Advanced Applied Econometrics
Week 9 - Static discrete choice with market-level data

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Organisation

- My part has three problem sets (to be completed in groups of max. 2 participants)
- The third problem set is graded.
- The 3 best of 4 graded problem sets will count in the course.
- Felix's first problem set counts as one of the 4.

Organisation

- June 21: recap demand estimation, discuss details, practical session (BLP)
- Read (again) Berry et al. (1995) and Nevo (2000)
- Hand in second problem set

- June 26 (Wednesday, another change in date): dynamic discrete choice

Plan for today

- Structural econometrics in Industrial Organization
- Demand for differentiated products: Berry (1994), Berry et al. (1995)
- Practical session: logit demand with aggregate data

Structural econometrics in Industrial Organization

- Highly recommended slides by Phil Haile on "Models, Measurement, and the Language of Empirical Economics" to read at: `https://sites.google.com/view/philhaile/home/teaching`
- "Structural: estimate features of a data generating process (i.e., a model) that are (assumed to be) invariant to the policy changes or other counterfactuals of interest"
- Fun fact: they are generative models because structural models specify and estimate the data-generating process, thus can generate $p(X, Y)$.

Structural econometrics in Industrial Organization

- IO applications: study the behavior of firms in markets, their strategic interactions, and the implications on profits and consumer welfare.

Recent policy discussions on inflation and markups

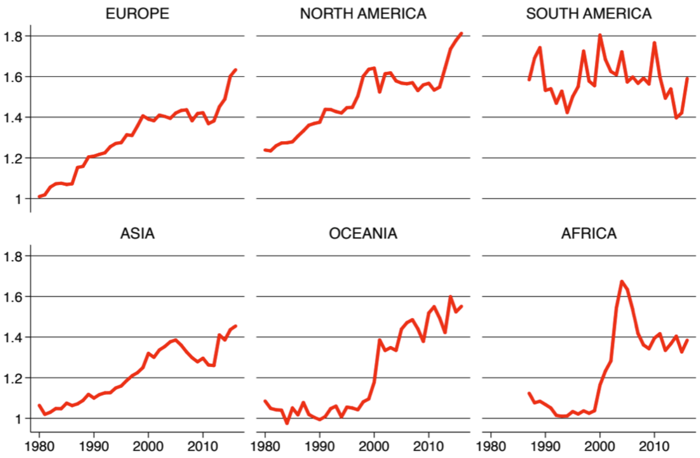
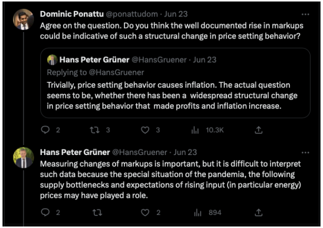


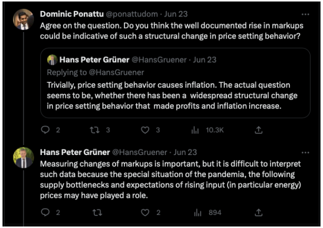
Figure 5: Global Regions

De Loecker, Jan and Jan Eeckhout (2021), "Global Market Power", working paper

Recent policy discussions on inflation and markups



Recent policy discussions on inflation and markups



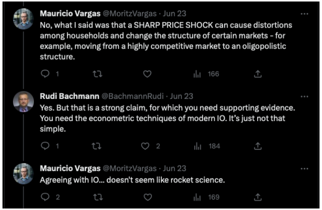
Dominic Ponattu @ponattudom · Jun 23
Agree on the question. Do you think the well documented rise in markups could be indicative of such a structural change in price setting behavior?

> **Hans Peter Grüner** @HansGruener · Jun 23
> Replying to @HansGruener
> Trivially, price setting behavior causes inflation. The actual question seems to be, whether there has been a widespread structural change in price setting behavior that made profits and inflation increase.

2 3 3 10.3K

Hans Peter Grüner @HansGruener · Jun 23
Measuring changes of markups is important, but it is difficult to interpret such data because the special situation of the pandemia, the following supply bottlenecks and expectations of rising input (in particular energy) prices may have played a role.

