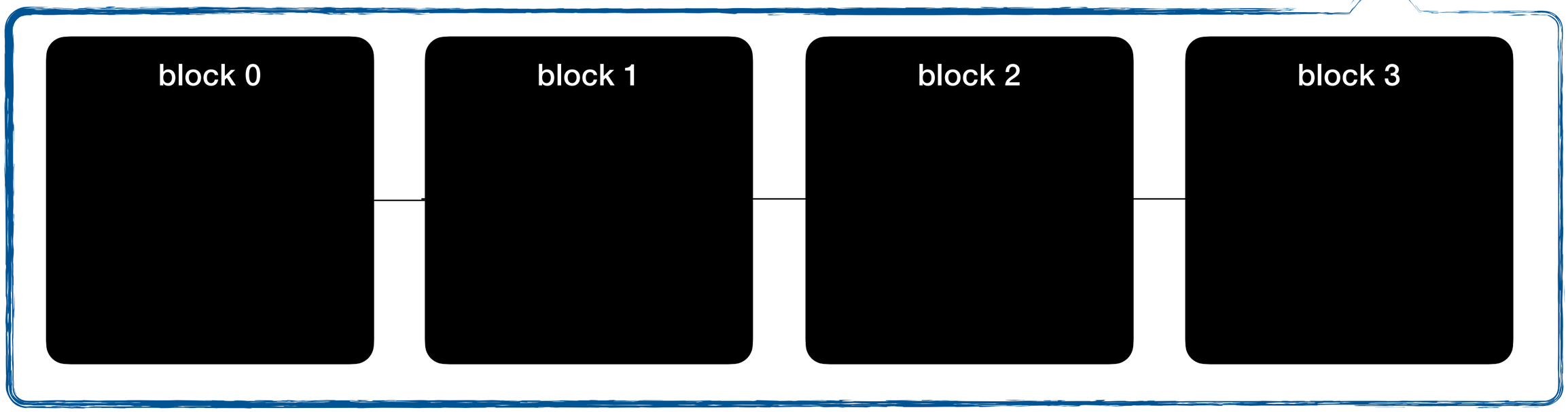

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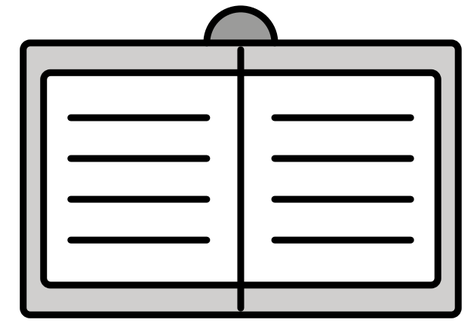

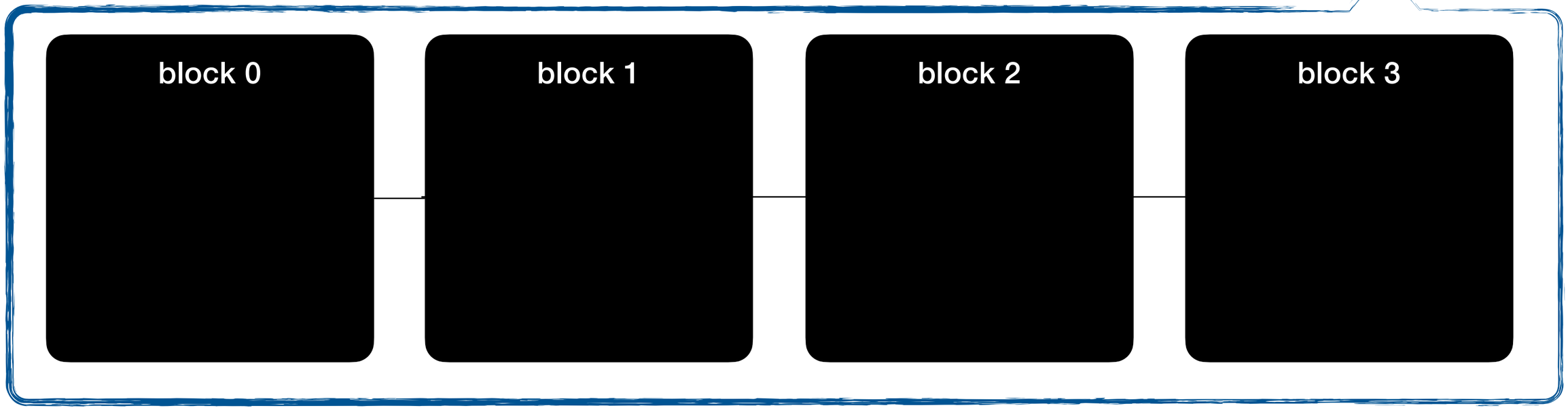

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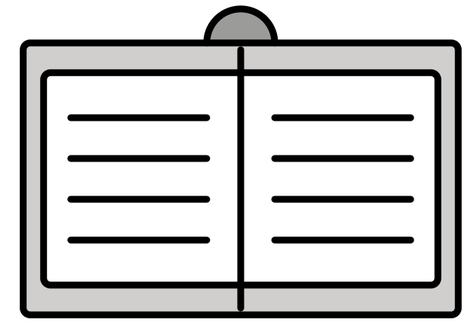

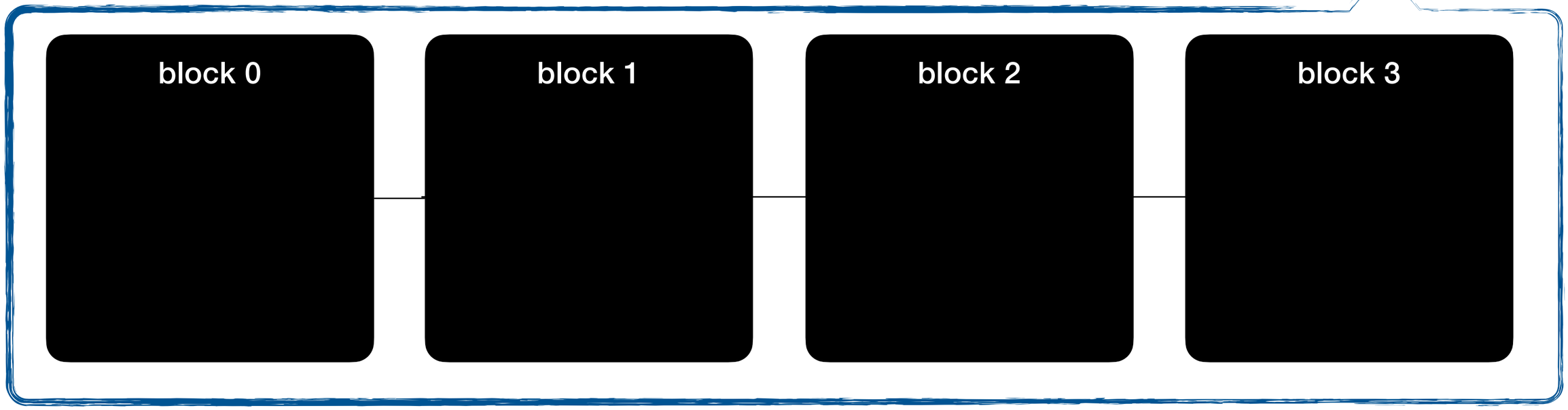

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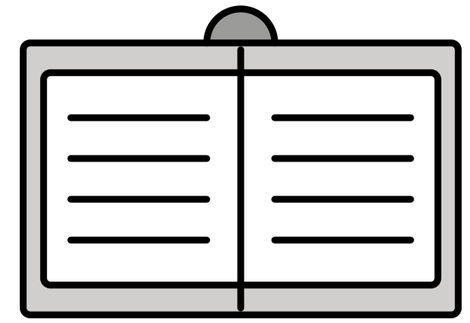

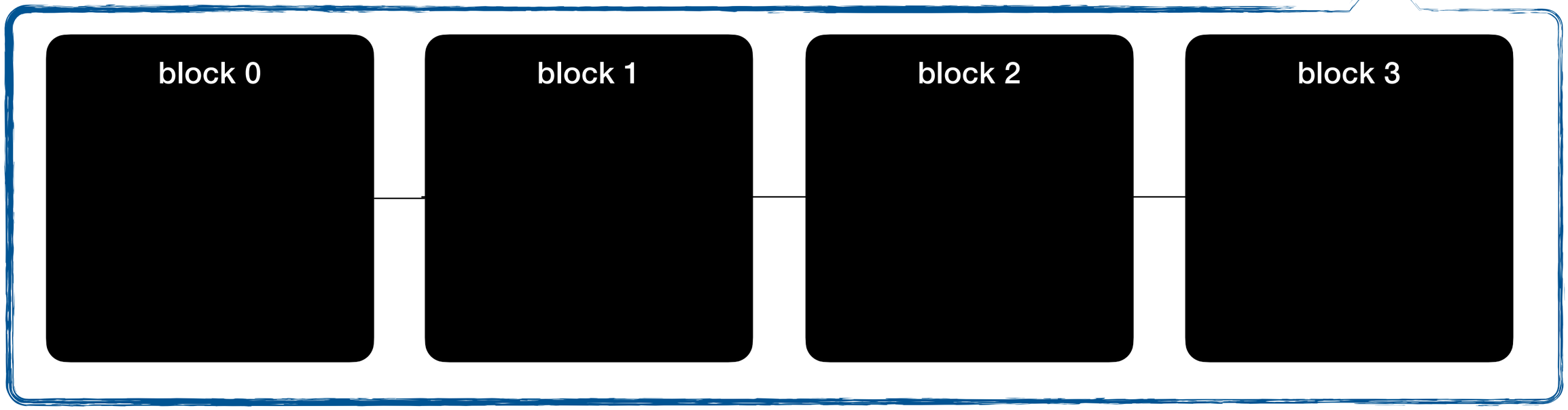

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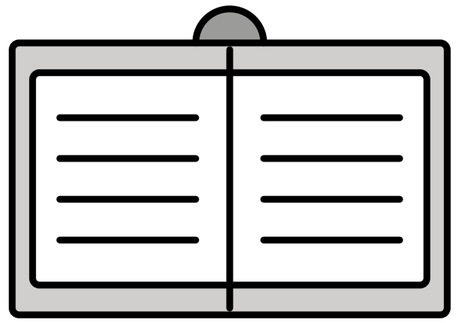

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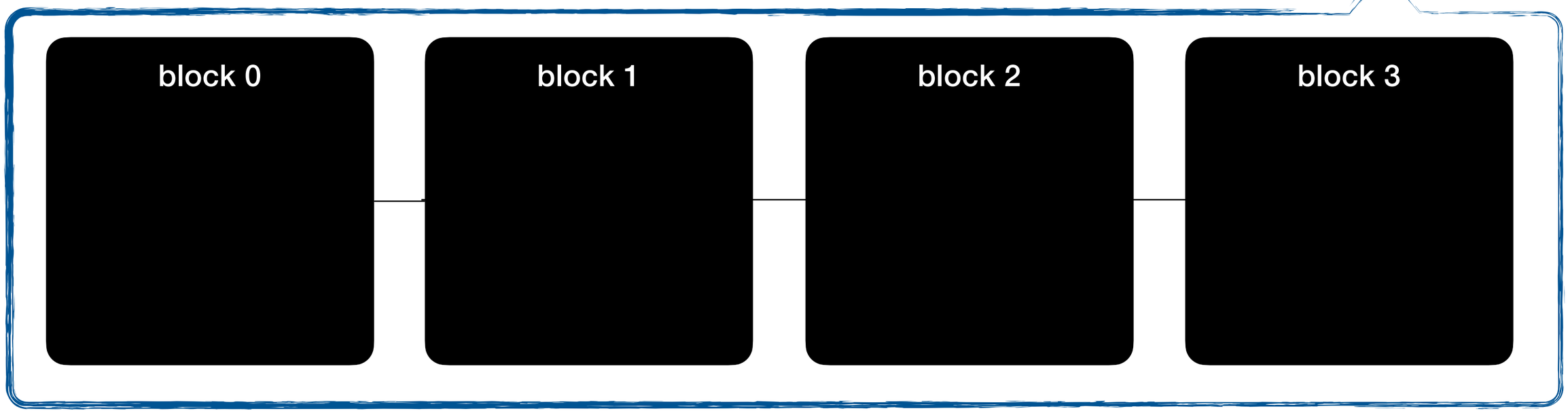

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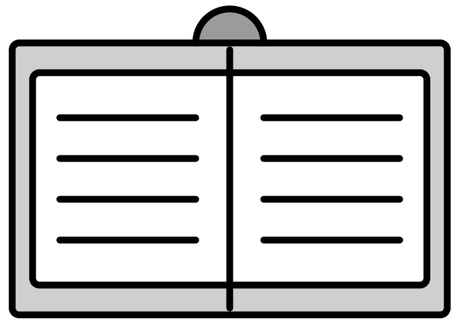

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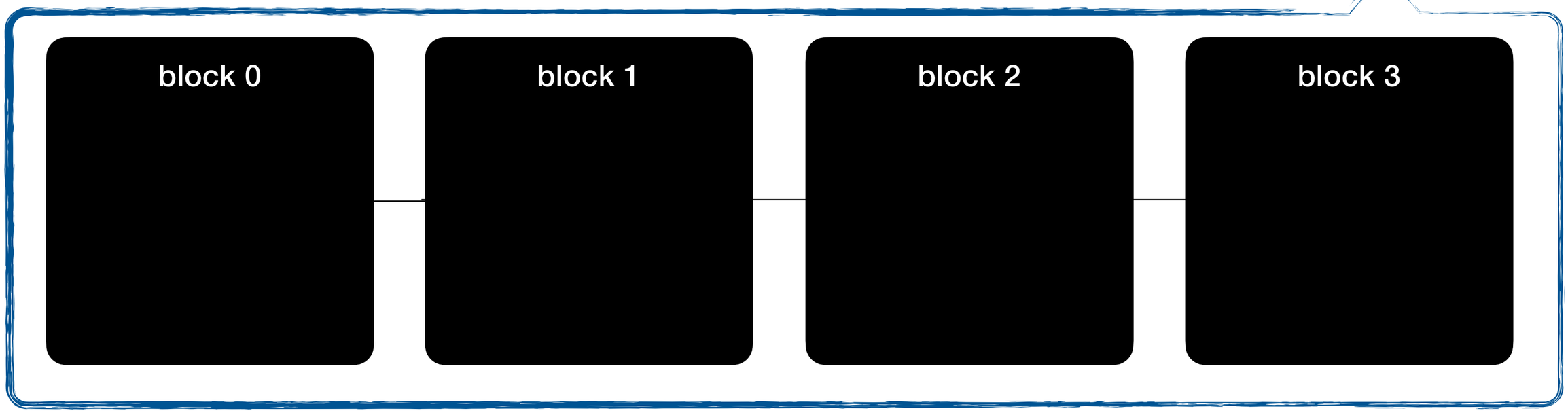

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how can we implement this property using a cryptographic object?

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- Any size input
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“avalanche effect”

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(you can find it eventually, 2^256 ~ 10^78 operations)

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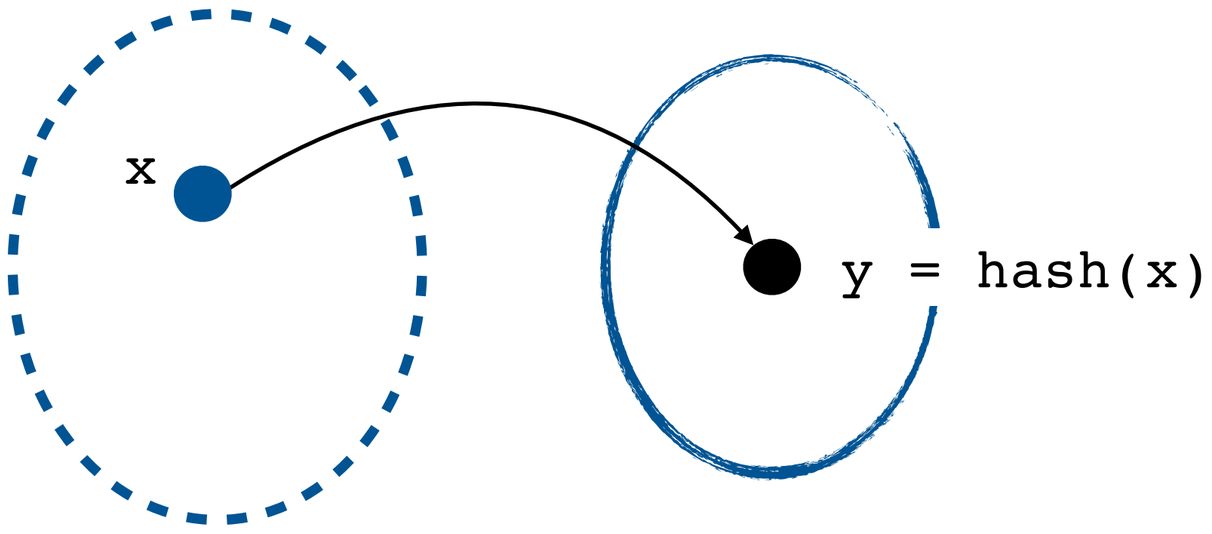

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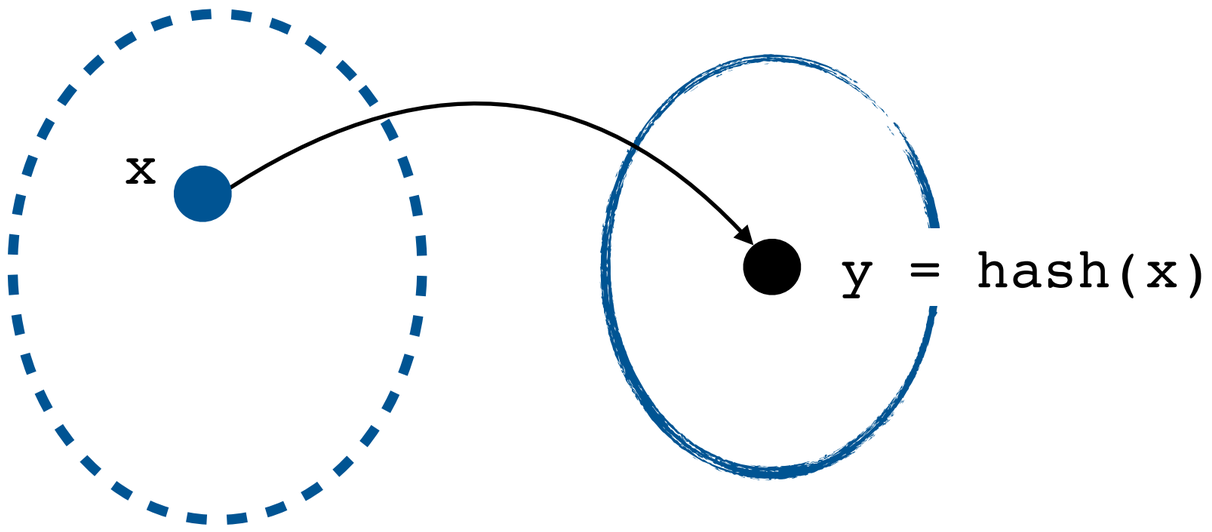

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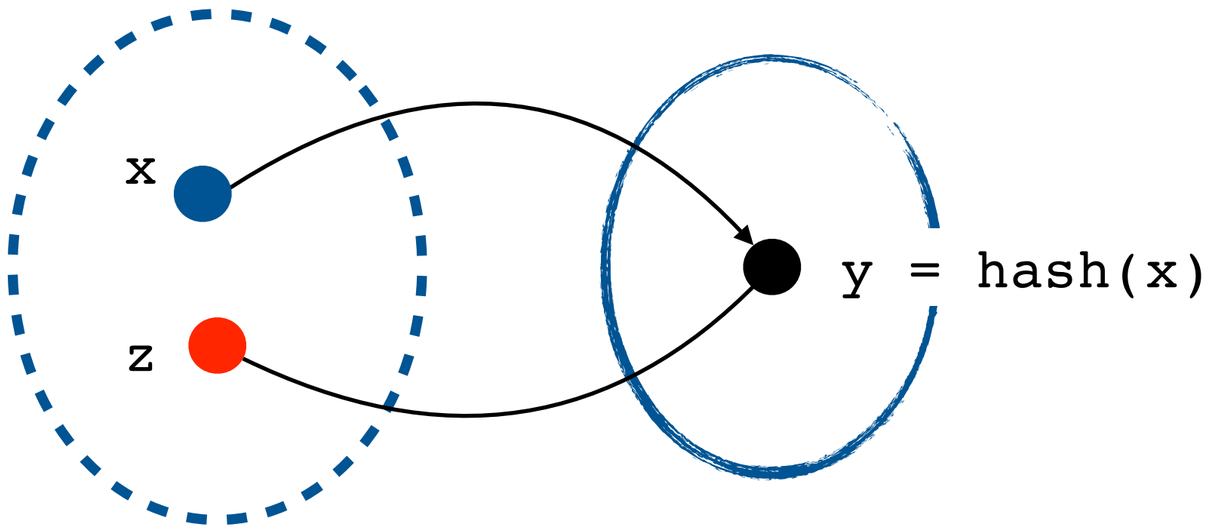

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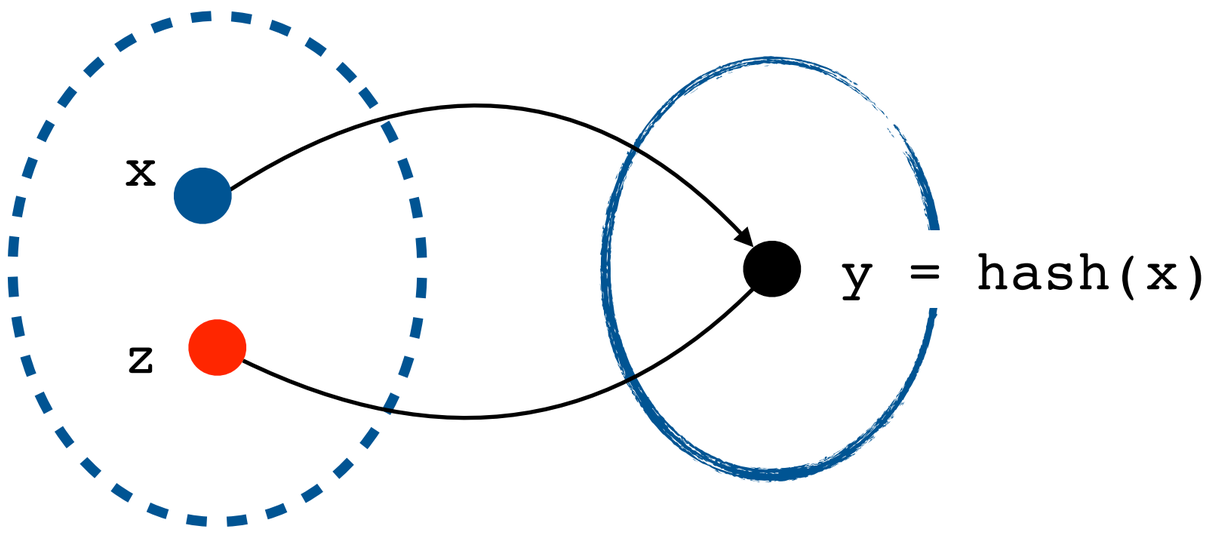
Second preimage resistance

Collision resistance

(you can find it eventually, in less than 2^256, expected ~ 2^128)



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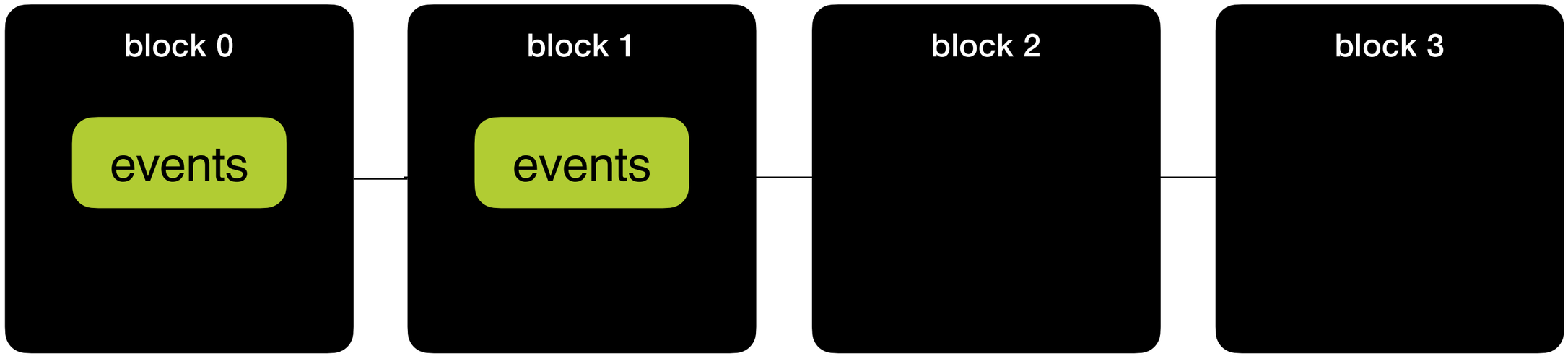

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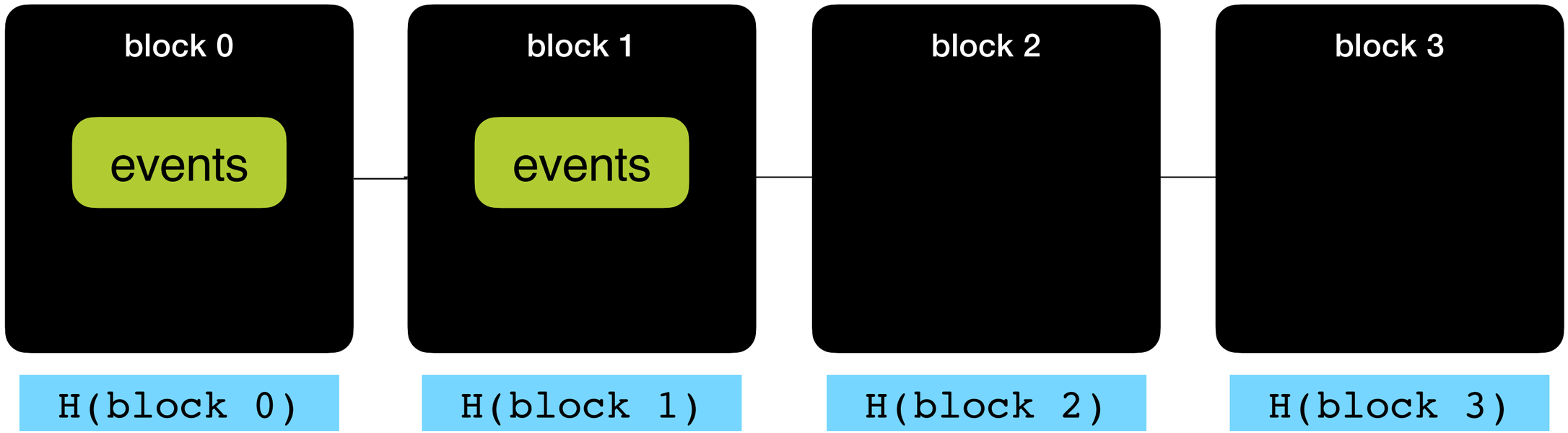

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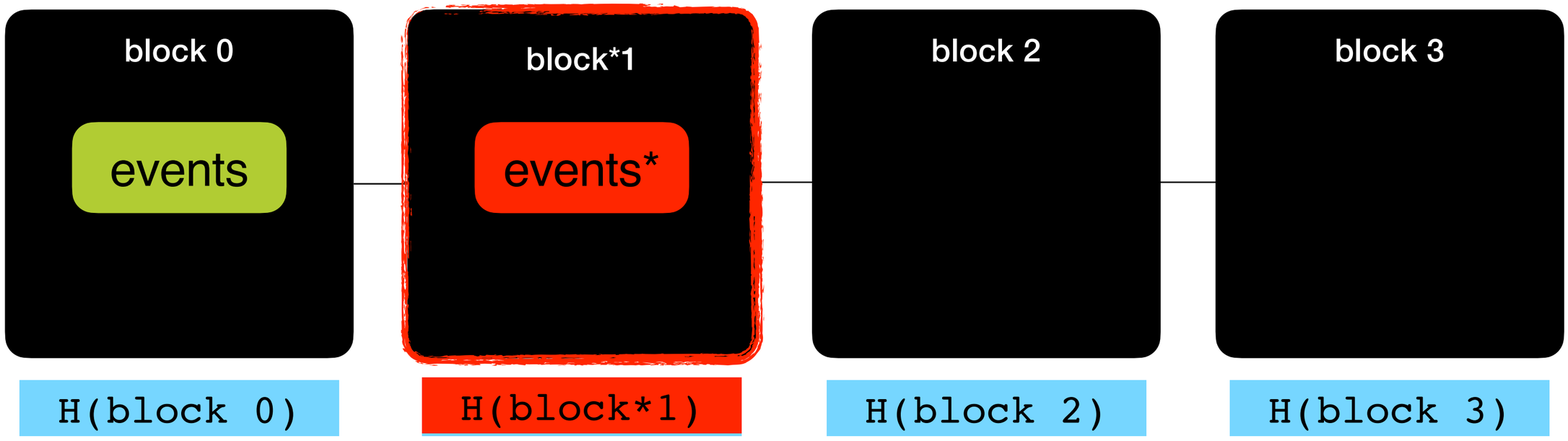

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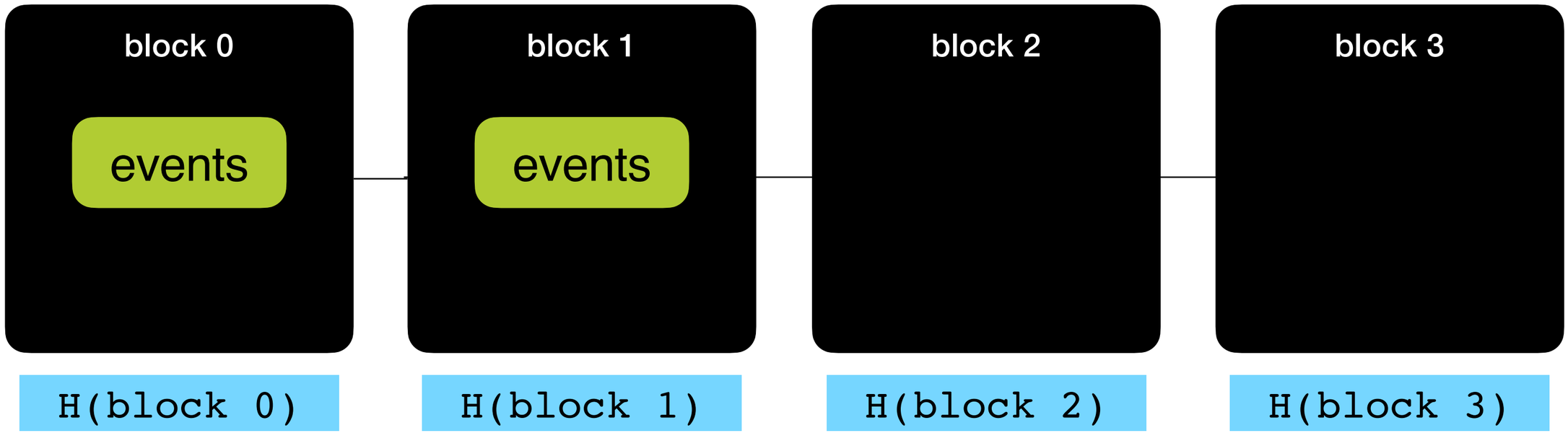

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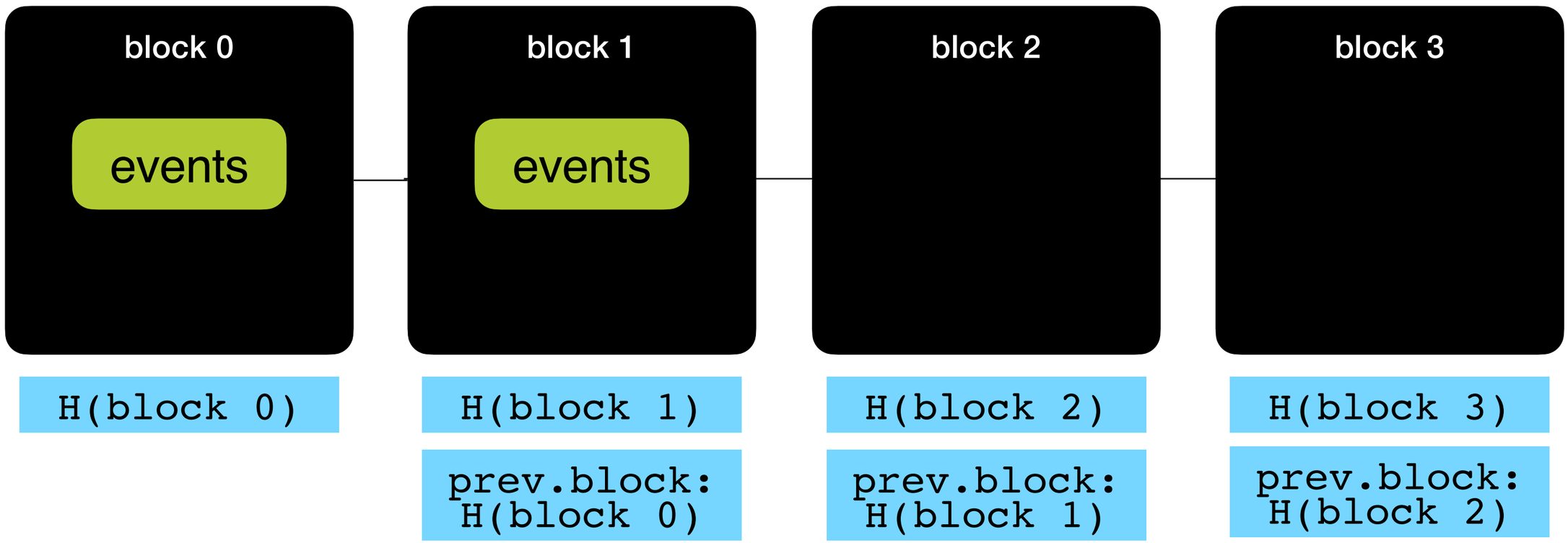

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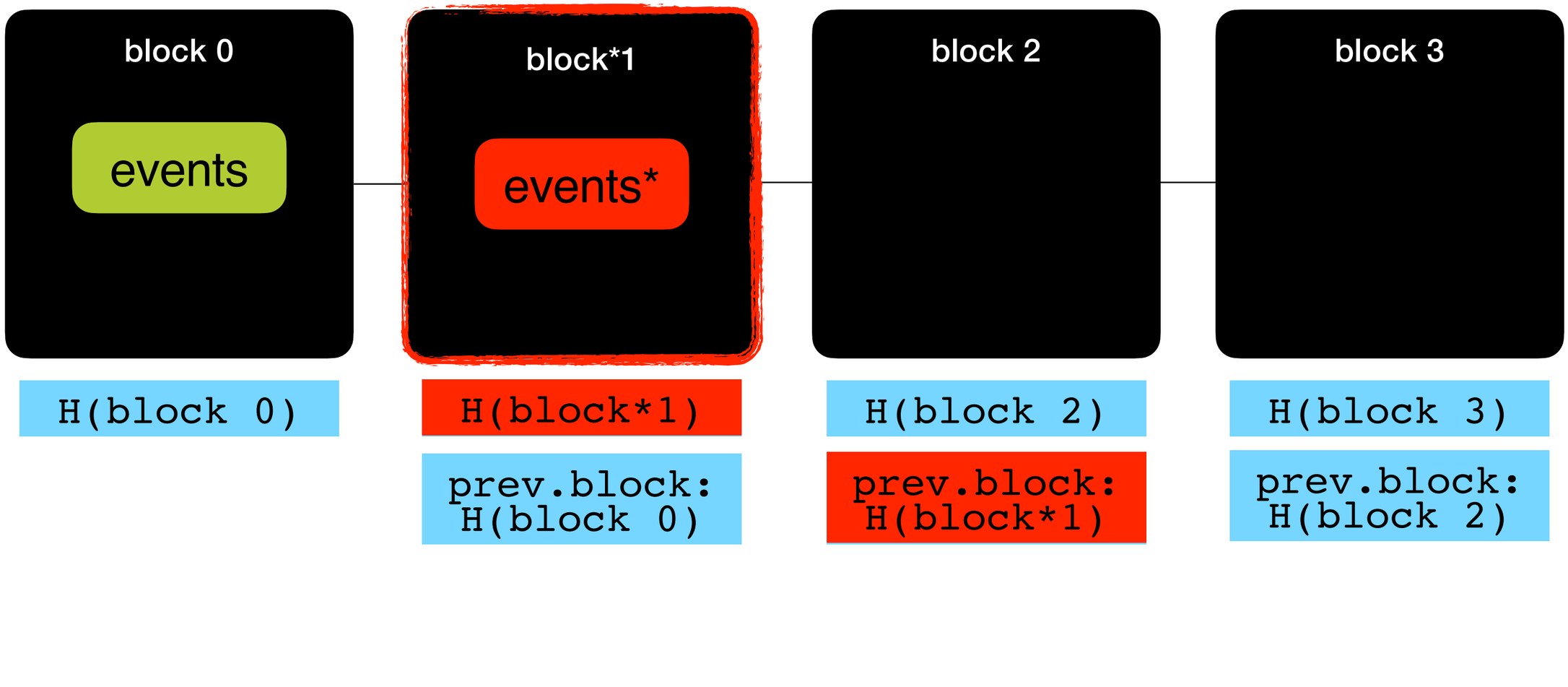

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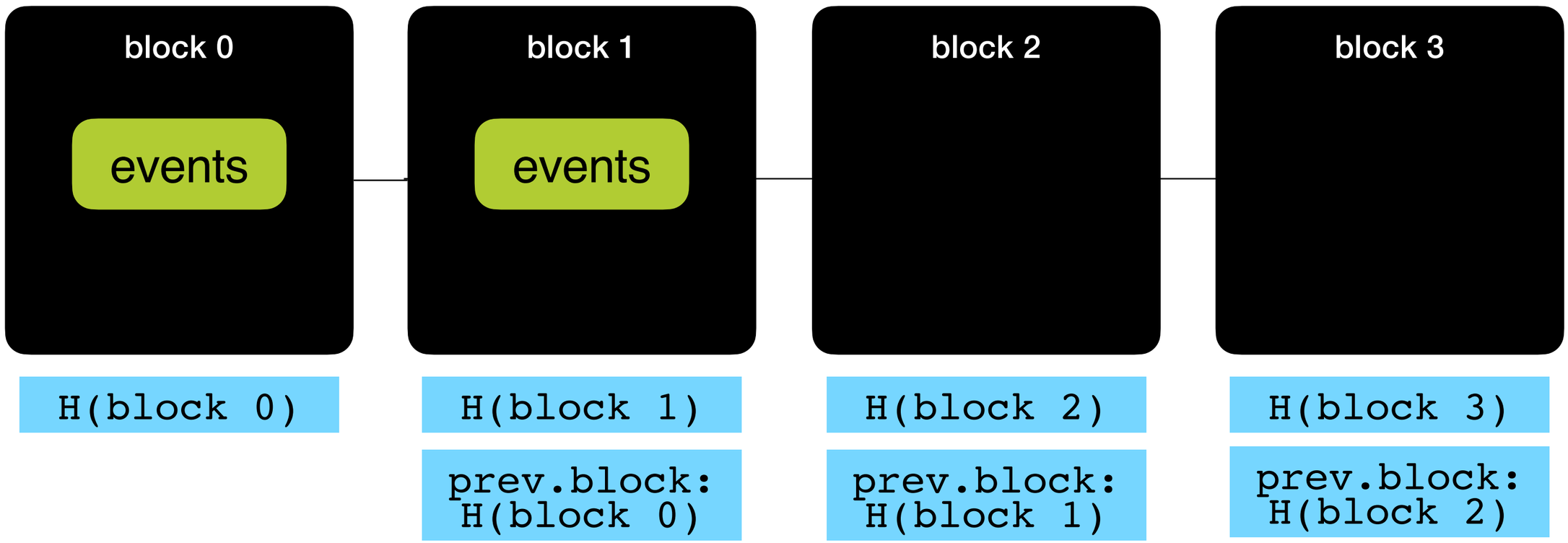

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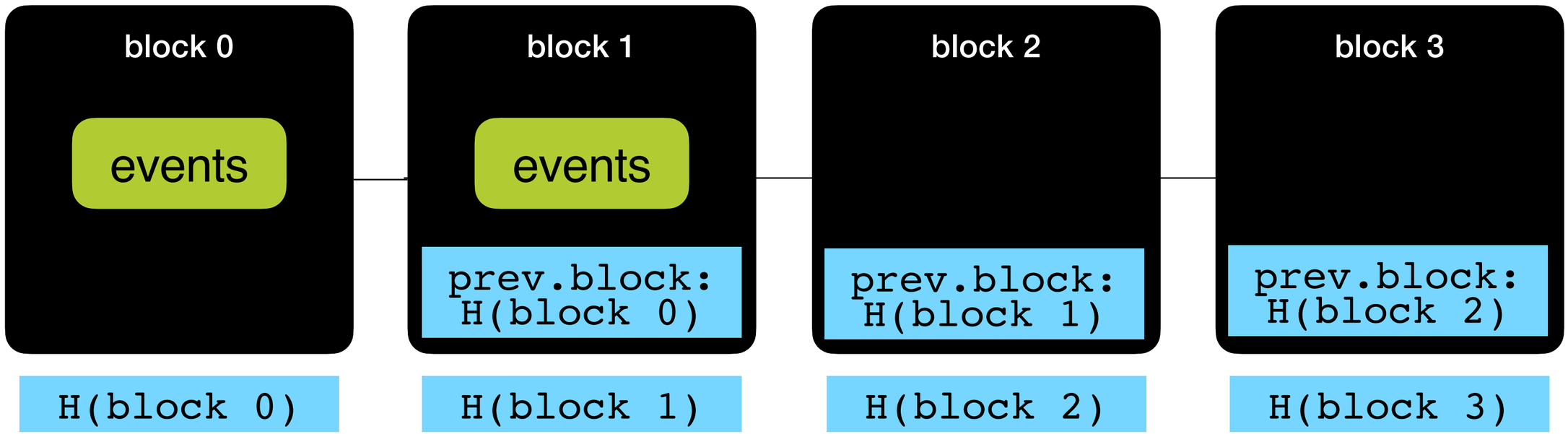

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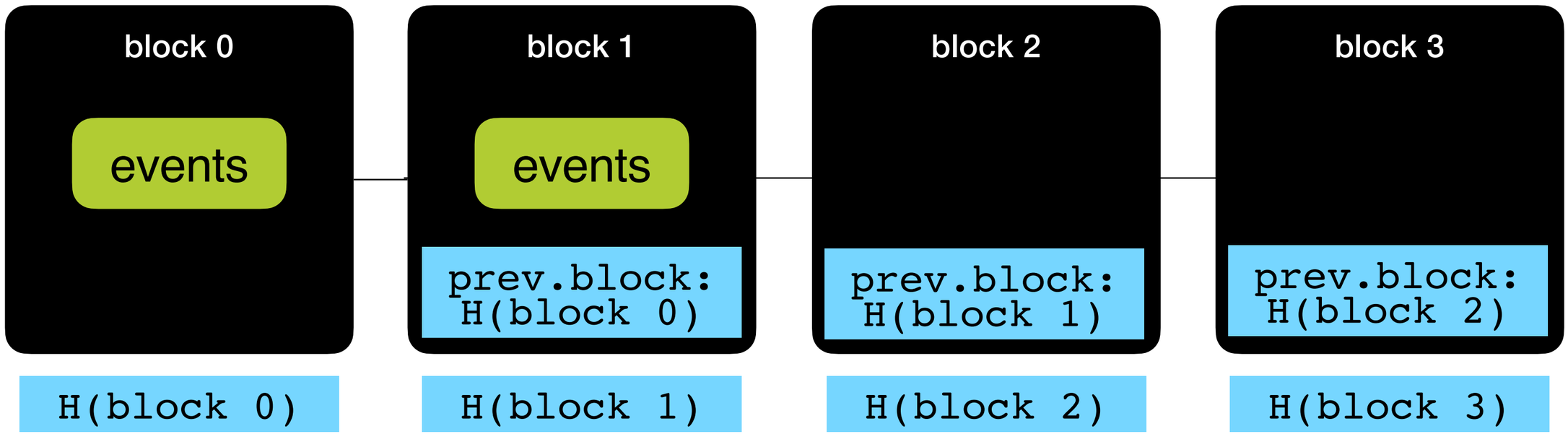
use the hash function to chain blocks



