**Advanced Econometrics**

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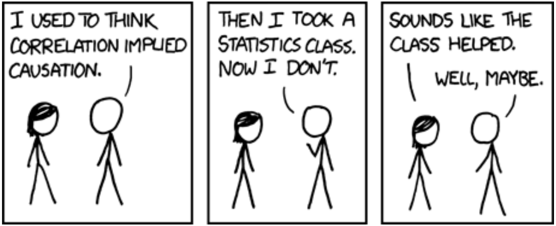


## Panel Methods

- Panel data: we observe the same units (individuals, firms, countries, schools, etc.) over several time periods
- Often our outcome variable depends on unobserved factors which are also correlated with our explanatory variable of interest
- If these omitted variables are constant over time, we can use panel data estimators to consistently estimate the effect of our explanatory variable
- Main estimators for panel data:
  - Pooled OLS
  - Fixed effects estimator
  - Random effects estimator

## Panel setup

- Let $y$ and $x \equiv (x_1, x_2, \dots, x_k)$ be observable random variables and $c$ be an unobservable random variable
- We are interested in the partial effects of variable $x_j$ in the population regression function
$$E[y|x_1, x_2, \dots, x_k, c]$$
- We observe a sample of $i = 1, 2, \dots, N$ cross-sectional units for $t = 1, 2, \dots, T$ time periods (a balanced panel)
    - For each unit $i$, we denote the observable variables for all time periods as $\{(y_{it}, x_{it}) : t = 1, 2, \dots, T\}$
    - $x_{it} \equiv (x_{it1}, x_{it2}, \dots, x_{itk})$ is a $1 \times K$ vector
- Typically assume that cross-sectional units are i.i.d. draws from the population: $\{y_i, x_i, c_i\}_{i=1}^N \sim i.i.d.$ (cross-sectional independence)
    - $y_i \equiv (y_{i1}, y_{i2}, \dots, y_{iT})'$ and $x_i \equiv (x_{i1}, x_{i2}, \dots, x_{iT})$
    - Consider asymptotic properties with $T$ fixed and $N \rightarrow \infty$

## Panel setup

Single unit:

$$y_i = \begin{pmatrix} y_{i1} \\ \vdots \\ y_{it} \\ \vdots \\ y_{iT} \end{pmatrix}_{T\times 1} \qquad X_i = \begin{pmatrix} X_{i,1,1} & X_{i,1,2} & X_{i,1,j} & \dots & X_{i,1,K} \\ \vdots & \vdots & \vdots & & \vdots \\ X_{i,t,1} & X_{i,t,2} & X_{i,t,j} & \dots & X_{i,t,K} \\ \vdots & \vdots & \vdots & & \vdots \\ X_{i,T,1} & X_{i,T,2} & X_{i,T,j} & \dots & X_{i,T,K} \end{pmatrix}_{T\times K}$$

Panel with all units:

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_N \end{pmatrix}_{NT\times 1} \qquad X = \begin{pmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{pmatrix}_{NT\times K}$$

## Unobserved effects model: Farm output

- For a randomly drawn cross-sectional unit $i$, the model is given by

$$y_{it} = x_{it}\beta + c_i + \varepsilon_{it}, \ \ t = 1, 2, \ldots, T$$

  - $y_{it}$: output of farm $i$ in year $t$
  - $x_{it}$ : $1 \times K$ vector of variable inputs for farm $i$ in year $t$, such as labor, fertilizer, etc. plus an intercept
  - $\beta$ : $K \times 1$ vector of marginal effects of variable inputs
  - $c_i$: sum of all time-invariant inputs known to farmer $i$ (but unobserved for the researcher), e.g., soil quality, managerial ability, etc.
    - often called the unobserved effect, unobserved heterogeneity, etc
  - $\varepsilon_{it}$: time-varying unobserved inputs, such as rainfall, unknown to the farmer at the time the decision on the variable inputs $x_{it}$ is made
    - often called the idiosyncratic error
- What happens when we regress $y_{it}$ on $x_{it}$?

# Pooled OLS

- When we ignore the panel structure and regress $y_{it}$ on $x_{it}$ we get

$$y_{it} = x_{it}\beta + v_{it}; \;\; t = 1, 2, \ldots, T$$

  with composite error $v_{it} \equiv c_i + \varepsilon_{it}$
- Main assumption to obtain consistent estimates for $\beta$ is:
  - $E[v_{it}|x_{i1}, x_{i2}, \ldots, x_{iT}] = E[v_{it}|x_{it}] = 0$ for $t = 1, 2, \ldots, T$
    - $x_{it}$ are strictly exogenous: the composite error $v_{it}$ in each time period is uncorrelated with the past, current and future regressors
    - But: labour input $x_{it}$ likely depends on soil quality $c_i$ and so we have omitted variable bias and $\widehat{\beta}$ is not consistent
  - No correlation between $x_{it}$ and $v_{it}$ implies no correlation between unobserved effect $c_i$ and $x_{it}$ for all $t$
    - Violations are common: whenever we omit a time-constant variable that is correlated with the regressors (heterogeneity bias)
  - Additional problem: $v_{it}$ are serially correlated for same $i$ since $c_i$ is present in each $t$ and thus pooled OLS standard errors are invalid

## Unobserved effects model: program evaluation

- Program evaluation model:

$$y_{it} = prog_{it}\beta + c_i + \varepsilon_{it}; t = 1, 2, \ldots, T$$

  - $y_{it}$: log wage of individual $i$ in year $t$
  - $prog_{it}$: indicator coded 1 if individual $i$ participants in training program at $t$ and 0 otherwise
  - $\beta$: effect of program
  - $c_i$: sum of all time-invariant unobserved characteristics that affect wages, such as ability, etc.
- What happens when we regress $y_{it}$ on $prog_{it}$? $\widehat{\beta}$ not consistent since $prog_{it}$ is likely correlated with $c_i$ (e.g., ability)
- Always ask: is there a time-constant unobserved variable ($c_i$) that is correlated with the regressors? If yes, then pooled OLS is problematic

## Fixed effect regression

- Our unobserved effects model is:

$$y_{it} = x_{it}\beta + c_i + \varepsilon_{it};\, t = 1, 2, \ldots, T$$

- If we have data on multiple time periods, we can think of $c_i$ as **fixed effects** or "nuisance parameters" to be estimated
- OLS estimation with fixed effects yields

$$(\widehat{\beta}, \widehat{c}_1, \ldots, \widehat{c}_N) = \underset{b, m_1, \ldots, m_N}{\operatorname{argmin}} \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - x_{it} b - m_i)^2$$

this amounts to including $N$ farm dummies in regression of $y_{it}$ on $x_{it}$

## Derivation: fixed effects regression

$$(\widehat{\beta}, \widehat{c}_1, \ldots, \widehat{c}_N) = \underset{b, m_1, \ldots, m_N}{\operatorname{argmin}} \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - x_{it} b - m_i)^2$$

The first-order conditions (FOC) for this minimization problem are:

$$\sum_{i=1}^{N} \sum_{t=1}^{T} x_{it}' (y_{it} - x_{it} \widehat{\beta} - \widehat{c}_i) = 0$$

and

$$\sum_{t=1}^{T} (y_{it} - x_{it} \widehat{\beta} - \widehat{c}_i) = 0$$

for $i = 1, \ldots, N$.