2 2 894



Mauricio Vargas @MoritzVargas · Jun 23
No, what I said was that a SHARP PRICE SHOCK can cause distortions among households and change the structure of certain markets - for example, moving from a highly competitive market to an oligopolistic structure.

1 166

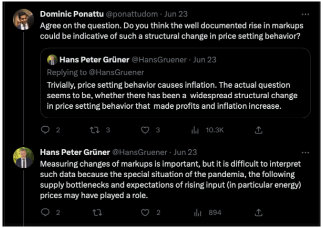
Rudi Bachmann @BachmannRudi · Jun 23
Yes. But that is a strong claim, for which you need supporting evidence. You need the econometric techniques of modern IO. It's just not that simple.

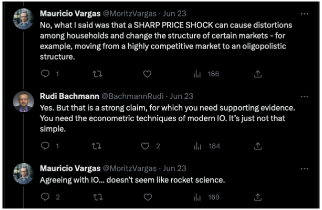
1 2 184

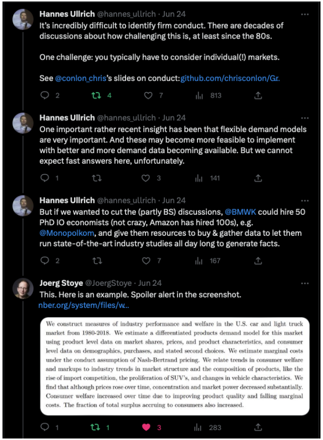
Mauricio Vargas @MoritzVargas · Jun 23
Agreeing with IO... doesn't seem like rocket science.

2 169

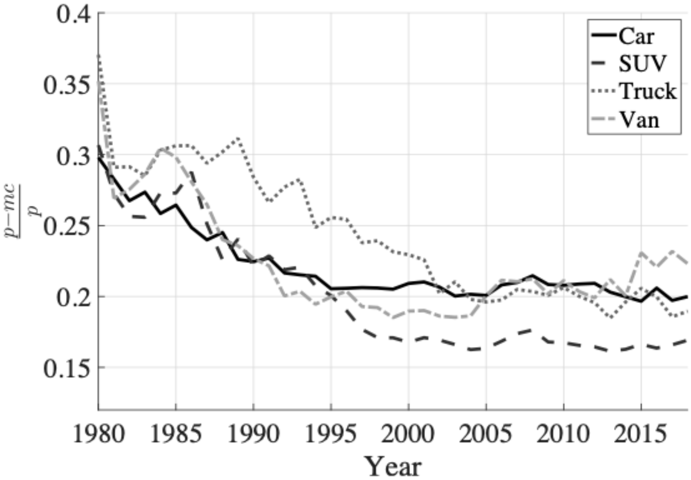
Recent policy discussions on inflation and markups







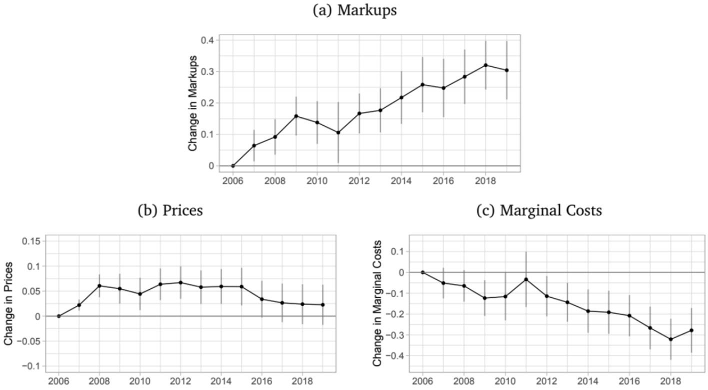
Recent policy discussions on inflation and markups



(a) Markups by Vehicle Style

Grieco, Paul, Charles Murry, and Ali Yurukoglu (2024), "The Evolution of Market Power in the US Auto Industry", Quarterly Journal of Economics