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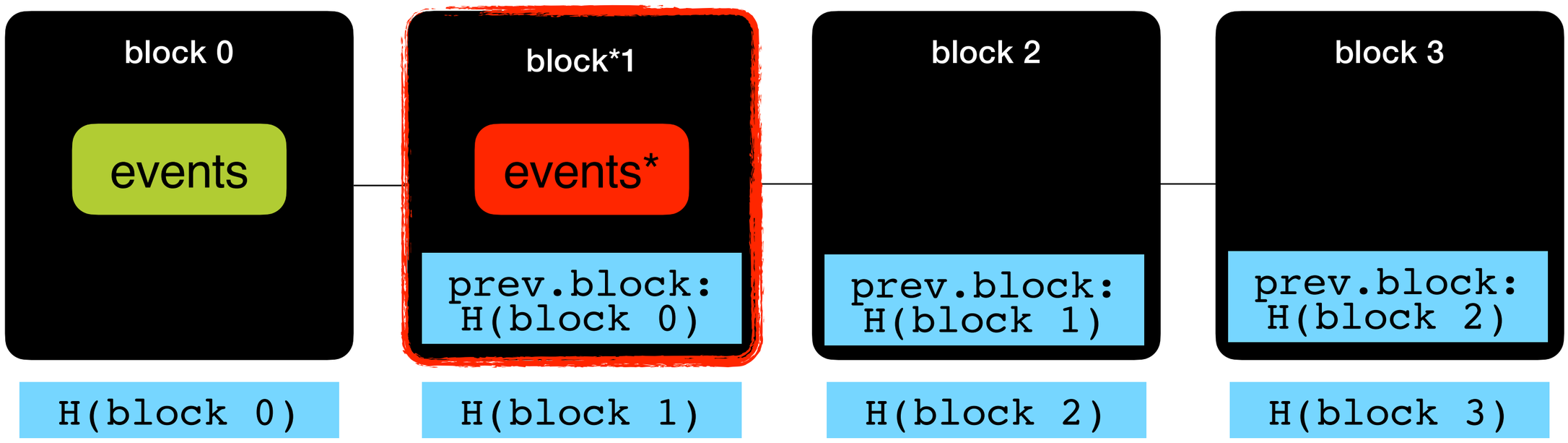

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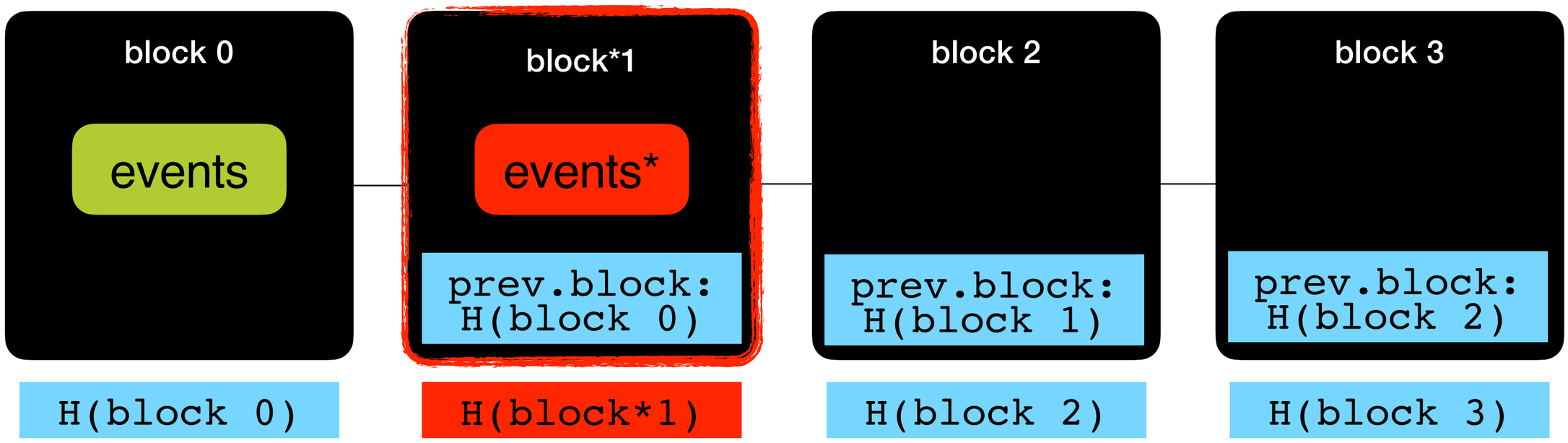

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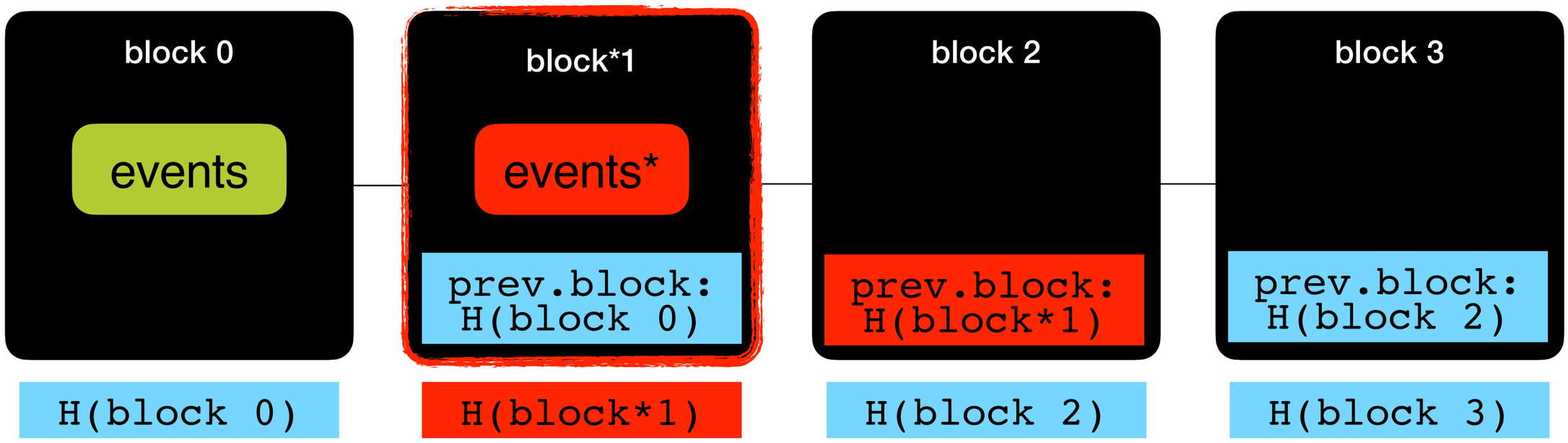

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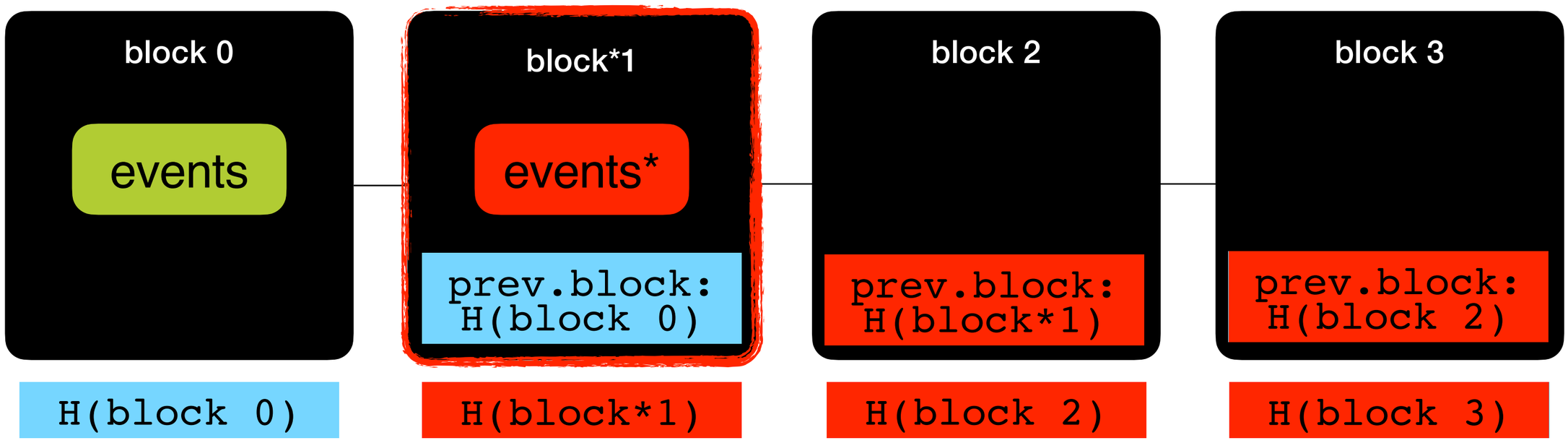

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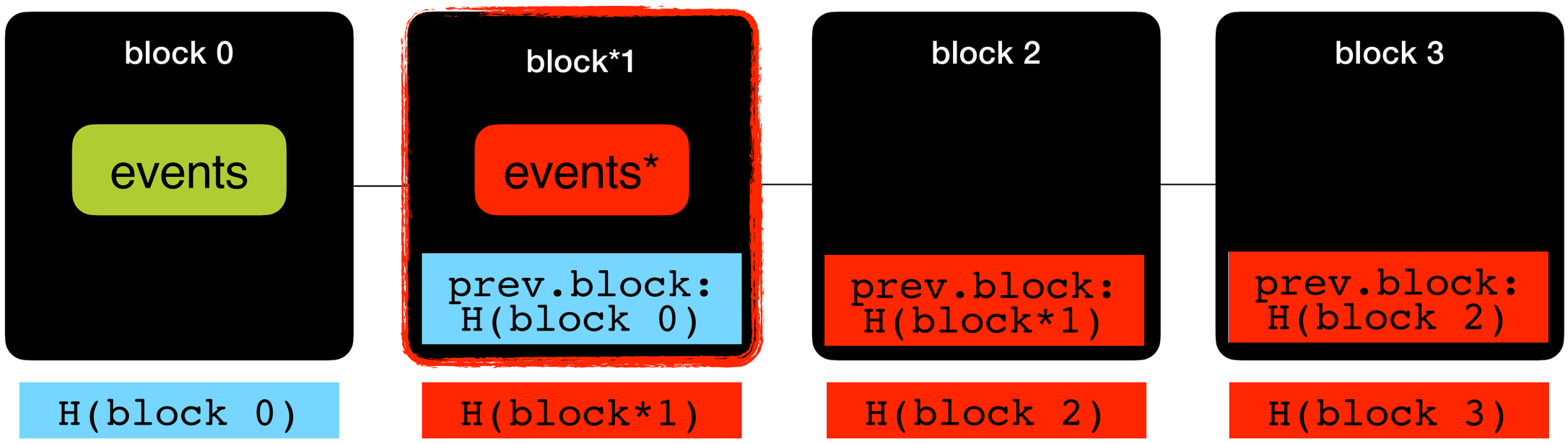

⇨ Any change to an ‘old’ block affects all following blocks

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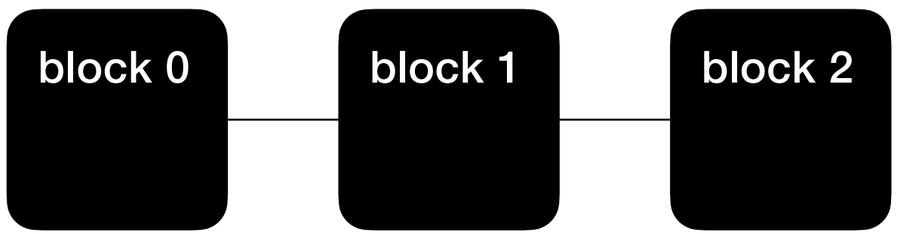

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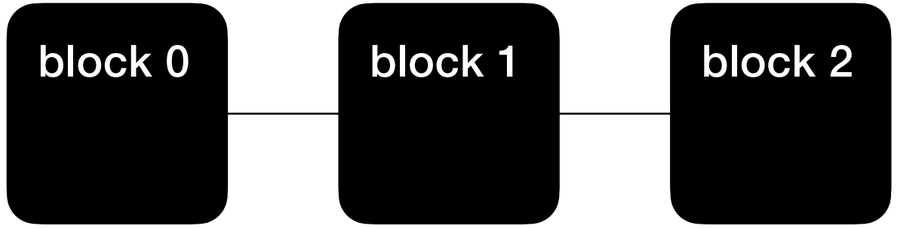

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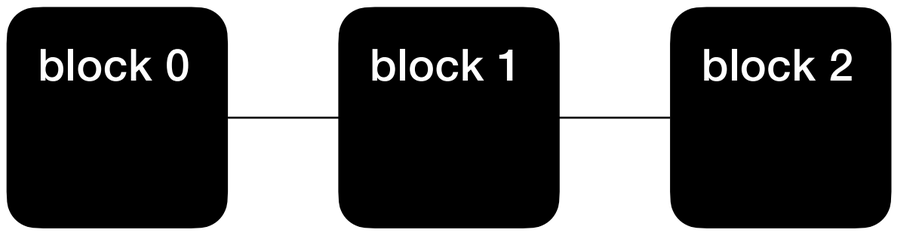

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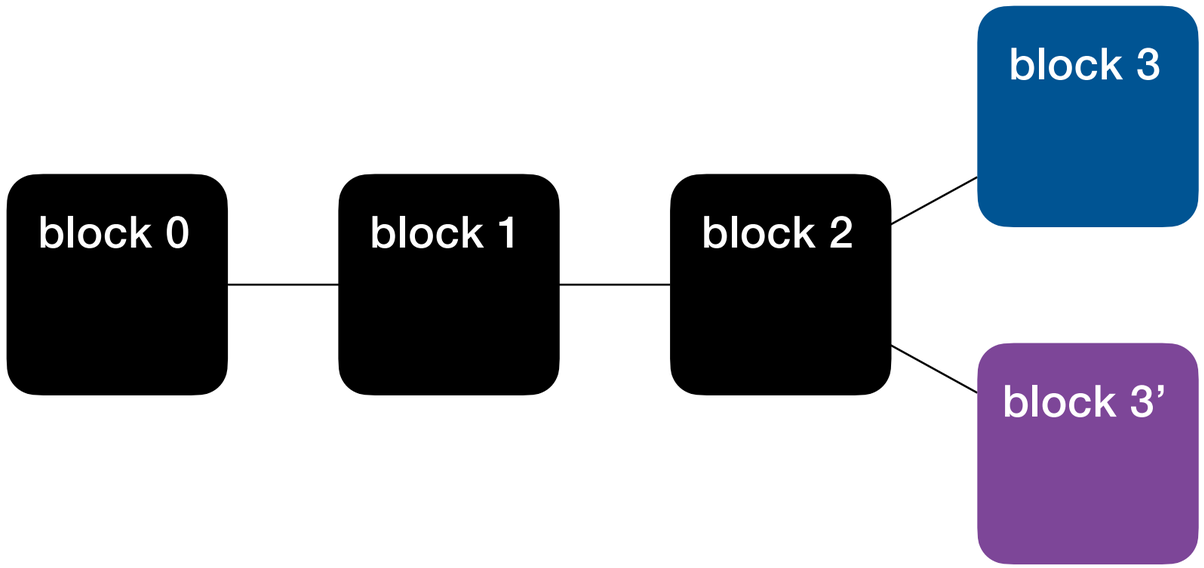

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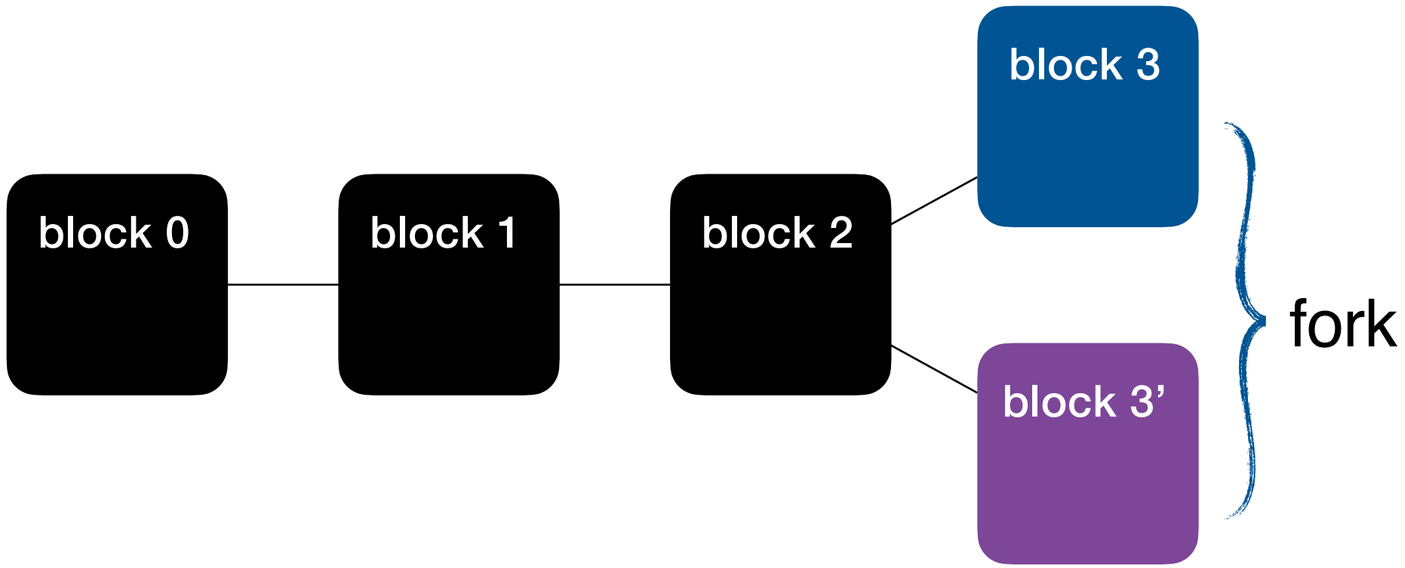

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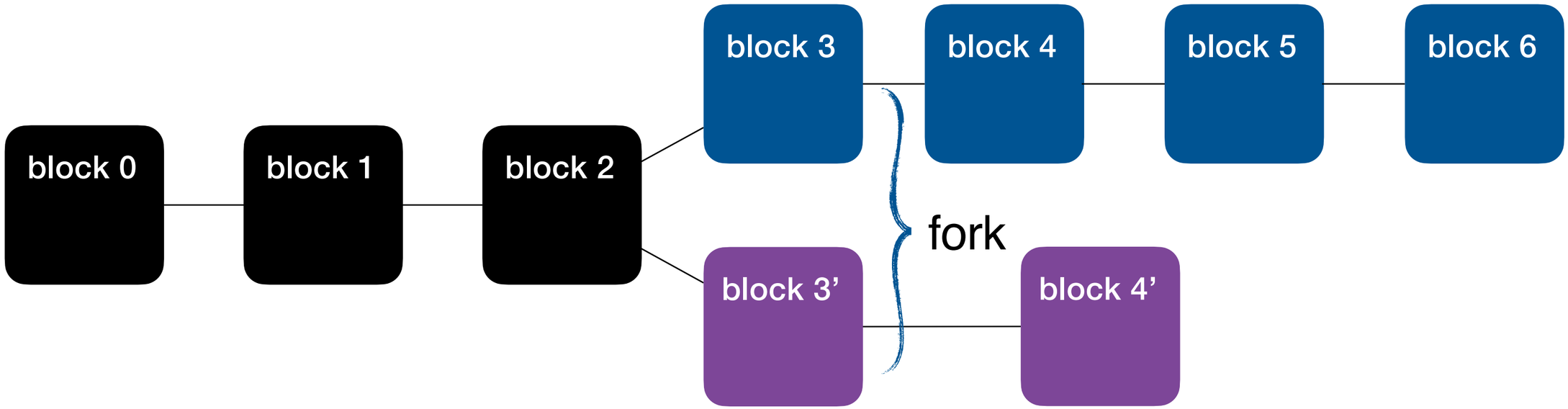

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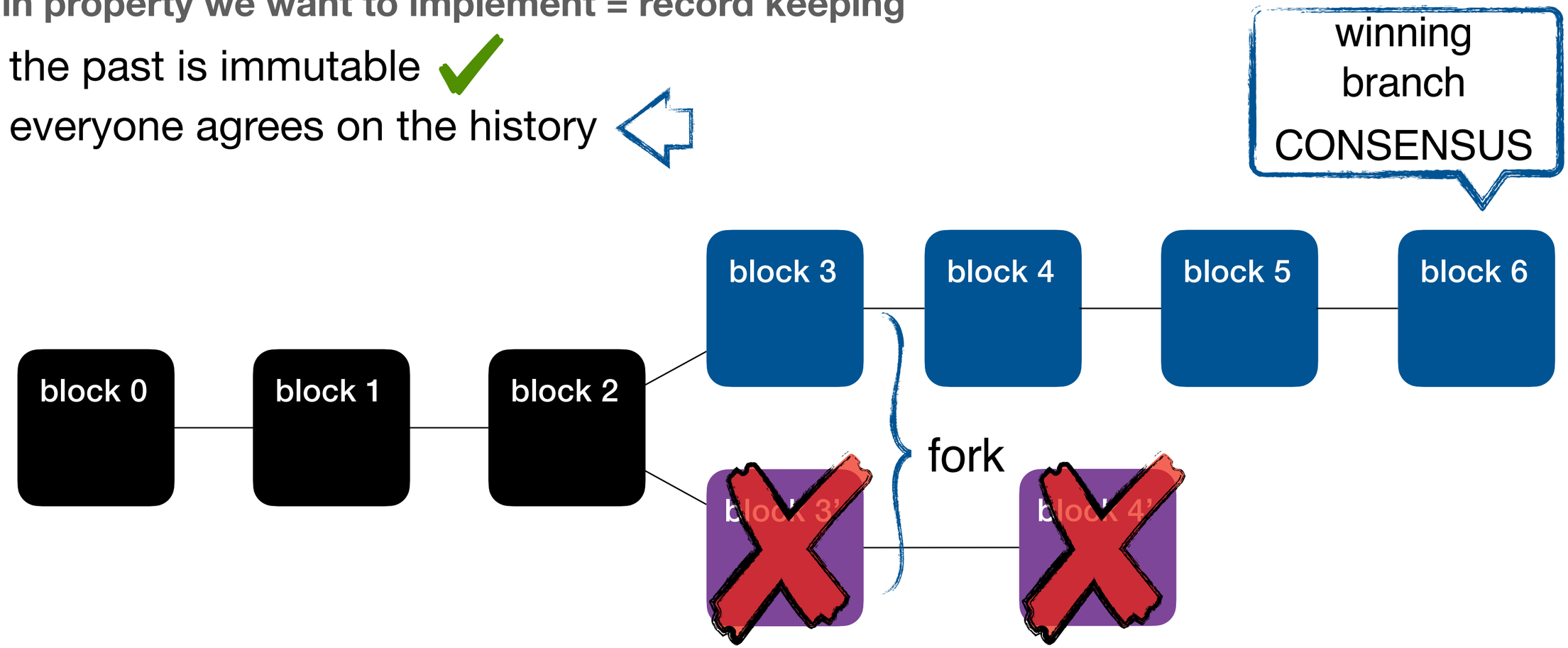

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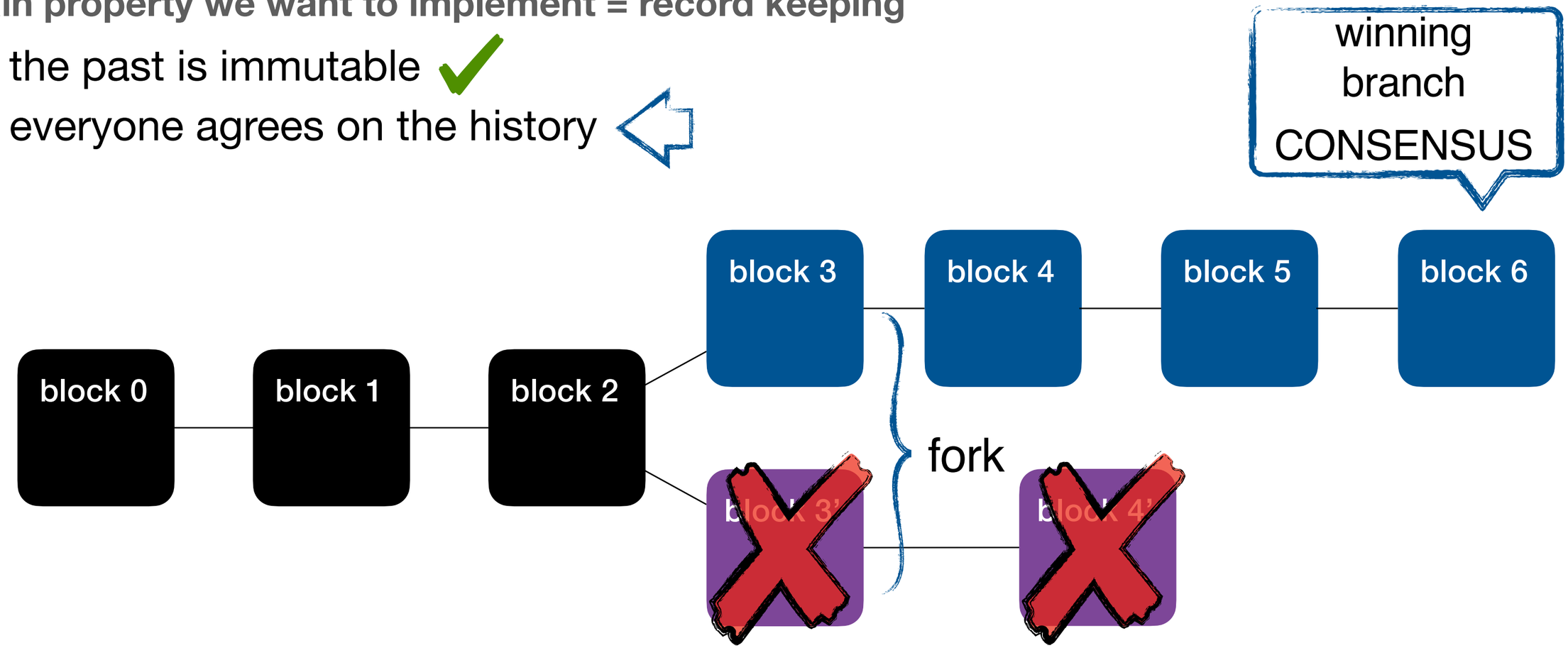

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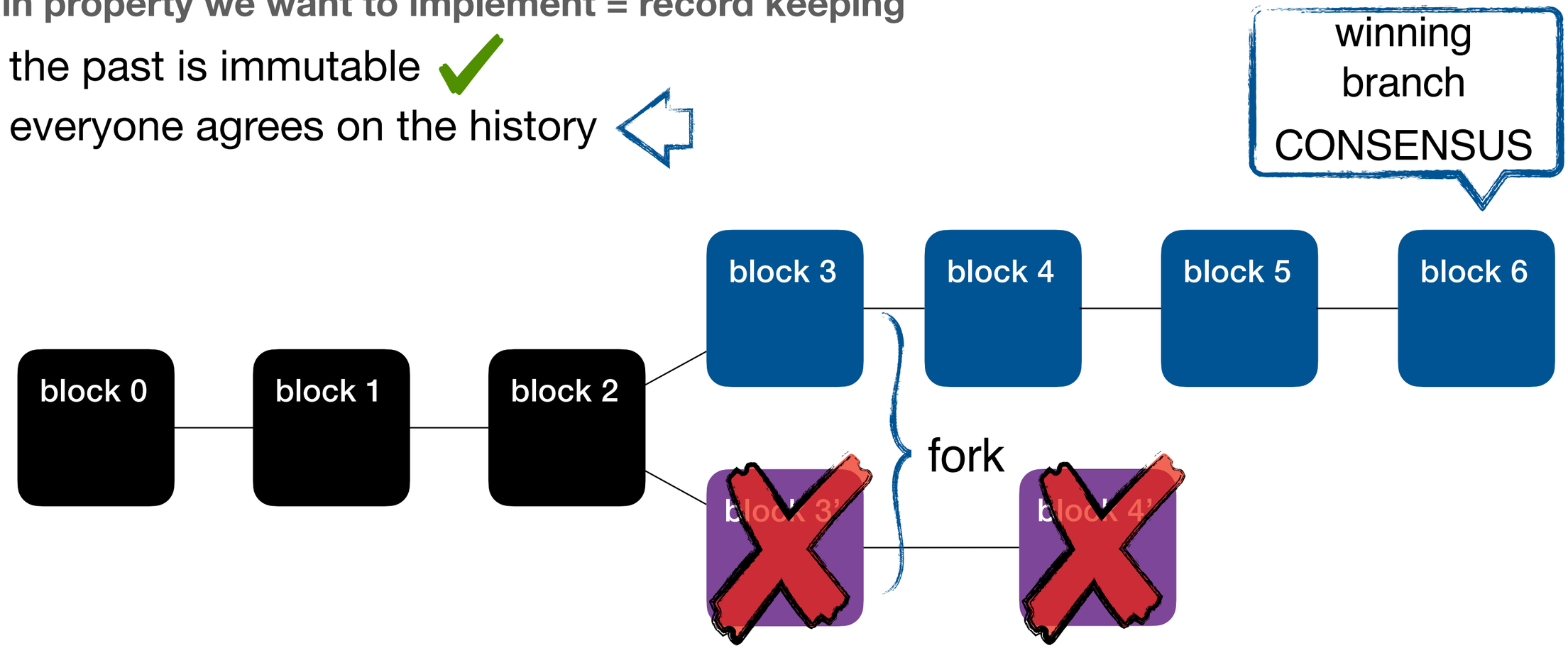

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Proof of Work

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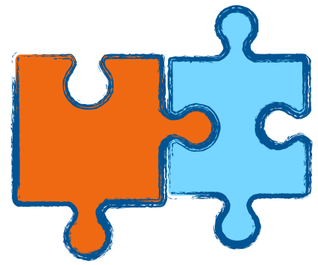

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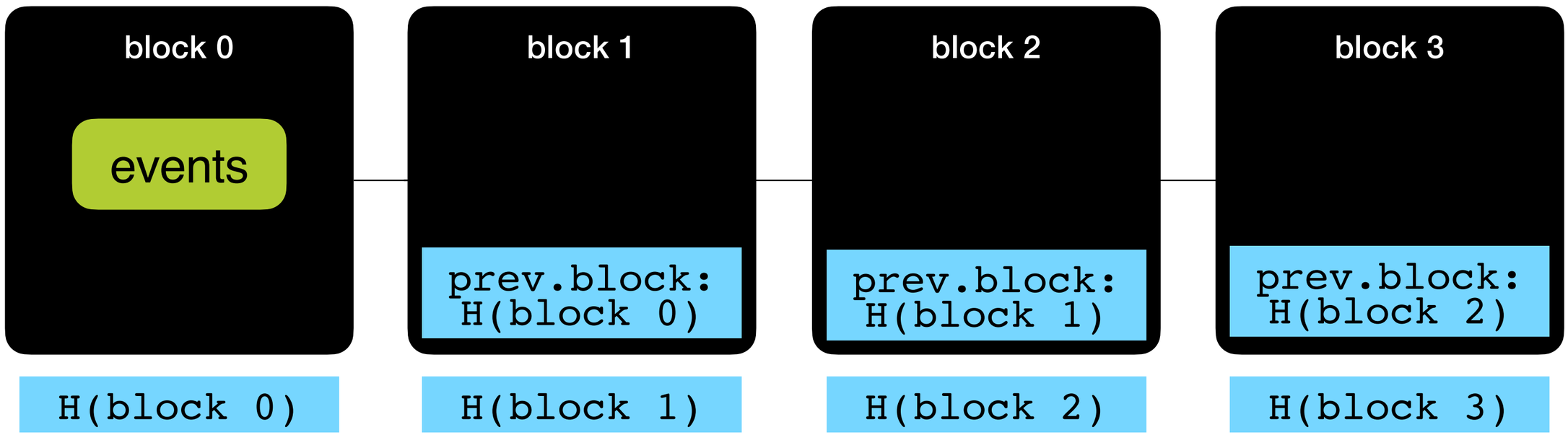

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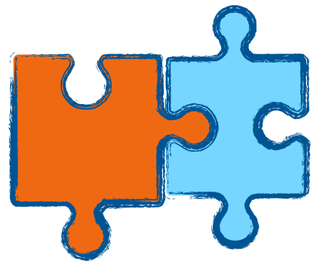

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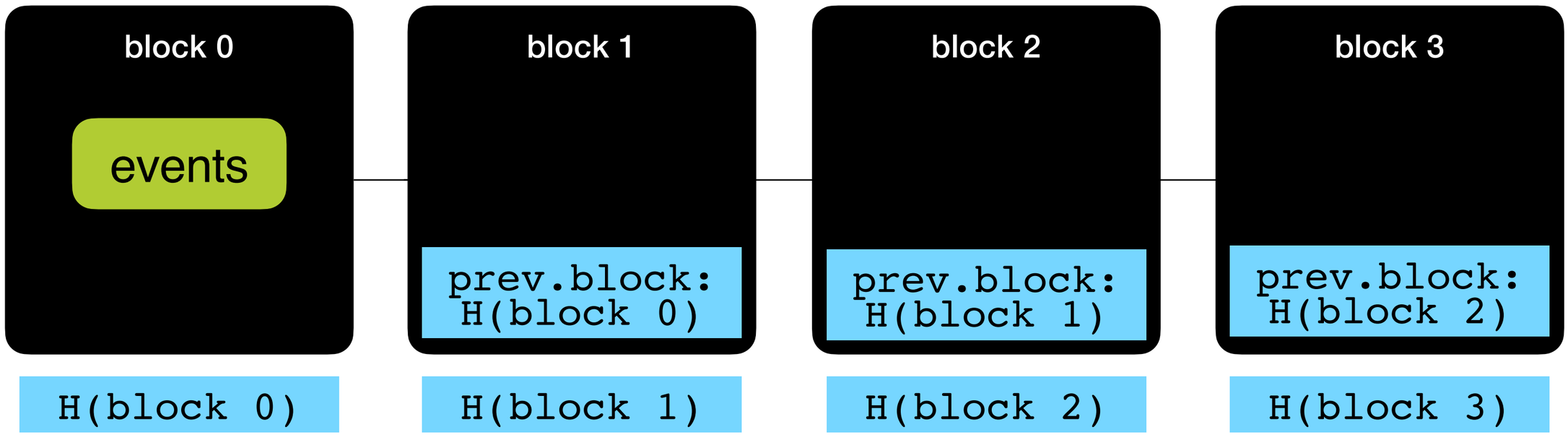

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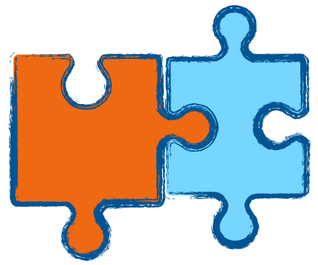

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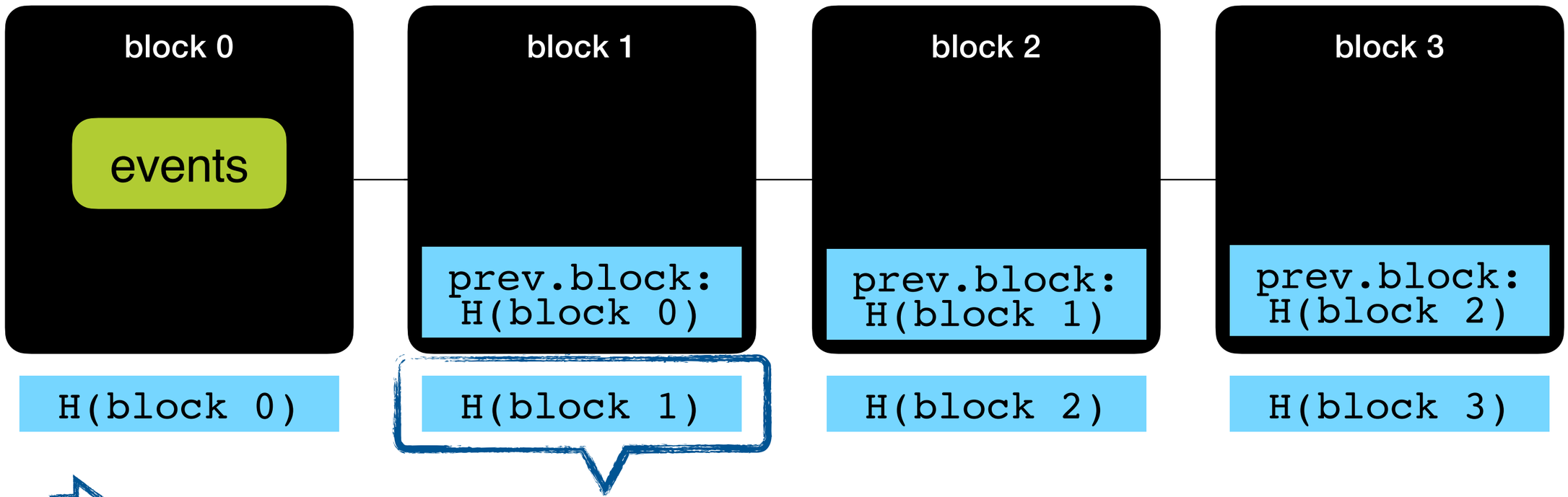

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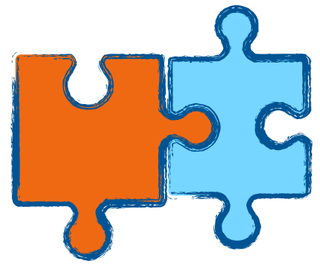

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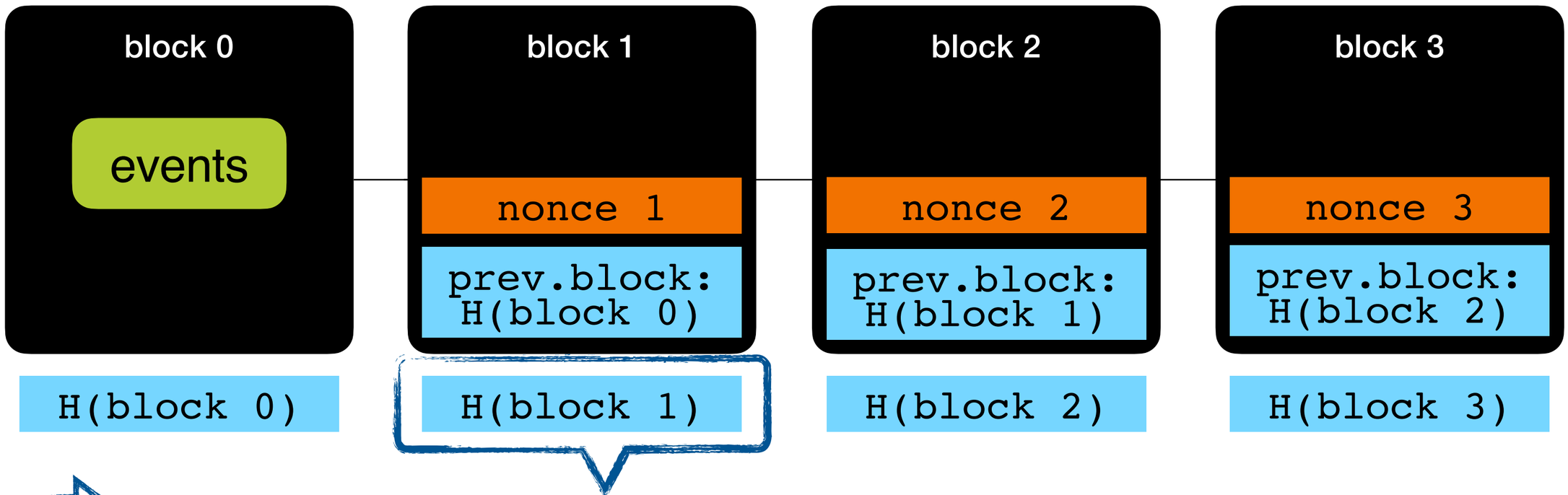

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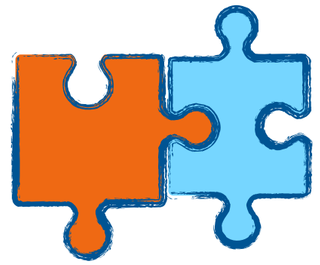

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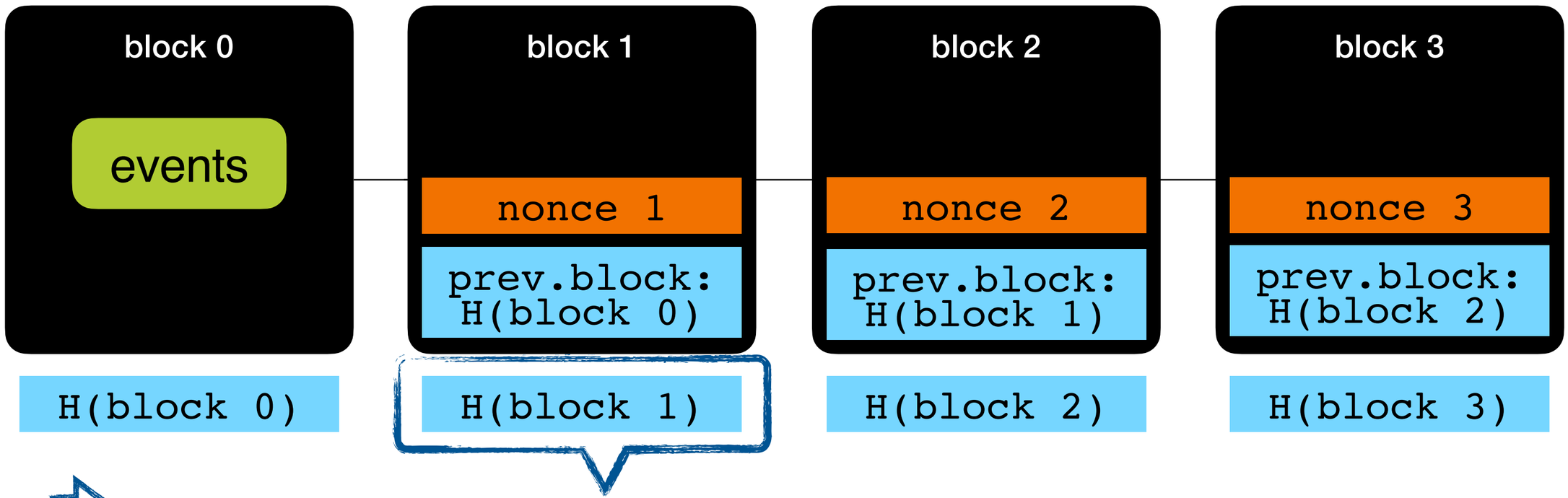

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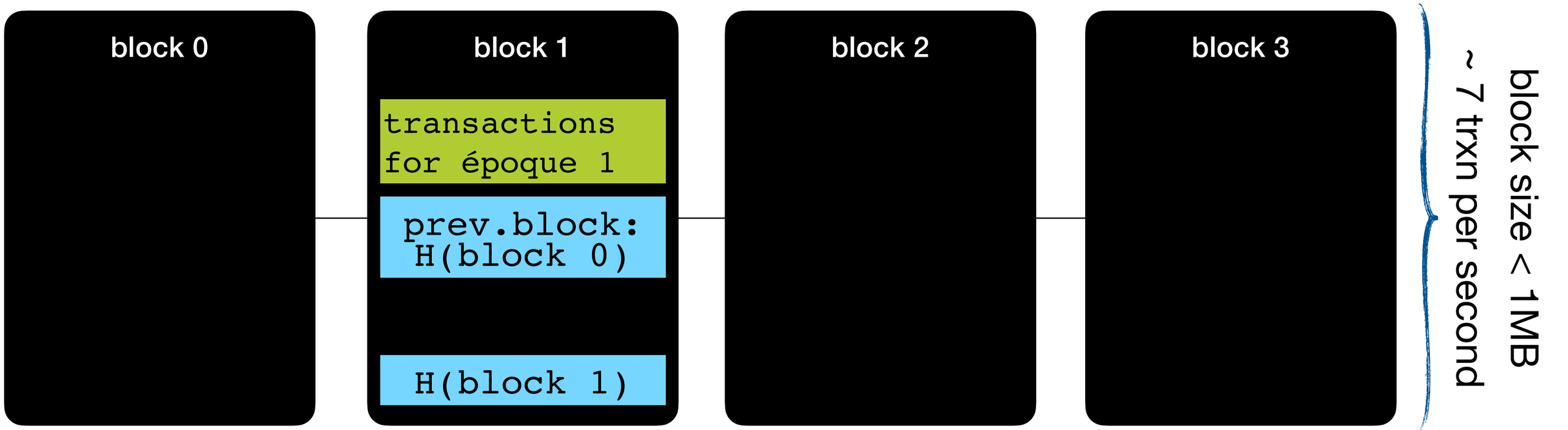
```
sha256("Advanced Web Security")                = f61d24aff60c141a0c0606cad04d7b4776db9bef9c33c0ea38766f48b5edf8ac
sha256("Advanced Web Security"||878140116400) = 00000d5999cf256d9b8d29a28433223ef51051c956bb1e3d22fcbf7208cf34f
```

Example: Bitcoin

set by the puzzle difficulty
(currently 19 leading zeroes)

époque ~ 10 mins



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how large is the bitcoin chain by NOW?