## Derivation: fixed effects regression

Therefore, for $i = 1, \ldots, N$,

$$\widehat{c}_i = \frac{1}{T}\sum_{t=1}^{T}(y_{it} - x_{it}\widehat{\beta}) = \bar{y}_i - \bar{x}_i\widehat{\beta},$$

where

$$\bar{x}_i \equiv \frac{1}{T}\sum_{t=1}^{T} x_{it}; \bar{y}_i \equiv \frac{1}{T}\sum_{t=1}^{T} y_{it}$$

Plug this result into the first FOC to obtain:

$$\widehat{\beta} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it} - \bar{x}_i)'(x_{it} - \bar{x}_i)\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it} - \bar{x}_i)'(y_{it} - \bar{y})\right)$$

$$\widehat{\beta} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\ddot{x}_{it}'\ddot{x}_{it}\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\ddot{x}_{it}'\ddot{x}_{it}\right)$$

with time-demeaned variables $\ddot{x}_{it} \equiv x_{it} - \bar{x}, \ddot{y}_{it} \equiv y_{it} - \bar{y}_i$

## Fixed effects regression

Running a regression with the time-demeaned variables $\ddot{y}_{it} \equiv y_{it} - \bar{y}_i$ and $\ddot{x}_{it} \equiv x_{it} - \bar{x}$ is numerically equivalent to a regression of $y_{it}$ on $x_{it}$ and unit specific dummy variables.

Even better, the regression with the time demeaned variables is consistent for $\beta$ even when $Cov[x_{it}, c_i] \neq 0$ because time-demeaning eliminates the unobserved effects

$$\begin{array}{rcl} y_{it} & = & x_{it}\beta + c_i + \varepsilon_{it} \\ \bar{y}_i & = & \bar{x}_i\beta + c_i + \bar{\varepsilon}_i \\ \hline (y_{it} - \bar{y}_i) & = & (x_{it} - \bar{x})\beta + (c_i - c_i) + (\varepsilon_{it} - \bar{\varepsilon}_i) \\ \ddot{y}_{it} & = & \ddot{x}_{it}\beta + \ddot{\varepsilon}_{it} \end{array}$$

## Fixed effects regression: main results

- Identification assumptions:
  1. $E[\varepsilon_{it}|x_{i1}, x+i2, \ldots, x_{iT}, c_i] = 0; t = 1, 2, \ldots, T$
     - regressors are strictly exogenous conditional on the unobserved effect
     - allows $x_{it}$ to be arbitrarily related to $c_i$
  2. $rank\left(\sum_{t=1}^{T} E[\ddot{x}_{it}'\ddot{x}_{it}]\right) = K$
     - regressors vary over time for at least some $i$ and not collinear
- Fixed effects estimator
  1. Demean and regress $\ddot{y}_{it}$ on $\ddot{x}_{it}$ (need to correct degrees of freedom)
  2. Regress $y_{it}$ on $x_{it}$ and unit dummies (dummy variable regression)
  3. Regress $y_{it}$ on $x_{it}$ with canned fixed effects routine
     - STATA: `xtreg y x, fe i(PanelID)`
- Properties (under assumptions 1-2):
  - $\widehat{\beta}_{FE}$ is consistent: $\underset{N\to\infty}{plim}\,\widehat{\beta}_{FE,N} = \beta$
  - $\widehat{\beta}_{FE}$ is unbiased conditional on $\mathbf{X}$

## Fixed effects regression: main issues

- Inference:
  - Standard errors have to be "clustered" by panel unit (e.g., farm) to allow correlation in the $\varepsilon_{it}$'s for the same $i$.
    - STATA: `xtreg , fe i(PanelID) cluster( PanelID )`
  - Yields valid inference as long as number of clusters is reasonably large
- Typically we care about $\beta$, but unit fixed effects $c_i$ could be of interest
  - $\widehat{c}_i$ from dummy variable regression is unbiased but not consistent for $c_i$ (based on fixed $T$ and $N \to \infty$)
  - `xtreg , fe` routine demeans the data before running the regression and therefore does not estimate $\widehat{c}_i$
    - intercept shows average $\widehat{c}_i$ across units
    - we can recover $\widehat{c}_i$ using $\widehat{c}_i = \bar{y}_i - \bar{x}_i \widehat{\beta}$
    - `predict c_i, u`

## Example: Direct Democracy and Naturalizations

- Do minorities fare worse under direct democracy than under representative democracy?
- Hainmueller and Hangartner (2012) examine data on naturalization requests of immigrants in Switzerland, where municipalities vote on naturalization applications in:
  - referendums (direct democracy)
  - elected municipality councils (representative democracy)
- Annual panel data from 1,400 municipalities for the 1991-2009 period
  - $y_{it}$ : naturalization rate $= \frac{no.naturalizations_{it}}{eligibleforeignpopulation_{i,t-1}}$
  - $x_{it}$ : 1 if municipality used representative democracy, 0 if municipality used direct democracy in year $t$

# Naturalization Panel Data

```
. des muniID muni_name year nat_rate repdem

              storage  display     value
variable name   type   format      label      variable label
-------------------------------------------------------------------------------------
muniID          float  %8.0g                  municipality code
muni_name       str43  %43s                   municipality name
year            float  %ty                    year
nat_rate        float  %9.0g                  naturalization rate (percent)
repdem          float  %9.0g                  1 representative democracy, 0 direct
```

# Panel Data Long Format

```
. list muniID muni_name year nat_rate repdem in 31/40
```

| | muniID | muni_name | year | nat_rate | repdem |
|---|---|---|---|---|---|
| 31. | 2 | Affoltern A.A. | 2002 | 4.638365 | 0 |
| 32. | 2 | Affoltern A.A. | 2003 | 4.844814 | 0 |
| 33. | 2 | Affoltern A.A. | 2004 | 5.621302 | 0 |
| 34. | 2 | Affoltern A.A. | 2005 | 4.387827 | 0 |
| 35. | 2 | Affoltern A.A. | 2006 | 8.115358 | 1 |
| 36. | 2 | Affoltern A.A. | 2007 | 7.067371 | 1 |
| 37. | 2 | Affoltern A.A. | 2008 | 8.977719 | 1 |
| 38. | 2 | Affoltern A.A. | 2009 | 6.119704 | 1 |
| 39. | 3 | Bonstetten | 1991 | .8333334 | 0 |
| 40. | 3 | Bonstetten | 1992 | .8403362 | 0 |

# Pooled OLS

```
. reg nat_rate repdem , cl(muniID)

Linear regression                                      Number of obs =    4655
                                                       F(  1,   244) =  130.04
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.0748
                                                       Root MSE      =    3.98

                               (Std. Err. adjusted for 245 clusters in muniID)
------------------------------------------------------------------------------
             |               Robust
    nat_rate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      repdem |   2.503318   .2195202    11.40   0.000     2.070921    2.935714
       _cons |   2.222683     .10088    22.03   0.000     2.023976    2.421389
------------------------------------------------------------------------------
```

# Decompose within and between variation

```
. tsset  muniID year ,  yearly
       panel variable:  muniID (strongly balanced)
        time variable:  year, 1991 to 2009
                delta:  1 year

. xtsum  nat_rate
```

| Variable | | Mean | Std. Dev. | Min | Max | Observations |
|---|---|---|---|---|---|---|
| nat_rate | overall | 2.938992 | 4.137305 | 0 | 24.13793 | N = 4655 |
| | between | | 1.622939 | 0 | 7.567746 | n = 245 |
| | within | | 3.807039 | -3.711323 | 24.80134 | T = 19 |