Recent policy discussions on inflation and markups

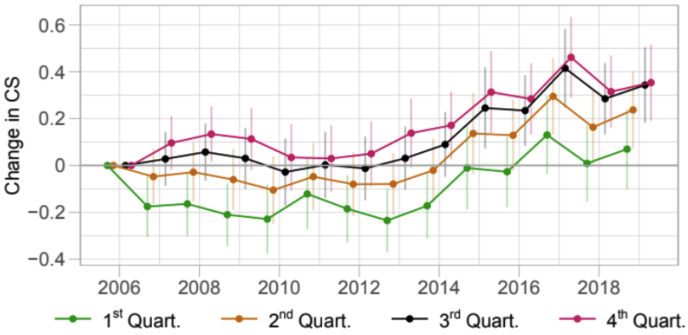


Notes: This figure shows coefficients and 95 percent confidence intervals of a regressions of the log of the Lerner index, real prices, and real marginal costs at the product-chain-DMA-quarter-year level on year dummies controlling for product-chain-DMA and quarter fixed effects. The year 2006 is the base category.

Döpper, Hendrik, Alexander MacKay, Nathan H. Miller, and Joel Stiebale (2023), "Rising Markups and the Role of Consumer Preferences", R&R Journal of Political Economy

Recent policy discussions on inflation and markups

Figure 10: Consumer Surplus Over Time By Income Group



Döpper, Hendrik, Alexander MacKay, Nathan H. Miller, and Joel Stiebale (2023), "Rising Markups and the Role of Consumer Preferences", R&R Journal of Political Economy

Structural econometrics in Industrial Organization

- IO applications: study the behavior of firms in markets, their strategic interactions, and the implications on profits and consumer welfare.
- Why use economic theory for structural assumptions in IO (and elsewhere)?
  E.g. why not simply regress prices on number of firms, or market shares on prices?
- Counterfactuals for ex-ante (welfare) evaluation of mergers, antitrust cases, regulation (e.g. price or entry barriers), subsidies, etc.

→ Require a flexible, well-specified <u>demand</u> system

Typical IO Models

How is economic theory used as structure in IO applications?

1. Model of consumer behavior (demand), product differentiation

2. Model for firms costs, economies of scale, economies of scope, entry costs, investment costs

Conduct

3. Equilibrium model of static competition, price (Bertrand), quantity (Cournot), bargaining, collusion, etc.

4. Equilibrium model of market entry-exit

5. Equilibrium model of dynamic competition, investment, advertising, quality, product characteristics, stores, etc.

Plan for today

- Structural econometrics in Industrial Organization
- Demand for differentiated products: Berry (1994), Berry et al. (1995)
- Practical session: logit demand with aggregate data

Typical Structure of IO Models

For this course:

1. Model of consumer behavior (demand)
2. Model for firms costs
3. Equilibrium model of static competition

Demand for differentiated products: discrete choice models

- Empirical observation:
  - Products mostly not homogenous
  - Firms typically set prices strategically
- Then, how to model and estimate demand for differentiated products
- Aggregation to market-level important. In 1990s, move from
  - representative consumer to
  - micro-modelling of consumer preferences.

Demand for differentiated products: discrete choice models

- One way: system of linear demand equations

$$q = D(p, x, \theta), \text{ with } D(p) = Ap$$

- J products
- Flexible demand system but number of parameters to be estimated: $J \times J$ matrix of price coefficients

Demand for differentiated products: discrete choice models

- Discrete choice models as models of product demand:
  - Characteristics space to reduce dimensionality (McFadden, 1974)
- Berry (1994): foundation for discrete choice demand estimation using market-level data, single-product firm oligopoly pricing
- Berry et al. (1995): first application of the proposed framework, multi-product firm oligopoly pricing, proof of contraction mapping

Demand for differentiated products: discrete choice models

- In markets with imperfect competition, primitives of the model
  - Product characteristics
  - Consumer preferences
  - Equilibrium notion (price setting, quantity setting, bargaining, etc.)