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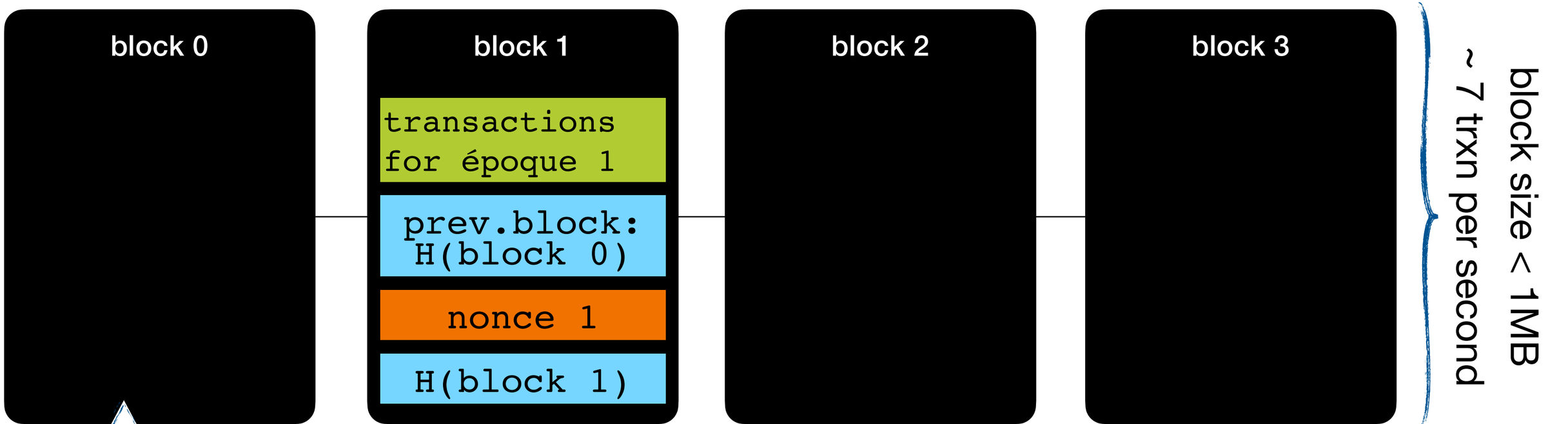

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- Proof of Work (Cryptographic Puzzles)
- Example: Bitcoin

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- Mining Blocks
- Merkle Trees
- Block Reword System

How To Circulate Money

- Transactions
- ECDSA

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- Freshness of the Genesis Block
- 51% Attack
- Proof of Work VS Proof of Stake

Advanced Tools

- Proofs of Knowledge
- zkSNARKs
- MPC

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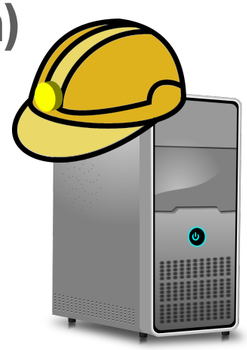

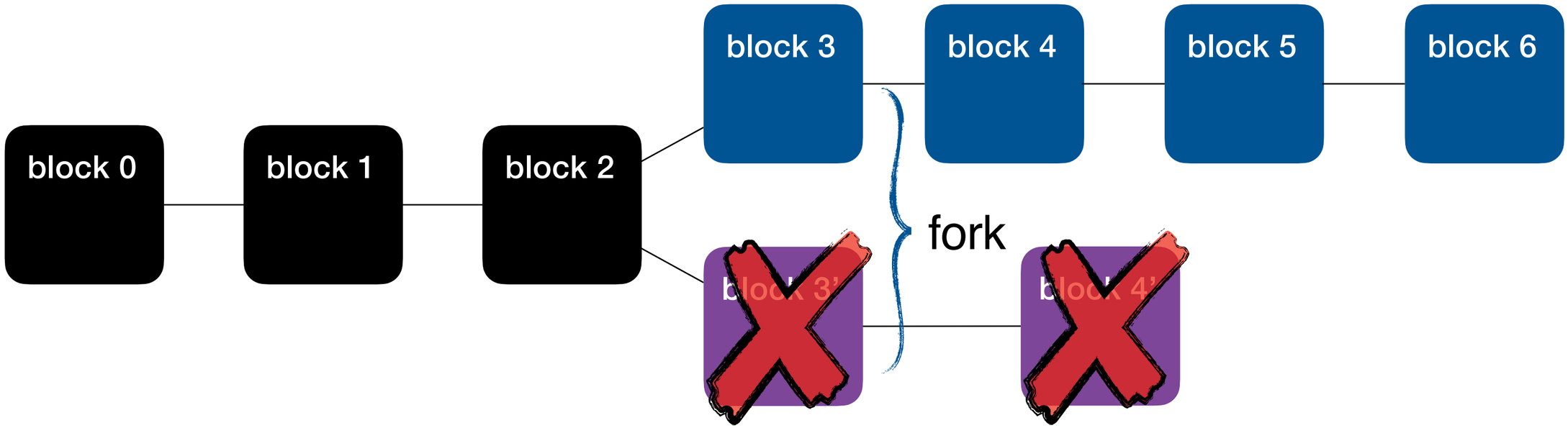

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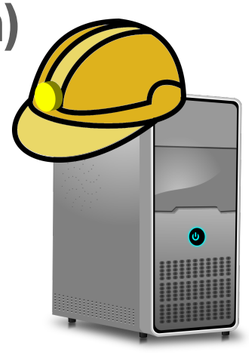

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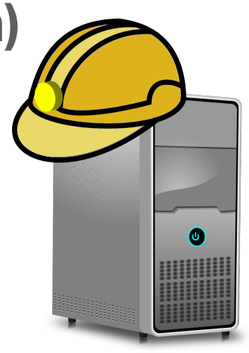

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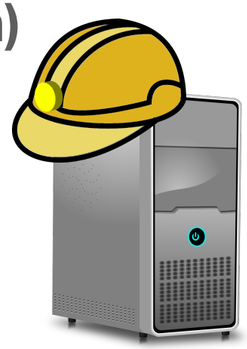

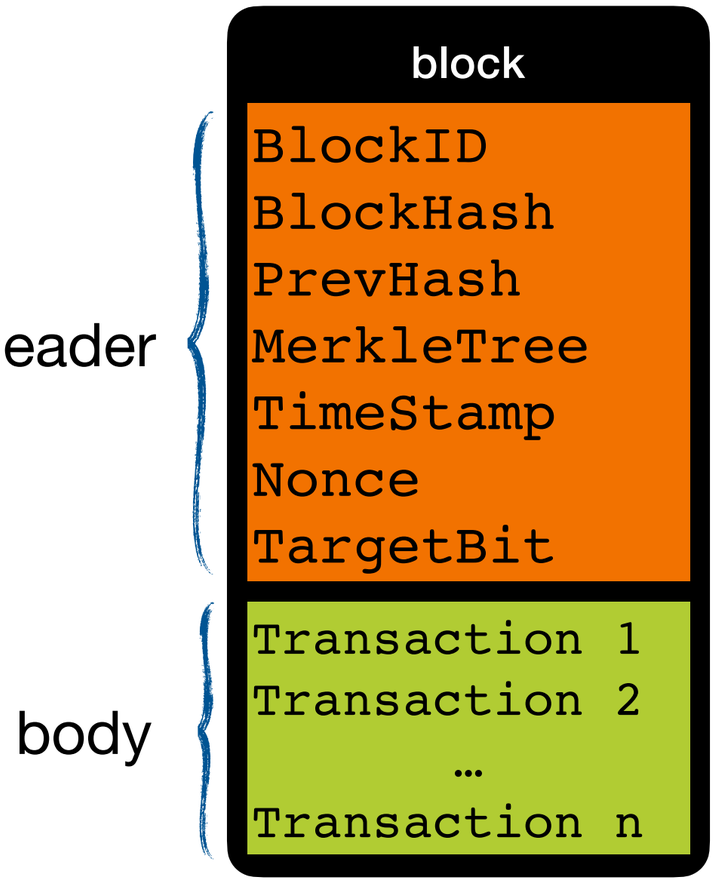

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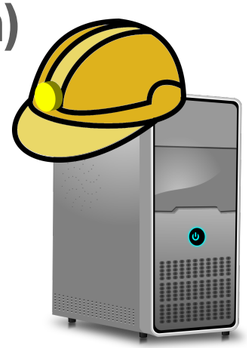

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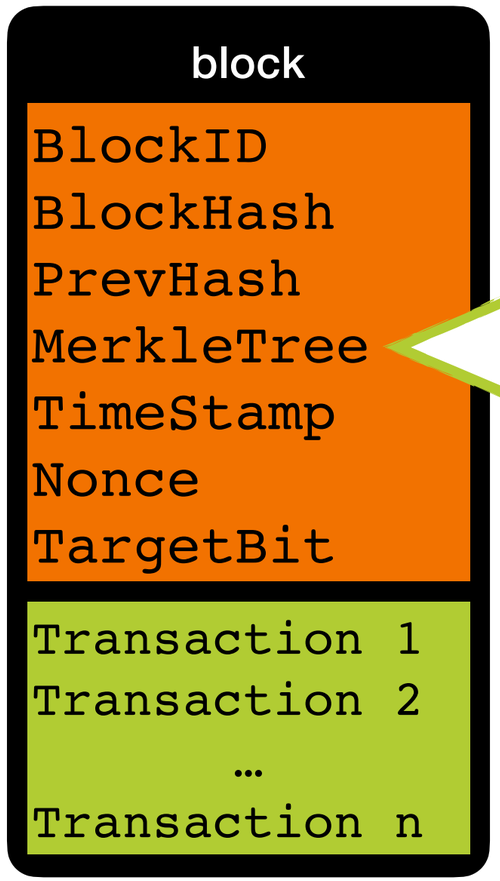

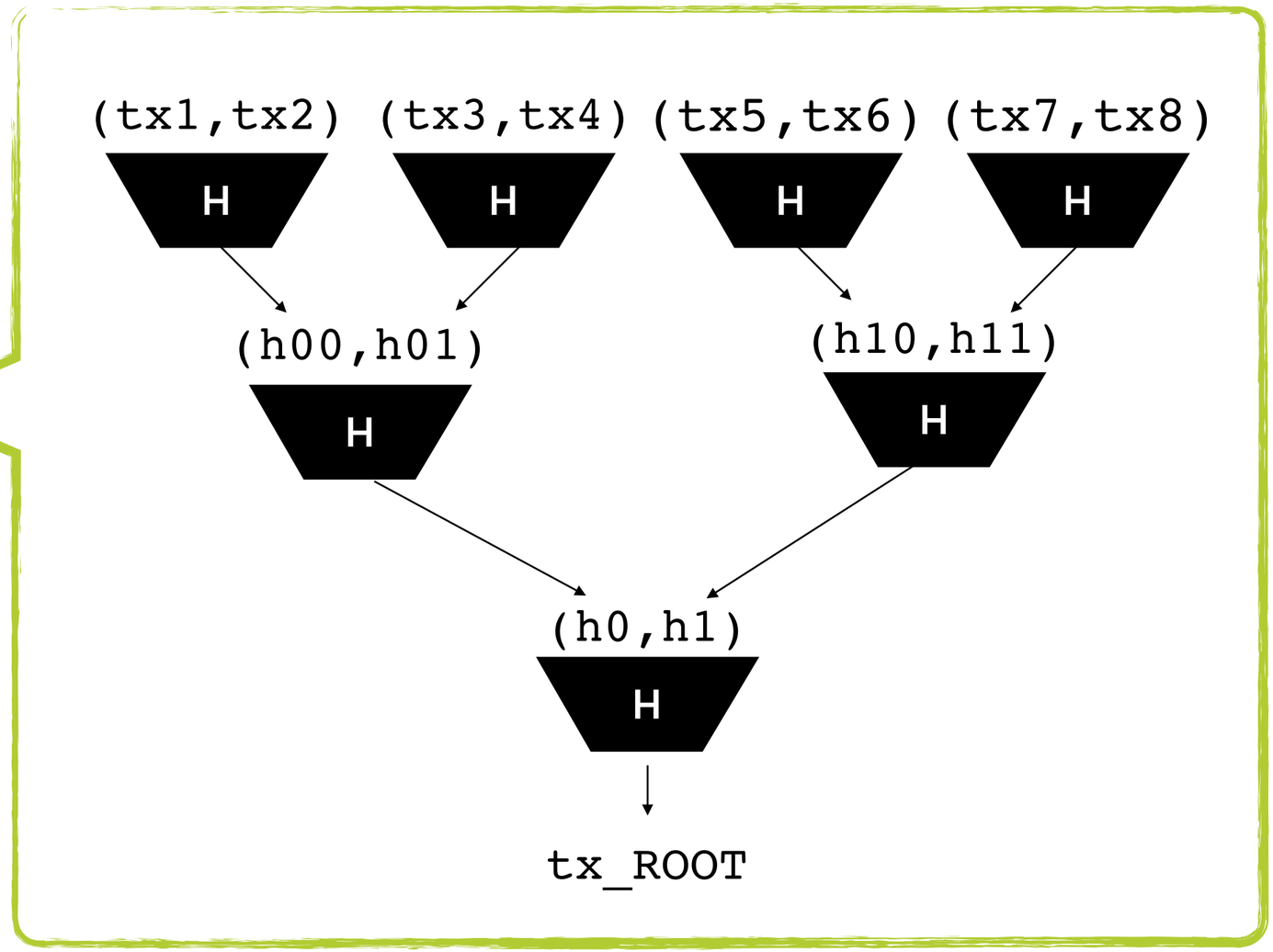

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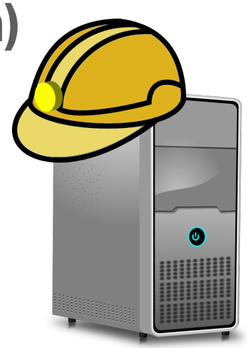
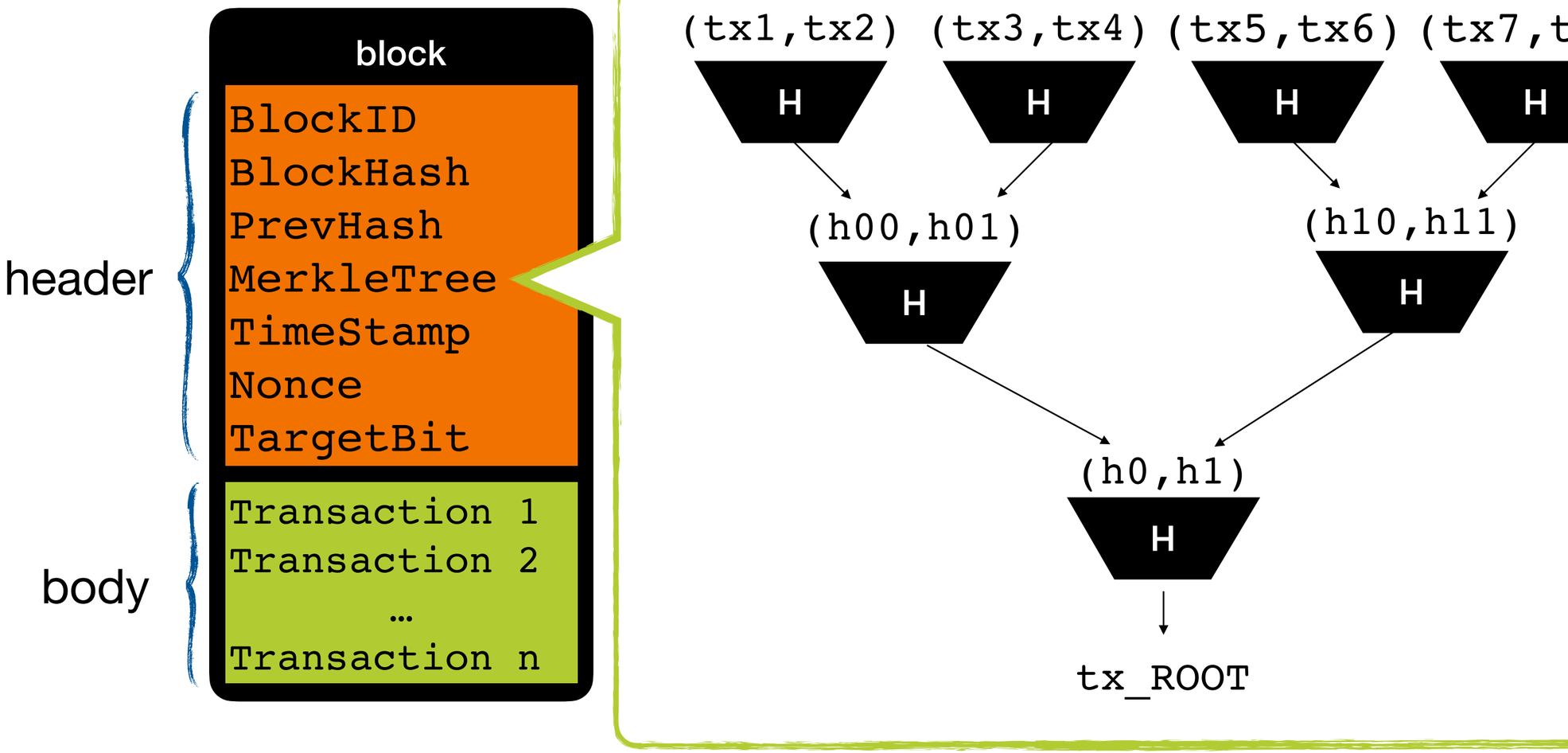
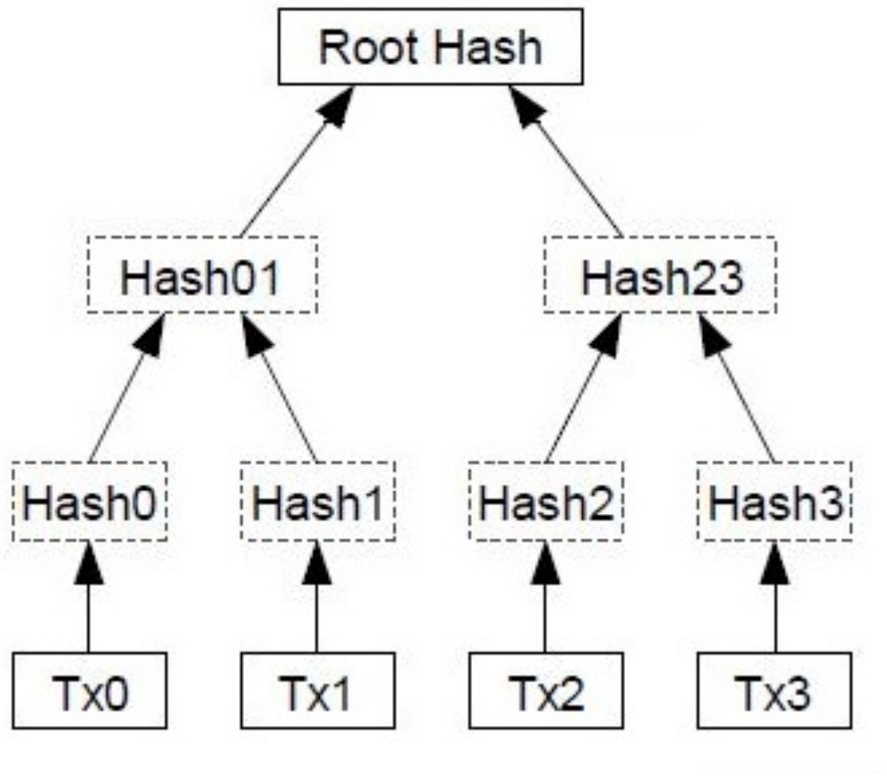




mine = “find `nonce` s.t. `H(PrevHash || tx_ROOT || nonce)` starts with 19 zeroes”

Merkel's Tree

Merkle Tree



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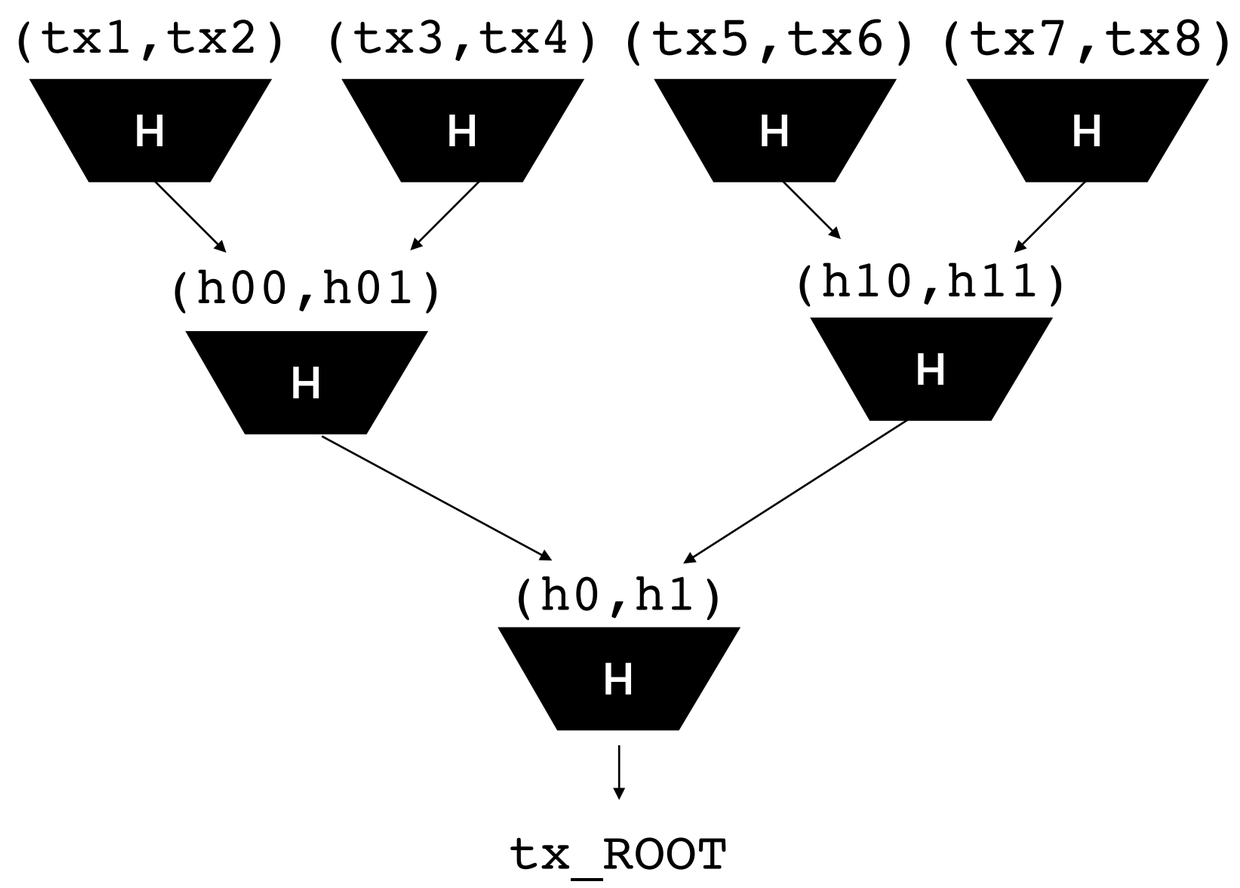

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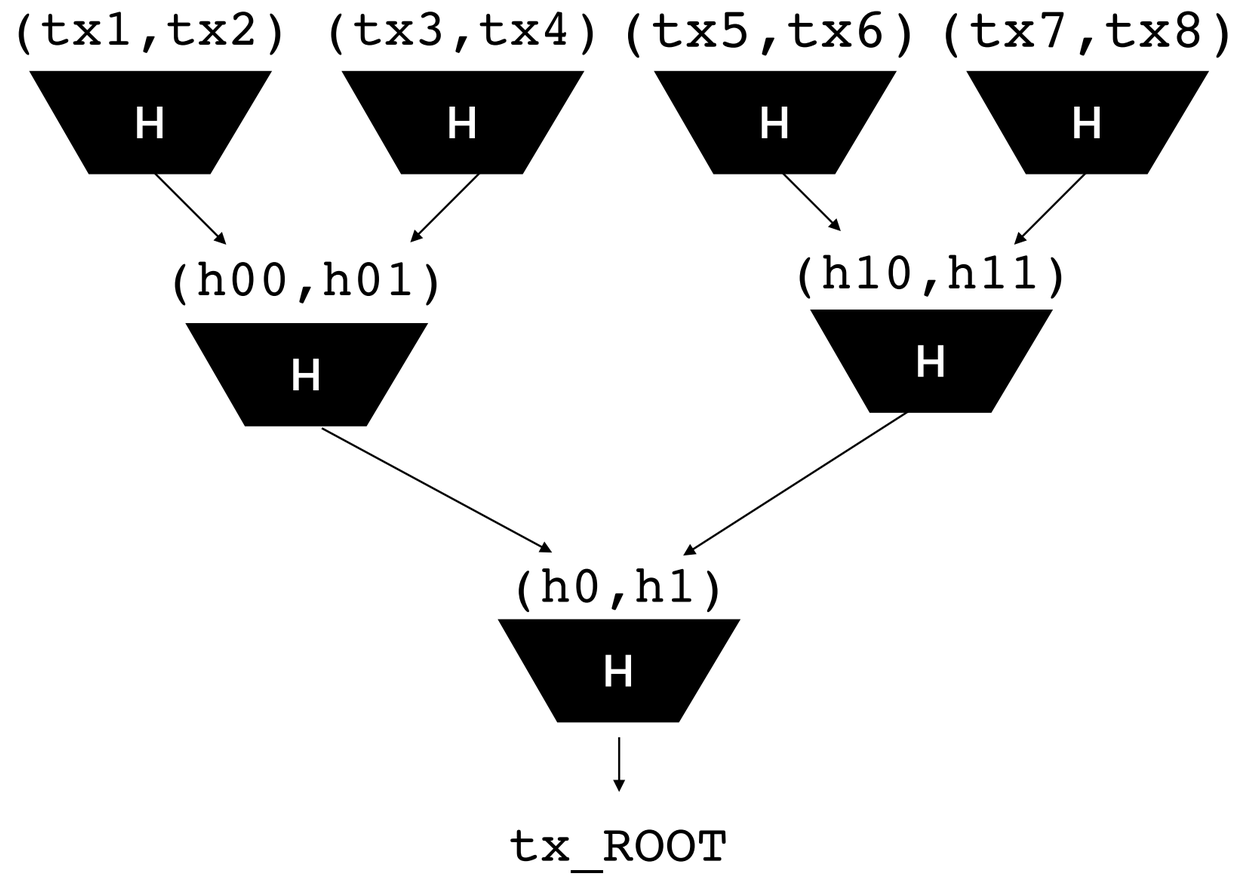
1) The root is always of the same size



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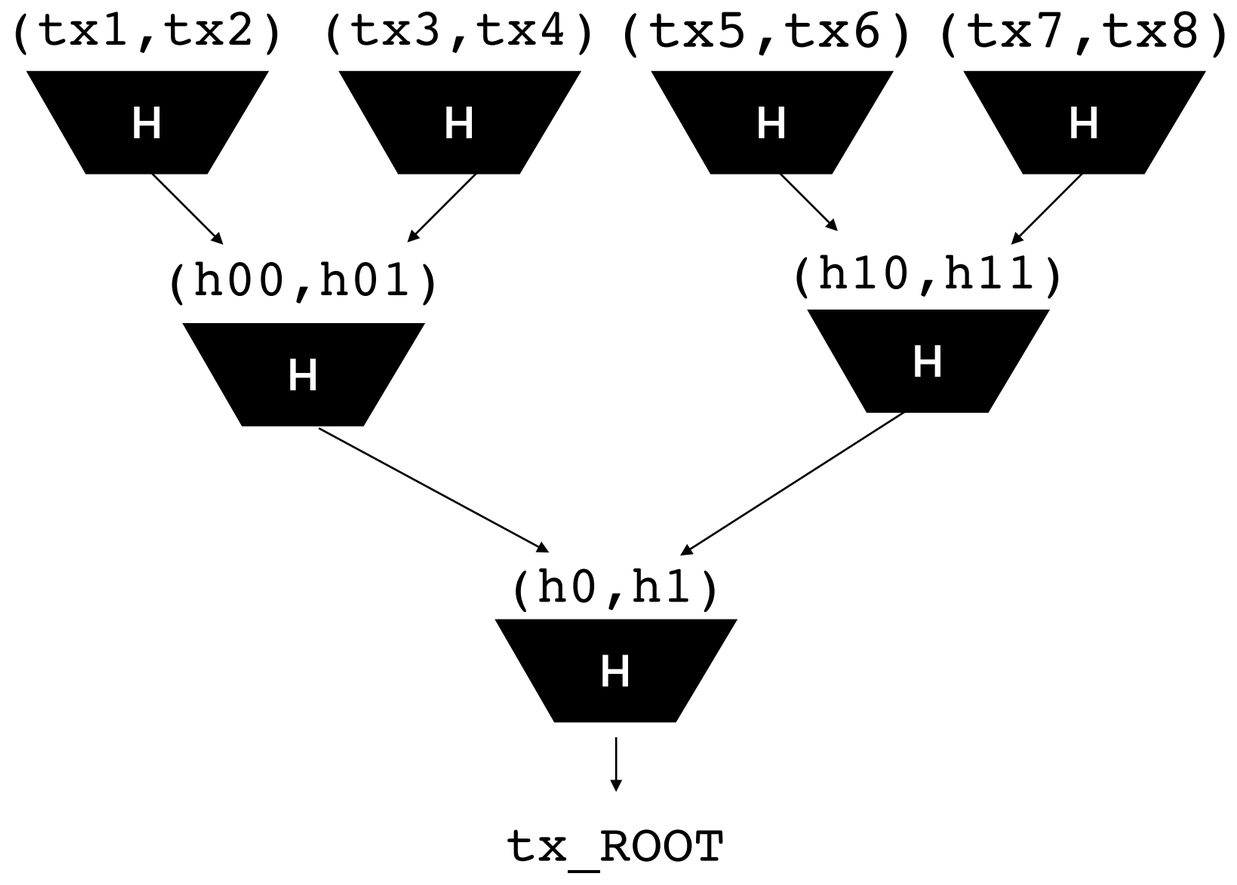

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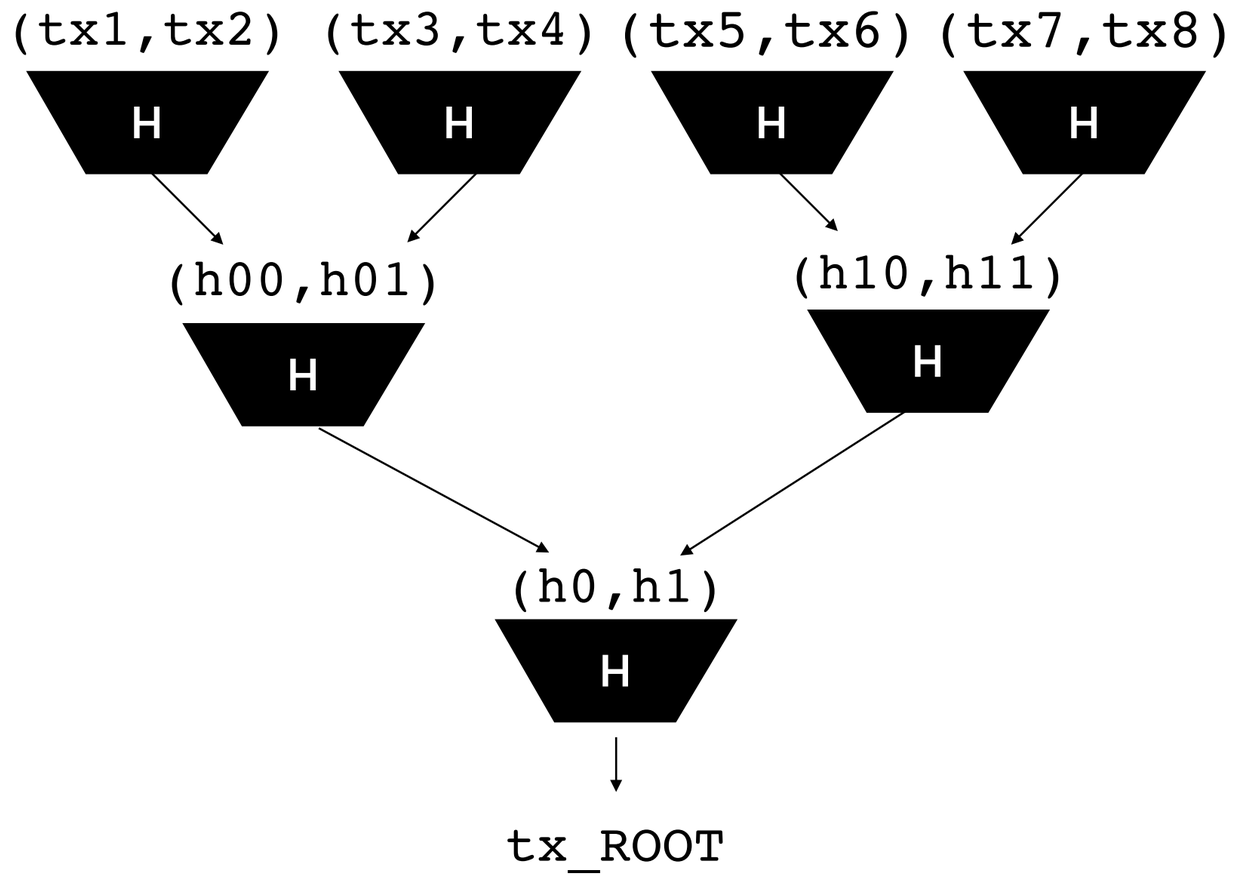

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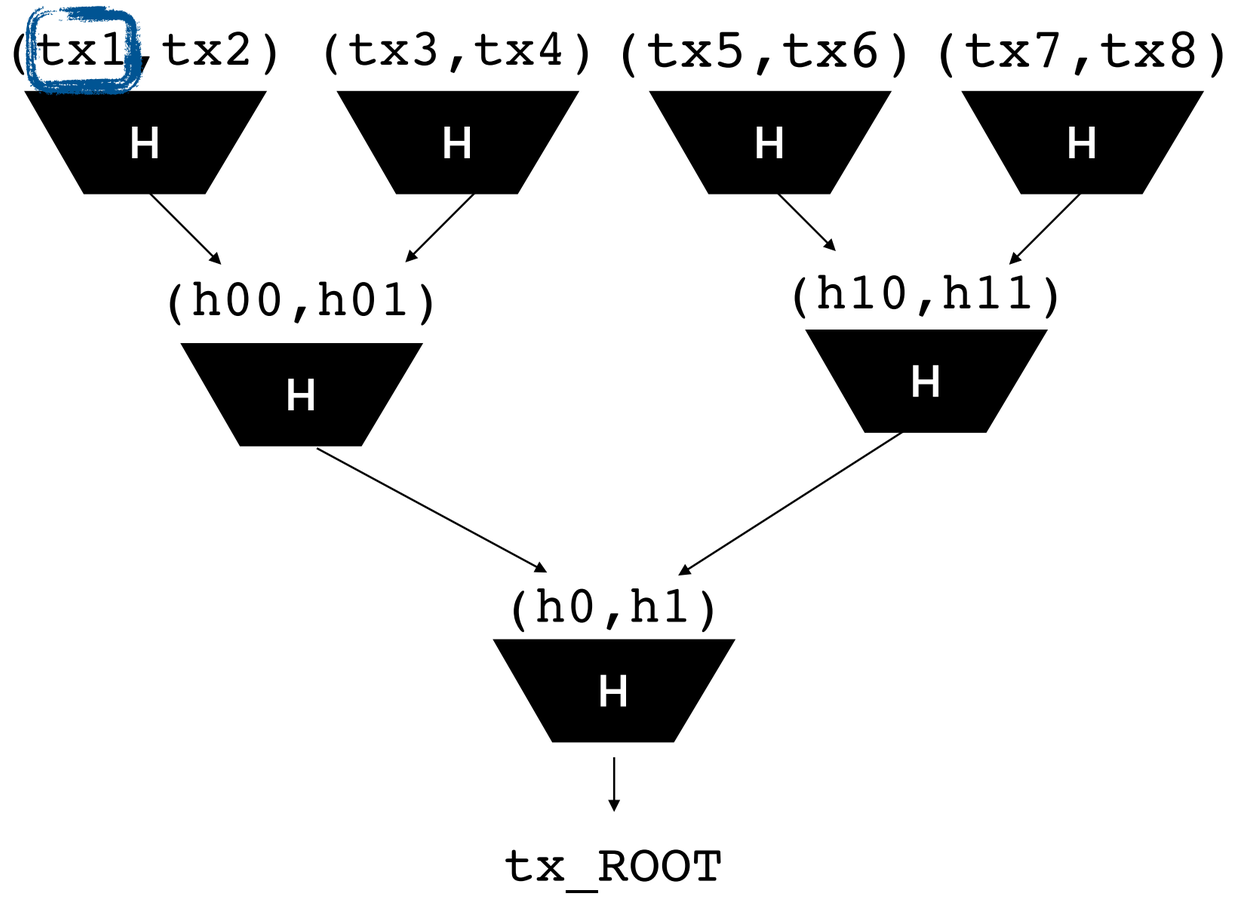

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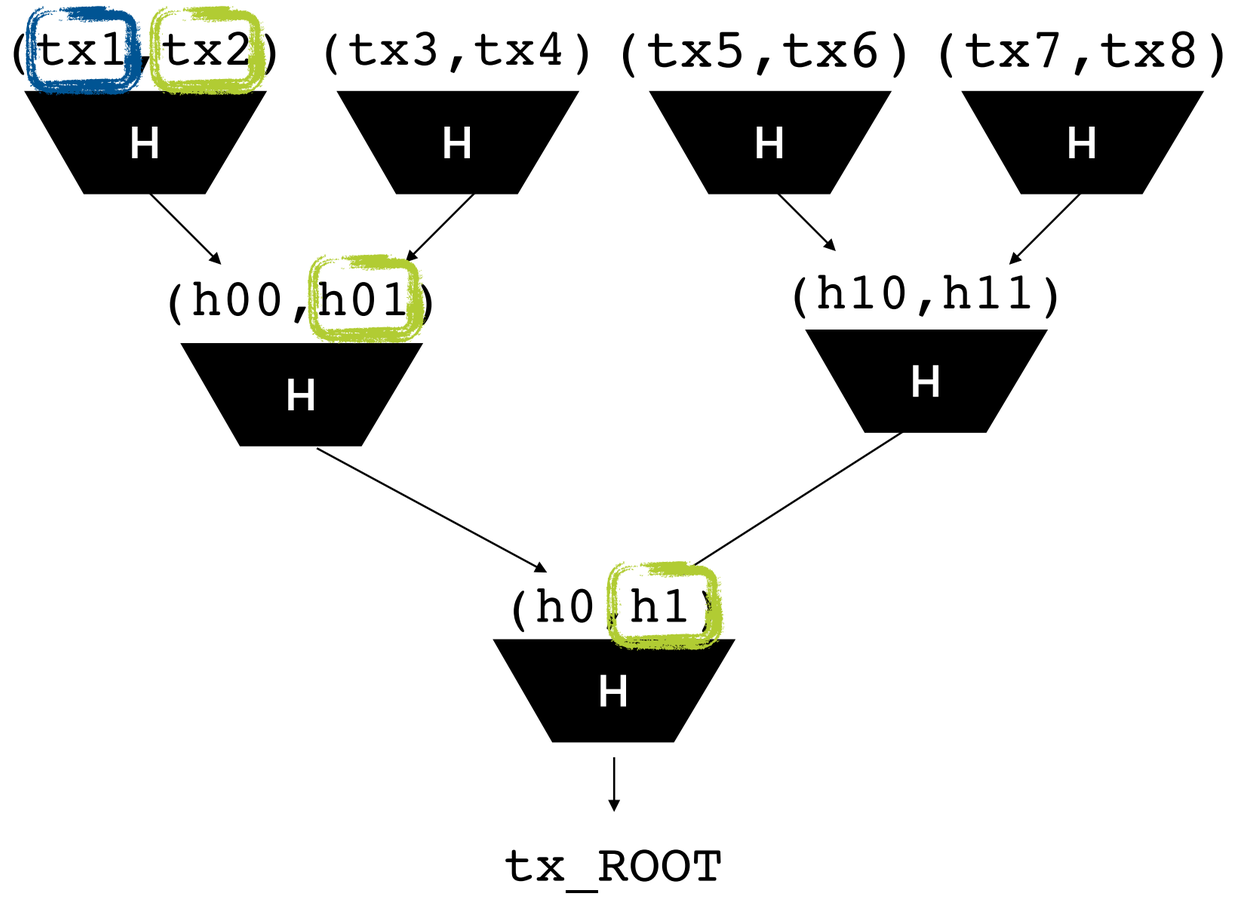

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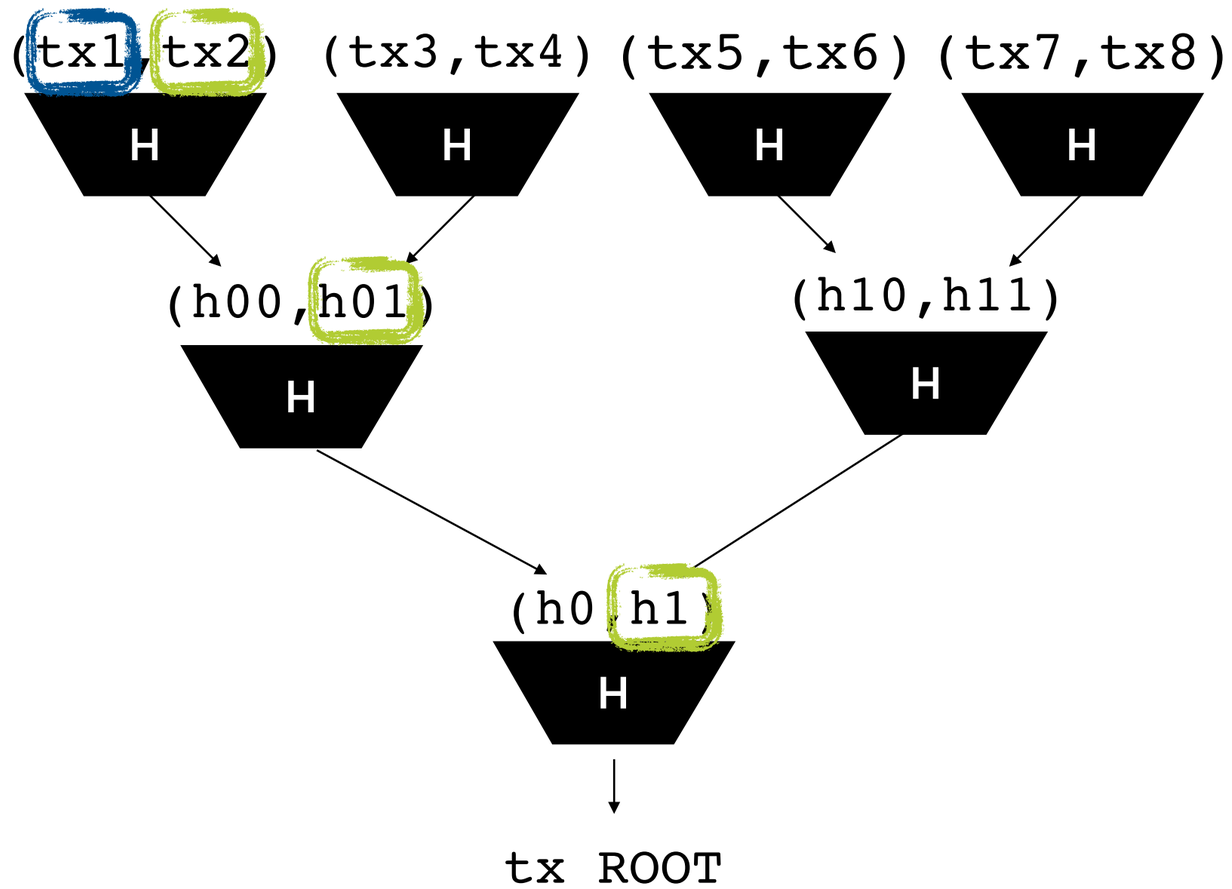
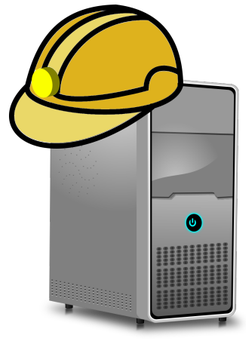