# Time-demeaning for fixed effects: $y_{it} \to \ddot{y}_{it}$

```
. * get municipality means
. egen means_nat_rate = mean(nat_rate) , by(muniID)

. * compute deviations from means
. gen  dm_nat_rate = nat_rate - means_nat_rate

. list muniID muni_name year nat_rate means_nat_rate dm_nat_rate in 20/40 ,ab(20)
```

| | muniID | muni_name | year | nat_rate | means_nat_rate | dm_nat_rate |
|---|---|---|---|---|---|---|
| 20. | 2 | Affoltern A.A. | 1991 | .2173913 | 3.595932 | -3.37854 |
| 21. | 2 | Affoltern A.A. | 1992 | .9473684 | 3.595932 | -2.648563 |
| 22. | 2 | Affoltern A.A. | 1993 | 1.04712 | 3.595932 | -2.548811 |
| 23. | 2 | Affoltern A.A. | 1994 | .8342023 | 3.595932 | -2.761729 |
| 24. | 2 | Affoltern A.A. | 1995 | 2.002002 | 3.595932 | -1.59393 |
| 25. | 2 | Affoltern A.A. | 1996 | 1.7769 | 3.595932 | -1.819031 |
| 26. | 2 | Affoltern A.A. | 1997 | 1.862745 | 3.595932 | -1.733186 |
| 27. | 2 | Affoltern A.A. | 1998 | 2.054155 | 3.595932 | -1.541776 |
| 28. | 2 | Affoltern A.A. | 1999 | 2.402135 | 3.595932 | -1.193796 |

# Fixed effects regression with demeaned data

```
. egen means_repdem   = mean(repdem) , by(muniID)

. gen  dm_repdem   = repdem   - means_repdem

.
. * regression with demeaned data
. reg dm_nat_rate dm_repdem , cl(muniID)

Linear regression                                      Number of obs =    4655
                                                       F(  1,   244) =  265.18
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.1052
                                                       Root MSE      =  3.6017

                              (Std. Err. adjusted for 245 clusters in muniID)
------------------------------------------------------------------------------
             |               Robust
 dm_nat_rate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   dm_repdem |     3.0228   .1856244    16.28   0.000     2.657169    3.388431
       _cons |   6.65e-10   5.81e-09     0.11   0.909    -1.08e-08    1.21e-08
------------------------------------------------------------------------------
```

# Fixed effects regression with canned routine

```
. xtreg nat_rate repdem , fe cl(muniID) i(muniID)

Fixed-effects (within) regression               Number of obs      =      4655
Group variable: muniID                          Number of groups   =       245

R-sq:  within  = 0.1052                         Obs per group: min =        19
       between = 0.0005                                        avg =      19.0
       overall = 0.0748                                        max =        19

                                                F(1,244)           =    265.18
corr(u_i, Xb)  = -0.1373                        Prob > F           =    0.0000

                              (Std. Err. adjusted for 245 clusters in muniID)
------------------------------------------------------------------------------
             |               Robust
    nat_rate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      repdem |     3.0228   .1856244    16.28   0.000     2.657169    3.388431
       _cons |   2.074036   .0531153    39.05   0.000     1.969413    2.178659
-------------+----------------------------------------------------------------
     sigma_u |  1.7129711
     sigma_e |    3.69998
         rho |  .17650677   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

# Fixed effects regression with dummies

```
. reg nat_rate repdem i.muniID, cl(muniID)

Linear regression                               Number of obs =     4655
                                                F(  0,    244) =        .
                                                Prob > F      =        .
                                                R-squared     =   0.2423
                                                Root MSE      =      3.7

