Labor Problem Set 2

Juli 2024

Many years after Robinson made it back to the mainland, Titanic is caught in a terrible storm, crashes into a huge rock hidden under water next to Robinson's island and is crippled forever by irreparable damage with all supplies on board lost.

All 1000 people on board manage to get to the island. In their last communication with the mainland, they are told that rescue will come after exactly 60 days. Each day each individual can choose to either lay in a hammock under the palm trees (Hammock, H), work the land (Land, L), or go fishing (Fishing, F). The individual's utility function is given by:

$$u(c_t, l_t; \theta) = \frac{c_t^{\mu}}{\mu} exp \begin{cases} 0, & \text{if } a_t = H, \\ \theta_L, & \text{if } a_t = L, \\ \theta_F, & \text{if } a_t = F, \end{cases}$$

$\theta_{a_t,i}$ is the individual specific disutility associated with the respective activity a_t. Each individual is either the play-hard, or rather the work-hard type. Given the individuals type, θ takes on different values for working the land and fishing. In particular:

$$\begin{aligned} \theta_{L,workhard} &= -0.12 \\ \theta_{L,playhard} &= -0.24 \\ \theta_{F,workhard} &= -0.19 \\ \theta_{F,playhard} &= -0.26. \end{aligned}$$

It is known that 20% of the individuals are the play-hard type.

For individuals who choose to work the land or go fishing, consumption for the day is given by:

$$\begin{aligned} ln\, c_t &= \gamma_{s,0} + \gamma_{s,1} ln(e_t + 1) + \xi_t, \\ e_t &= e_L * 0.75 + e_F, \end{aligned}$$

where e_L and e_F measure the total days in land work and finishing experience accumulated up to day t. The subscript s refers to the age group an individual belongs to, either old or young. The fraction of youngsters is 0.3. Individuals who lay in the hammock drink coconut milk which is equivalent to 2.0 units of consumption.

The distribution of the fluctuations to consumption when engaging in a productive activity are given by:

$$\begin{pmatrix} \xi_L \\ \xi_F \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.025 & 0 \\ 0 & 0.0625 \end{pmatrix}\right]$$

Finally, the remaining parameters of the model are given by:

Parameter	Value
discount factor	0.98
μ	0.6
γ_{0young}	.3446
γ_{0old}	.2064
γ_{1young}	.5363
γ_{1old}	.4223

Questions:

1. Simulate the development of the island economy for 10 days. (3p)

Bonus: Extend your simulation to the full period of 60 days until rescue arrives. How long does your code run for? 2p if less than 3 min.

2. Plot the choice rates of each activity and average consumption in the population by day. (1p)

3. How would your plots change if the disutility levels of the play-hard type were instead given by $\theta_{L,playhard} = -0.29$ and $\theta_{F,playhard} = -0.31$, respectively. Verify your intuition by constructing new plots. (1p)

4. Imagine that you found a ledger where the community kept record of everyone's choice of activity and their consumption over the 10 days spent on the island. Describe in a few words how you would estimate the parameters of the model using the method of moments and the information from the ledger. (2p)

5. The actually observed mean consumption per day is recorded in the file "moments_obs.txt". It turns out that, apart from the constants in the consumption equation, the model fits the data well; Use i) the simulated moments of mean daily consumption from part 2., ii) the observed moments provided, iii) your strategy from part 4., and iv) [0.25, 0.25] as starting values to estimate $\gamma_{s,0}$ Report your estimates. (3p)

NOTE: This problem set is graded. Each person/group has to submit a solution, if ECTS and a grade for the course are desired. Please send your code to mblesch@diw.de by Sunday, August 4th, 23PM. Make sure that your code is well commented and include brief answers to the questions in the comments, where suitable. Include all members of your group in cc as well as in the header of the notebook.