Demand for differentiated products: Berry (1994)

- Assume consumer i's linear utility from buying product j:

$$u_{ij} = \delta_j + \varepsilon_{ij}$$

  where $\delta_j = x_j\beta - \alpha p_j + \xi_j$.

- Assumption: each consumer buys one unit of utility-maximizing good.

- If ε_{ij} is extreme value distributed, the probability that consumer i chooses product j is:

$$s_j = \frac{exp(\delta_j)}{1 + \sum_{m=1}^{J} exp(\delta_m)}$$

- Role of the outside good. What is a reasonable market definition?

Demand for differentiated products: Berry (1994)

- With strategic price setting:
  p_j correlated with unobserved characteristic ξ_j $\rightarrow$ endogeneity problem

- Idea: s_j can be inverted such that

$$\ln(s_j) - \ln(s_0) = \delta_j = x_j\beta - \alpha p_j + \xi_j$$

- Market share inversion allows for linear IV estimation

Demand for differentiated products: Berry (1994)

- Fixed effects
- Instruments - do we have good ones (for price)?
- What about endogeneity of other product characteristics?

Instruments

- Good instruments might be the regressors
  (derivative of the moment function with respect to the parameters)
- Cost shifters uncorrelated with the demand shock (rarely observed)...
- Bresnahan (1987), BLP (1995): Assume product characteristics exogenous. Use
  - observed product characteristics; sums of the same characteristics of other products offered by that firm; sums of the same characteristics of products offered by other firms
  - Intuition: closeness in characteristics space, in oligopoly markups decreases with the presence of good substitutes
- Gandhi and Houde (2019): Differentiation IVs
  - Polynomial of differences in observed product characteristics

Demand for differentiated products: Berry (1994)

- Homogenous consumers (α, β) strong assumption
  $\rightarrow$ representative consumer up to i.i.d. ε_{ij}

- Leads to substitution patterns depending only on market shares

Unobserved preference heterogeneity

- Random coefficients: allow consumer to have heterogenous preferences over certain characteristics, i.e. β_i

$$u_{ij} = x_j\beta_i - \alpha p_j + \xi_j + \varepsilon_{ij}$$

- Berry (1994) inversion to obtain linear equation no longer possible

- Market shares, as we did not integrate out all individual errors:

$$s_j = \int ... \int (s_{ij} dF_1(\alpha|\theta) ... dF_{K+1}(\beta_k|\theta))$$

- Now can estimate completely flexible substitution patterns (market share derivatives depend on all characteristics in u_{ij})

Unobserved preference heterogeneity

- Assume that individual i can be described by a vector of unobserved characteristics $(\nu_i, \varepsilon_{i0}, ..., \varepsilon_{iJ})$.
- We can re-write consumer i's utility:

$$u_{ij} = \delta_j(x_j, p_j, \xi_j; \theta_1) + \mu_{ij}(x_j, p_j, \nu_i, \theta_2) + \varepsilon_{ij}$$

$\rightarrow$ Partition utility into mean utility δ_j and consumer-specific μ_{ij}

- Vector $\theta_1 = (\alpha, \beta)$ contains linear and $\theta_2 = (\sigma)$ nonlinear parameters

Unobserved preference heterogeneity

- For simplicity, assume that consumers only have heterogenous preferences over one characteristic in x_j, so that $\theta_2 = (\sigma)$ is a scalar.
- Nonlinear part of consumer utility:

$$\mu_{ij}(x_j, p_j, \nu_i, \theta_2) = x_j \nu_i \sigma$$

  where the distribution of ν_i is assumed. Typically, $N(0, 1)$.
- If consumer demographics are observed (e.g. age, income), empirical or fitted distributions

Simulation

- Idiosyncratic terms ε_{ij} integrated out (EV distribution assumption)
- But due to ν_i no closed form solution for the integral
- Numerically invert the system of equation $s_j = s_j(x, p, \delta_j, \theta_2)$ to obtain δ_j given θ_2
- Two numerical ingredients needed:

$\rightarrow$ Numerical integration / simulation to compute market shares

$\rightarrow$ Contraction mapping to compute δ_j

Integration to compute market shares

- Take s draws from the distribution of ν for each observation
- Given s draws of ν, σ, and δ, compute the market shares:

$$s_j = \frac{1}{ns}\sum_{i=1}^{ns} s_{ij} = \frac{1}{ns}\sum_{1}^{ns} \frac{exp(\delta_j + x_j \nu_i \sigma)}{1 + \sum_{m=1}^{J} exp(\delta_m + x_m \nu_i \sigma)}$$

Integration to compute market shares

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- Monte Carlo simulation
  - Pseudo-random draws
  - Quasi-random - e.g. Halton, modified Latin hypercube sampling
- Quadrature: Gauss-Hermite for (univariate) normal distribution, unbounded support

Intermission: Newton's Method

- Idea: Approximate (univariate) nonlinear function $f(x)$ by its tangent at x_n, where $x_{n+1} = x_n + \epsilon$:
$$y = f'(x_n)(x_{n+1} - x_n) + f(x_n)$$
- Finding the root of this tangent is easy, set $y = 0$ and solve for x_{n+1}:
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
- Iterating until $x_{n+1} \approx x_n$ yields the (local) root of f(x)
- Optimization: Apply Newton's method to $f'(x)$, then check $f''(x)$

Intermission: Newton's Method

```
# Newton's root-finding method
def newton_sqrt(a, tol=1e-12):

    if a < 0:
        raise ValueError("Number a should ...
            be positive.")

    x_t = a / 2
    x_t1 = a / 2 - 1
    i = 0

    while abs(x_t1 - x_t) > tol:
        x_t = x_t1
        f_t = a - x_t**2
        f_t_derivative = -2 * x_t
        x_t1 = x_t - f_t / f_t_derivative
        i += 1
        print(f"Value at iteration {i} is ...
            {x_t1:.10f}.")

    return x_t1
```

Find $\sqrt{a}$, $\rightarrow$ find $x = \sqrt{a}$.

$\rightarrow x^2 = a$
$\rightarrow f(x) = a - x^2 = 0$

Newton's method is a contraction mapping if $f''(x)$ is continuous, $f'(x) \neq 0 \ \forall \ x \in \mathbb{R}$, and $q \in (0, 1)$ such that $|f(x)f''(x)| \leq q|f'(x)|^2 \ \forall x \in \mathbb{R}$

Contraction mapping for δ_j

- Given σ, compute vector δ that equates model predicted to observed market shares
- Contraction mapping

$$\delta^{h+1} = \delta^h + ln(s) - ln(s(p, x, \delta^h, \sigma)), h = 0, ..., H,$$

  where H is the smallest integer such that $\|\delta^H - \delta^{H-1}\|$is smaller than some tolerance level, and δ^H is the approximation to δ.
- Compute vector $s(x, p, \delta, \sigma)$ at each iteration

Estimation

- Nested simulated GMM procedure
- IV estimation. Population moment conditions $E[Z\xi(\theta^*)] = 0$
- Using δ_j, computed at each parameter iteration in the GMM objective function, obtain: $\xi_j = \delta_j - (x_j\beta - \alpha p_j)$

Estimation

- GMM objective function:
$$f = \xi(\theta)' Z \Phi^{-1} Z' \xi(\theta),$$
where Φ^{-1} is a consistent estimate of $E[Z'\xi\xi' Z]$

- Search $\theta = (\alpha, \beta, \sigma)$ that minimizes the objective function

- Note: finding the nonlinear parameter σ requires a nonlinear search.