                                (Std. Err. adjusted for 245 clusters in muniID)
```

| nat_rate | Coef. | Robust Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| repdem | 3.0228 | .1906916 | 15.85 | 0.000 | 2.647188 | 3.398412 |
| muniID | | | | | | |
| 2 | 1.367365 | 5.17e-14 | 2.6e+13 | 0.000 | 1.367365 | 1.367365 |
| 3 | 1.292252 | 5.17e-14 | 2.5e+13 | 0.000 | 1.292252 | 1.292252 |
| 9 | 1.284652 | 5.17e-14 | 2.5e+13 | 0.000 | 1.284652 | 1.284652 |
| 10 | 1.271783 | 5.17e-14 | 2.5e+13 | 0.000 | 1.271783 | 1.271783 |
| 13 | .3265469 | 5.17e-14 | 6.3e+12 | 0.000 | .3265469 | .3265469 |

## Applying fixed effects

- We can use fixed effects for other data structures to restrict comparisons to within unit variation
  - Matched pairs
    - Twin fixed effects to control for unobserved effects of family background
  - Cluster fixed effects in hierarchical data
    - School fixed effects to control for unobserved effects of school

## Problems that even fixed effects do not solve

$$y_{it} = x_{it}\beta + c_i + \varepsilon_{it};\ t = 1, 2, \ldots, T$$

- Where $y_{it}$ is murder rate and $x_{it}$ is police spending per capita
- What happens when we regress $y$ on $x$ and city fixed effects?
  - $\widehat{\beta}_{FE}$ inconsistent unless strict exogeneity conditional on $c_i$ holds
    - $E[\varepsilon_{it}|x_{i1}, x_{i2}, \ldots, x_{iT}, c_i] = 0; t = 1, 2, \ldots, T$
    - implies $\varepsilon_{it}$ uncorrelated with past, current and future regressors
- Most common violations
  1. Time-varying omitted variables
     - Economic boom leads to more police spending and less murders
     - Can include time-varying controls, but avoid post-treatment bias (i.e., collider)
  2. Simultaneity
     - if city adjusts police based on past murder rate, then spending$_{t+1}$ is correlated with $\varepsilon_t$ (since higher $\varepsilon_t$ leads to higher murder rate at $t$)
     - strictly exogenous $x$ cannot react to what happens to $y$ in the past or the future!
- Fixed effects do not obviate need for good research design!

## Random Effects

- Reconsider our unobserved effects model:

$$y_{it} = x_{it}\beta + c_i + \varepsilon_{it}, \quad t = 1, 2, \ldots, T$$

  - Cannot use the fixed effects regression to estimate effects of time-constant regressors in $x_{it}$ (eg., soil quality, farm location, etc.)
    - Since fixed effect estimator allows $c_i$ to be correlated with $x_{it}$, we cannot distinguish the effects of time-invariant regressors from the time-invariant unobserved effect $c_i$
  - Need orthogonality assumption: $Cov[x_{it}, c_i] = 0; \quad t = 1, \ldots, T$
    - Strong assumption: Unobserved effects $c_i$ are uncorrelated with each explanatory variable in $x_{it}$ in each time period
    - For example if we include soil quality in $x_{it}$ we have to assume it is uncorrelated with all other time-invariant inputs

## Random effects assumptions

$$y_{it} = x_{it}\beta + c_i + \varepsilon_{it}; \quad t = 1, \ldots, T$$

1. $E[\varepsilon_{it}|x_i, c_i] = 0;\ \ t = 1, 2, \ldots, T$: explanatory variables are strictly exogenous conditional on the unobserved effect
2. $E[c_i|x_i] = E[c_i] = 0$: unobserved effects $c_i$ are uncorrelated with regressors
3. rank $E[X_i'\Omega X_i] = K$: no collinearity among regressors
   - $\Omega = E[v_i v_i']$ : the variance matrix of the composite error $v_{it} = c_i + \varepsilon_{it}$
4. We typically also assume that $\Omega$ takes a special form:
   - $E[\varepsilon_i \varepsilon_i'|x_i] = \sigma_\varepsilon^2 \mathbf{I}_T$ : idiosyncratic errors are homoskedastic for all $t$ and serially uncorrelated
   - $E[c_i^2|x_i] = \sigma_c^2$: unobserved effect $c_i$ is homoscedastic

# Random effects assumptions

$$y_{it} = x_{it}\beta + c_i + \varepsilon_{it}; \quad t = 1, \ldots, T$$

1. $E[\varepsilon_{it}|x_i, c_i] = 0; \ \ t = 1, 2, \ldots, T$: explanatory variables are strictly exogenous conditional on the unobserved effect
2. $E[c_i|x_i] = E[c_i] = 0$: unobserved effects $c_i$ are uncorrelated with regressors
3. rank $E[X_i'\Omega X_i] = K$: no collinearity among regressors
   - $\Omega = E[v_i v_i']$ : the variance matrix of the composite error $v_{it} = c_i + \varepsilon_{it}$
4. We typically also assume that $\Omega$ takes a special form:
   - $E[\varepsilon_i \varepsilon_i'|x_i] = \sigma_\varepsilon^2 \mathbf{I}_T$ : idiosyncratic errors are homoskedastic for all $t$ and serially uncorrelated
   - $E[c_i^2|x_i] = \sigma_c^2$: unobserved effect $c_i$ is homoscedastic

Assumption 4 implies $\Omega = E[v_i v_i'|x_i] = \begin{pmatrix} \sigma_c^2 + \sigma_\varepsilon^2 & \sigma_c^2 & \ldots & \sigma_c^2 \\ \sigma_c^2 & \sigma_c^2 + \sigma_\varepsilon^2 & \ldots & \vdots \\ \vdots & & \ddots & \sigma_c^2 \\ \sigma_c^2 & & & \sigma_c^2 + \sigma_\varepsilon^2 \end{pmatrix}_{T \times T}$

# Random effects assumptions

$$y_{it} = x_{it}\beta + c_i + \varepsilon_{it}; \quad t = 1, \ldots, T$$

1. $E[\varepsilon_{it}|x_i, c_i] = 0; \;\; t = 1, 2, \ldots, T$: explanatory variables are strictly exogenous conditional on the unobserved effect
2. $E[c_i|x_i] = E[c_i] = 0$: unobserved effects $c_i$ are uncorrelated with regressors
3. rank $E[X_i'\Omega X_i] = K$: no collinearity among regressors
   - $\Omega = E[v_i v_i']$ : the variance matrix of the composite error $v_{it} = c_i + \varepsilon_{it}$
4. We typically also assume that $\Omega$ takes a special form:
   - $E[\varepsilon_i \varepsilon_i'|x_i] = \sigma_\varepsilon^2 \mathbf{I}_T$ : idiosyncratic errors are homoskedastic for all $t$ and serially uncorrelated
   - $E[c_i^2|x_i] = \sigma_c^2$: unobserved effect $c_i$ is homoscedastic

- Given assumptions 1-3, pooled OLS is consistent, since composite error $v_{it}$ is uncorrelated with $x_{it}$ for all $t$
- However, pooled OLS ignores the serial correlation in $v_{it}$

# Random effects assumptions

$$y_{it} = x_{it}\beta + c_i + \varepsilon_{it}; \quad t = 1, \ldots, T$$

1. $E[\varepsilon_{it}|x_i, c_i] = 0; \; t = 1, 2, \ldots, T$: explanatory variables are strictly exogenous conditional on the unobserved effect
2. $E[c_i|x_i] = E[c_i] = 0$: unobserved effects $c_i$ are uncorrelated with regressors
3. rank $E[X_i'\Omega X_i] = K$: no collinearity among regressors
   - $\Omega = E[v_i v_i']$ : the variance matrix of the composite error $v_{it} = c_i + \varepsilon_{it}$
4. We typically also assume that $\Omega$ takes a special form:
   - $E[\varepsilon_i \varepsilon_i'|x_i] = \sigma_\varepsilon^2 \mathbf{I}_T$ : idiosyncratic errors are homoskedastic for all $t$ and serially uncorrelated
   - $E[c_i^2|x_i] = \sigma_c^2$: unobserved effect $c_i$ is homoscedastic

- Random effects estimator $\widehat{\beta}_{RE}$ exploits this serial correlation in a generalized least squares (GLS) framework
  - $\widehat{\beta}_{RE}$ is consistent under assumption 1-3: $\underset{N\to\infty}{plim}\, \widehat{\beta}_{RE,N} = \beta$
  - $\widehat{\beta}_{RE}$ is asymptotically efficient given assumption 4 (in the class of estimators consistent under $E[v_i|x_i] = 0$)

# Random effects estimator

- Consider the transformation parameter

$$\lambda = 1 - \left( \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + T\sigma_c^2} \right)^{\frac{1}{2}} \; with \; 0 \leq \lambda \leq 1$$

  - $\sigma_\varepsilon^2 = Var[\varepsilon_{it}]$: variance of idiosyncratic error
  - $\sigma_c^2 = Var(c_i)$: Variance of unobserved effect
- $\widehat{\beta}_{RE}$ is equivalent to pooled OLS on:

$$\begin{aligned} y_{it} - \bar{y}_i &= (x_{it} - \lambda \bar{x}_i)\beta + (v_{it} - \lambda \bar{v}_i), \forall i, t \\ \tilde{y}_{it} &= \tilde{x}_{it}\beta + \tilde{v}_{it} \end{aligned}$$

  - As $\lambda \to 1$, $\widehat{\beta}_{RE} \to \widehat{\beta}_{FE}$
  - As $\lambda \to 0$, $\widehat{\beta}_{RE} \to \widehat{\beta}_{Pooled\ OLS}$
    - $\lambda \to 1$ as $T \to \infty$ or if variance of $c_i$ is large relative to variance of $\varepsilon_{it}$
- $\lambda$ can be estimated from data $\widehat{\lambda} = 1 - (\widehat{\sigma}_\varepsilon^2 / (\widehat{\sigma}_\varepsilon^2 + T\widehat{\varepsilon}_c^2))^{\frac{1}{2}}$
- Usually wise to cluster the standard errors since assumption 4 is strong

# Random effects regression

```
. xtreg nat_rate repdem , re cl(muniID) i(muniID)

Random-effects GLS regression                   Number of obs      =      4655
Group variable: muniID                          Number of groups   =       245

R-sq:  within  = 0.1052                         Obs per group: min =        19
       between = 0.0005                                        avg =      19.0
       overall = 0.0748                                        max =        19

                                                Wald chi2(1)       =    227.99
corr(u_i, X)   = 0 (assumed)                    Prob > chi2        =    0.0000

                               (Std. Err. adjusted for 245 clusters in muniID)
------------------------------------------------------------------------------
             |               Robust
    nat_rate |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      repdem |   2.859397   .1893742    15.10   0.000      2.48823    3.230564
       _cons |   2.120793   .0972959    21.80   0.000     1.930096    2.311489
-------------+----------------------------------------------------------------
     sigma_u |  1.3866768
     sigma_e |    3.69998
         rho |   .1231606   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

## Summary: Fixed effects, random effects, Pooled OLS

- Main assumptions
  1. Regressors are strictly exogenous conditional on the time-invariant unobserved effects
  2. Regressors are uncorrelated with the time-invariant unobserved effects
- Results
  - Fixed effects estimator is consistent given assumption 1, but rules out time-invariant regressors
  - Random effects estimators and pooled OLS are consistent under assumptions 1-2, and allow for time-invariant regressors
  - Given homoskedasticity assumptions (random effects assumption 4), the random effects estimator is asymptotically efficient
- Assumption 2 is strong so fixed effects are typically more credible
  - Often the main reason for using panel data is to rule out all time-invariant unobserved confounders!