π = (tx1,tx2,h01,h1)

π proves that `tx1` is in the block this can be verified fast without the need to process the entire block

Block Reward System (in Bitcoin)



Main property we want to implement = record keeping

1) the past is immutable
2) everyone agrees on the history
3) all transactions on chain are valid

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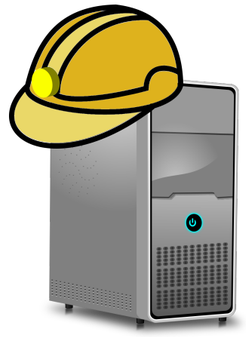

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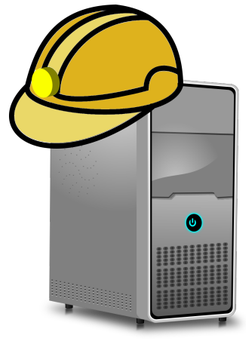

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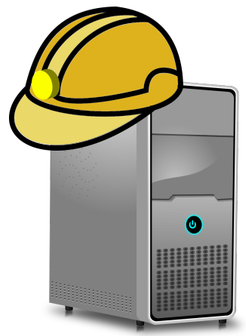

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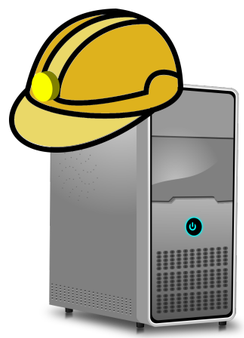

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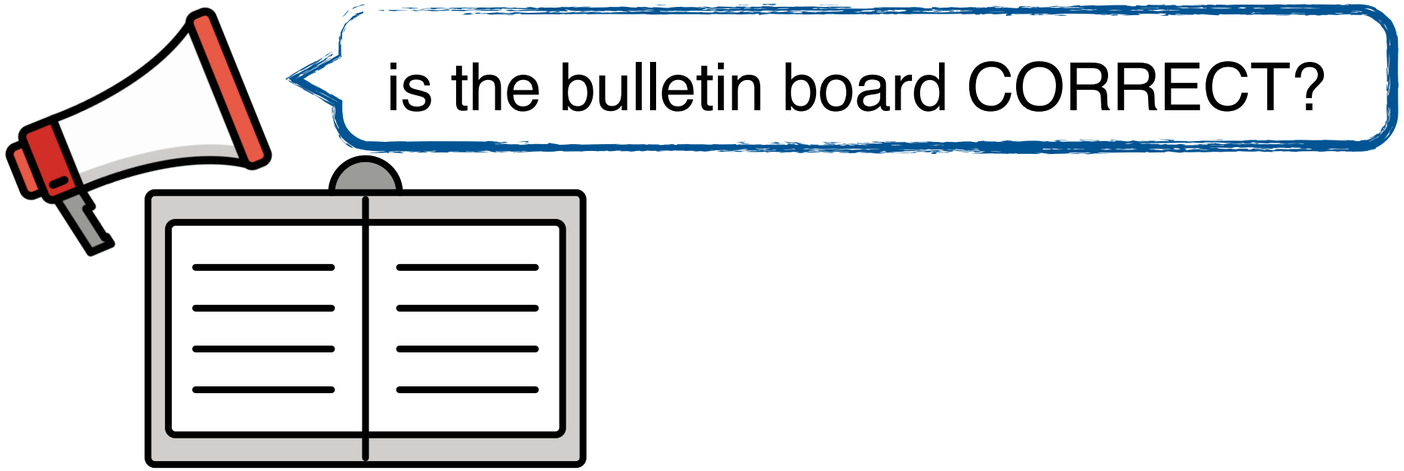

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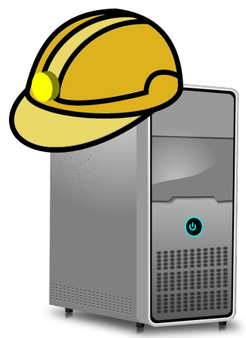

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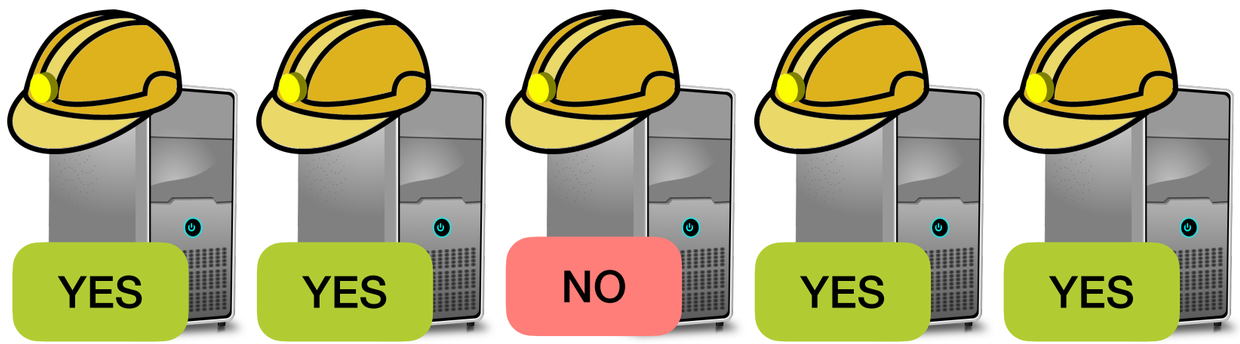
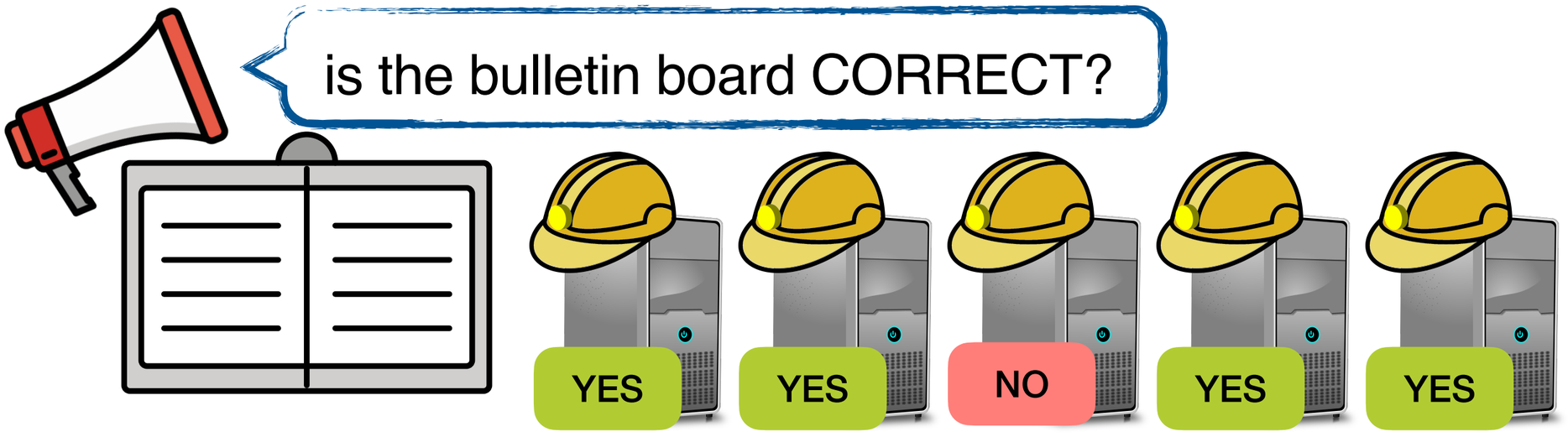


Consensus in Bitcoin



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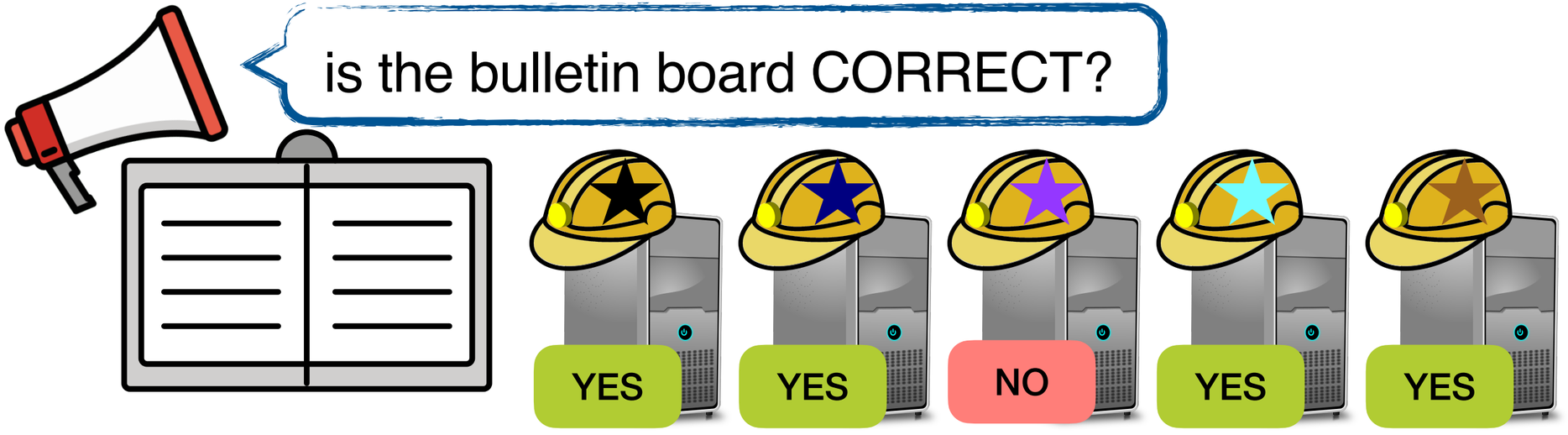

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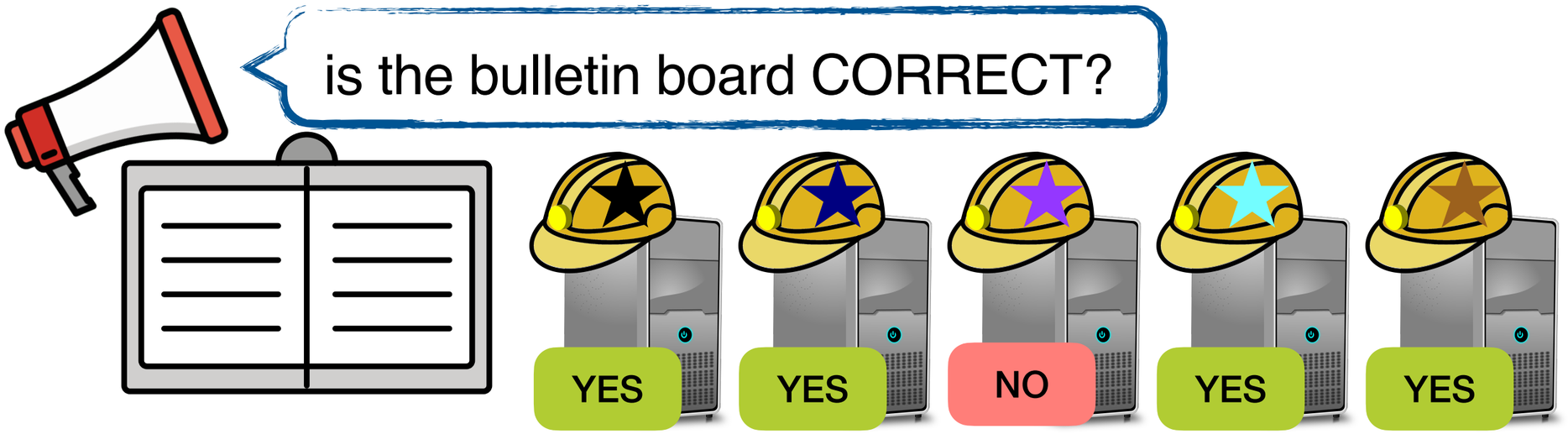

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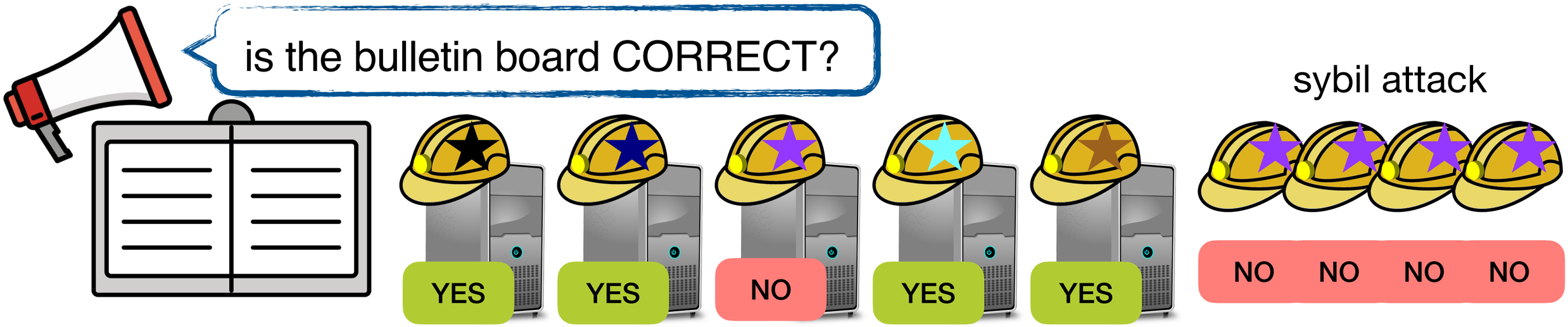

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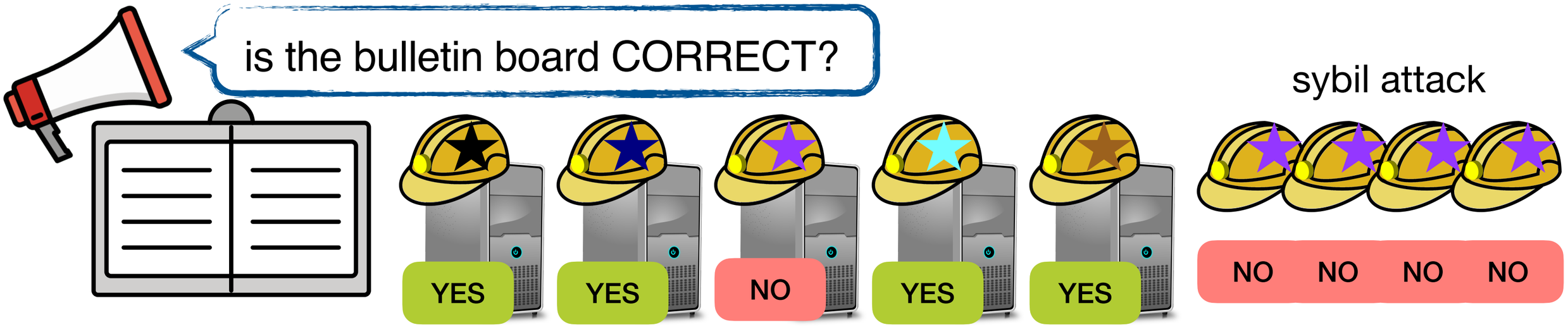

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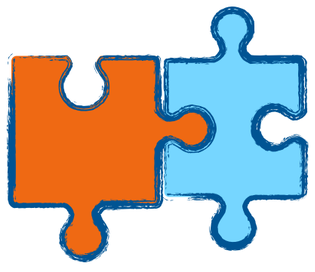

PoW prevents sybil attacks
(creating multiple entities does not create additional computational power)

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- Hash Functions
- Proof of Work (Cryptographic Puzzles)
- Example: Bitcoin

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- Mining Blocks
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- Transactions
- ECDSA

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- Freshness of the Genesis Block
- 51% Attack
- Proof of Work VS Proof of Stake

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- zkSNARKs
- MPC

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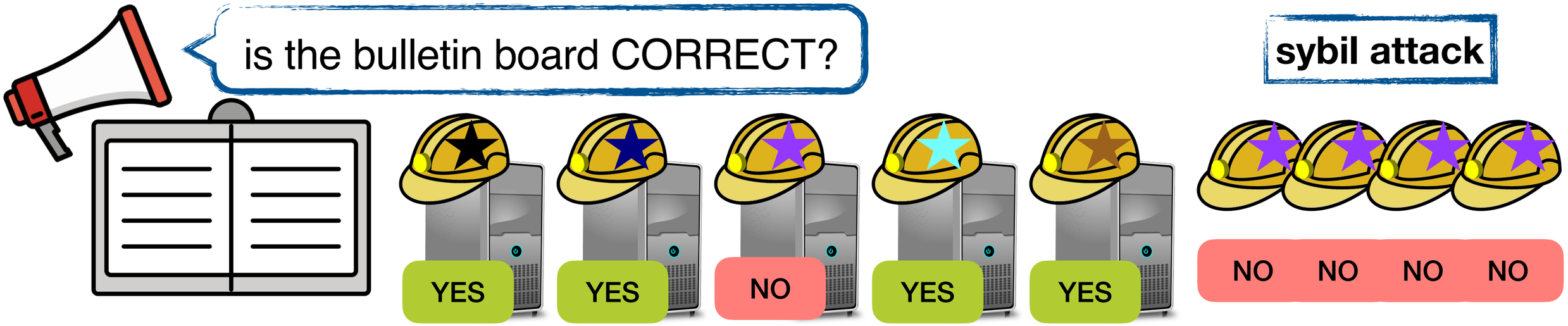

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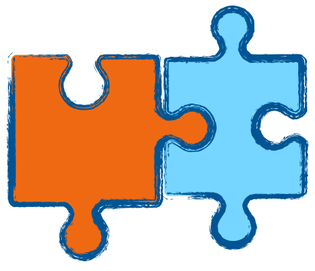

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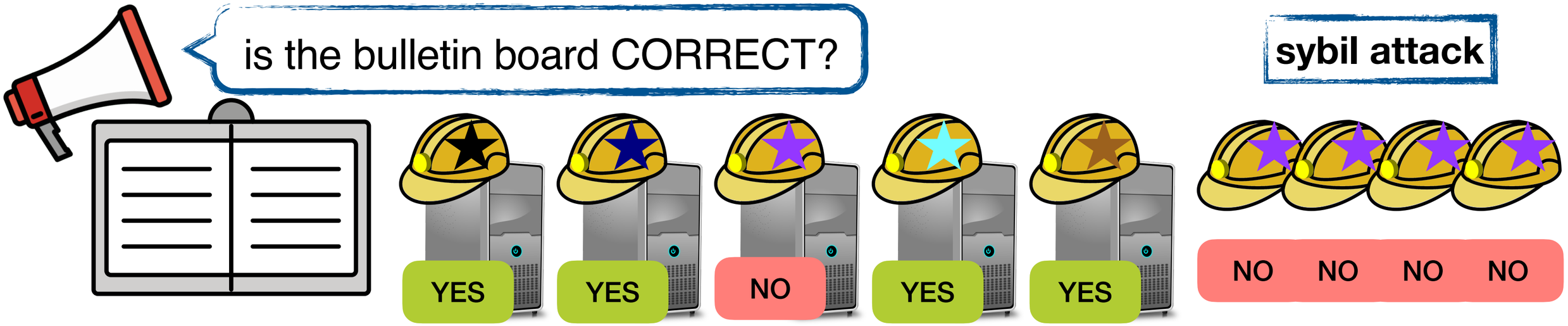

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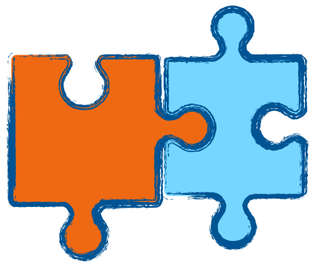

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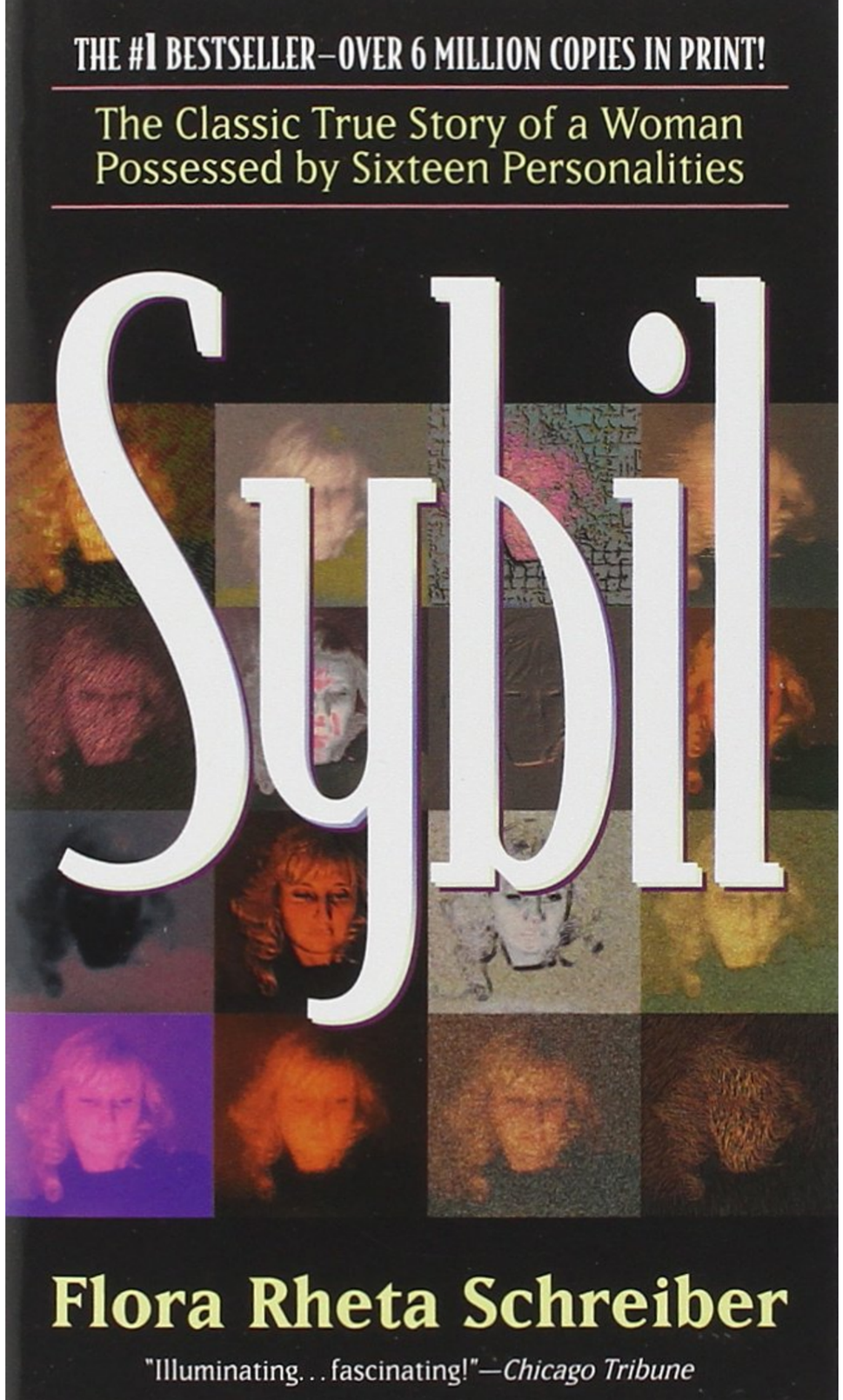
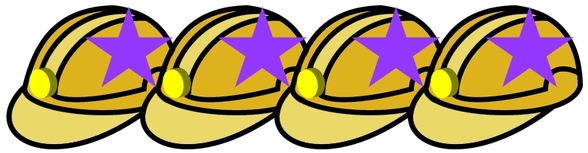


sybil attack



NO NO NO NO

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```
KeyGen (sec.par) ⇨ (sk, pk)
  d ←$— [0 ... n-1]
  sk = d
  pk = Q = d*G
```

```
Sign (sk, msg) ⇨ sgn
  k ←$— [0 ... n-1]
  R = k*G
  r = R_x mod n
  z = sha256(msg)
  s = inv(k)·(z + d·r) mod n
  sgn = (r, s)
```

```
Verify(pk, msg, sgn) ⇨ {0, 1}
  z = sha256(msg)
  T = [z·inv(s) mod n]*G
  P = [inv(s)·r mod n]*Q
  if R == T+P return 1
  else return 0
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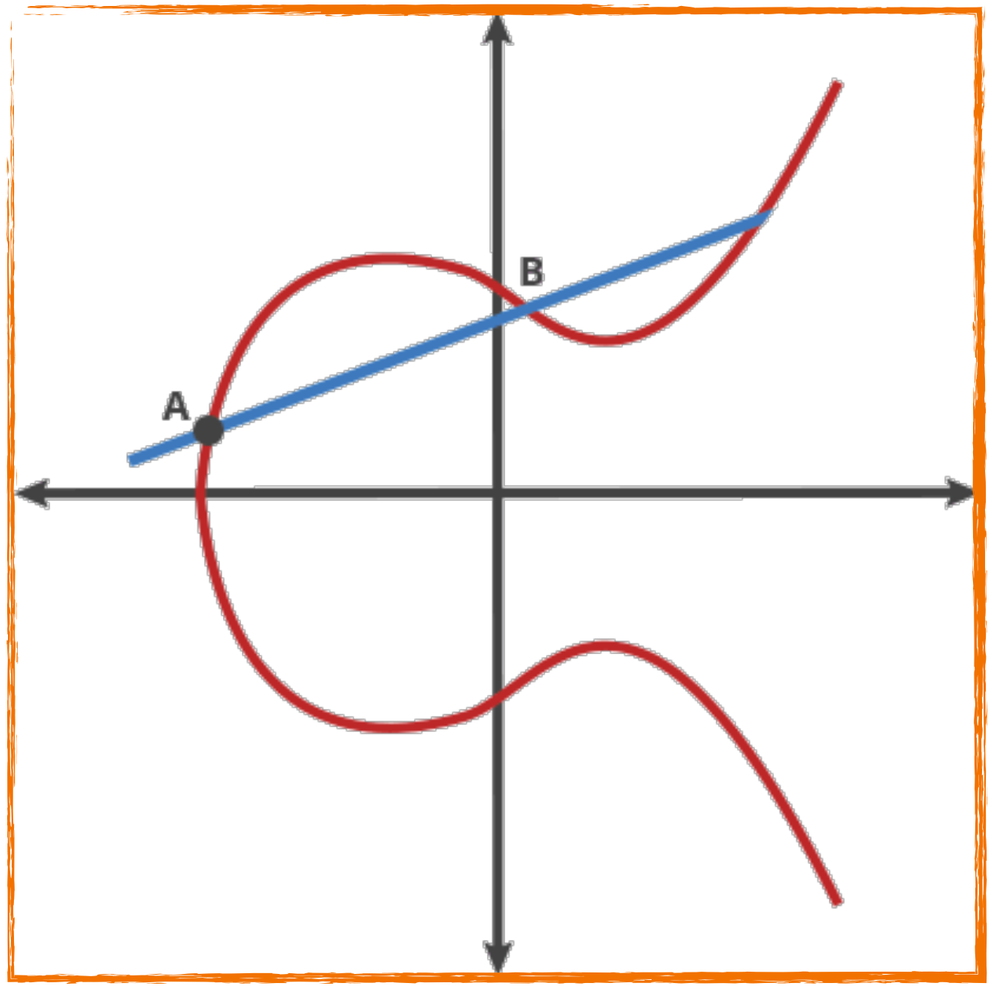

ECDSA - the Good

- ★ Shorter keys and better security than the RSA signature scheme
- ★ Non-malleable
- ★ IoT friendly
- ★ In wide adoption (TLS, DigiCert (Symantec), Sectigo (Comodo) …)

ECDSA - the Bad

PS3 hacked through poor cryptography implementation

A group of hackers named fail0verflow revealed in a presentation how they ...

CASEY JOHNSTON - 12/30/2010, 6:25 PM

{* SECURITY *}

Android bug batters Bitcoin wallets

Old flaw, new problem

Richard Chirgwin Mon 12 Aug 2013 // 00:43 UTC

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ECDSA - What's Next?

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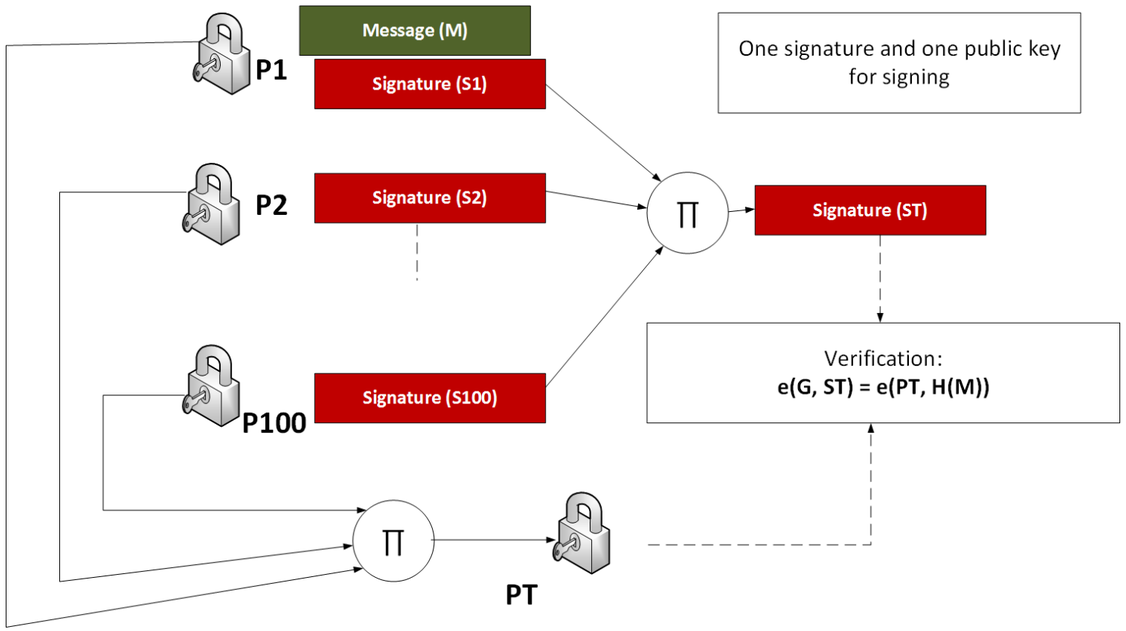
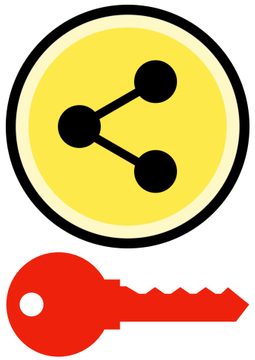
Threshold Signatures





LBS signature

Schnorr signature

ECDSA - What's Next?

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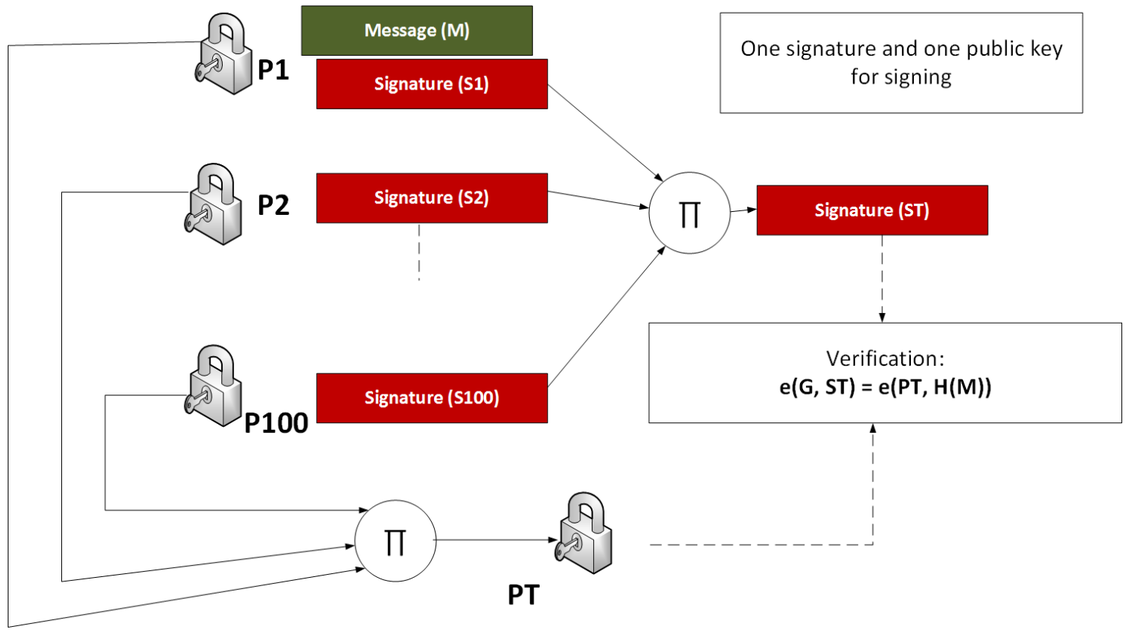

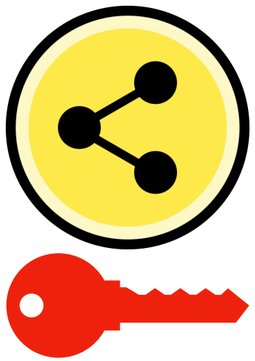

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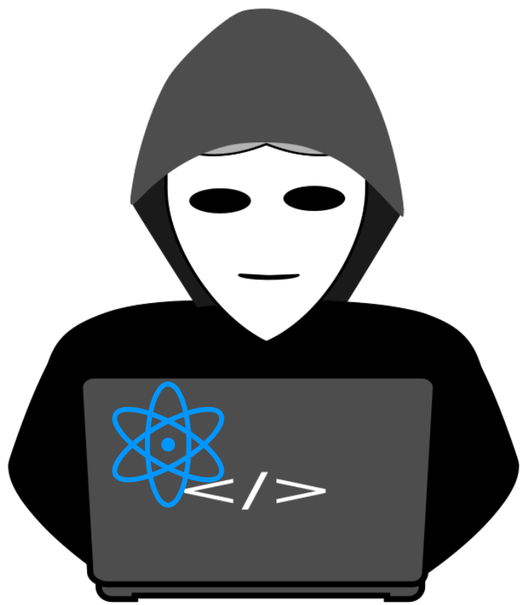
Post Quantum Secure Signatures

FALCON

Fast-Fourier Lattice-based Compact Signatures over NTRU



DILITHIUM

Lecture Agenda

How To Set Up a Bulletin Board

- Hash Functions
- Proof of Work (Cryptographic Puzzles)
- Example: Bitcoin

How To Generate Money

- Mining Blocks
- Merkle Trees
- Block Reword System

How To Circulate Money

- Transactions
- ECDSA

Some Problems With Blockchains

- Freshness of the Genesis Block
- 51% Attack
- PoW Vs PoS
- The Byzantine Fault Tolerance Problem

Advanced Tools

- Zero Knowledge Proofs
- Schnorr Interactive PoK
- Fiat-Shamir Heuristic (NIZK)

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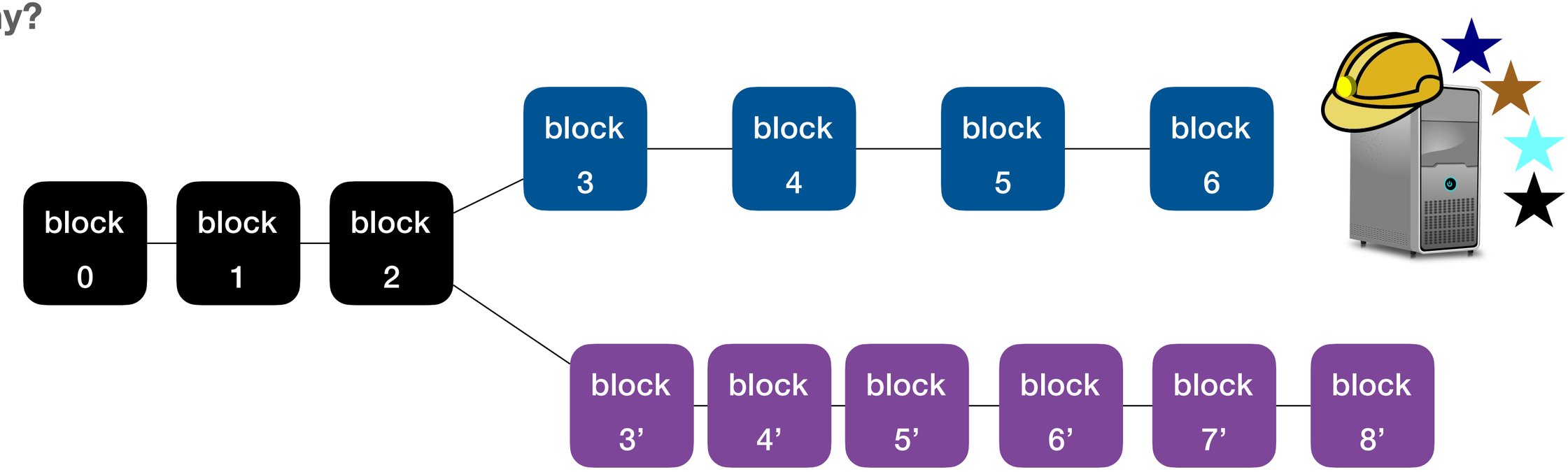

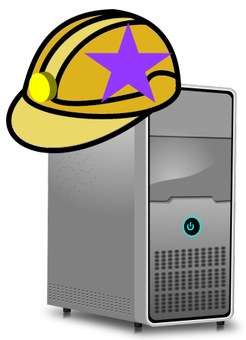

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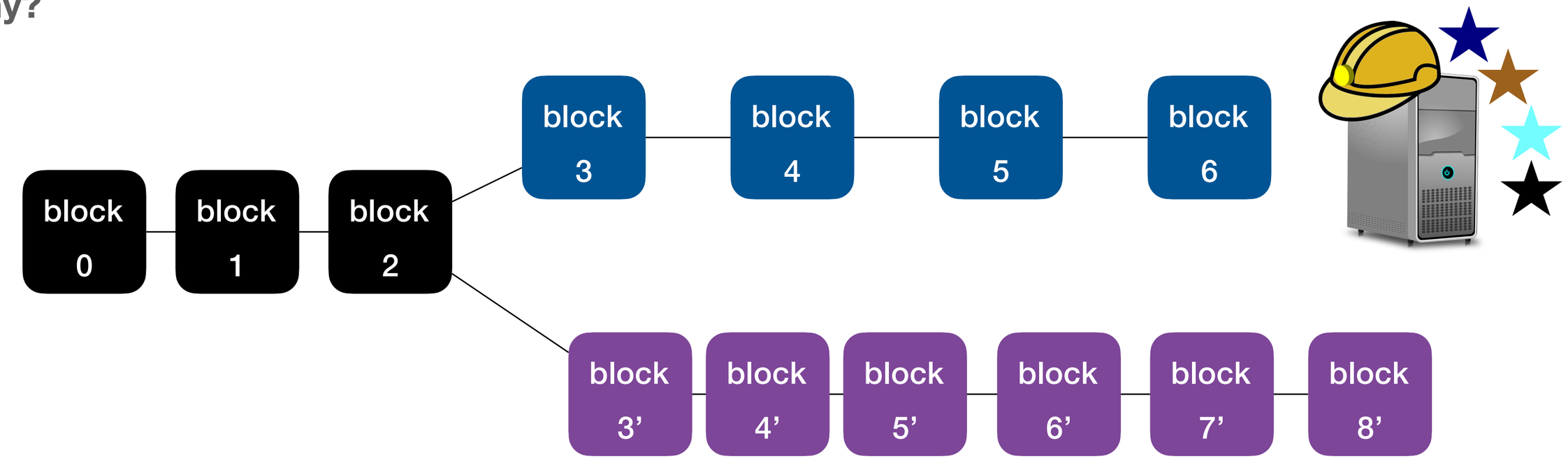

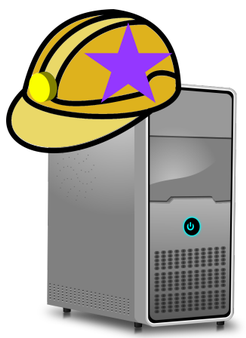

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The difficulty **drops** dramatically, so can quickly produce a chain longer than the valid one and publish it

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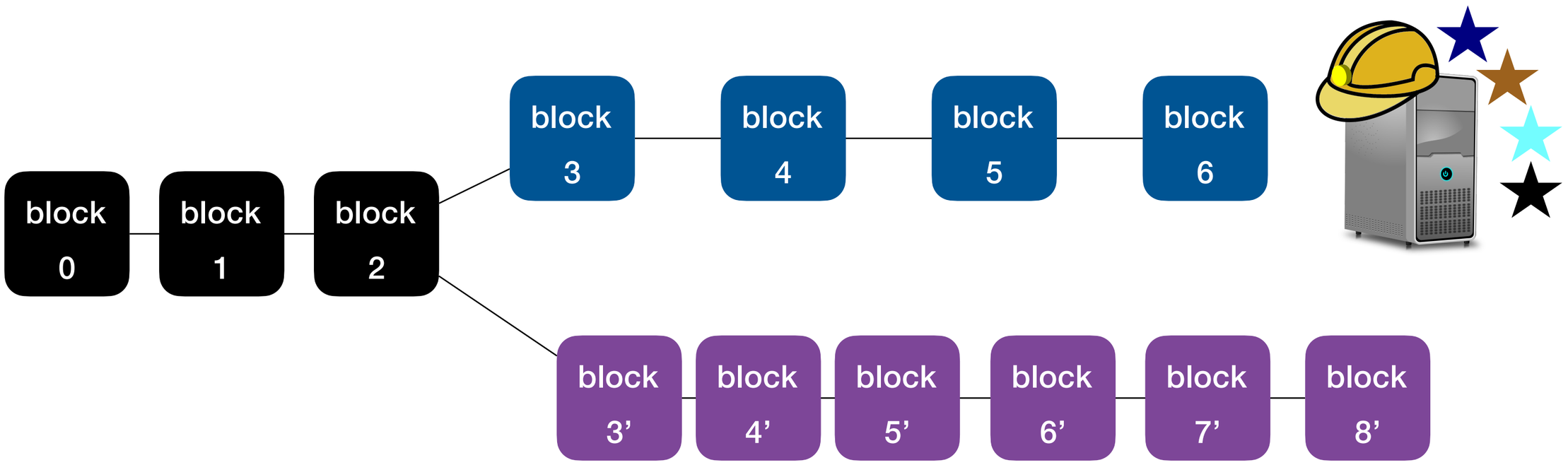