Equilibrium Model

- How are prices determined in oligopolistic markets with differentiated products?
- Increased efficiency of demand estimates by adding structure on the supply side
- Assume prices are result of firms' optimization problem
- Use information from first order conditions for estimation
- Single-product vs. multi-product firms
- Require cost function and equilibrium notion

Equilibrium Model

Berry (1994) single-product firms

- Profits of firm j are given by $\Pi_j(\mathrm{p}) = p_j q_j(\mathrm{p}) - C_j(q_j(\mathrm{p}))$
- Conduct assumption: compete à la Nash in prices
- The first-order condition is:
$$q_j + (p_j - mc_j)\frac{\partial q_j}{\partial p_j} = 0$$
- which can be rewritten as
$$p_j = mc_j + b_j(\mathrm{p})$$
where $b_j(\mathrm{p}) = \frac{-q_j}{\partial q_j \backslash \partial p_j}$ is the *markup*

Multiproduct Firms

BLP (1995) look at the more general case of a multiproduct firm producing a subset $\Im_f$ of the J products

- Profits of firm f are given by:

$$\Pi_f = \sum_{j \in \Im_f} (p_j - mc_j) M s_j(p, x, \theta_1, \theta_2)$$

- The usual assumption is that firms compete à la Nash in prices

Multiproduct Firms

- The first-order condition is:

$$s_j(p,x,\theta_1,\theta_2) + \sum_{r \in \Im_f} (p_r - mc_r) \frac{\partial s_r(p,x,\theta_1,\theta_2)}{\partial p_j} = 0$$

- which can be rewritten in vector notation as:

$$p_j = mc_j + \Delta(p,x,\theta_1,\theta_2)^{-1} s(p,x,\theta_1,\theta_2)$$

where the (j,r) element of the J by J matrix $\Delta(p,x,\theta_1,\theta_2)$ is

$$\Delta_{jr} \begin{cases} 1 \times \frac{-\partial s_r}{\partial p_j} = \frac{-\partial s_r}{\partial p_j} & \text{if r and j are produced by the same firm} \\ 0 \times \frac{-\partial s_r}{\partial p_j} = 0 & \text{otherwise} \end{cases}$$

Multiproduct Firms: note on conduct

- Rewrite profit maximization problem:

$$\Pi_f = \sum_{j \in \Im_f} (p_j - mc_j) M s_j(p, x, \theta_1, \theta_2) + \kappa_{fg} \sum_{j \in \Im_g} (p_j - mc_j) M s_j(p, x, \theta_1, \theta_2)$$

- The first-order condition is then:

$$s_j(p, x, \theta_1, \theta_2) + \sum_{r \in (\Im_f, \Im_g)} \kappa_{fg} (p_r - mc_r) \frac{\partial s_r(p, x, \theta_1, \theta_2)}{\partial p_j} = 0$$

- Instead of 0's and 1's, now $\kappa_{fg} \in [0, 1]$ represents how much firm f cares about profits of g
- In reality, $\kappa_{fg} \in (0, 1)$, but evidence that $\kappa_{fg} > 0$ not necessarily "anti-competitive" – just deviation from static Bertrand pricing
- Berry and Haile (2014): κ_{fg} can be identified using IV that shifts demand but not supply ("rotate demand")

Multiproduct Firms

- For simplicity, assume that marginal costs are constant and take the form:

$$ln(mc) = w_j\gamma + \omega_j \tag{1}$$

- The equation to be estimated is therefore

$$ln(p - b(p, x, \theta_1, \theta_2)) = w_j\gamma + \omega_j$$

  where w_j are observed and ω_j unobserved product characteristics

- The markup $b(p, x, \theta_1, \theta_2) = \Delta(p, x, \theta_1, \theta_2)^{-1}s(p, x, \theta_1, \theta_2)$ depends on demand parameter and, through the equilibrium price, on the unobserved cost component ω

- We therefore need some instruments, which are uncorrelated with ω as well as with the structural demand error ξ

GMM estimation

- Estimate demand and supply simultaneously. Define additional instruments $z_j = (x_j, w_j)$.
- Additional moment conditions

$$E[\xi_j|z] = E[\omega_j|z] = 0$$

where $\omega = mc - [x, w]\gamma$

Problem set