## Hausman test

| | $\widehat{\beta}_{RE}$ | $\widehat{\beta}_{FE}$ |
|---|---|---|
| $H_0 : Cov[x_{it}, c_i] = 0$ | Consistent and efficient | Consistent |
| $H_1 : Cov[x_{it}, c_i] \neq 0$ | Inconsistent | Consistent |

Then,

- Under $H_0, \widehat{\beta}_{RE} - \widehat{\beta}_{FE}$ should be close to zero
- Under $H_1, \widehat{\beta}_{RE} - \widehat{\beta}_{FE}$ should be different from zero
- It can be shown that in large samples, under $H_0$, the test statistic

  $$(\widehat{\beta}_{FE} - \widehat{\beta}_{RE})'(\widehat{Var}[\beta_{FE}] - \widehat{Var}[\beta_{RE}])^{-1}(\widehat{\beta}_{FE} - \widehat{\beta}_{RE}) \xrightarrow{d} \chi^2_k$$

  where $k$ is the number of time-varying regressors.
- We may reject the null hypothesis of "random effects" and stick with the less efficient, but consistent fixed effects specification

# Random effects regression

```
. quietly: xtreg nat_rate repdem , fe i(muniID)

. estimates store FE

.
. quietly: xtreg nat_rate repdem , re i(muniID)

. estimates store RE

. hausman FE RE

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |       FE           RE         Difference          S.E.
-------------+----------------------------------------------------------------
      repdem |     3.0228      2.859397        .1634027         .0304517
------------------------------------------------------------------------------
                          b = consistent under Ho and Ha; obtained from xtreg
           B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =       28.79
```

## Hausman test

- Hausman test does not test if the fixed effect model is correct; the test assumes that the fixed effects estimator is consistent!
- Conventional Hausman test assumes homoskedastic model and does not allow for clustering
- There are Haumsman like tests that allow for clustered standard errors

```
. * hausman test with clustering
. quietly: xtreg nat_rate repdem , re i(muniID) cl(muniID)

. xtoverid

Test of overidentifying restrictions: fixed vs random effects
Cross-section time-series model: xtreg re  robust cluster(muniID)
Sargan-Hansen statistic  26.560  Chi-sq(1)    P-value = 0.0000
```

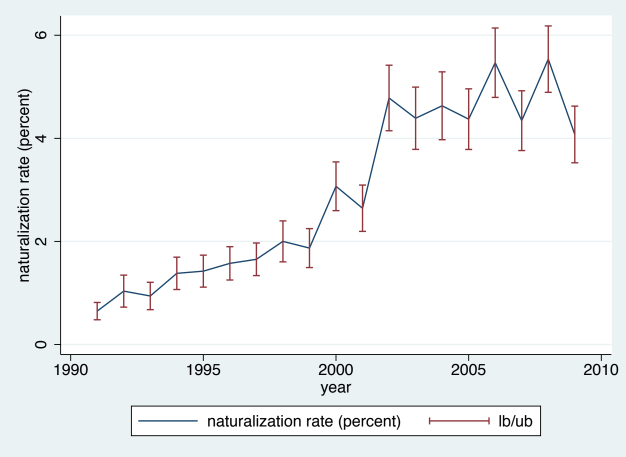
## Adding Time Effects

- Reconsider our unobserved effects model:

$$y_{it} = x_{it}\beta + c_i + \varepsilon_{it}, \;\; t = 1, 2, \ldots, T$$

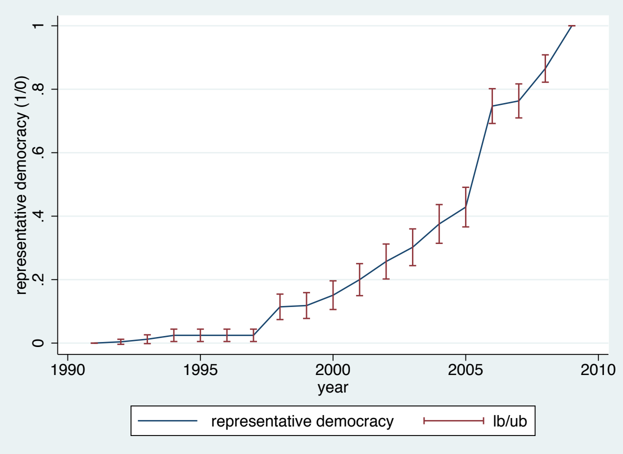
  - Fixed effects assumption: $E[\varepsilon_{it}|x_i, c_i] = 0; t = 1, 2, \ldots, T$: regressors are strictly exogenous conditional on the unobserved effect
  - Typical violation: Common shocks that affect all units in the same way and are correlated with $x_{it}$
    - Trends in farming technology or climate affect productivity
    - Trends in immigration inflows affect naturalization rates
  - We can allow for such common shocks by including time effects into the model

# Random effects regression




```
xtgraph nat_rate
```

# Random effects regression




```
xtgraph repdem
```

## Fixed effects: adding time effects

- Linear time trend:

$$y_{it} = x_{it}\beta + c_i + t + \varepsilon_{it}; \;\; t = 1, 2, \ldots, T$$

  - Linear time trend common to all units

- Time fixed effects:

$$y_{it} = x_{it}\beta + c_i + t_t + \varepsilon_{it}; \;\; t = 1, 2, \ldots, T$$

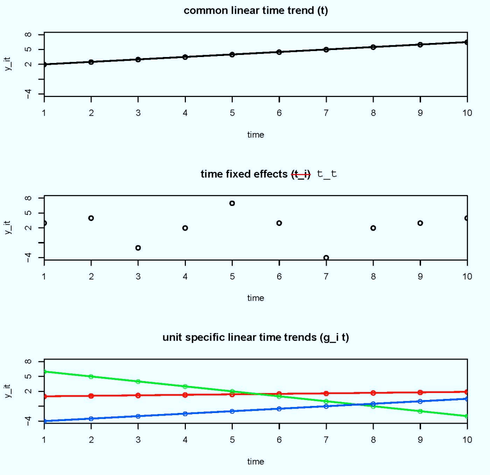
  - Common shock in each time period
  - Generalized difference-in-difference model

- Unit specific linear time trends:

$$y_{it} = x_{it}\beta + c_i + g_i \cdot t + t_t + \varepsilon_{it}; \;\; t = 1, 2, \ldots, T$$