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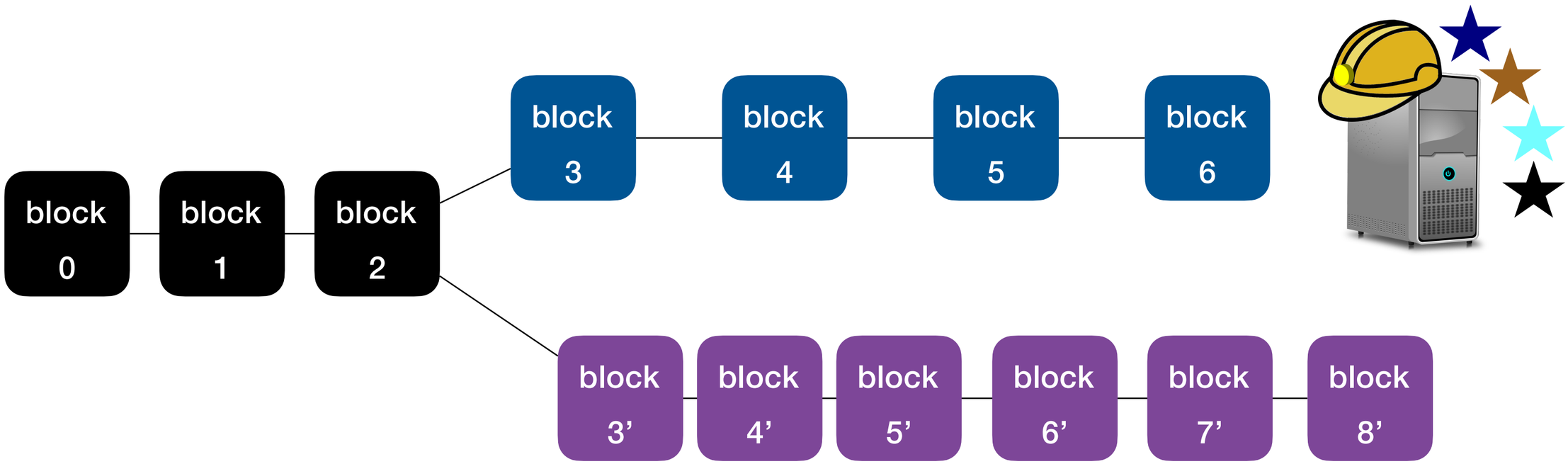

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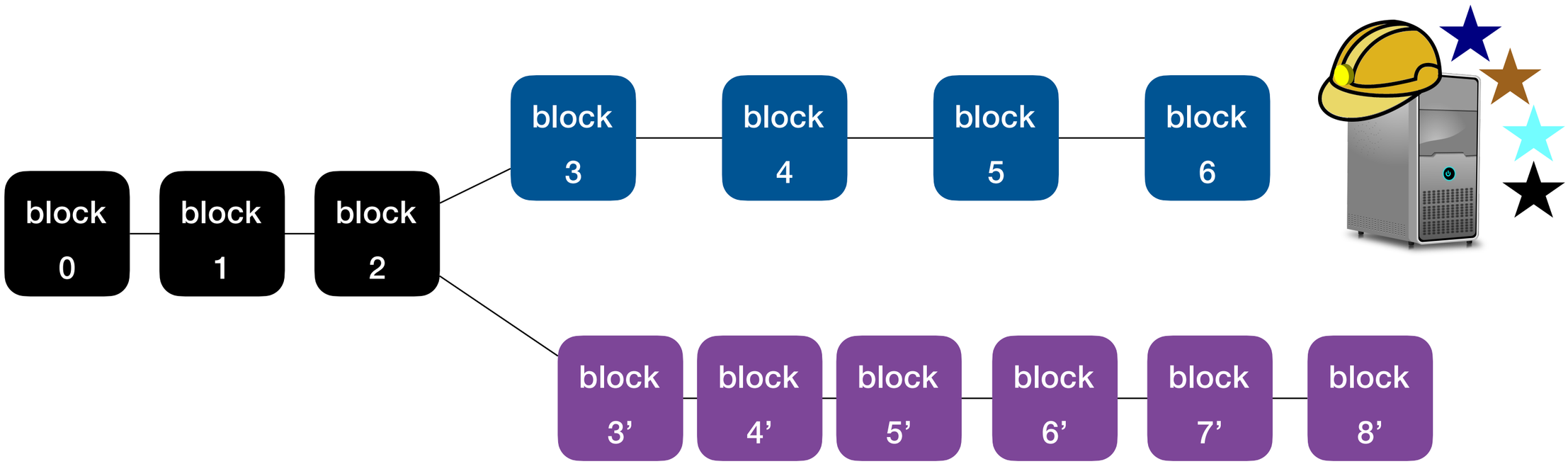

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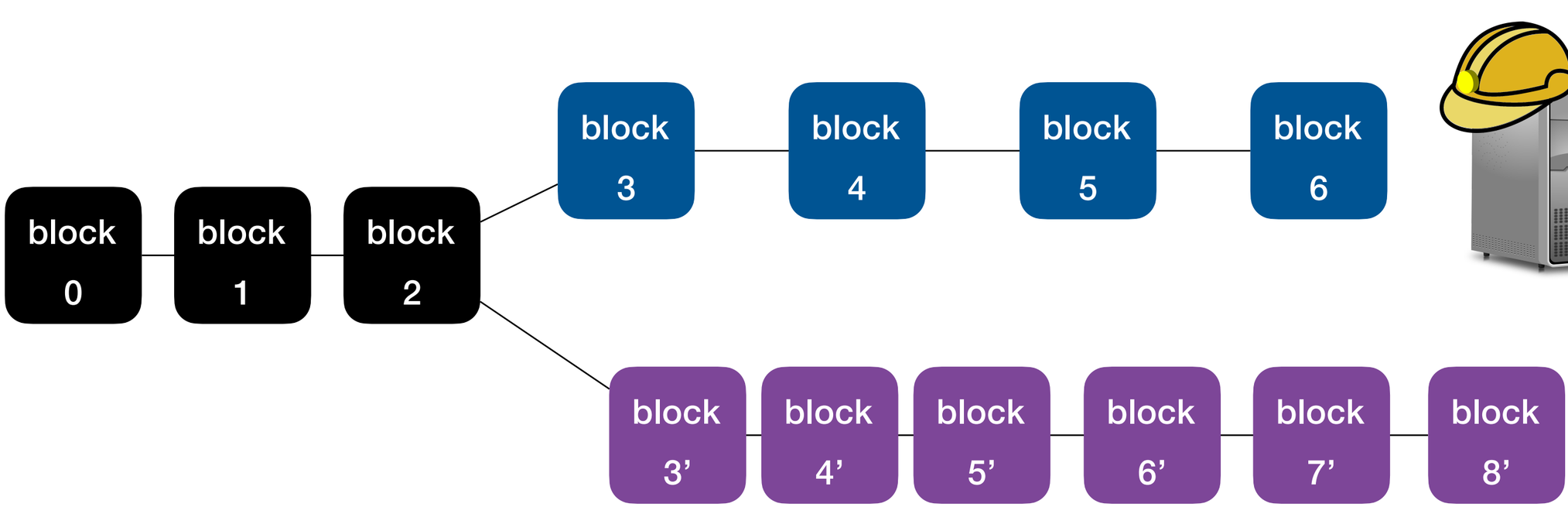

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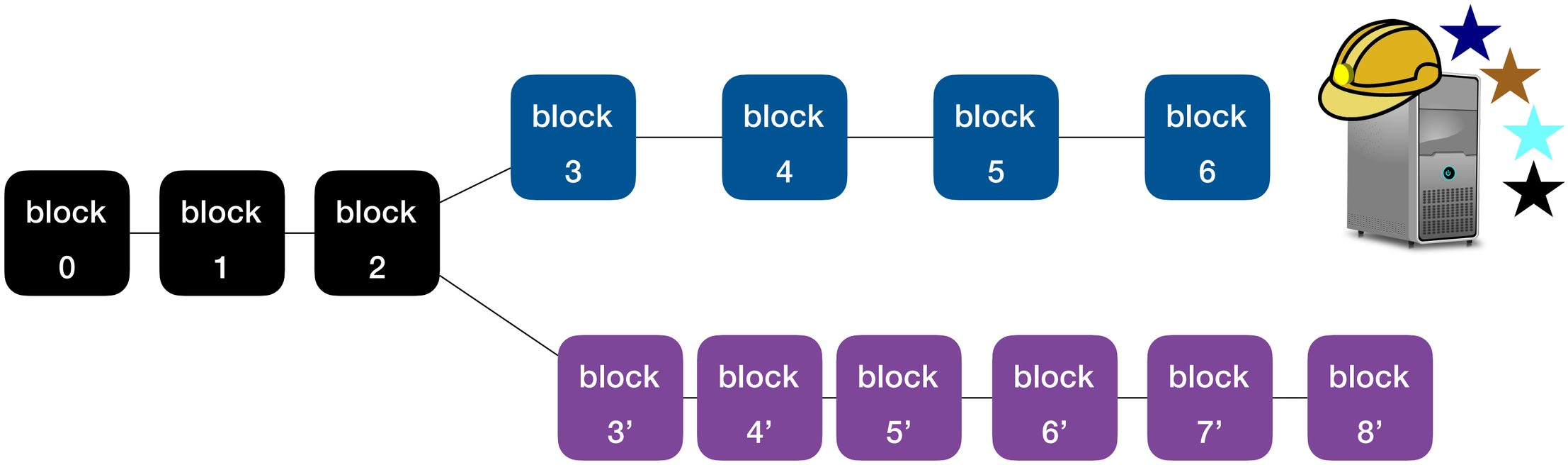

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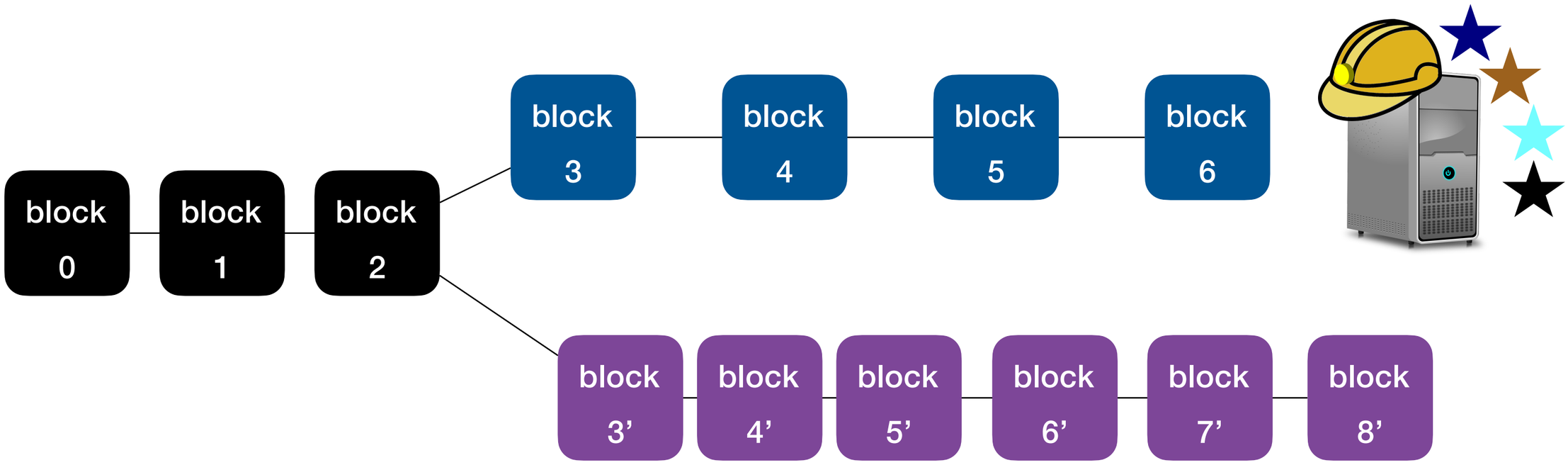

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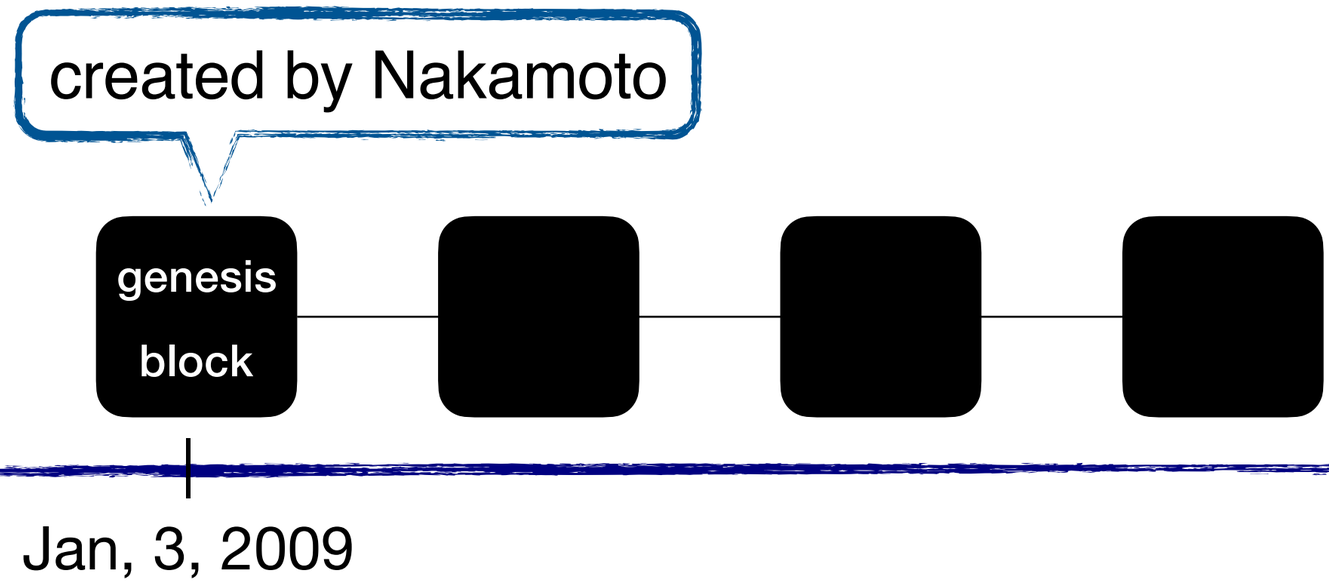
Freshness of the Genesis Block



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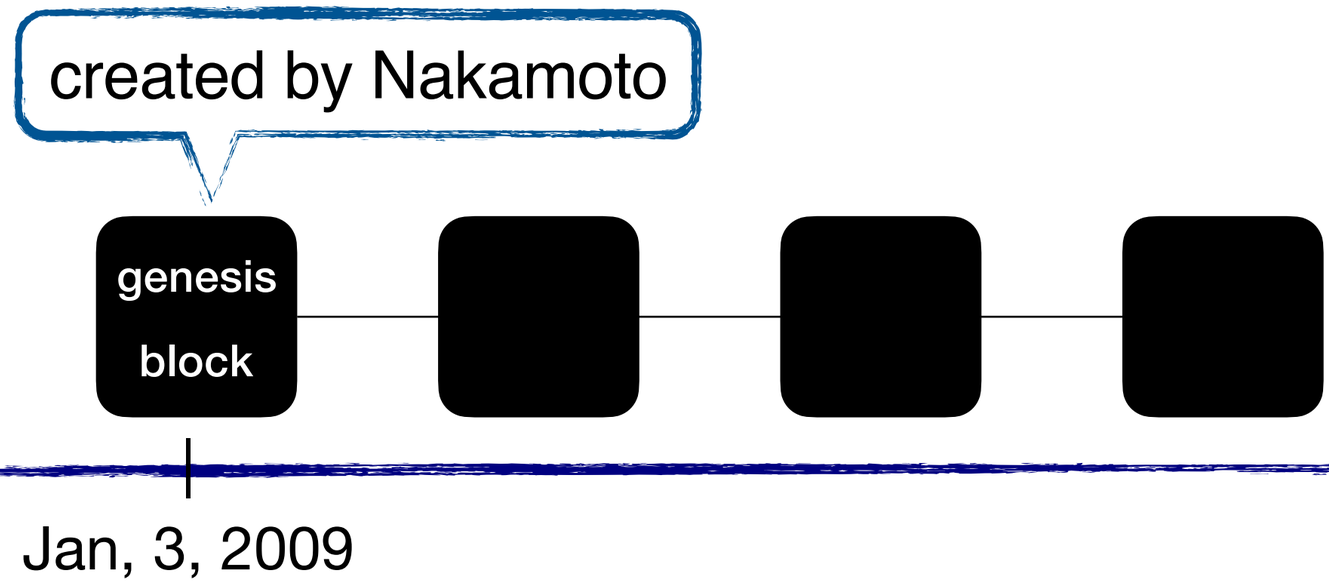

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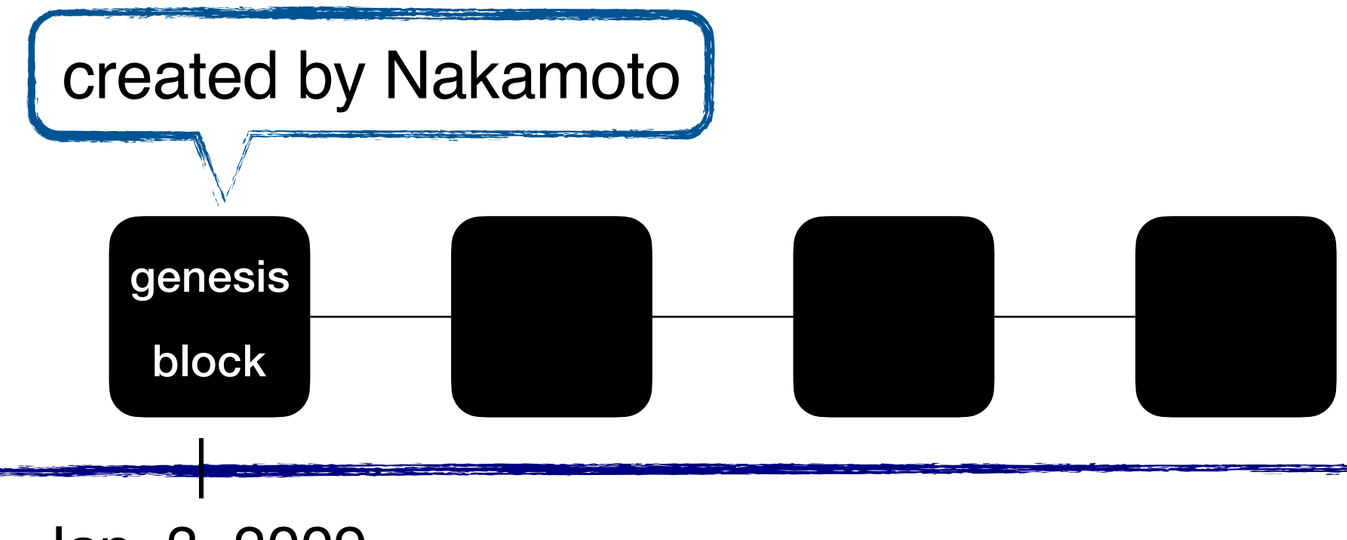
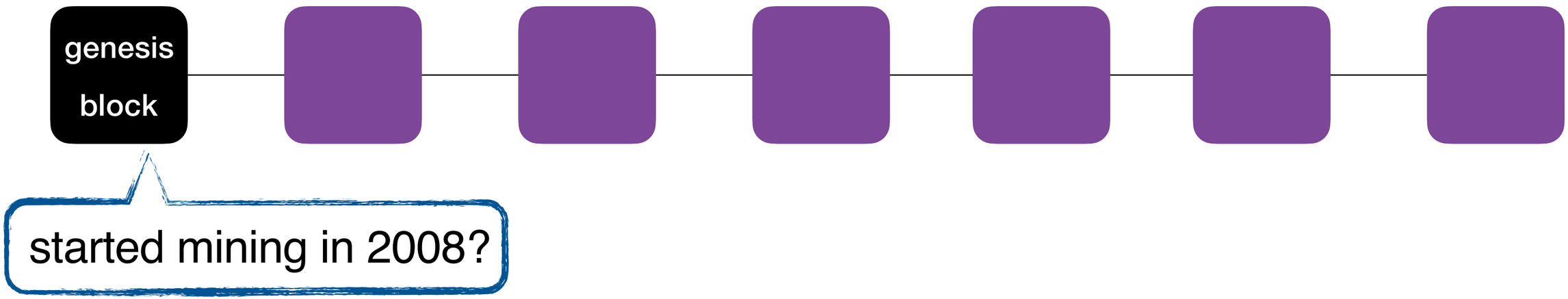
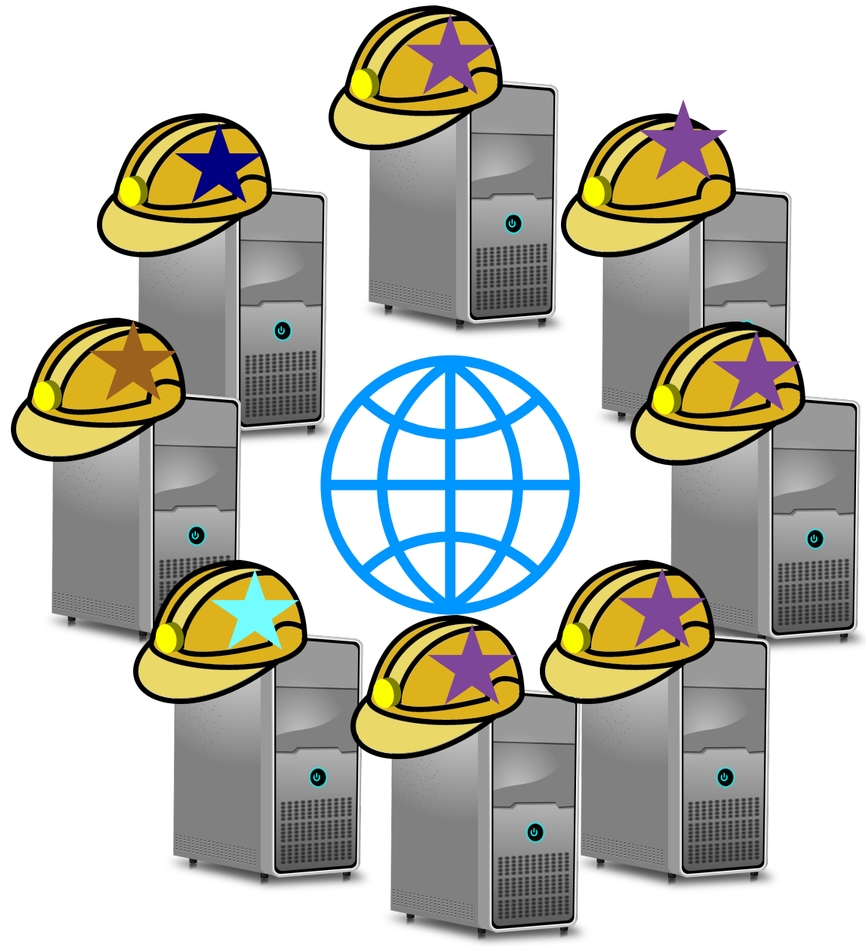


what if in reality



The 51% Attack



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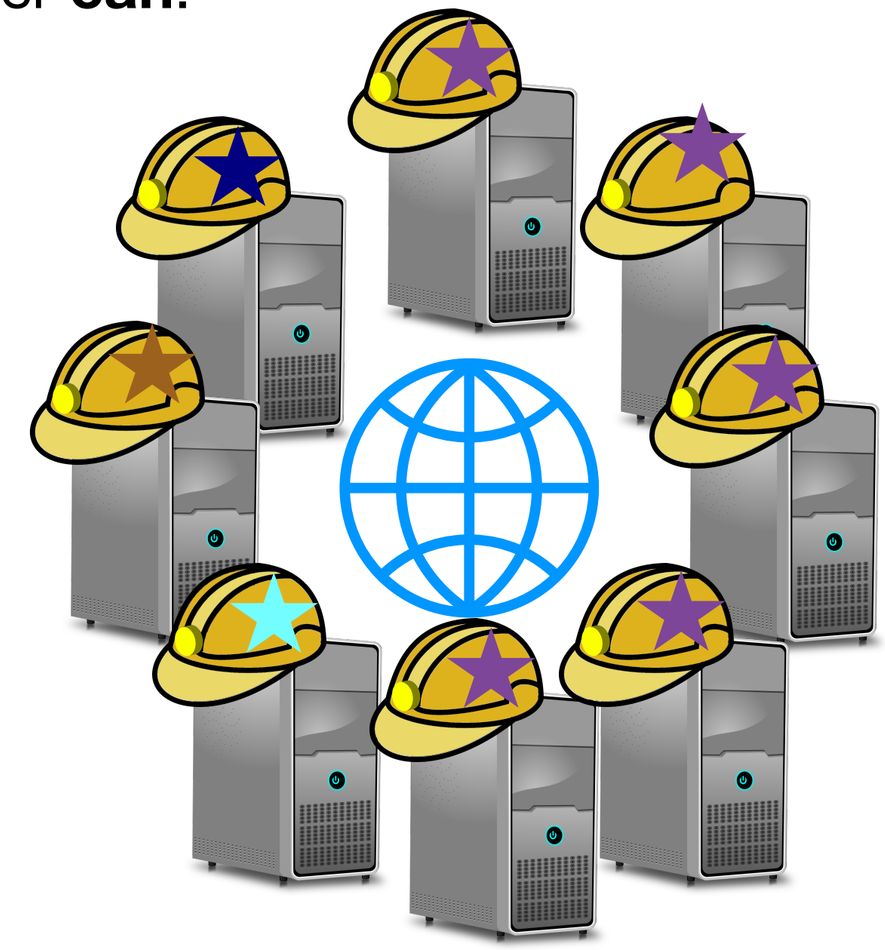

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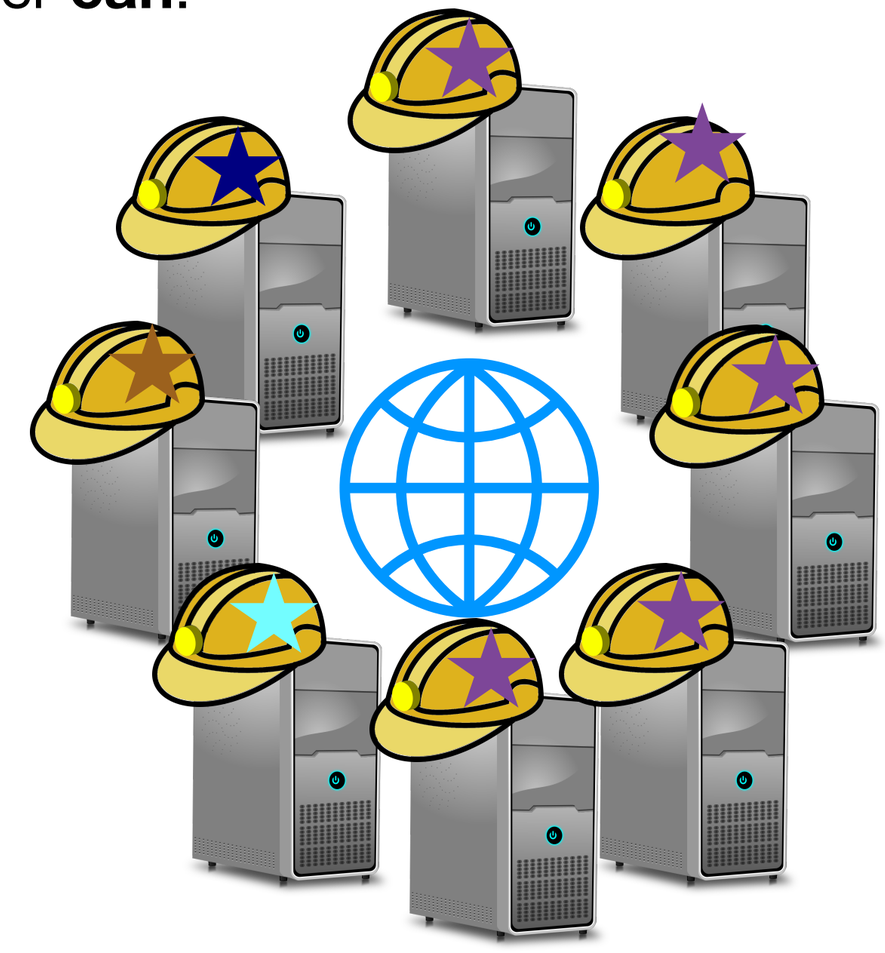

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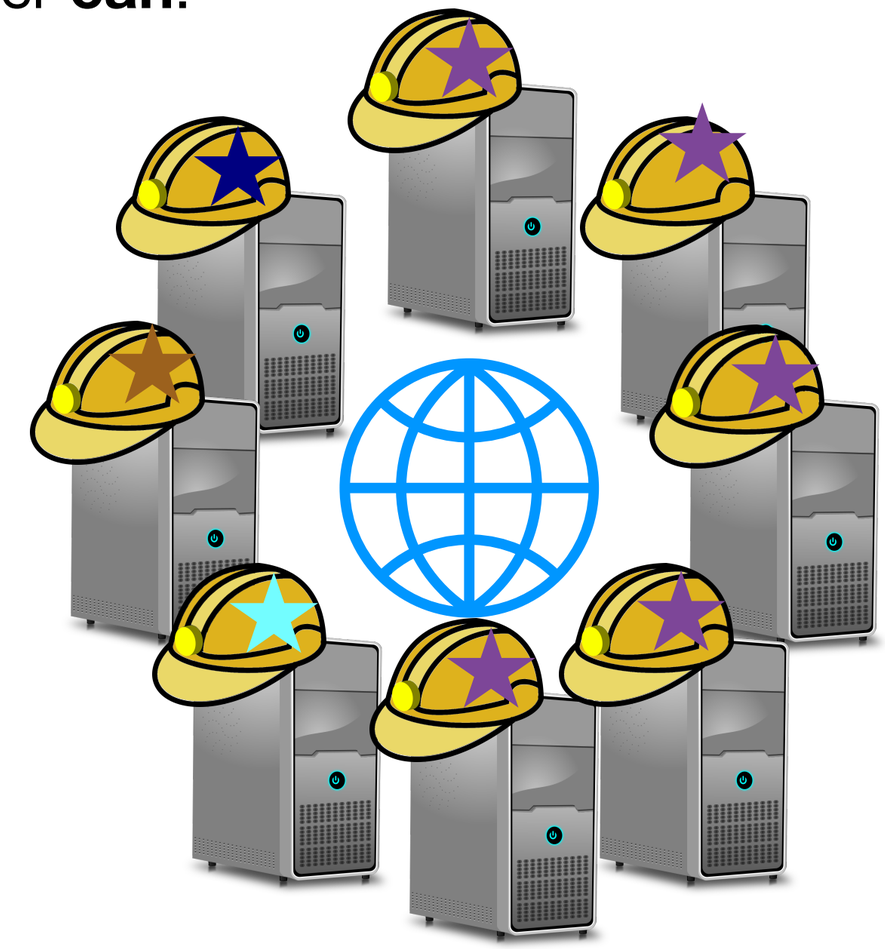

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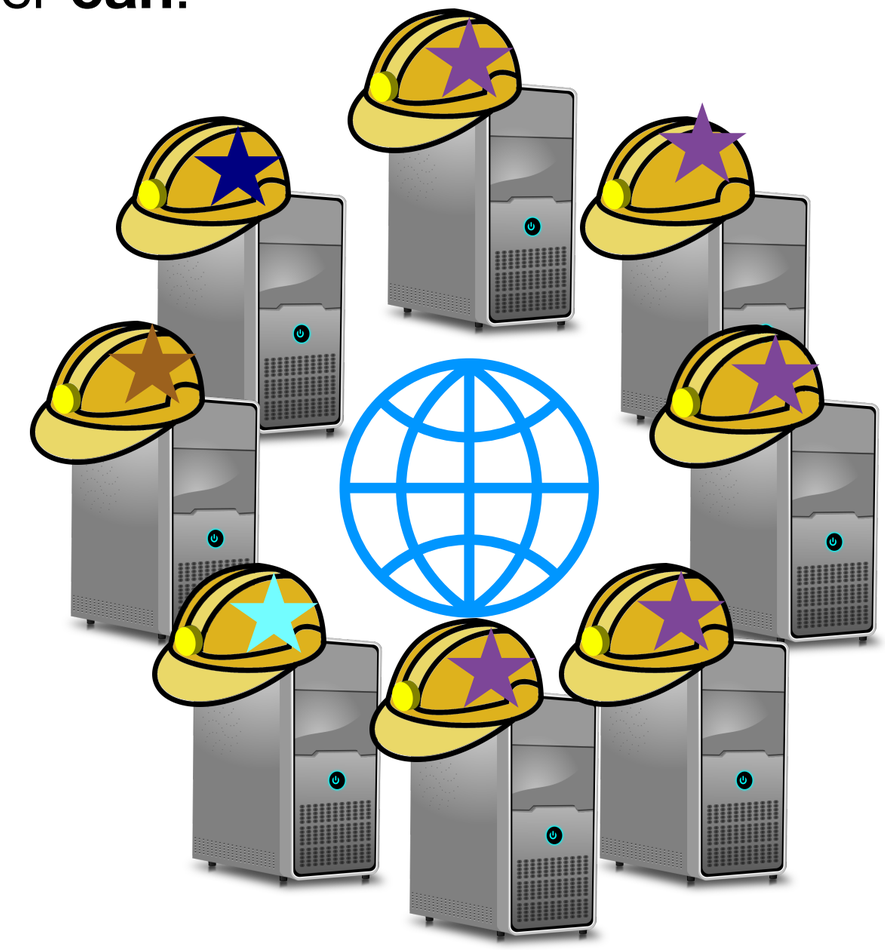

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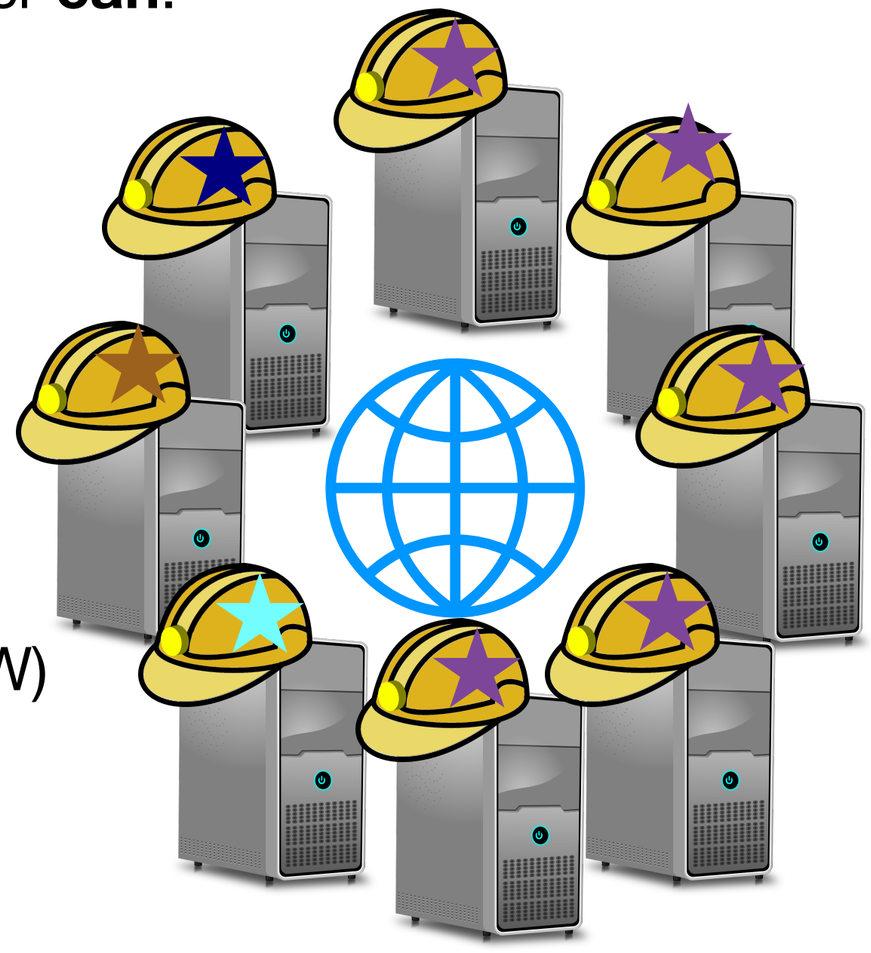

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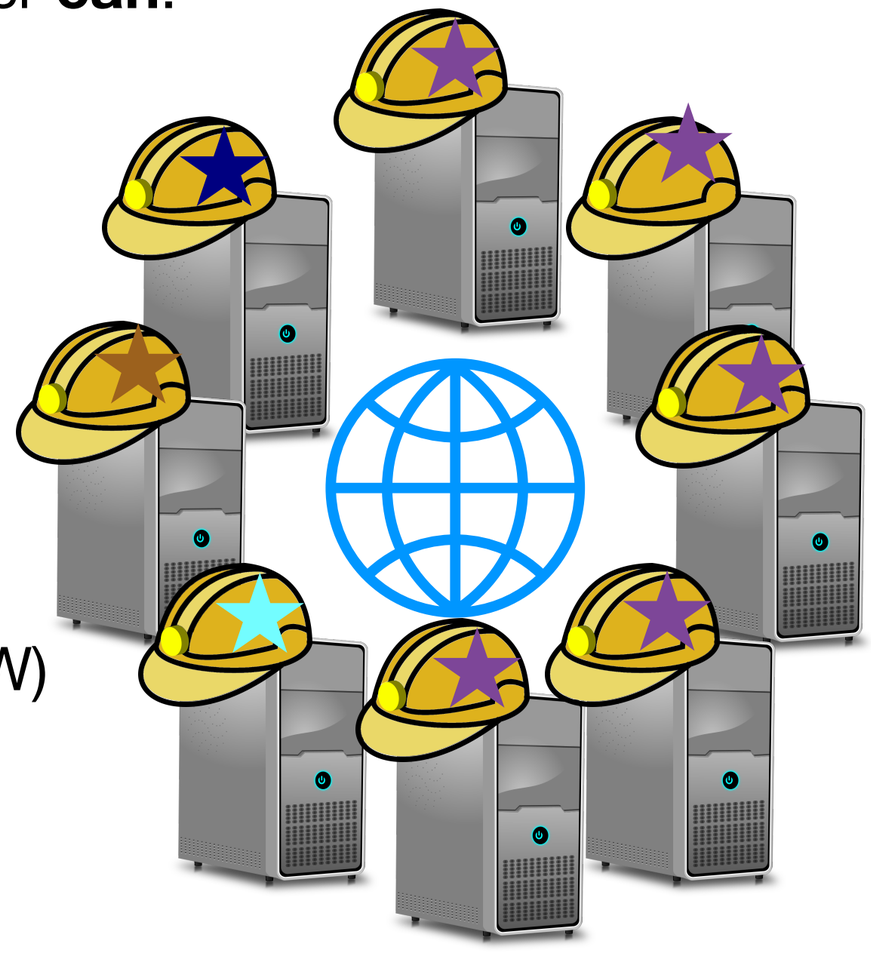

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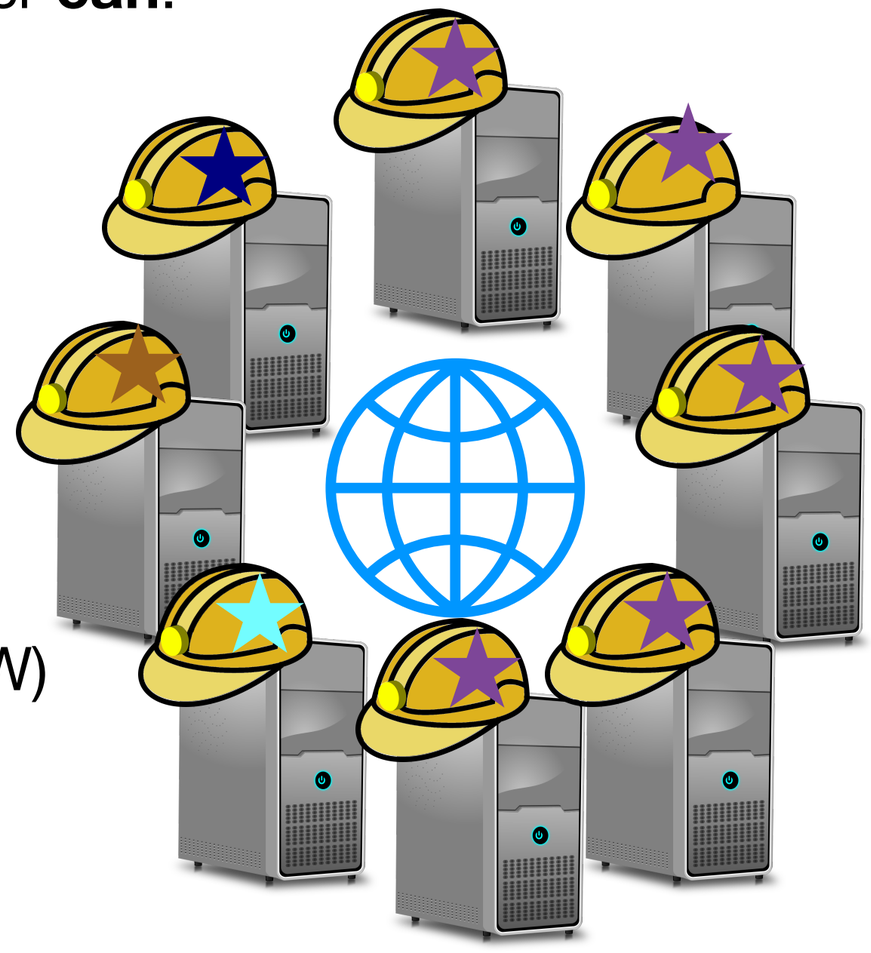

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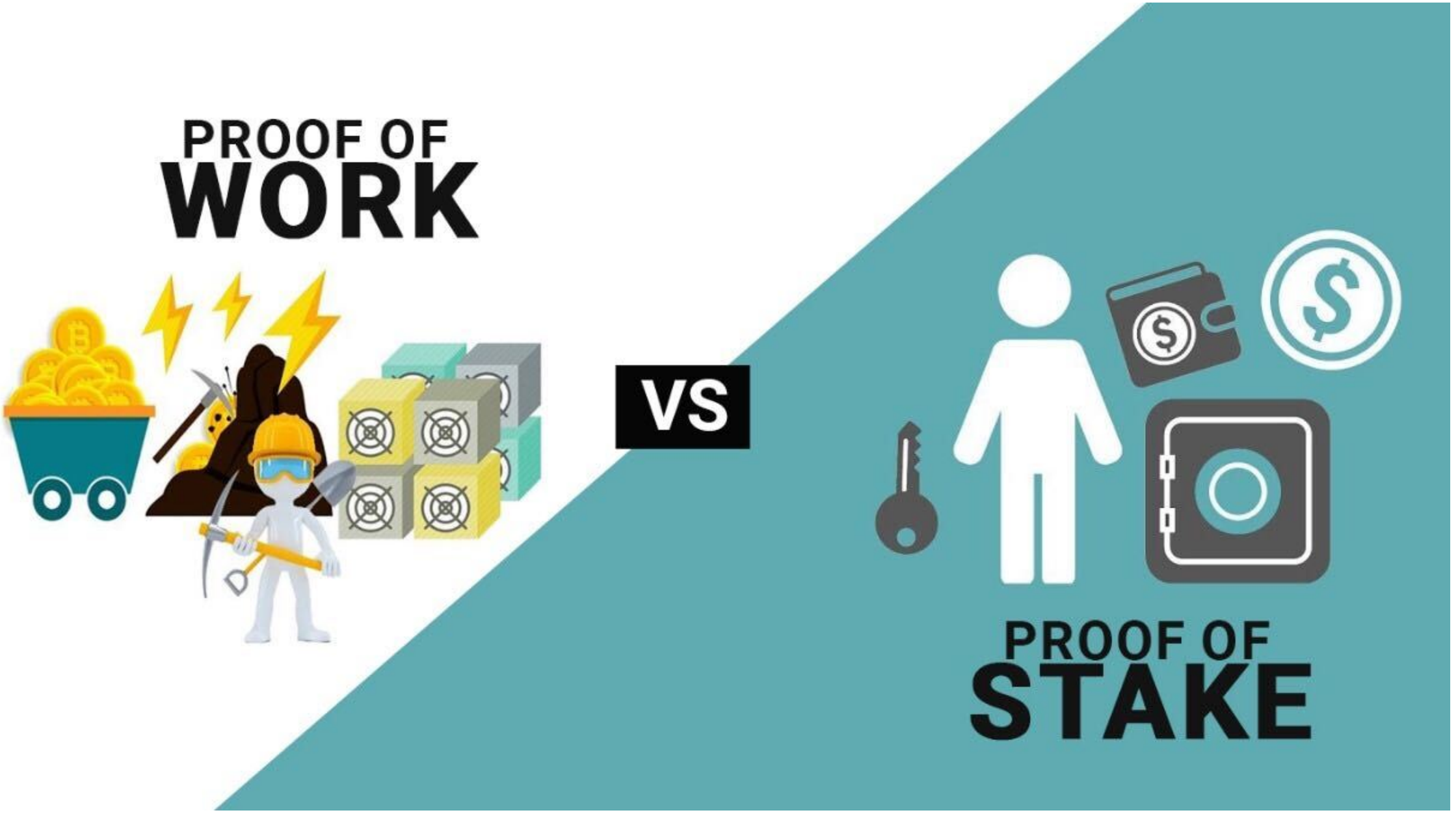
The Selfish Miner Attack

Produce a new block and keep it to oneself:

- Honest miners waste their effort to mine blocks that will never make it to the blockchain

Alternative Consensus Mechanisms



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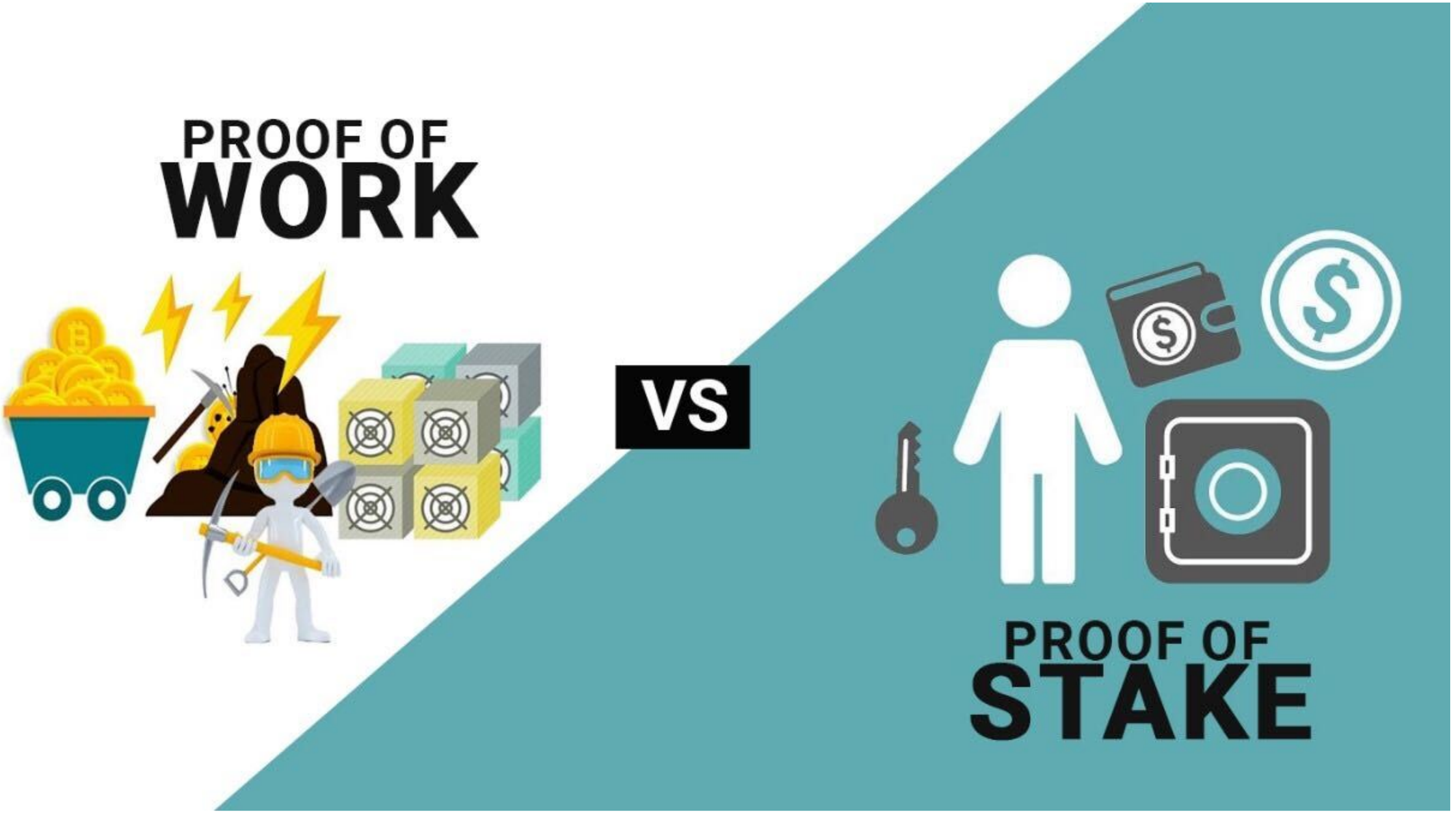
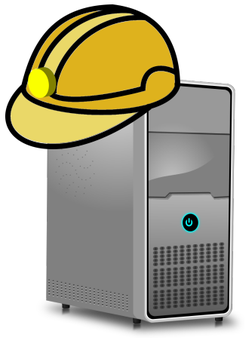


why do we need consensus?