  - Linear time trends that vary by unit

# Modeling time effects



# Fixed effects: adding time effects

```
. egen time = group(year)

. list muniID muni_name year time in 20/40 ,ab(20)
```

| | muniID | muni_name | year | time |
|---|---|---|---|---|
| 20. | 2 | Affoltern A.A. | 1991 | 1 |
| 21. | 2 | Affoltern A.A. | 1992 | 2 |
| 22. | 2 | Affoltern A.A. | 1993 | 3 |
| 23. | 2 | Affoltern A.A. | 1994 | 4 |
| 24. | 2 | Affoltern A.A. | 1995 | 5 |
| 25. | 2 | Affoltern A.A. | 1996 | 6 |
| 26. | 2 | Affoltern A.A. | 1997 | 7 |
| 27. | 2 | Affoltern A.A. | 1998 | 8 |
| 28. | 2 | Affoltern A.A. | 1999 | 9 |
| 29. | 2 | Affoltern A.A. | 2000 | 10 |
| 30. | 2 | Affoltern A.A. | 2001 | 11 |

# Fixed effects: linear time trend

```
. xtreg nat_rate repdem time , fe cl(muniID) i(muniID)

Fixed-effects (within) regression               Number of obs      =      4655
Group variable: muniID                          Number of groups   =       245

R-sq:  within  = 0.1604                         Obs per group: min =        19
       between = 0.0005                                        avg =      19.0
       overall = 0.1350                                        max =        19

                                                F(2,244)           =    247.57
corr(u_i, Xb)  = -0.0079                        Prob > F           =    0.0000

                                (Std. Err. adjusted for 245 clusters in muniID)
------------------------------------------------------------------------------
             |               Robust
    nat_rate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      repdem |   .8247928   .2590615     3.18   0.002     .3145106    1.335075
        time |   .2313692   .0171752    13.47   0.000     .1975386    .2651997
       _cons |   .3892908   .1309232     2.97   0.003     .1314069    .6471747
-------------+----------------------------------------------------------------
     sigma_u |  1.6271657
     sigma_e |   3.584409
         rho |  .17086519   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

# Fixed effects: year fixed effects

```
. xtreg nat_rate repdem i.time , fe cl(muniID) i(muniID)

Fixed-effects (within) regression               Number of obs      =      4655
Group variable: muniID                          Number of groups   =       245

R-sq:  within  = 0.1885                         Obs per group: min =        19
       between = 0.0005                                        avg =      19.0
       overall = 0.1575                                        max =        19

                                                F(19,244)          =     31.48
corr(u_i, Xb)  = -0.0168                        Prob > F           =    0.0000

                               (Std. Err. adjusted for 245 clusters in muniID)
```

| nat_rate | Coef. | Robust Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| repdem | 1.203658 | .3031499 | 3.97 | 0.000 | .6065335 | 1.800783 |
| time | | | | | | |
| 2 | .3829173 | .1723225 | 2.22 | 0.027 | .0434879 | .7223468 |
| 3 | .2789777 | .1514124 | 1.84 | 0.067 | -.0192644 | .5772198 |
| 4 | .7034078 | .167466 | 4.20 | 0.000 | .3735443 | 1.033271 |

# Fixed effects: unit specific time trends

```
. xtreg nat_rate repdem muniID#c.time i.time  , fe cl(muniID) i(muniID)
note: 19.time omitted because of collinearity

Fixed-effects (within) regression               Number of obs      =      4655
Group variable: muniID                          Number of groups   =       245

R-sq:  within  = 0.2650                         Obs per group: min =        19
       between = 0.5185                                        avg =      19.0
       overall = 0.2864                                        max =        19

                                                F(18,244)          =         .
corr(u_i, Xb)  = -0.3963                        Prob > F           =         .

                                (Std. Err. adjusted for 245 clusters in muniID)
```

| nat_rate | Coef. | Robust Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| repdem | .9865241 | .322868 | 3.06 | 0.002 | .3505601 | 1.622488 |
| muniID#c.time | | | | | | |
| 1 | .333343 | .024298 | 13.72 | 0.000 | .2854823 | .3812036 |
| 2 | .2914274 | .024298 | 11.99 | 0.000 | .2435667 | .339288 |
| 3 | .248985 | .024298 | 10.25 | 0.000 | .2011244 | .2968457 |

# Unit specific time trends often eliminate "results"

TABLE 5.2.3
Estimated effects of labor regulation on the performance of firms in Indian states

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Labor regulation (lagged) | −.186 | −.185 | −.104 | .0002 |
| | (.064) | (.051) | (.039) | (.020) |
| Log development expenditure per capita | | .240 | .184 | .241 |
| | | (.128) | (.119) | (.106) |
| Log installed electricity capacity per capita | | .089 | .082 | .023 |
| | | (.061) | (.054) | (.033) |
| Log state population | | .720 | 0.310 | −1.419 |
| | | (.96) | (1.192) | (2.326) |
| Congress majority | | | −.0009 | .020 |
| | | | (.01) | (.010) |
| Hard left majority | | | −.050 | −.007 |
| | | | (.017) | (.009) |
| Janata majority | | | .008 | −.020 |
| | | | (.026) | (.033) |
| Regional majority | | | .006 | .026 |
| | | | (.009) | (.023) |
| State-specific trends | No | No | No | Yes |
| Adjusted $R^2$ | .93 | .93 | .94 | .95 |

*Notes*: Adapted from Besley and Burgess (2004), table IV. The table reports regression DD estimates of the effects of labor regulation on productivity. The

## Distributed Lag model

$$y_{it} = x_{it}\beta_0 + x_{i,t-1}\beta_1 + x_{i,t-2}\beta_2 + c_i + \varepsilon_{it}, \;\; t = 1, 2, \ldots, T$$

- Model recognizes the effect of change in $x$ may occur with a late
    - effect of new tax credit for children on fertility rate
- Interpretation of coefficients:
    - Consider **temporary increase** in $x_{it}$ from level $m$ to $m+1$ at $t$ which lasts only one period
        - $y_{t-1} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
        - $y_t = (m+1)\beta_0 + m\beta_1 + m\beta_2 + c_i$
        - $y_{t+1} = m\beta_0 + (m+1)\beta_1 + m\beta_2 + c_i$
        - $y_{t+2} = m\beta_0 + m\beta_1 + (m+1)\beta_2 + c_i$
        - $y_{t+3} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$

## Distributed Lag model

$$y_{it} = x_{it}\beta_0 + x_{i,t-1}\beta_1 + x_{i,t-2}\beta_2 + c_i + \varepsilon_{it}, \;\; t = 1, 2, \ldots, T$$

- Model recognizes the effect of change in $x$ may occur with a late
  - effect of new tax credit for children on fertility rate
- Interpretation of coefficients:
  - Consider **temporary increase** in $x_{it}$ from level $m$ to $m+1$ at $t$ which lasts only one period
    - $y_{t-1} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
    - $y_t = (m+1)\beta_0 + m\beta_1 + m\beta_2 + c_i$
    - $y_{t+1} = m\beta_0 + (m+1)\beta_1 + m\beta_2 + c_i$
    - $y_{t+2} = m\beta_0 + m\beta_1 + (m+1)\beta_2 + c_i$
    - $y_{t+3} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
- $\beta_0 = y_t - y_{t-1}$ immediate change in $y$ due to temporary one-unit increase in $x$ (impact propensity)

## Distributed Lag model

$$y_{it} = x_{it}\beta_0 + x_{i,t-1}\beta_1 + x_{i,t-2}\beta_2 + c_i + \varepsilon_{it}, \ \ t = 1, 2, \ldots, T$$

- Model recognizes the effect of change in $x$ may occur with a late
  - effect of new tax credit for children on fertility rate
- Interpretation of coefficients:
  - Consider **temporary increase** in $x_{it}$ from level $m$ to $m+1$ at $t$ which lasts only one period
    - $y_{t-1} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
    - $y_t = (m+1)\beta_0 + m\beta_1 + m\beta_2 + c_i$
    - $y_{t+1} = m\beta_0 + (m+1)\beta_1 + m\beta_2 + c_i$
    - $y_{t+2} = m\beta_0 + m\beta_1 + (m+1)\beta_2 + c_i$
    - $y_{t+3} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
- $\beta_1 = y_{t+1} - y_t$ change in $y$ one period after temporary one-unit increase in $x$

## Distributed Lag model

$$y_{it} = x_{it}\beta_0 + x_{i,t-1}\beta_1 + x_{i,t-2}\beta_2 + c_i + \varepsilon_{it}, \;\; t = 1, 2, \ldots, T$$

- Model recognizes the effect of change in $x$ may occur with a late
  - effect of new tax credit for children on fertility rate
- Interpretation of coefficients:
  - Consider **temporary increase** in $x_{it}$ from level $m$ to $m+1$ at $t$ which lasts only one period
    - $y_{t-1} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
    - $y_t = (m+1)\beta_0 + m\beta_1 + m\beta_2 + c_i$
    - $y_{t+1} = m\beta_0 + (m+1)\beta_1 + m\beta_2 + c_i$
    - $y_{t+2} = m\beta_0 + m\beta_1 + (m+1)\beta_2 + c_i$
    - $y_{t+3} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
- $\beta_2 = y_{t+2} - y_{t-1}$ change in $y$ two periods after temporary one-unit increase in $x$

## Distributed Lag model

$$y_{it} = x_{it}\beta_0 + x_{i,t-1}\beta_1 + x_{i,t-2}\beta_2 + c_i + \varepsilon_{it}, \;\; t = 1, 2, \ldots, T$$

- Model recognizes the effect of change in $x$ may occur with a late
    - effect of new tax credit for children on fertility rate
- Interpretation of coefficients:
    - Consider **temporary increase** in $x_{it}$ from level $m$ to $m+1$ at $t$ which lasts only one period
        - $y_{t-1} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
        - $y_t = (m+1)\beta_0 + m\beta_1 + m\beta_2 + c_i$
        - $y_{t+1} = m\beta_0 + (m+1)\beta_1 + m\beta_2 + c_i$
        - $y_{t+2} = m\beta_0 + m\beta_1 + (m+1)\beta_2 + c_i$
        - $y_{t+3} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
- $\beta_3 = y_{t-1}$ change in $y$ is zero three periods after temporary one-unit increase in $x$

## Distributed Lag model

$$y_{it} = x_{it}\beta_0 + x_{i,t-1}\beta_1 + x_{i,t-2}\beta_2 + c_i + \varepsilon_{it}, \;\; t = 1, 2, \ldots, T$$

- Interpretation of coefficients:
    - Consider **permanent increase** in $x_{it}$ from level $m$ to $m+1$ at $t$, i.e., $(x_s = m, s < t$ and $x_s = m+1, s \geq t)$
        - $y_{t-1} = m\beta_0 + m\beta_1 + m\beta_2 + c_i$
        - $y_t = (m+1)\beta_0 + m\beta_1 + m\beta_2 + c_i$
        - $y_{t+1} = (m+1)\beta_0 + (m+1)\beta_1 + m\beta_2 + c_i$
        - $y_{t+2} = (m+1)\beta_0 + (m+1)\beta_1 + (m+1)\beta_2 + c_i$
        - $y_{t+3} = (m+1)\beta_0 + (m+1)\beta_1 + (m+1)\beta_2 + c_i$
- After one period $y$ has increased by $\beta_0 + \beta_1$, after two periods $y$ has increased by $\beta_0 + \beta_1 + \beta_2$ and there are no further increases after two periods
- Long-run increase in $y$ : $\beta_0 + \beta_1 + \beta_2$ (long-run propensity)

# Lagged effects of direct democracy

```
. xtreg nat_rate repdem L1.repdem L2.repdem  L3.repdem i.year, fe cl(muniID) i(muniID)

Fixed-effects (within) regression               Number of obs      =      3920
Group variable: muniID                          Number of groups   =       245

R-sq:  within  = 0.1536                         Obs per group: min =        16
       between = 0.0012                                        avg =      16.0
       overall = 0.1235                                        max =        16

                                                F(19,244)          =     21.63
corr(u_i, Xb)  = -0.0206                        Prob > F           =    0.0000

                                  (Std. Err. adjusted for 245 clusters in muniID)
```

| nat_rate | Coef. | Robust Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| repdem | | | | | | |
| --. | .6364802 | .3593924 | 1.77 | 0.078 | -.0714272 | 1.344388 |
| L1. | 1.201065 | .4233731 | 2.84 | 0.005 | .367133 | 2.034998 |
| L2. | -.1648692 | .4697434 | -0.35 | 0.726 | -1.090139 | .7604003 |
| L3. | -.5245206 | .4109918 | -1.28 | 0.203 | -1.334065 | .2850239 |

# Long-run effect of direct democracy

```
. lincom repdem + L1.repdem + L2.repdem + L3.repdem

 ( 1)  repdem + L.repdem + L2.repdem + L3.repdem = 0
```

| nat_rate | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| (1) | 1.294485 | .4426322 | 2.92 | 0.004 | .4226175 | 2.166353 |

## Lags and Leads model

$$y_{it} = x_{i,t+1}\beta_{-1} + x_{it}\beta_0 + x_{i,t-1}\beta_1 + x_{i,t-2}\beta_2 + c_i + \varepsilon_{it};\ \ t = 1, 2, \ldots, T$$

- Can use estimate of $\beta_{-1}$ to test for anticipation effects
  - Consider **temporary increase** in $x_{it}$ from level $m$ to $m+1$ at $t$
    - $y_{t-2} = \beta_{-1}m + m\beta_0 + m\beta_1 + m\beta_2 + c_i$
    - $y_{t-1} = \beta_{-1}(m+1) + m\beta_0 + m\beta_1 + m\beta_2 + c_i$
- Anticipation effect: $\beta_{-1} = y_{t-1} - y_{t-2}$ change in $y$ in period $t-1$, the period before the temporary one-unit increase in $x$
- Placebo test: if $x$ causes $y$, but $y$ does not cause $x$, then $\beta_{-1}$ should be close to zero

# Leads and Lags

```
. xtreg nat_rate F1.repdem repdem L1.repdem L2.repdem L3.repdem  i.year, fe cl(muniID) i(muniID)

Fixed-effects (within) regression               Number of obs      =      3675
Group variable: muniID                          Number of groups   =       245