Proof of Work (Recap)



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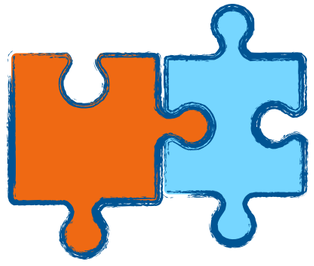

block

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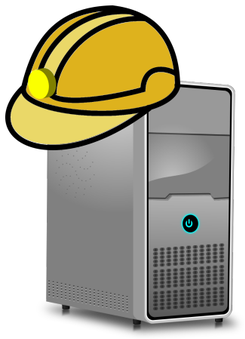

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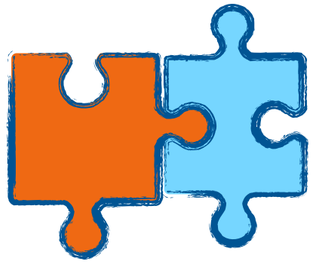

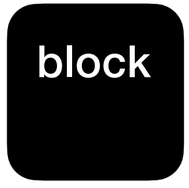

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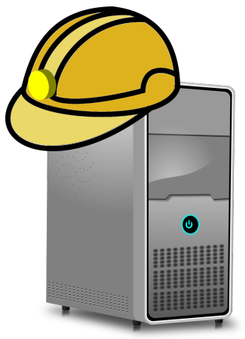

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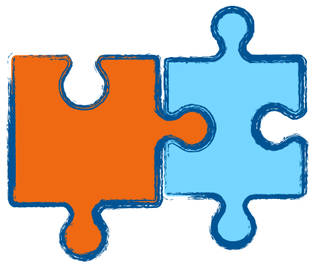

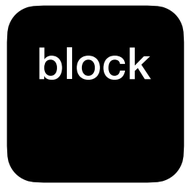

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Single Bitcoin Transaction Footprints

Carbon Footprint	Electrical Energy	Electronic Waste
873.58 kgCO2	1839.11 kWh	262.70 grams
Equivalent to the carbon footprint of **1,936,150** VISA transactions or **145,596** hours of watching Youtube.	Equivalent to the power consumption of an average U.S. household over **63.04** days.	Equivalent to the weight of **1.60** iPhones 12 or **0.54** iPads. (Find more info on e-waste here.)

Mining One Bitcoin	One BTC Worth of Gold
521,675 kWh	39,560 kWh
of electrical energy is used to mine a single Bitcoin.	of electrical energy is used to mine a BTC worth of gold.

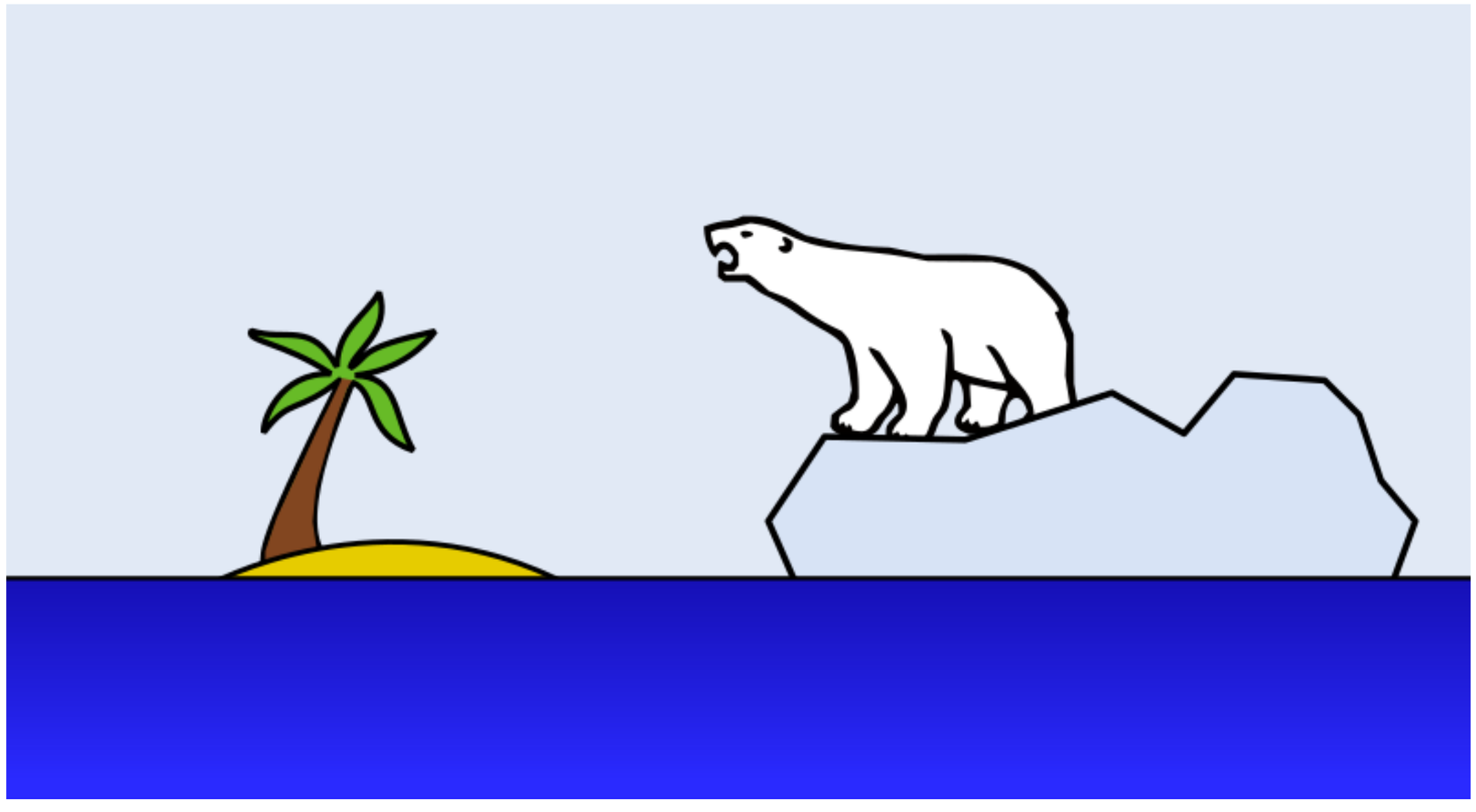
source: digiconomist.net

ENVIRONMENT

Rain Fell On The Peak Of Greenland's Ice Sheet For The First Time In Recorded History

August 20, 2021 · 11:00 AM ET

JOE HERNANDEZ



Proof of Stake

Idea: change the rule for selecting the lottery winner

Replace “work” with “stake” (computation power with digital cash)

The probability of being selected is directly proportional to one’s stake in the “bid”

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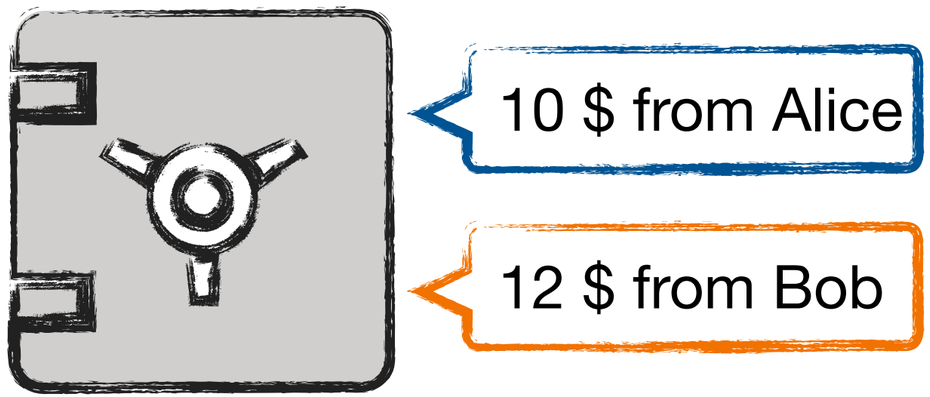

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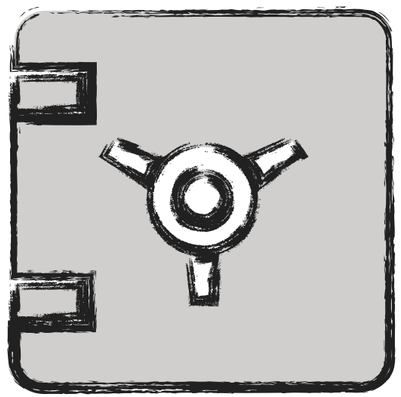

10 $ from Alice

12 $ from Bob

3 $ from Charlie

winner gets to check if all transactions in the block are valid, earns the transactions fees

Proof of Stake

Idea: change the rule for selecting the lottery winner

Replace “work” with “stake” (computation power with digital cash)

The probability of being selected is directly proportional to one’s stake in the “bid”

temporary lock stakes to determine
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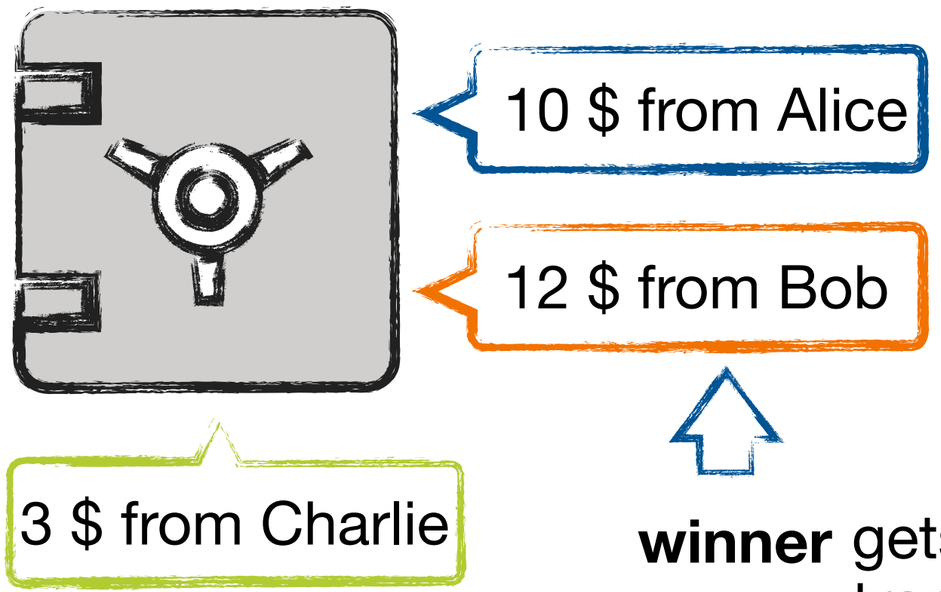

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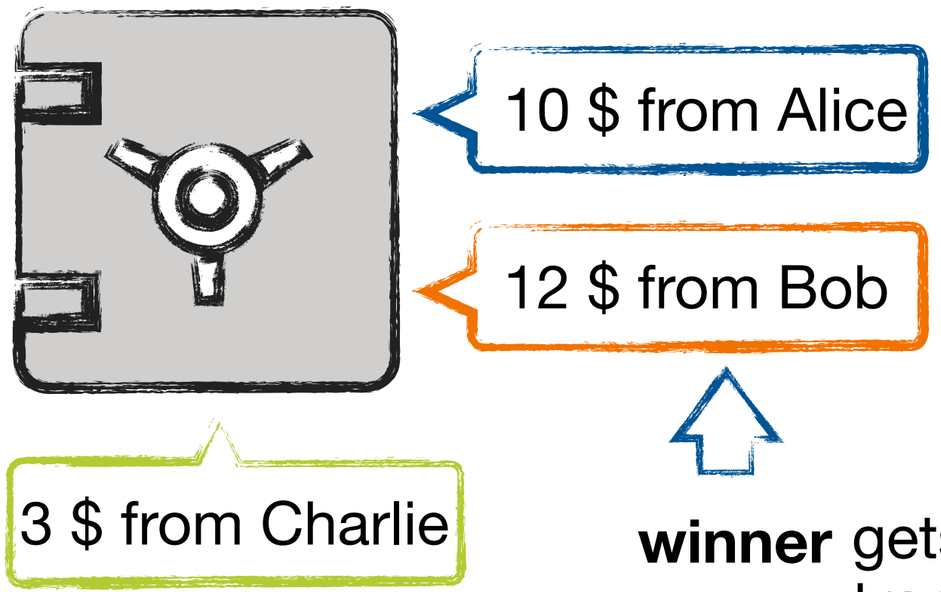


winner gets to check if all transactions in the block are valid, earns the transactions fees

How to enforce honest behaviours?

Validators lose part of their stake if they approve fraudulent transactions

Proof of Stake

EOS

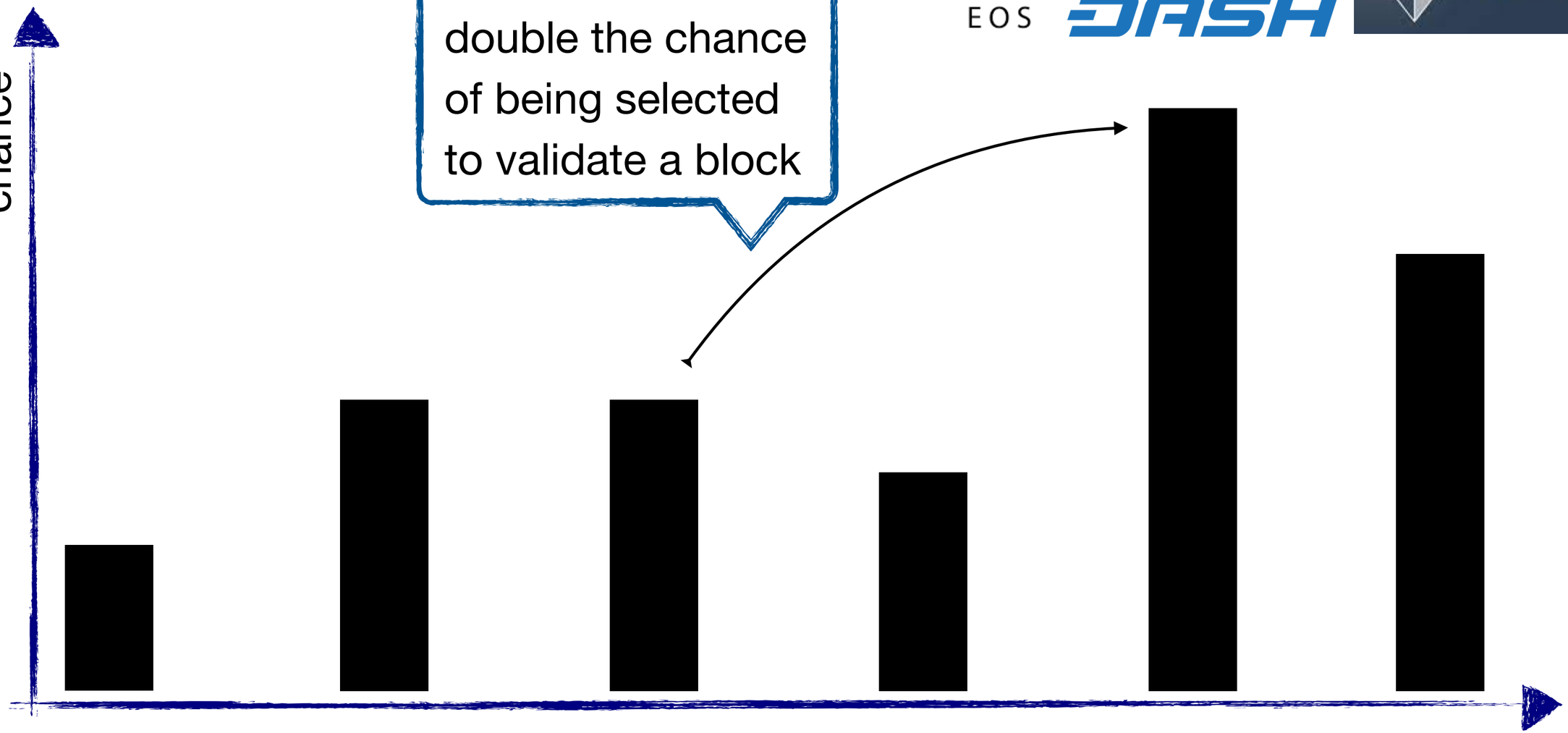
CARDANO

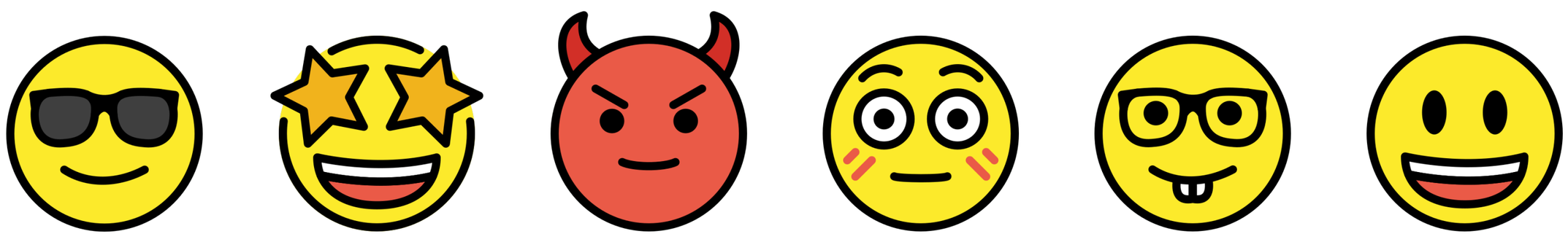
peercoin

DASH

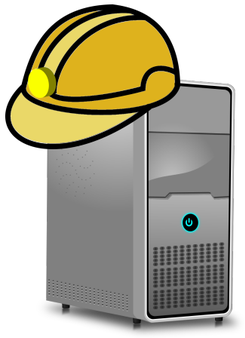
ETHEREUM v2.0

double the stake,
double the chance
of being selected
to validate a block

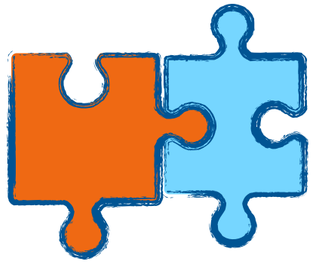


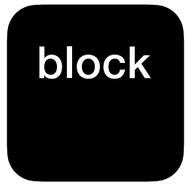


Proof of Work vs Proof of Stake

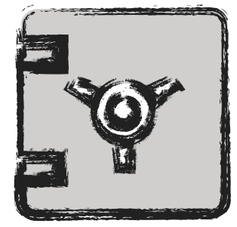


PoW





PoS



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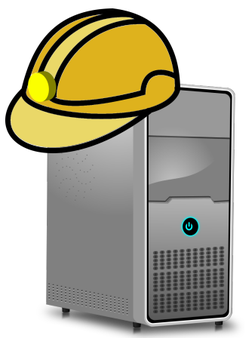

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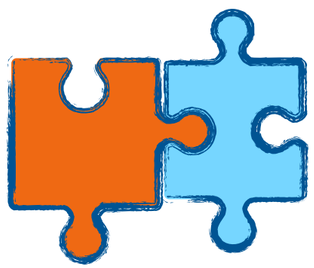

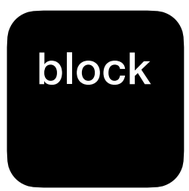

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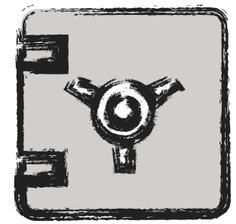

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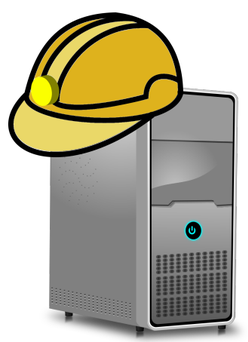

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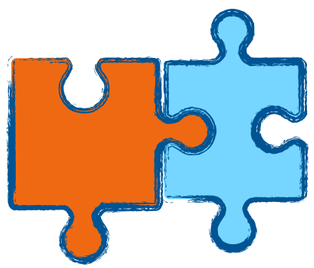

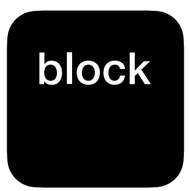

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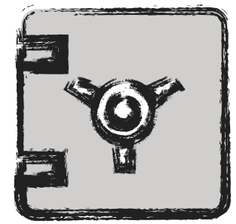

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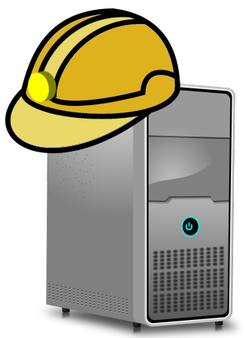

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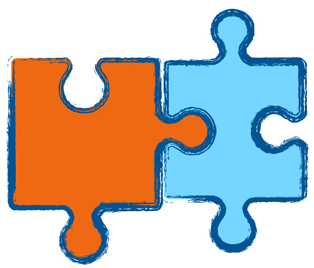


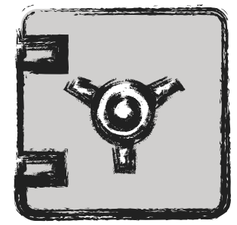
block

reward miners with best hardware (high computational power)

PoWs

Both selection mechanisms are *biased*

PoS



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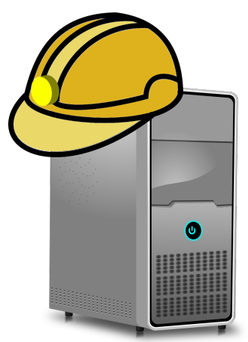

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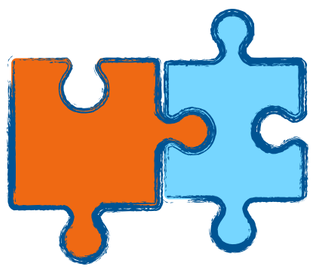

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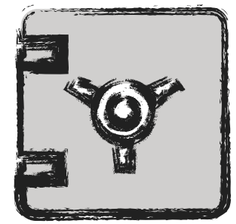

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reward miners with best hardware
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PoWs

Both selection mechanisms are *biased*

PoS

reward wealthier minters
(or who joined the coin first)

General View on the Problem

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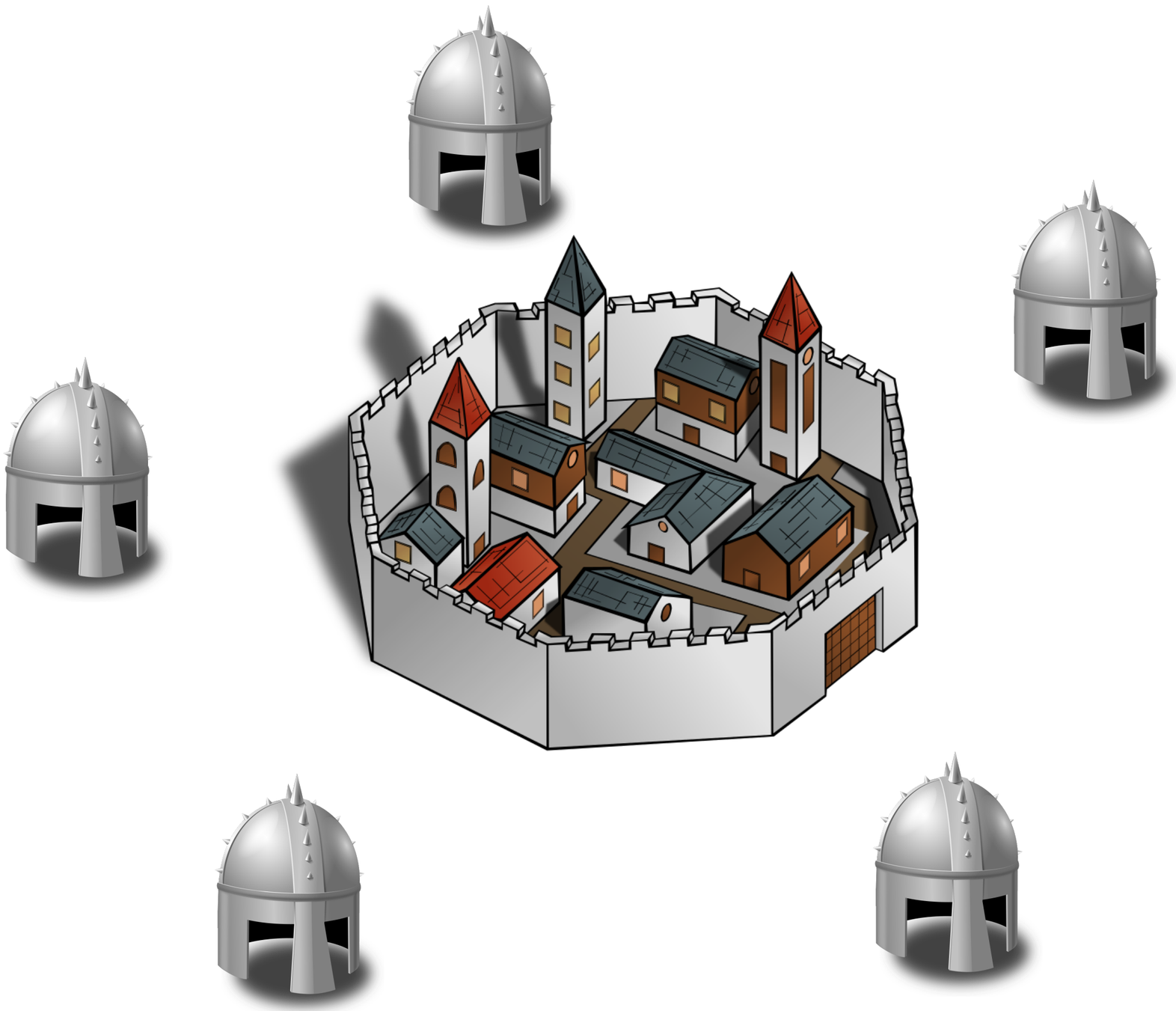

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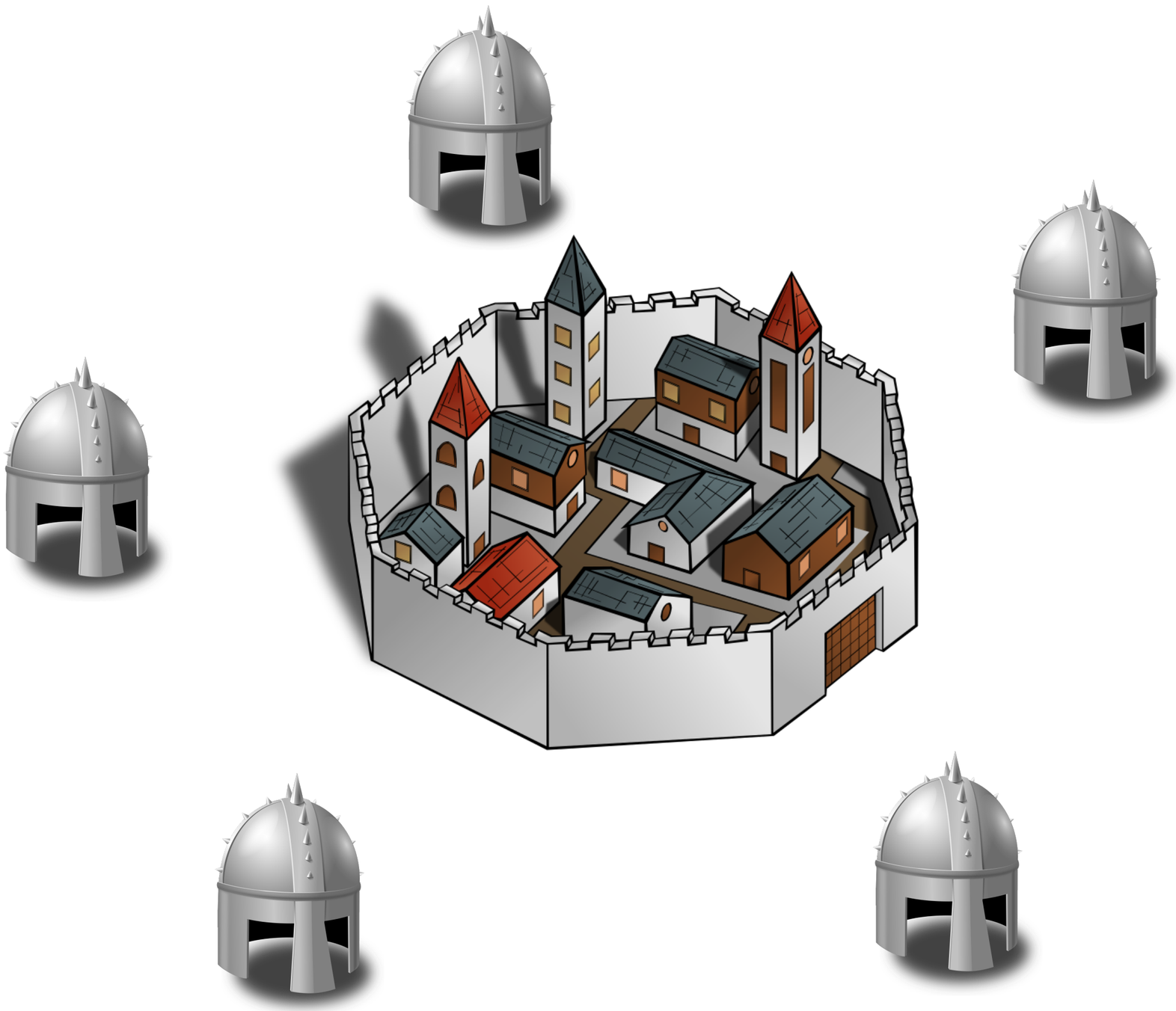

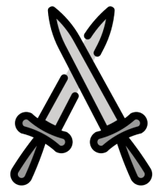

= attack

= retreat

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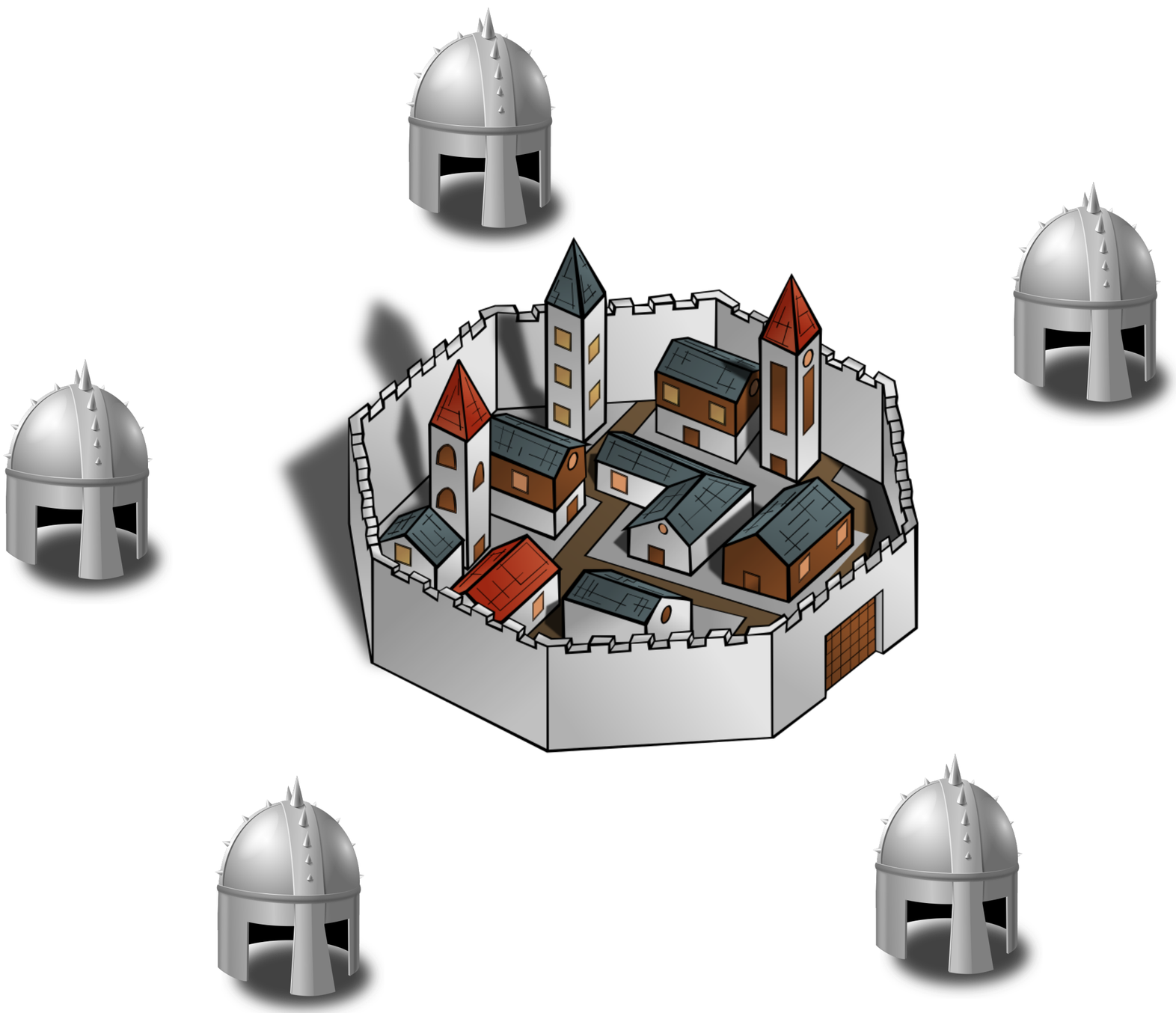

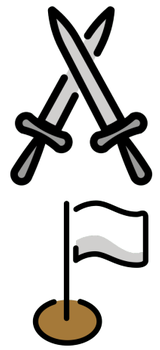

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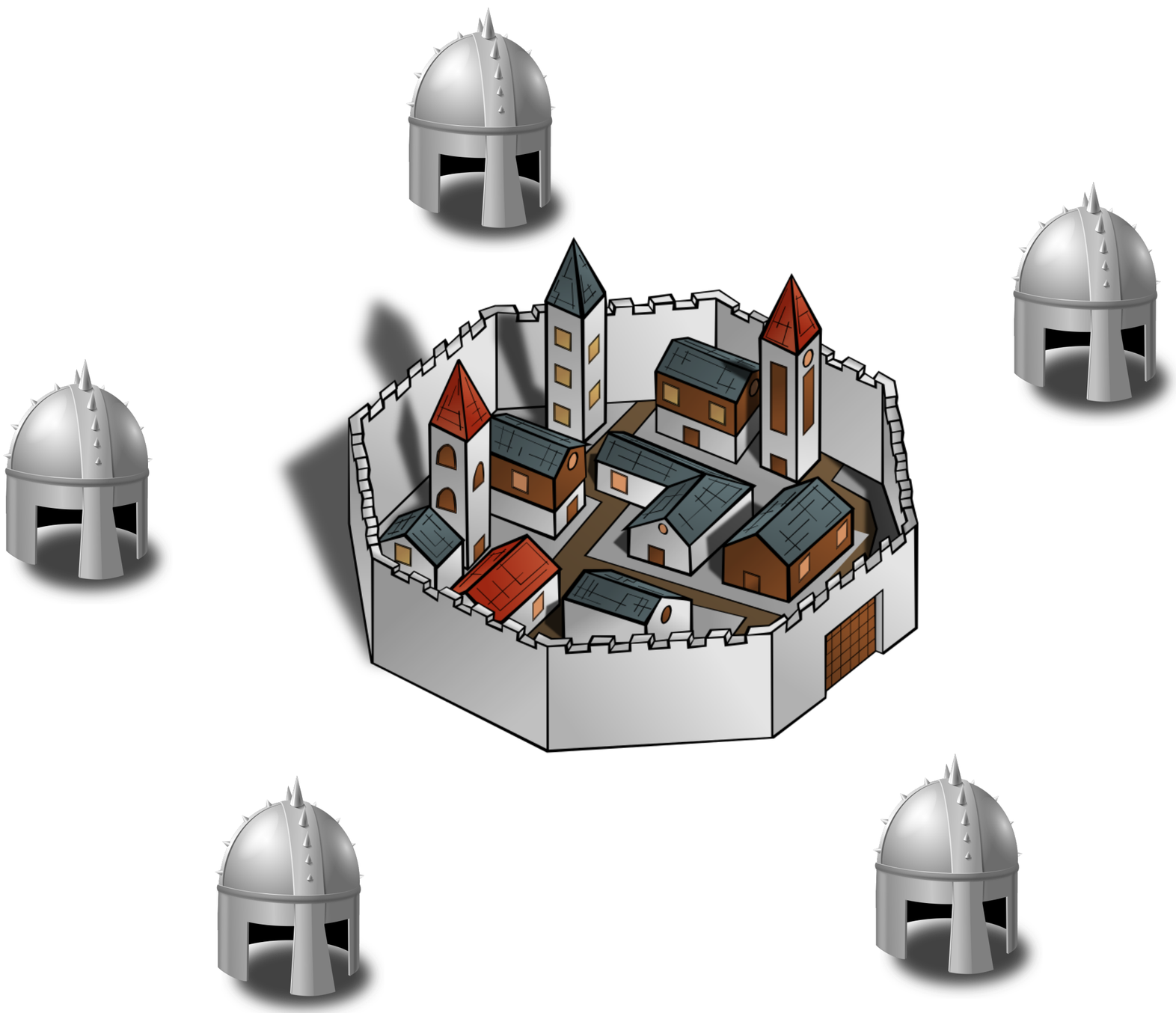

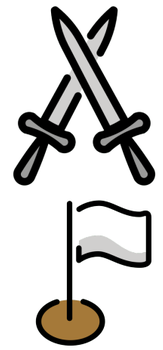

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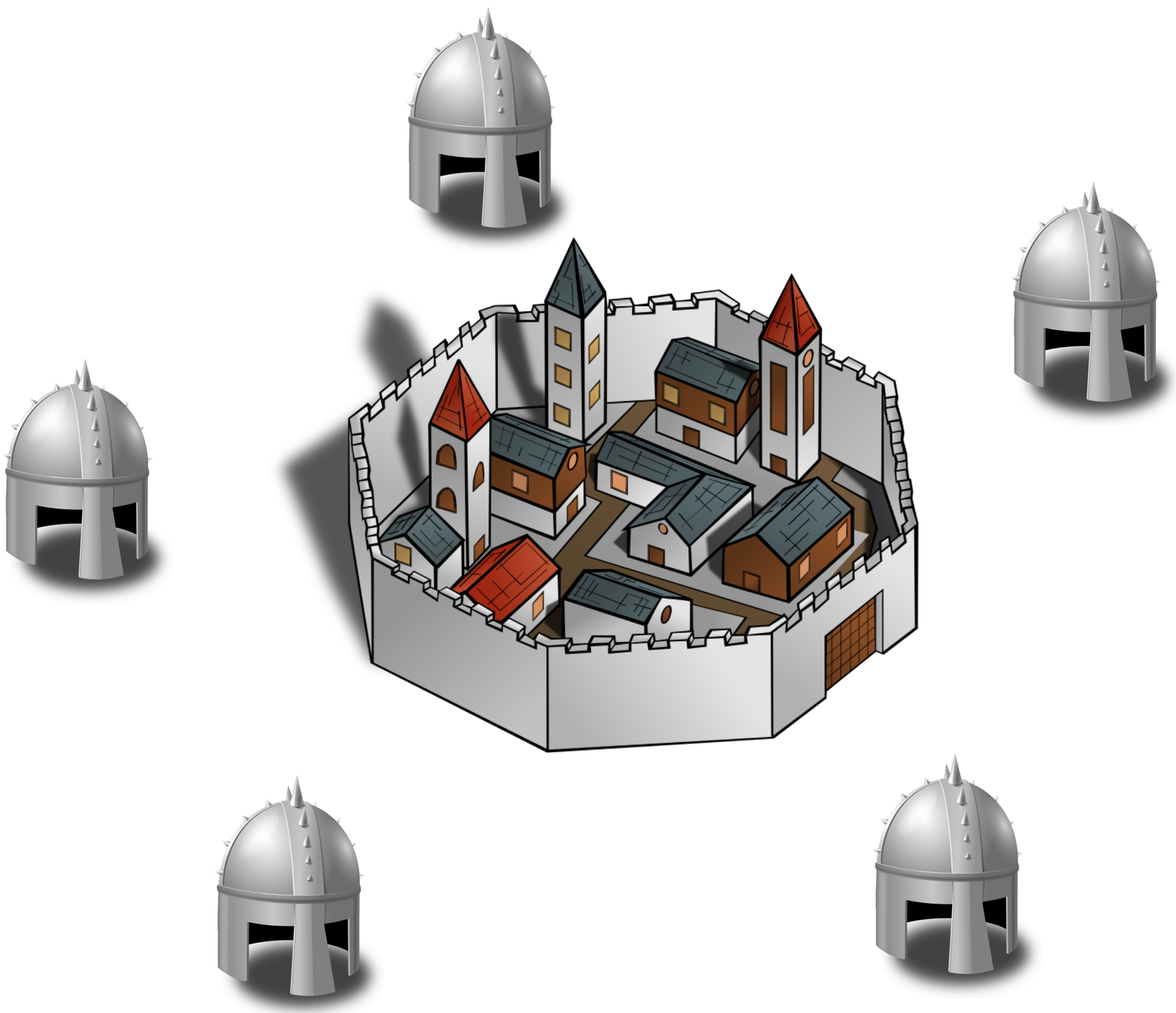

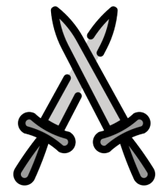

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(not secured)

there are traitors,

Generals with inconsistent behaviours

The Byzantine Fault Tolerance Agreement

The Byzantine Fault Tolerance Agreement

Setting:

The Byzantine Fault Tolerance Agreement

Setting:

★ A system of *n* parties

The Byzantine Fault Tolerance Agreement

Setting:

★ A system of *n* parties

★ With *t* dishonest parties

The Byzantine Fault Tolerance Agreement

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The Byzantine Fault Tolerance Agreement

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3. At the end of the protocol, all honest parties agree on the same value **y**.