R-sq:  within  = 0.1621                         Obs per group: min =        15
       between = 0.0010                                        avg =      15.0
       overall = 0.1269                                        max =        15

                                                F(19,244)          =     20.34
corr(u_i, Xb)  = -0.0353                        Prob > F           =    0.0000

                               (Std. Err. adjusted for 245 clusters in muniID)
------------------------------------------------------------------------------
             |               Robust
    nat_rate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      repdem |
         F1. |   .1707685   .3212906     0.53   0.596    -.4620886    .8036255
         --. |   .6975731   .4397095     1.59   0.114    -.1685376    1.563684
         L1. |   .8723962   .4619322     1.89   0.060    -.0374873     1.78228
         L2. |    .014941   .4583628     0.03   0.974    -.8879119    .9177939
         L3. |  -.2904252   .4108244    -0.71   0.480     -1.09964    .5187895
```

## The Autor Test

- Let $D_{it}$ be a binary indicator equaling 1 if unit $i$ **switched** from control to treatment between $t$ and $t-1$; 0 otherwise
  - Lags: $D_{i,t-1}$: unit switched between $t-1$ and $t-2$
  - Leads: $D_{i,t+1}$: unit switches between $t+1$ and $t$
- Include lags and leads into the fixed effects model:

$$y_{it} = D_{i,t+2}\beta_{-2} + D_{i,t+1}\beta_{-1} + D_{it}\beta_0 + D_{i,t-1}\beta_1 + D_{i,t-2}\beta_2 + c_i + \varepsilon_{it}$$

- Interpretation of coefficients:
  - Leads $\beta_{-1}, \beta_{-2}$, etc. test for anticipation effects
  - Switch $\beta_0$ tests for immediate effect
  - Lags $\beta_1, \beta_2$, etc. test for long-run effects
    - highest lag dummy can be coded 1 for all post-switch years

# Lags and Leads of Switch to Representative Democracy

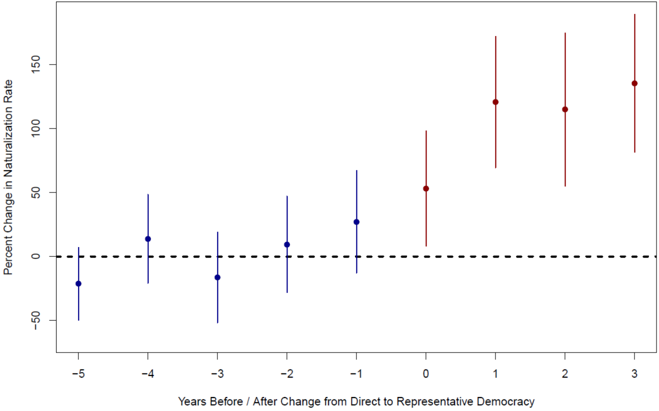
```
. list muni_name year repdem switch_t sw_lag1 sw_lag2 sw_lag3  ///
>                          sw_lead1 sw_lead2 sw_lead3 in 806/817
```

| | muni_n~e | year | repdem | switch_t | sw_lag1 | sw_lag2 | sw_lag3 | sw_lead1 | sw_lead2 | sw_lead3 |
|---|---|---|---|---|---|---|---|---|---|---|
| 806. | Stäfa | 1998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 807. | Stäfa | 1999 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 808. | Stäfa | 2000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 809. | Stäfa | 2001 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 810. | Stäfa | 2002 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 811. | Stäfa | 2003 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 812. | Stäfa | 2004 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 813. | Stäfa | 2005 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 814. | Stäfa | 2006 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 815. | Stäfa | 2007 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 816. | Stäfa | 2008 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 817. | Stäfa | 2009 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

# Dynamic Effect of Switching to Representative Democracy

```
. xtreg  nat_rate sw_lag3 sw_lag2 sw_lag1 switch_t ///
>        sw_lead1 sw_lead2 sw_lead3 sw_lead4 sw_lead5 i.year, fe cluster(muniID) i(muniID)

Fixed-effects (within) regression               Number of obs      =      4655
Group variable: muniID                          Number of groups   =       245

R-sq:  within  = 0.1913                         Obs per group: min =        19
       between = 0.0011                                        avg =      19.0
       overall = 0.1601                                        max =        19

                                                F(27,244)          =     23.76
corr(u_i, Xb)  = -0.0162                        Prob > F           =    0.0000

                              (Std. Err. adjusted for 245 clusters in muniID)
------------------------------------------------------------------------------
             |               Robust
    nat_rate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     sw_lag3 |   1.160345   .5080271     2.28   0.023     .1596665    2.161023
     sw_lag2 |   1.743682   .5395212     3.23   0.001      .680969    2.806396
     sw_lag1 |   1.881663   .4880099     3.86   0.000     .9204133    2.842913
    switch_t |   .7564792    .428627     1.76   0.079    -.0878019     1.60076
    sw_lead1 |   .2138757   .3899881     0.55   0.584    -.5542971    .9820485
    sw_lead2 |   .0843676   .3575292     0.24   0.814      -.61987    .7886051
    sw_lead3 |   .1440446   .3194086     0.45   0.652    -.4851054    .7731945
    sw_lead4 |   .0750194   .2990359     0.25   0.802    -.5140018    .6640405
    sw_lead5 |  -.0942415   .2599789    -0.36   0.717    -.6063307    .4178477
```

# Dynamic Effect of Switching to Representative Democracy



## Lagged Dependent Variable

$$y_{it} = \alpha y_{i,t-1} + c_i + \varepsilon_{it}, \;\; t = 1, 2, \ldots, T$$

- $y_{it}$ could be capital stock of firm $i$ at time $t$ and $\alpha$ the capital depreciation rate
- For simplicity, we assume that $\varepsilon_{it}$ are uncorrelated in time (as well as across individuals)
- Note that

$$y_{i,t-1} = \alpha y_{i,t-2} + c_i + \varepsilon_{i,t-1}$$

- So we have $Cov[y_{i,t-1}, c_i] \neq 0$ and therefore we need to include fixed effects $c_i$ into the regression
- Does this work though?

## Lagged dependent variable

With $T = 3$ we have

$$\begin{aligned} y_{i3} &= \alpha y_{i2} + c_i + \varepsilon_{i3} \\ y_{i2} &= \alpha y_{i1} + c_i + \varepsilon_{i2} \end{aligned}$$

and we can take time differences to eliminate $c_i$ (similar to fixed effects)

$$\begin{aligned} y_{i3} - y_{i2} &= \alpha(y_{i2} - y_{i1}) + (c_i - c_i) + (\varepsilon_{i3} - \varepsilon_{i2}) \\ \Delta y_{i3} &= \alpha \Delta y_{i2} + \Delta \varepsilon_{i3} \end{aligned}$$

Since $\varepsilon_{i2}$ affects both $\Delta y_{i2} = y_{i2} - y_{i1}$ and $\Delta \varepsilon_{i3} = \varepsilon_{i3} - \varepsilon_{i2}$ we get

$$Cov[\Delta y_{i2}, \Delta \varepsilon_{i3}] \neq 0 \text{ and thus still have endogeneity}$$

Models with fixed effects and lagged $y$ do not produce consistent estimators Might use past levels $y_{i1}$ as an instrument for $\Delta y_{i2}$, but this

requires strong assumptions (e.g., no serial correlation in $\varepsilon_{it}$)