BFT Consensus: History and Applications

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NASA

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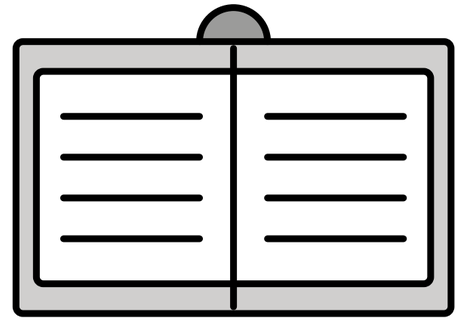
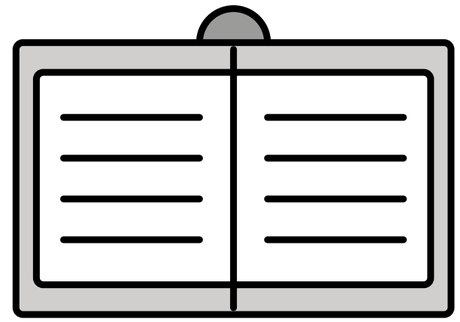


PoW / PoS ⇒ agree on a coherent global view of the ledger

BFT Consensus: History and Applications

NASA



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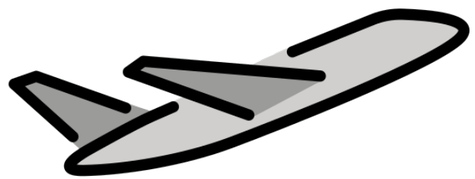

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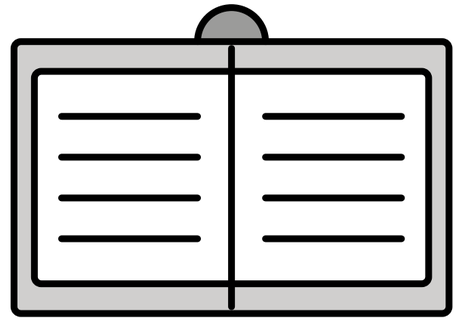

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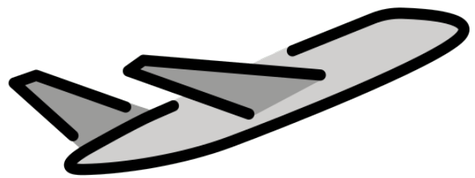

Lecture Agenda

How To Set Up a Bulletin Board

- Hash Functions
- Proof of Work (Cryptographic Puzzles)
- Example: Bitcoin

How To Generate Money

- Mining Blocks
- Merkle Trees
- Block Reword System

How To Circulate Money

- Transactions
- ECDSA

Some Problems With Blockchains

- Freshness of the Genesis Block
- 51% Attack
- PoW Vs PoS
- The Byzantine Fault Tolerance Problem

Advanced Tools

- Zero Knowledge Proofs
- Schnorr Interactive PoK
- Fiat-Shamir Heuristic (NIZK)

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$$H : \{0,1\}^* \rightarrow \{0,1\}^N$$

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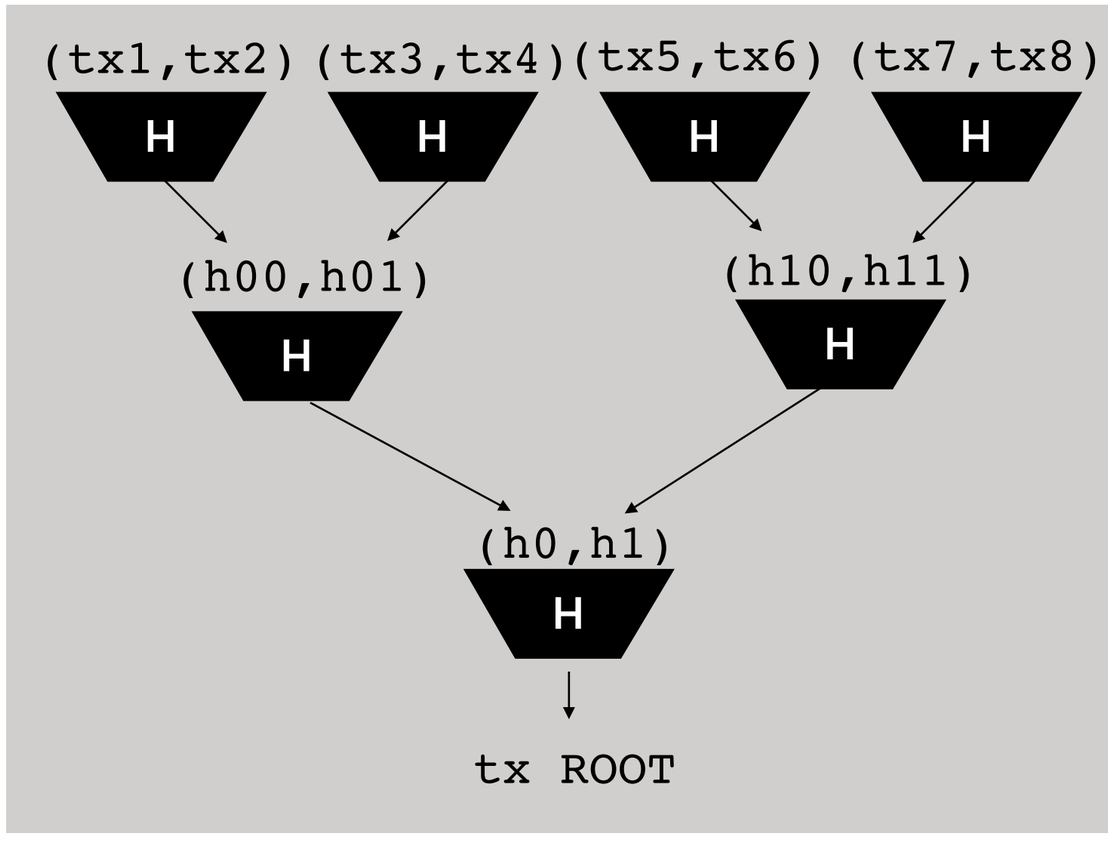
H_1(x)||H_2(x)   vs   H_1(x)
```



Lecture Agenda

How To Set Up a Bulletin Board

- Hash Functions
- Proof of Work (Cryptographic Puzzles)
- Example: Bitcoin

How To Generate Money

- Mining Blocks
- Merkle Trees
- Block Reword System

How To Circulate Money

- Transactions
- ECDSA

Some Problems With Blockchains

- Freshness of the Genesis Block
- 51% Attack
- PoW Vs PoS
- The Byzantine Fault Tolerance Problem

Advanced Tools

- Zero Knowledge Proofs
- Schnorr Interactive PoK
- Fiat-Shamir Heuristic (NIZK)

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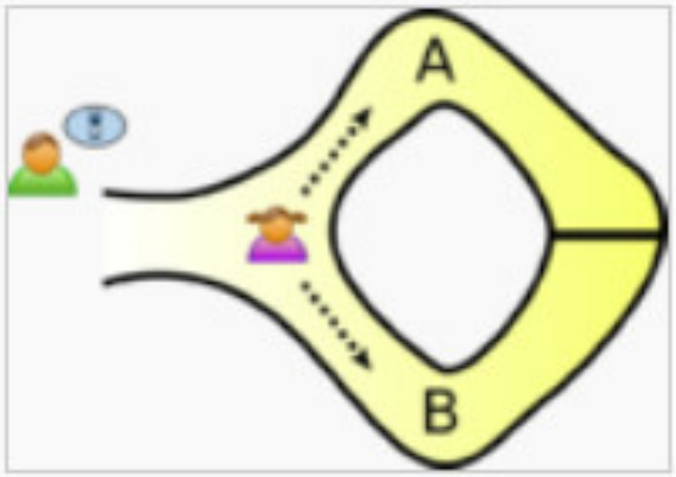
You can't have your cake and eat it too!

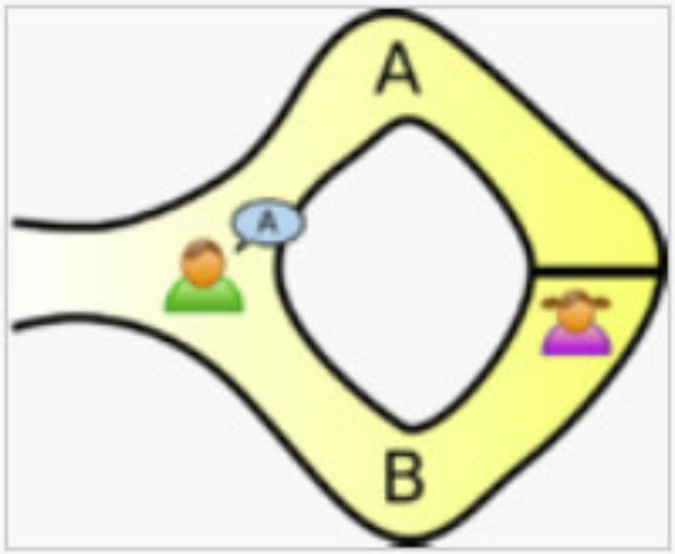
well, with crypto you do :)

zero knowledge proofs

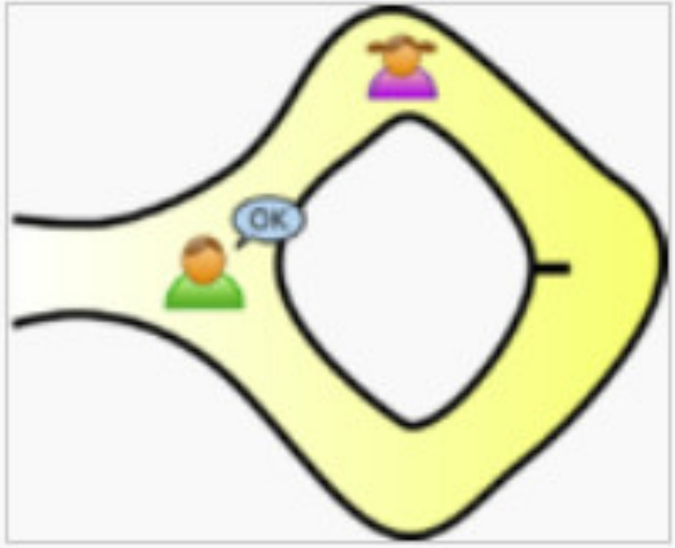
Zero Knowledge Proofs - a Metaphor



Peggy randomly takes either path A or B, while Victor waits outside



Victor chooses an exit path



Peggy reliably appears at the exit Victor names

How To Formalise This Into Math/Crypto?

How To Formalise This Into Math/Crypto?

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

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5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

x

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

w

The most general formalisation: $x \in L \text{ iff } \exists\, w \text{ s.t. } R(x,w) = 1$

How To Formalise This Into Math/Crypto?

5	3			7				
6			1	9	5			
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			4	1	9			5
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x

5	3	4	6	7	8	9	1	2
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w

set of all valid sudoku starters

The most general formalisation: $x \in L$ iff $\exists\ w$ s.t. $R(x,w) = 1$

How To Formalise This Into Math/Crypto?

5	3			7				
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4			8		3			1
7				2				6
	6					2	8	
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				8			7	9

x

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
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w

set of all valid sudoku starters

solution satisfies sudoku rules

The most general formalisation: $x \in L \text{ iff } \exists\ w \text{ s.t. } R(x,w) = 1$

How To Formalise This Into Math/Crypto?

Proof System

A process in which a prover probabilistically convinces a verifier of the correctness of a mathematical proposition

How To Formalise This Into Math/Crypto?

Zero Knowledge **Proof System**

A process in which a prover probabilistically convinces a verifier of the correctness of a mathematical proposition, and the verifier learns nothing else.

How To Formalise This Into Math/Crypto?

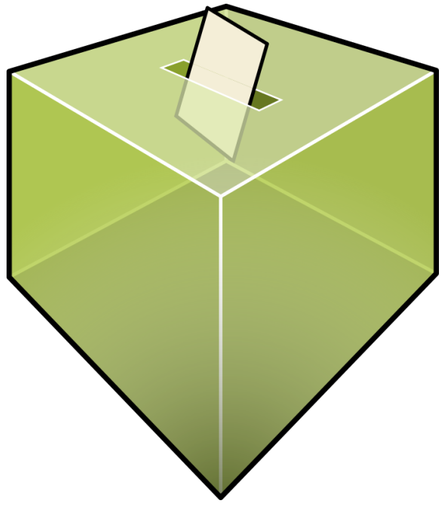
Zero Knowledge **Proof System**

A process in which a prover probabilistically convinces a verifier of the correctness of a mathematical proposition, and the verifier learns nothing else.

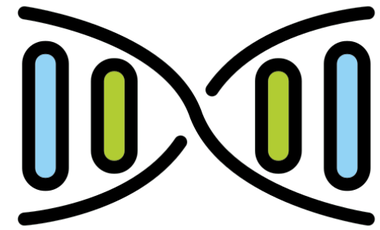
	statement	witness
factoring:	$\exists$ `p,q s.t. N = pq` $\wedge$ `p,q primes`	`(p,q)`
dLog:	$\exists$ `sk s.t. pk =` g^{sk}	`sk`
circuit satisfiability:	$\exists$ `w s.t.` $C_x(w)=1$	`w`

the language `L` is usually implicit in the application, what we make explicit is the relation `R`

Applications of Zero Knowledge Proofs

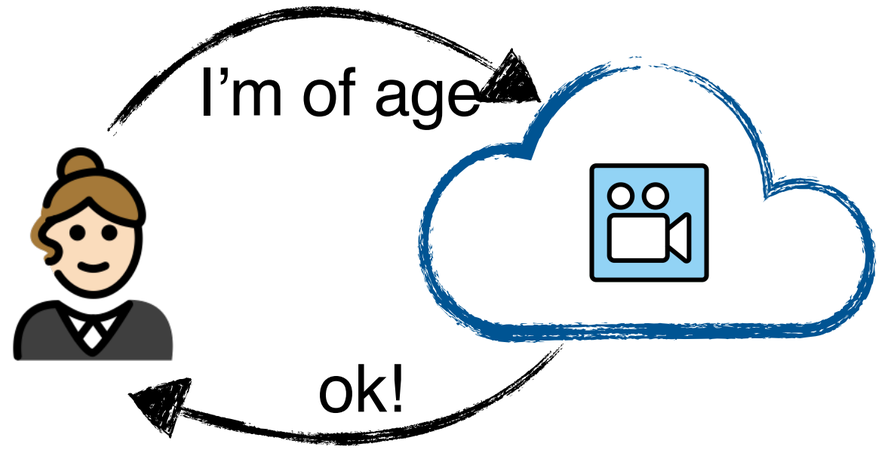


mixing and shufflers for electronic voting
& anonymous communication



DNA matches

anonymous transactions



anonymous credentials

Interactive Zero Knowledge Proofs (of Knowledge)

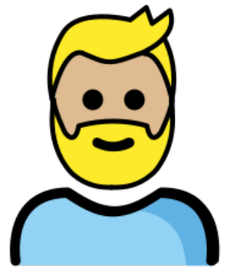
Interactive Zero Knowledge Proofs (of Knowledge)

Prover

unlimited computational power

may try to cheat

Verifier



bounded computational power

always honest

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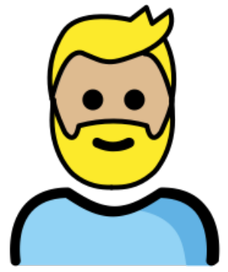

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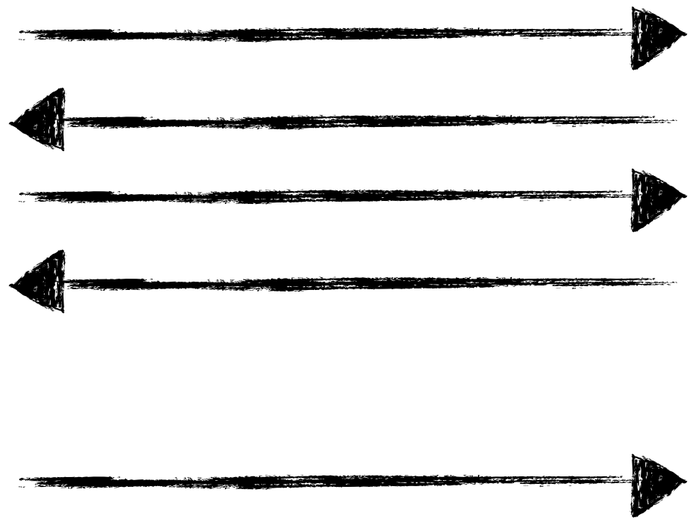
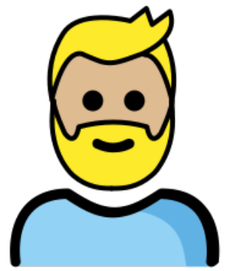


Verifier



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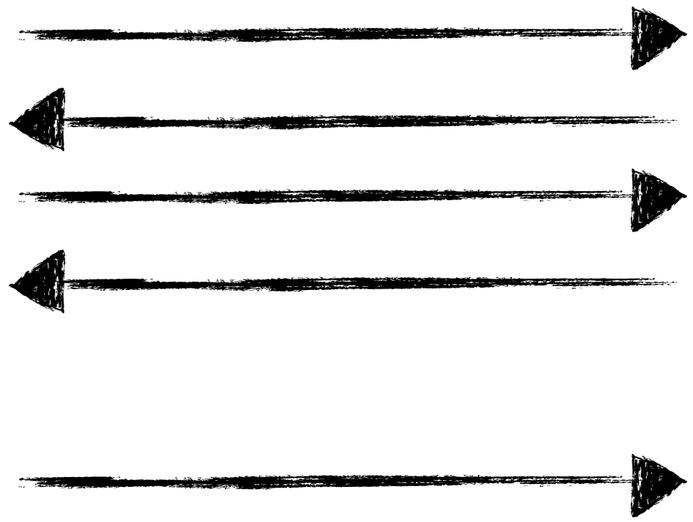

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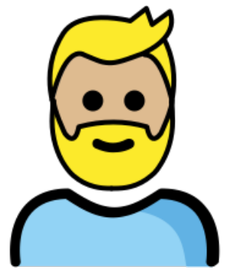

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YES /

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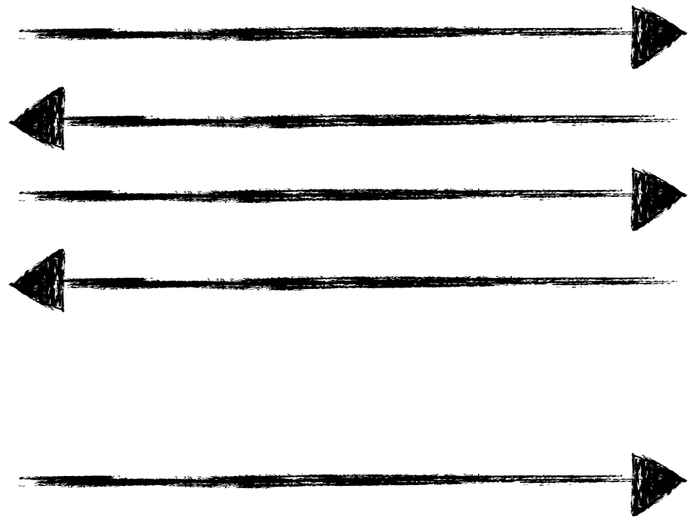

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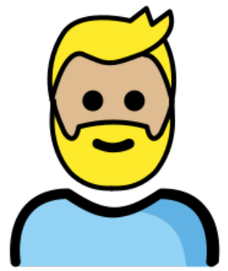

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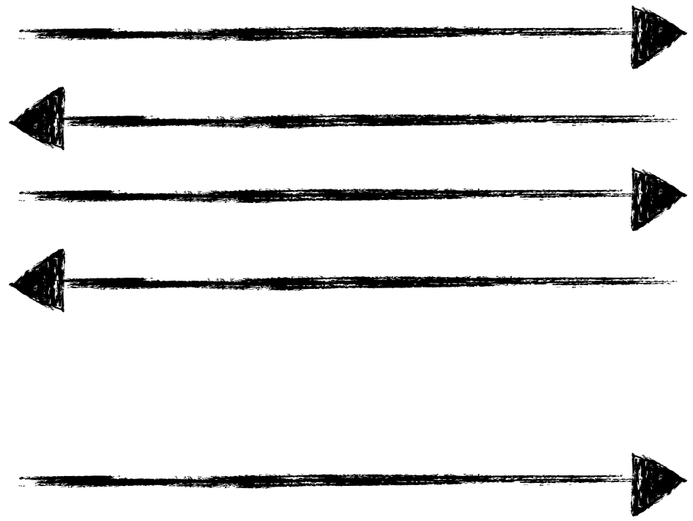

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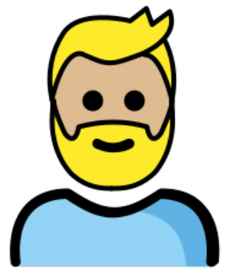

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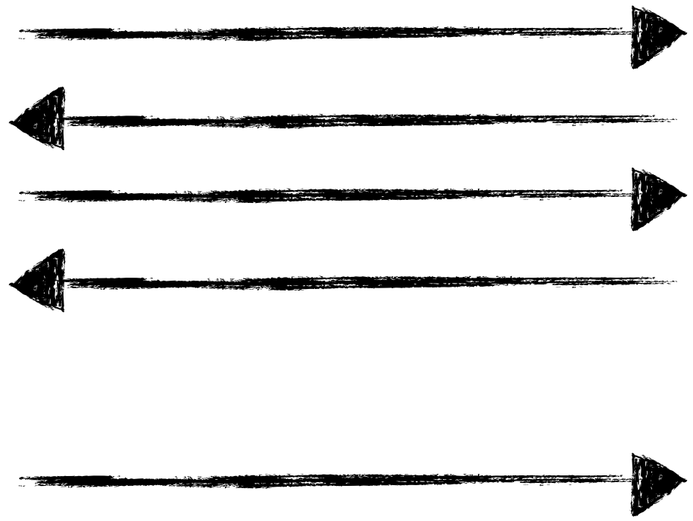

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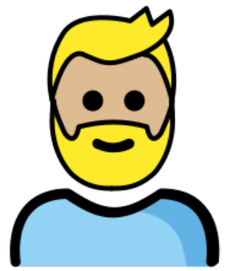

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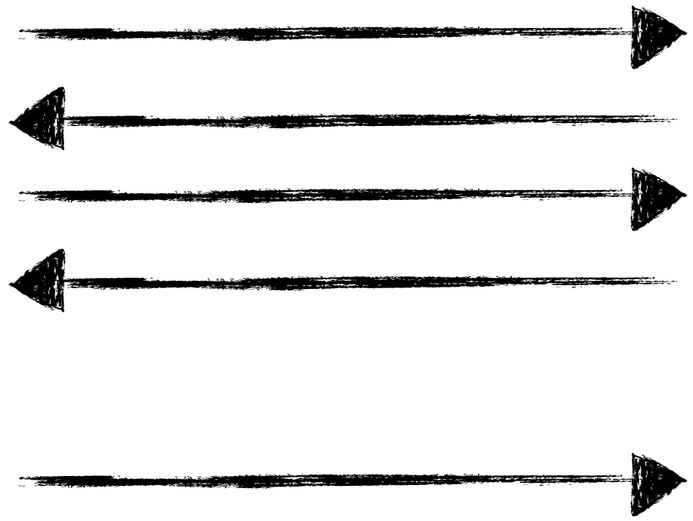

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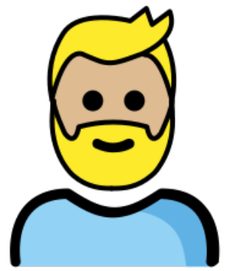

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Prover

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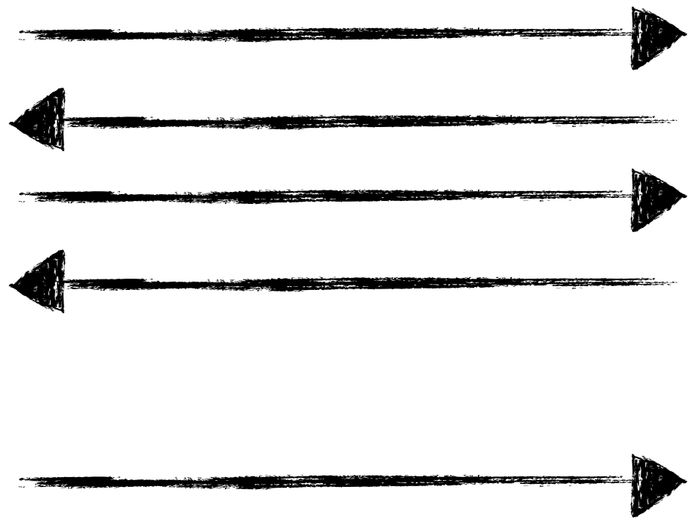
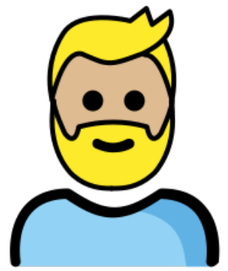


Verifier



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YES /

Properties

Completeness: if the Prover and the Verifier behave honestly, the proof will be accepted.

Soundness: the Prover cannot convince the Verifier of false statements.

Zero-Knowledge: the Verifier learns nothing beyond the truth of the statement.

Of Knowledge: the Verifier is convinced that the Prover knows a witness for the statement being true.

Schnorr Protocol

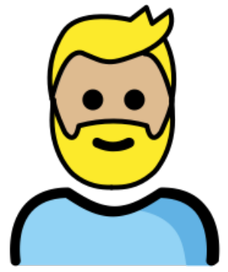
$L = \{ X \in \mathbb{G} \text{ such that } X = g^w, \exists\ w \in \mathbb{Z}_q \}$

Prover

(X, w)

Verifier

X



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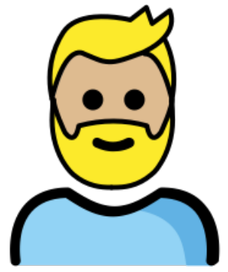

$r \leftarrow\$ \{0,1,..\ q-1\}$

$R = g^r \in \mathbb{G}$

Schnorr Protocol

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Prover

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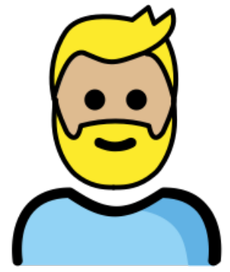

$r \leftarrow\$ -\{0,1,..\ q-1\}$

$R = g^r \in \mathbb{G}$

R

Schnorr Protocol

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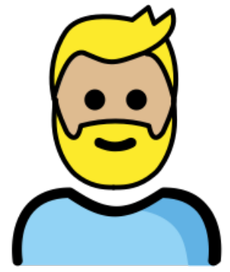

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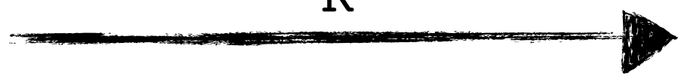

$c \leftarrow\$ \{0,1,..\ q-1\}$

Schnorr Protocol

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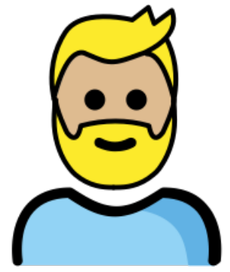
Prover

(X,w)

Verifier

X



$r \leftarrow\$ \{0,1,..\ q-1\}$

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R

c

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Schnorr Protocol

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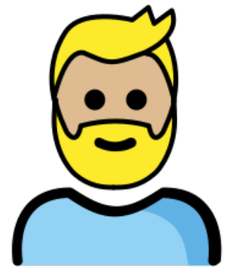

$r \leftarrow\$ \{0,1,..\ q-1\}$

$R = g^r \in \mathbb{G}$

R

c

c $\leftarrow\$ \{0,1,..\ q-1\}$

$z = c * w + r \in \mathbb{Z}_q$

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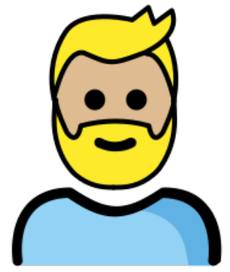
X



r ←$-{0,1,.. q-1}

R = $g^r \in \mathbb{G}$

R

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z = c * w + r $\in \mathbb{Z}_q$

z

Schnorr Protocol

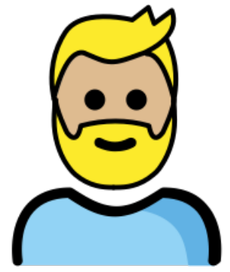
$L = \{ X \in \mathbb{G} \text{ such that } X = g^w, \exists\ w \in \mathbb{Z}_q \}$

Prover

(X,w)

Verifier

X



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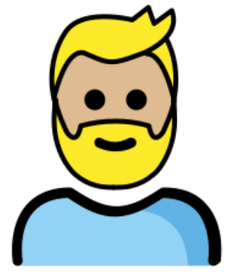

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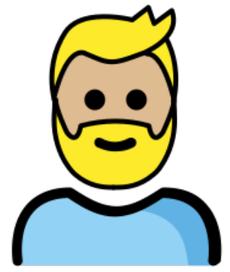

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Completeness: (easy computation)

Soundness: Follows from the Discrete Logarithm Assumption

Schnorr Protocol - Proving Zero Knowledge

“the Verifier learns nothing beyond the truth of the statement.”

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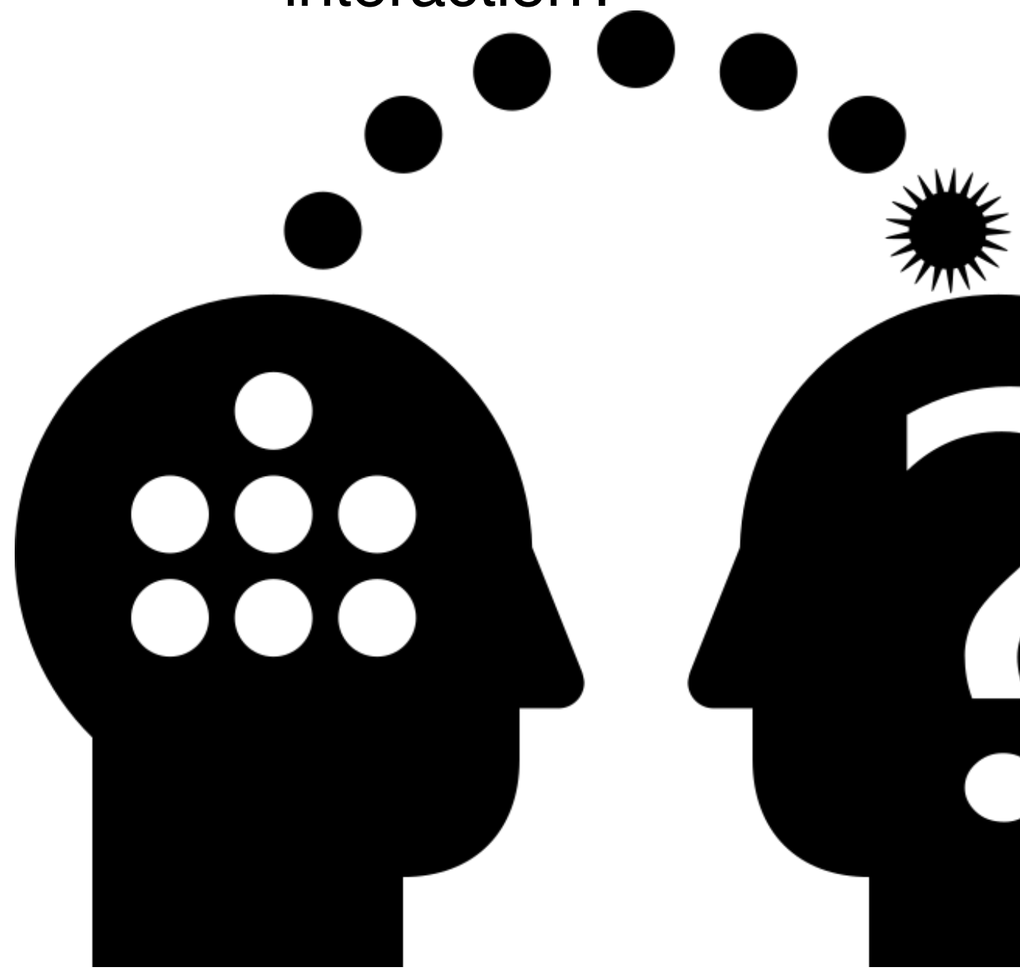
What does it mean to learn nothing from an interaction?



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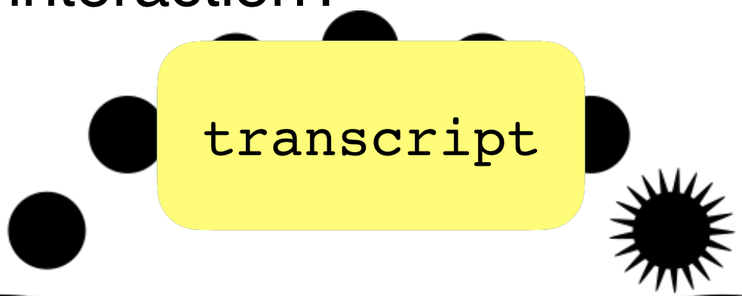

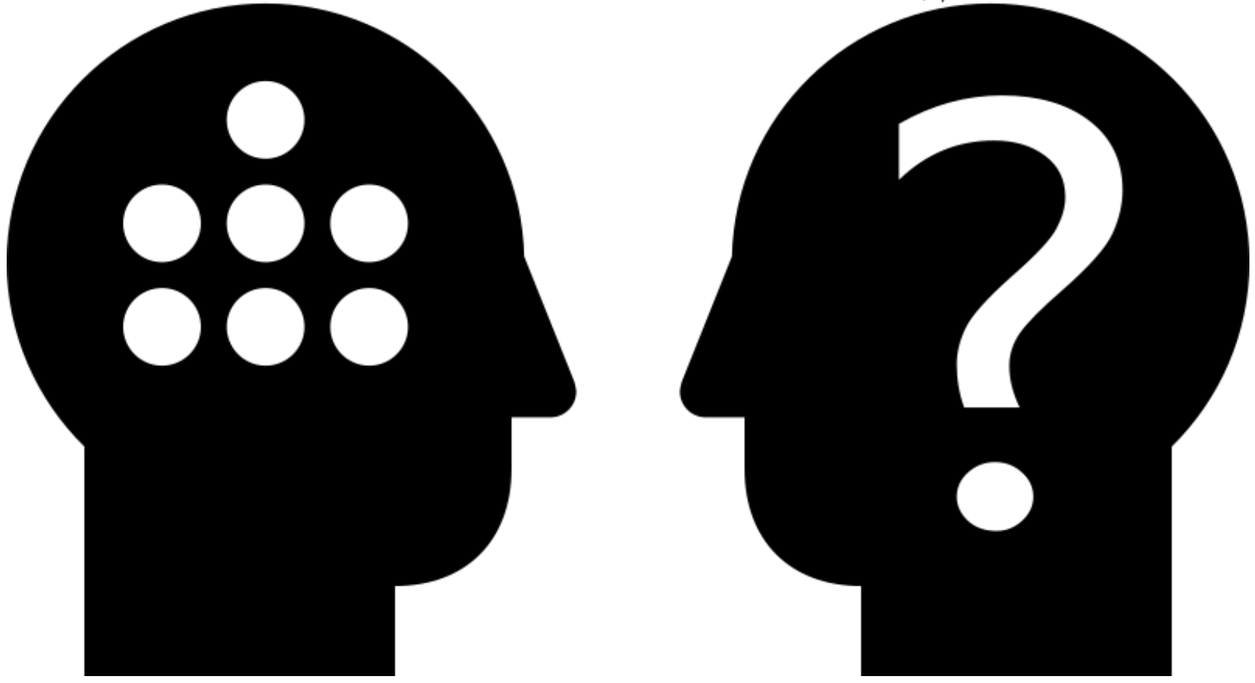

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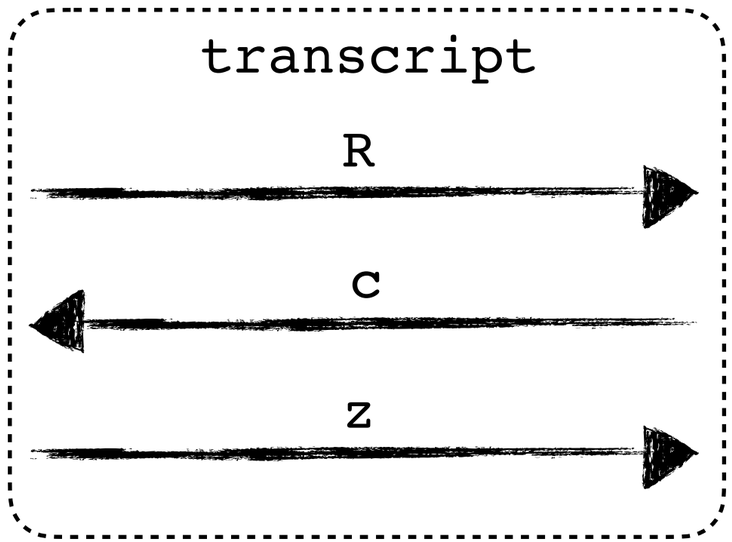

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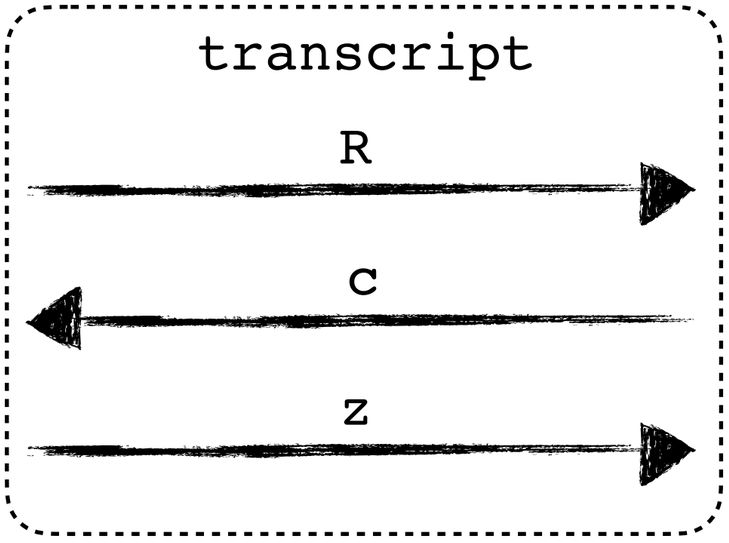
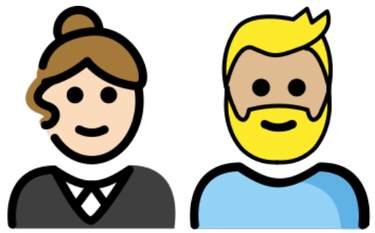


Real Interaction

(x,w)



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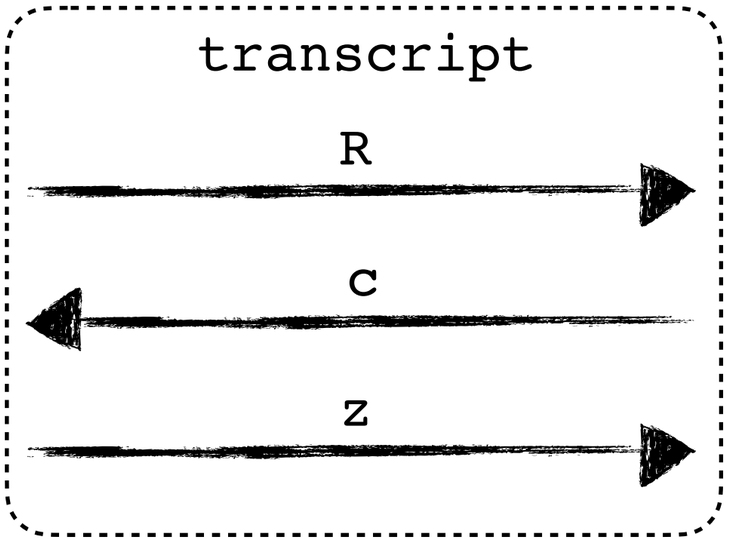

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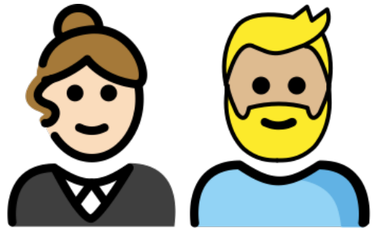

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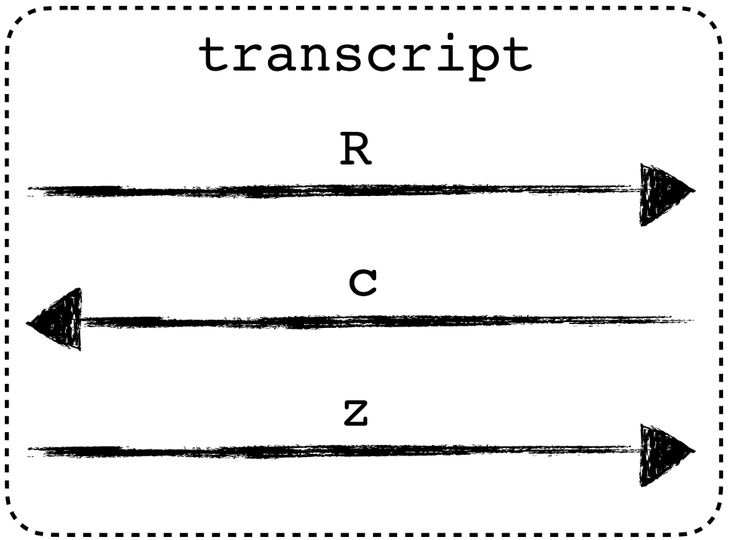

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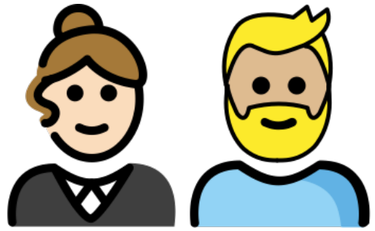

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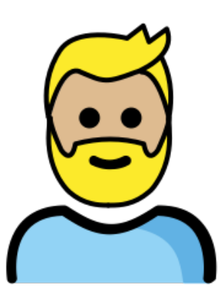

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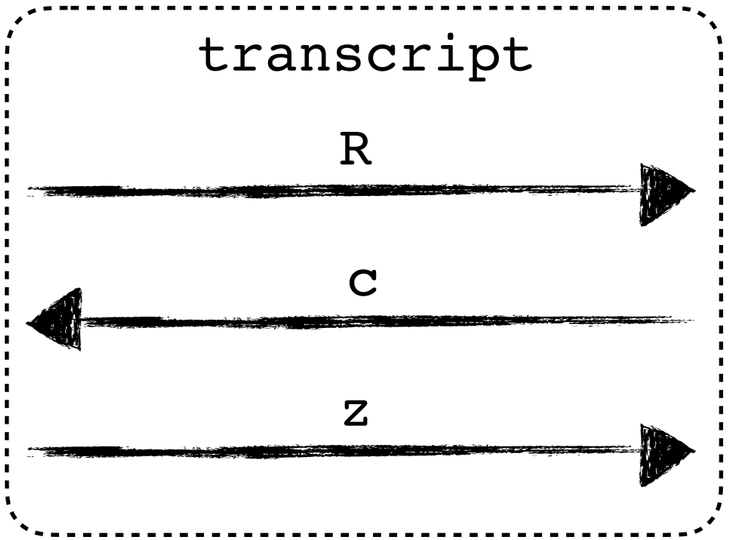

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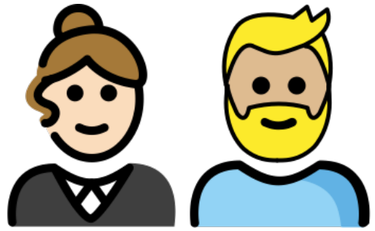

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the transcripts look the same to me

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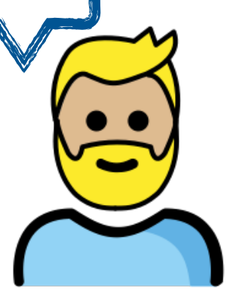

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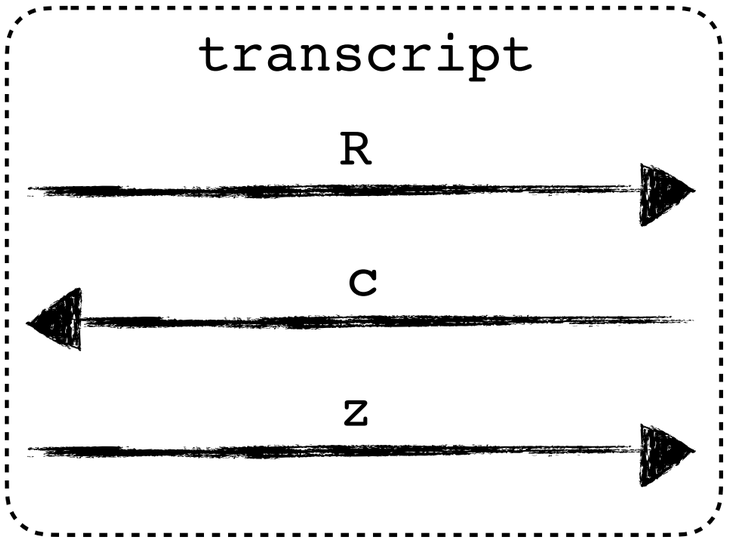

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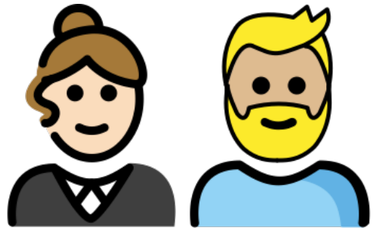

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$\{\text{View}_V[P(w) \leftrightarrow V(x)]\} \sim \{S(x)\}$

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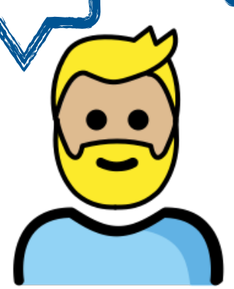

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Schnorr Protocol - Proving Knowledge

"it is possible to extract the witness from the prover"

Prover

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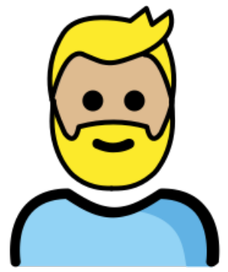

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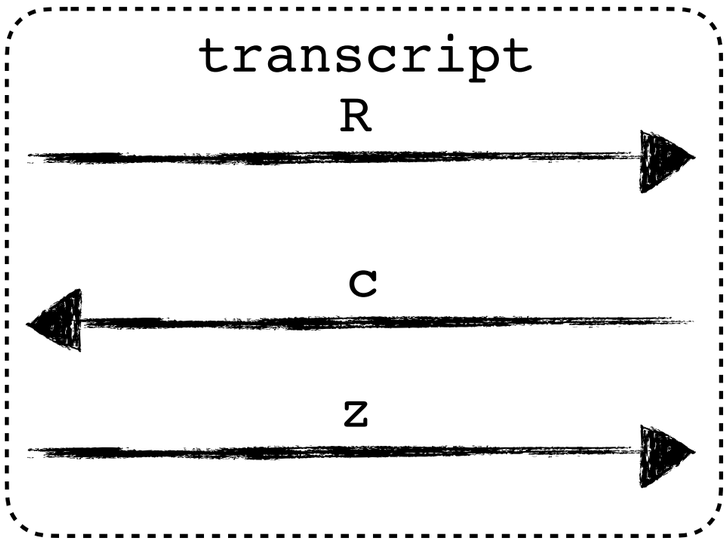

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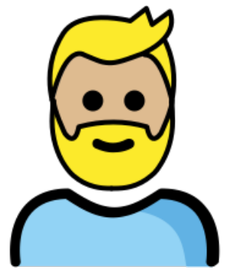

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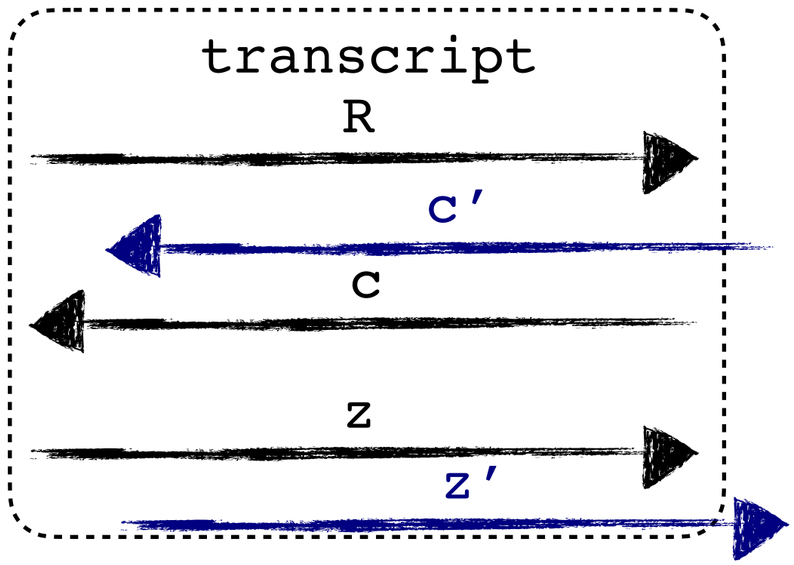

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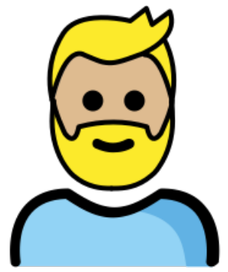

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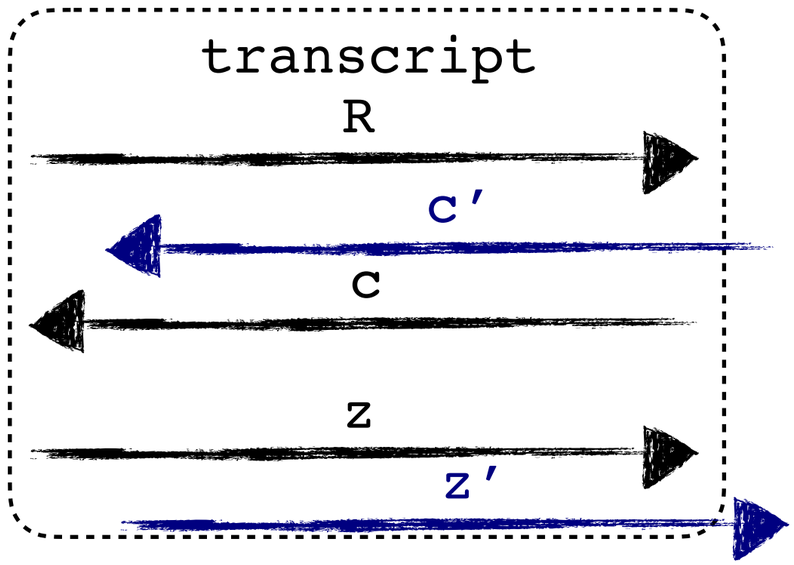

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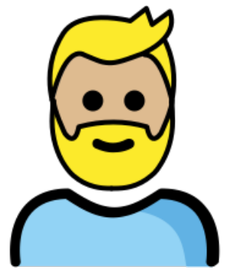

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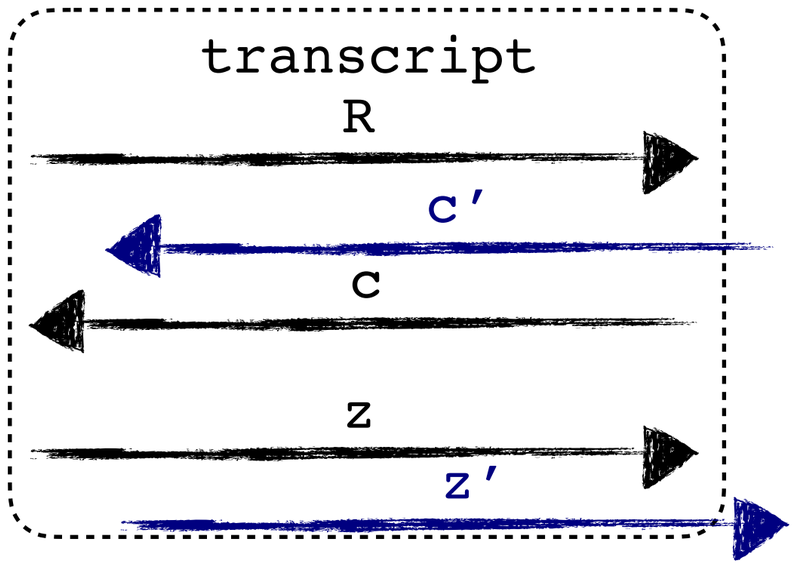

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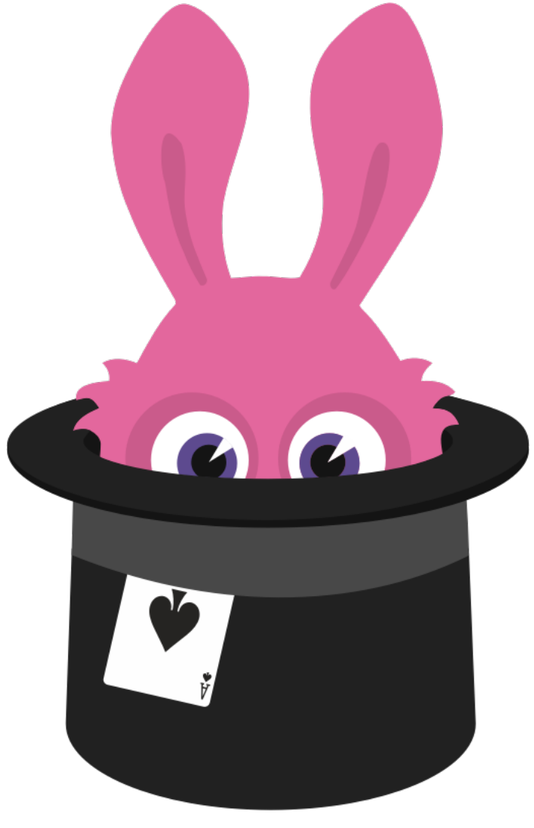
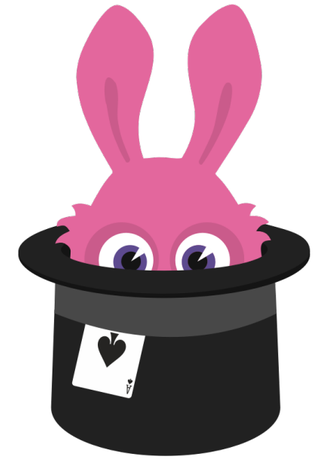
Have we seen something similar before?

w = z-z'/(c-c')

Can we construct a zero-knowledge proof system in which the proof is one single message?

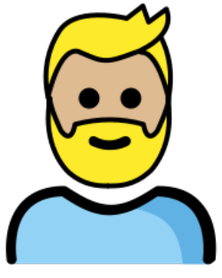


Non Interactive Zero Knowledge Proofs (NIZK)



Prover

π



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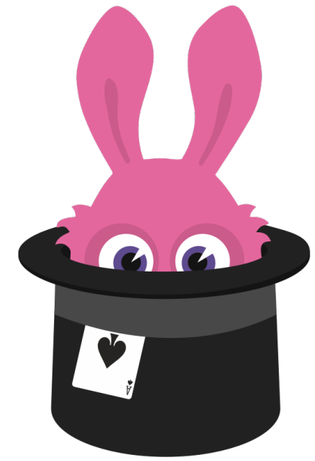

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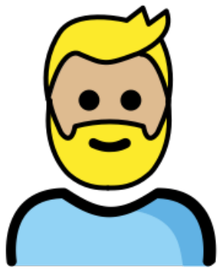

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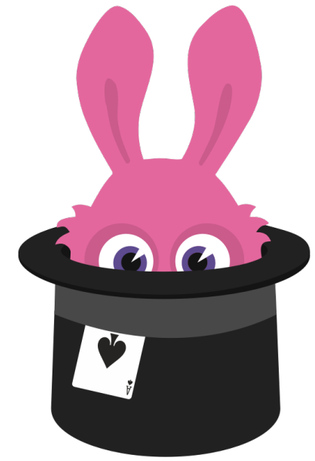

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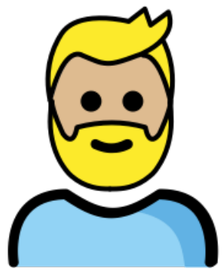

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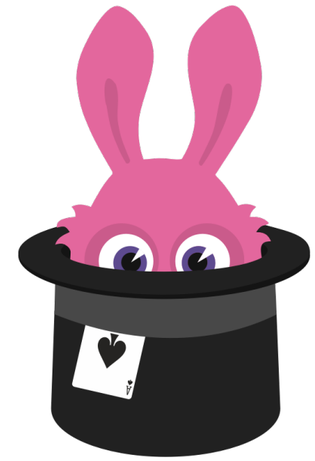
Simulator

Non Interactive Zero Knowledge Proofs (NIZK)



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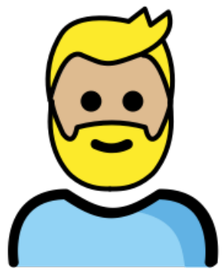

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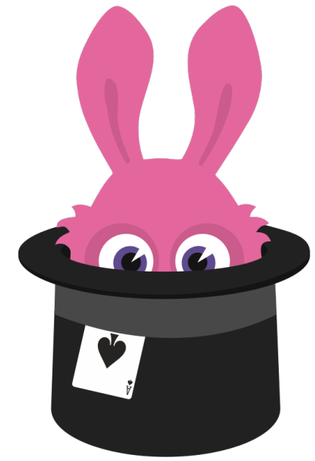
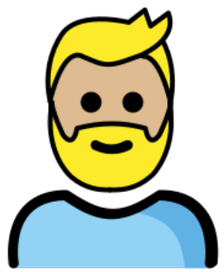
I can produce an accepting π without knowing w!

Non Interactive Zero Knowledge Proofs (NIZK)



Prover

π



Verifier

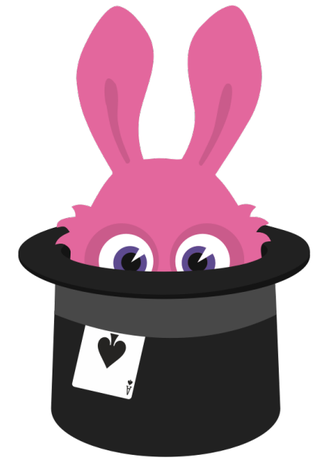
Zero-Knowledge: the Verifier learns nothing beyond the truth of the statement.

Simulator

I can produce an accepting π without knowing w!

NIZK proofs exist for trivial languages

Non Interactive Zero Knowledge Proofs (NIZK)



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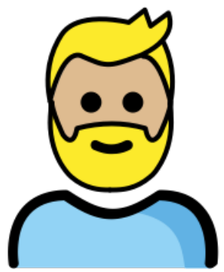
π



Verifier

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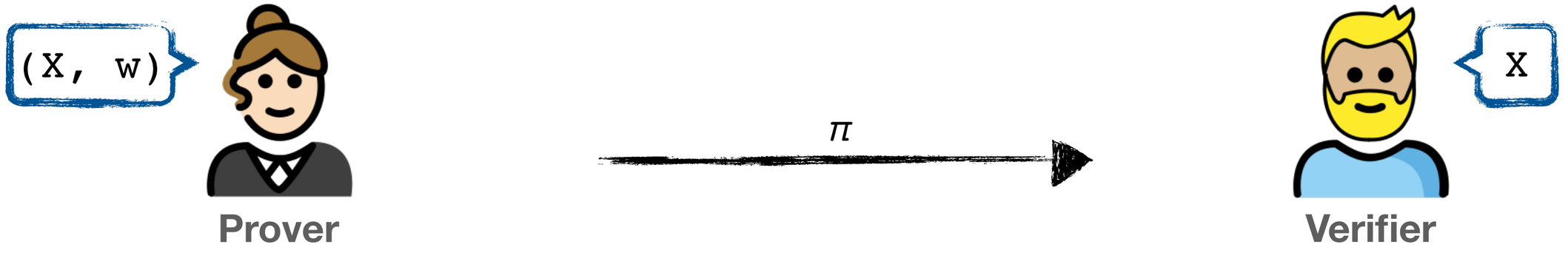
Simulator

I can produce an accepting π without knowing w!

NIZK proofs exist for trivial languages unless we make some extra assumptions

The Fiat-Shamir Heuristic

or how to turn an interactive protocol into non-int.



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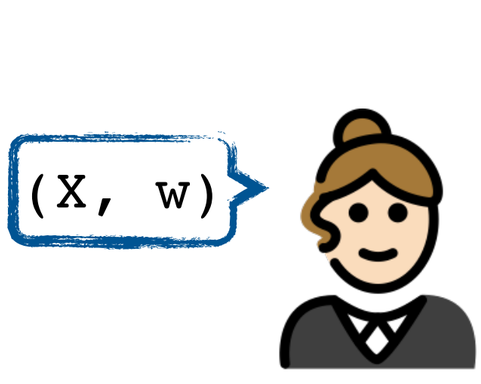

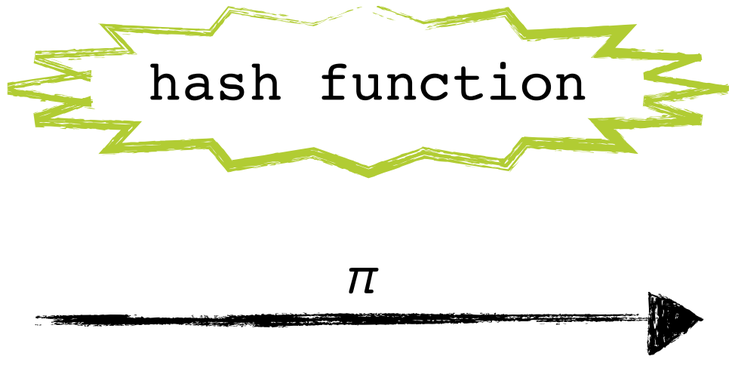

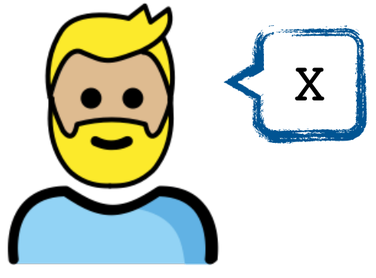

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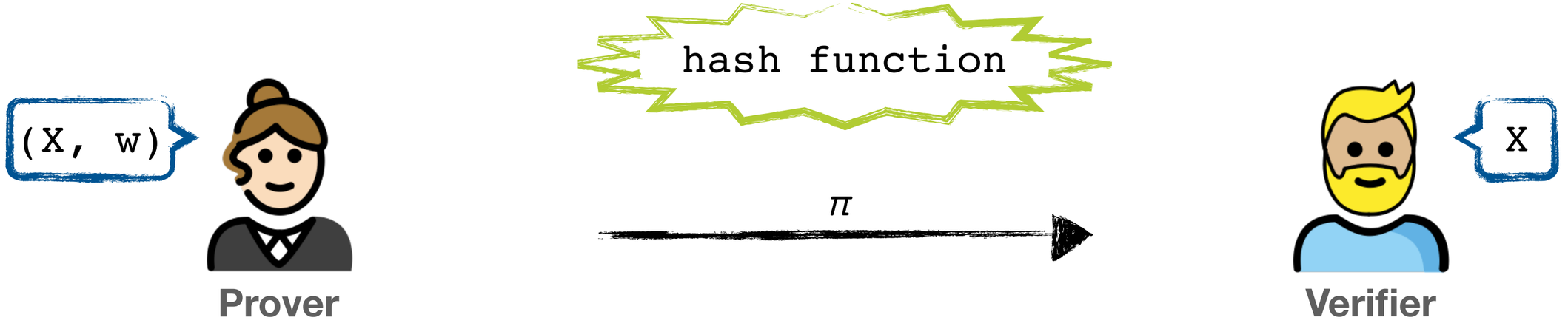

Non Interactive Schnorr Proof

The Fiat-Shamir Heuristic

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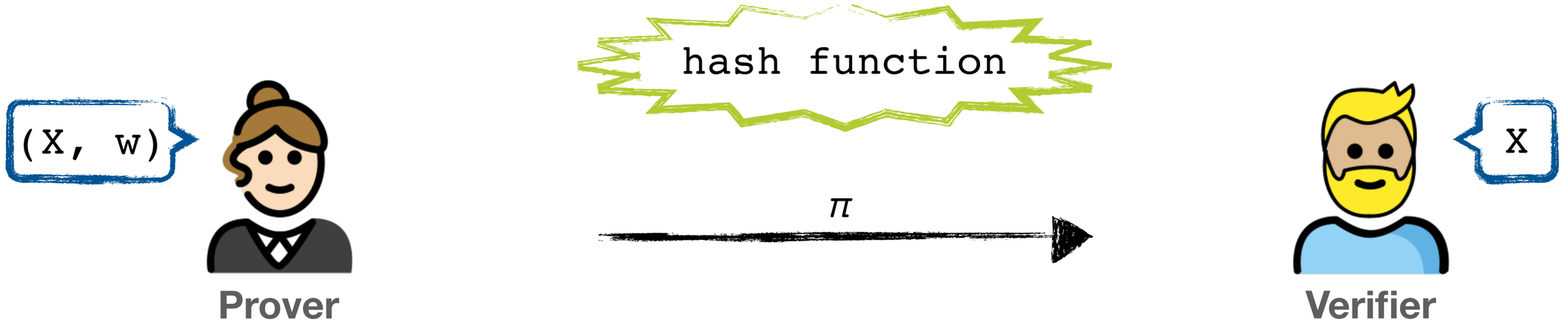
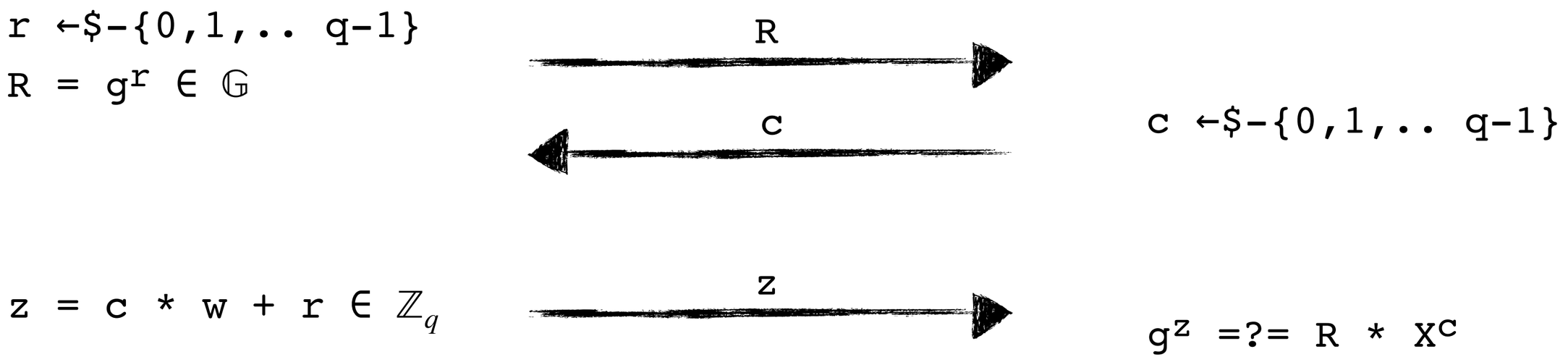


Non Interactive Schnorr Proof



The Fiat-Shamir Heuristic

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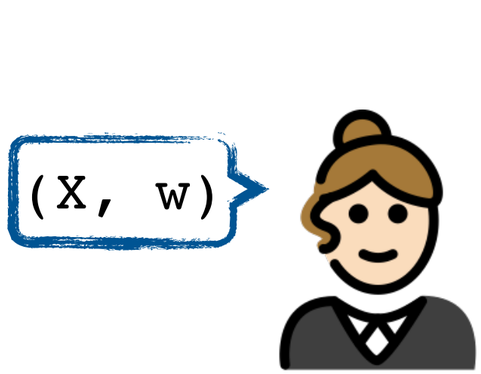
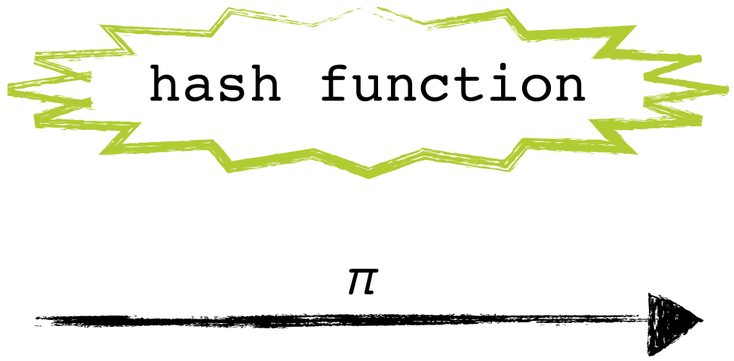
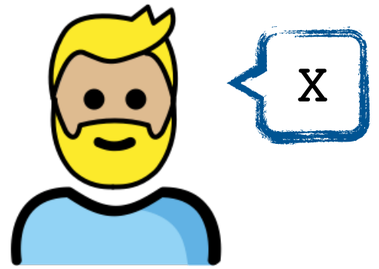


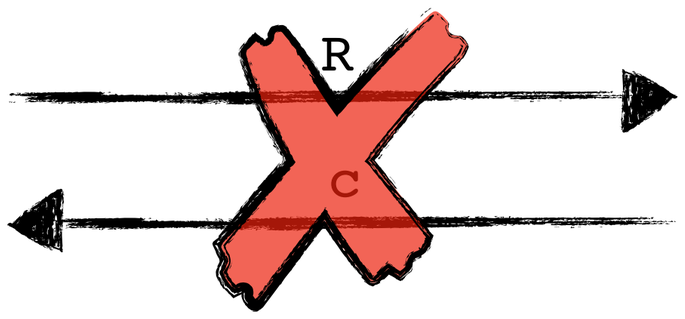




Non Interactive Schnorr Proof

$r \leftarrow\$ \{0,1,..\ q-1\}$

$R = g^r \in \mathbb{G}$



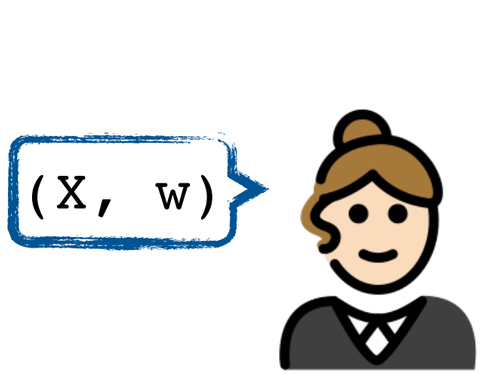
$c \leftarrow\$ \{0,1,..\ q-1\}$

$z = c * w + r \in \mathbb{Z}_q$

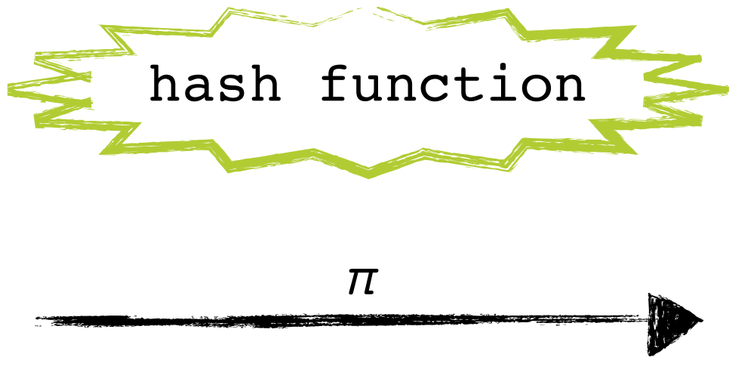
$g^z =?= R * X^c$

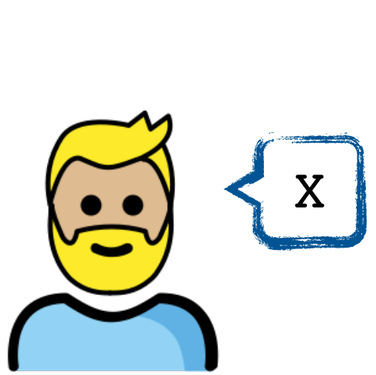
The Fiat-Shamir Heuristic

or how to turn an interactive protocol into non-int.



Prover





Verifier

Non Interactive Schnorr Proof

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$R = g^r \in \mathbb{G}$

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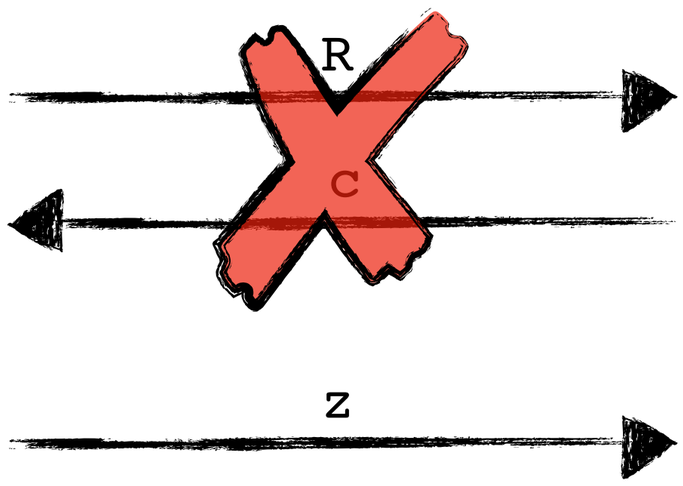

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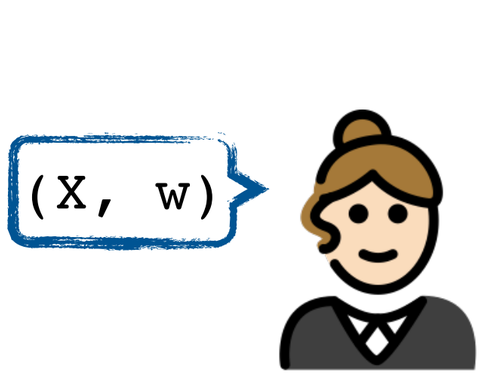

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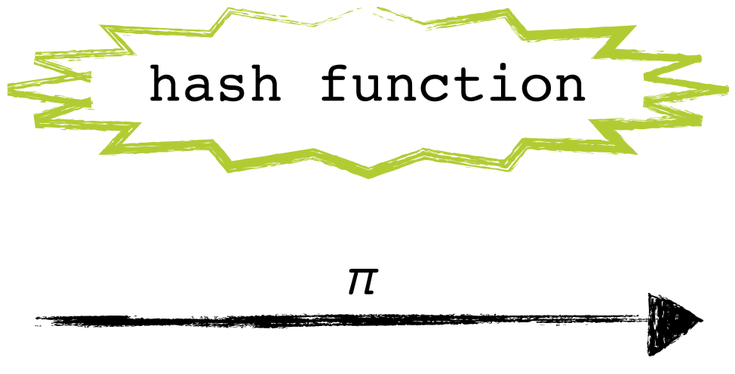

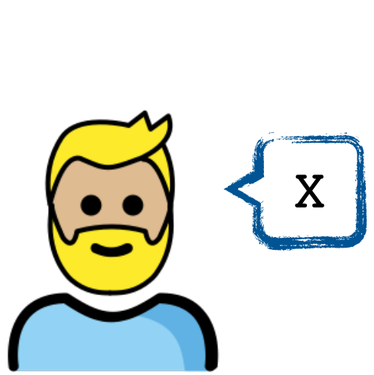

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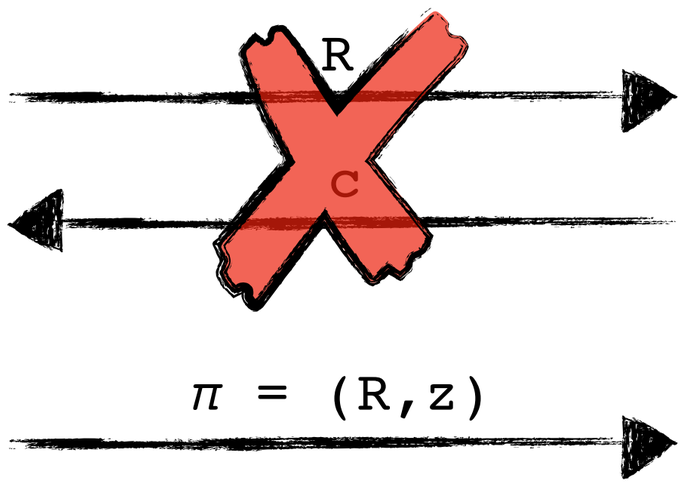

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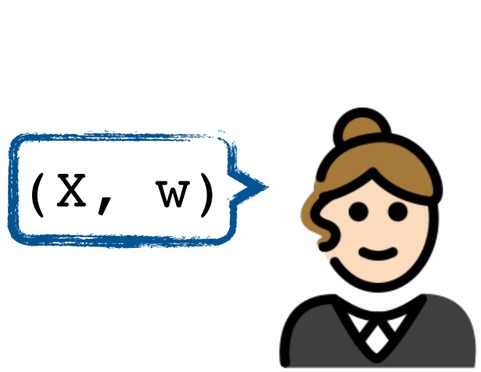

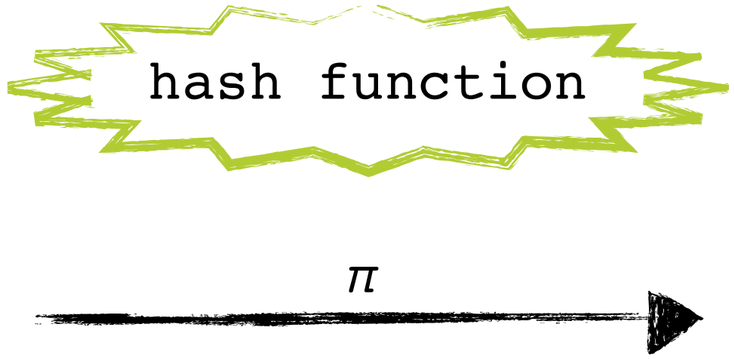

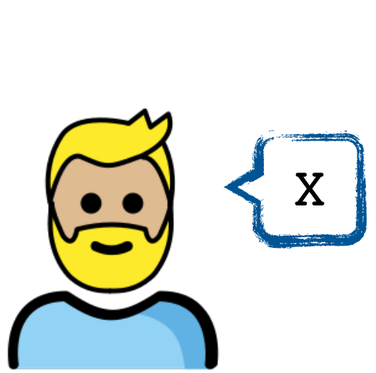

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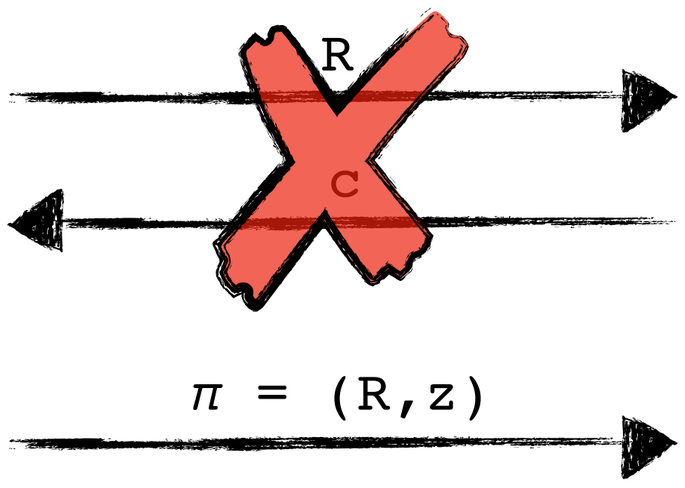

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Have We Seen This Before?

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c = Hash(g,X,R)
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```

```
Sign(sk, msg) ⇨ sgn
k ←$── [0 ... n-1]
R = k*G
r = R_x mod n
z = sha256(msg)
s = inv(k)·(z + d·r) mod n
sgn = (r, s)
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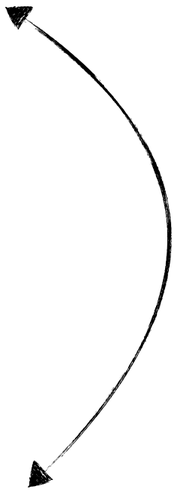

similar structure:

```
Sign(sk, msg) ⇨ sgn
k ←$— [0 ... n-1]
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r = R_x mod n
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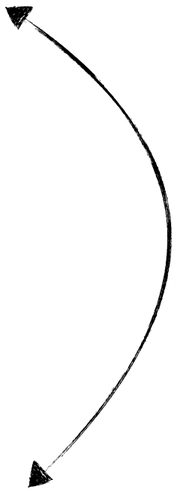

similar structure:

1. pick randomness

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similar structure:

1. pick randomness
2. generate new (unpredictable) randomness using the hash function

Have We Seen This Before?

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similar structure:

1. pick randomness
2. generate new (unpredictable) randomness using the hash function
3. use the secret and hide it with the randomness

Have We Seen This Before?

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similar structure:

1. pick randomness
2. generate new (unpredictable) randomness using the hash function
3. use the secret and hide it with the randomness
4. return a proof of knowledge of a secret value

Zk-SNARK

Zero Knowledge Succinct Non-interactive ARguments of Knowledge

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$|\pi| = o(|w|)$

Schnorr: π = (R,z), |π| > |w|

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$|\pi| = o(|w|)$

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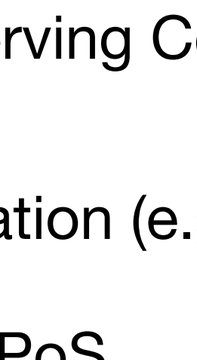
what?

Privacy-Preserving Coins (Zcash)

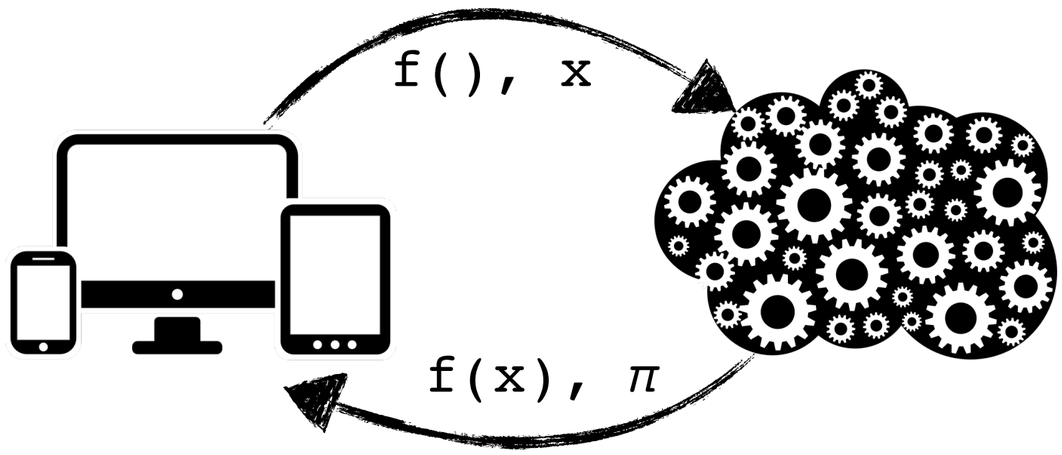
Ledger Verification (e.g. Coda)

Privacy-Preserving PoS
(e.g. Ouroboros Crypsinous)

Privacy-Preserving
Smart Contracts
(Hawk, Gyges)



Verifiable Computations



Verifiable Document Redacting

Zk-SNARK

Zero Knowledge Succinct Non-interactive ARguments of Knowledge

$|\pi| = o(|w|)$

Schnorr: π = (R,z), |π| > |w|

what?

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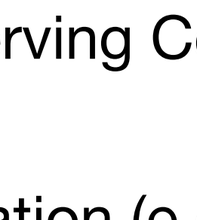

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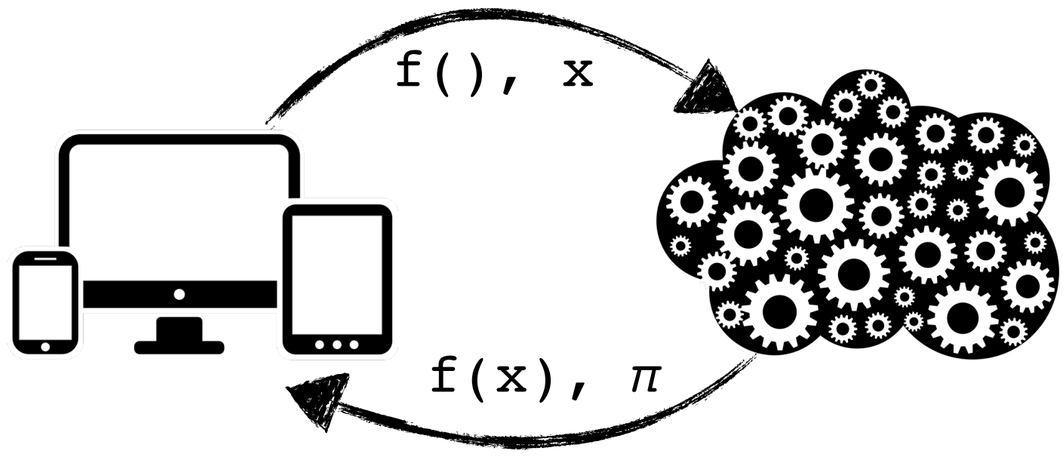

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how? elliptic curves, pairings, R1CS, QAP + non-falsifiable assumptions (KoE)