Structural Econometrics in Labor and IO.
Problem Set: Estimation of Static Differentiated Product Demand Models.

Description

Consider a scenario where only market level data on product demand are available: market shares, prices, product characteristics. We wish to estimate consumer preferences, compute the resulting price elasticities, and ultimately learn the firms' implied profit margins and marginal cost. To do so, we make use of a model of individual utility maximization and imperfect competition on the product market.[1]

Demand. We assume consumer i chooses one unit of product $j \in J$ or an outside good $j = 0$ (e.g. no purchase) to obtain utility

$$u_{ijt} = x_{jt}\beta_i + \alpha p_{jt} + \xi_{jt} + \varepsilon_{ijt} = \delta_{jt} + \mu_{jt}(\nu_i) + \varepsilon_{ijt} \tag{1}$$

where (x_{jt}, p_{jt}) are observable characteristics and price, ξ_{jt} is the unobservable characteristic, $\delta_{jt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt}$, $\mu_{jt}(\nu_i) = x_{jt}\sigma\nu_i$, and ε_{ijt} i.i.d. extreme value type 1. The utility of the outside good is normalized such that $\delta_{0t} = 0$. The assumption of utility-maximizing consumers and the distribution of ε_{ijt} yields the logit choice probabilities:

$$s_{jt}(\delta_t, \sigma) = \int \frac{\exp(\delta_{jt} + \mu_{jt}(\nu_i))}{1 + \sum_{l=1}^{J} \exp(\delta_{lt} + \mu_{lt}(\nu_i))} dP_\nu(\nu), \tag{2}$$

where P_ν is typically assumed to take a specific parametric form.

Supply. Assume that firms f maximize profits given by

$$\pi_f = \sum_{k \in F_f} (p_{kt} - c_{kt}) s_{kt} L_t, \tag{3}$$

where c_{kt} are marginal cost and L_t market size, so that the system of FOC for a Nash equilibrium:

$$s_{jt} + \sum_{k \in F_f} (p_{kt} - c_{kt}) \frac{\partial s_{kt}}{\partial p_{jt}} = 0 \tag{4}$$

For implementation, it will be useful to write expression (4) in vector notation as $s_t + \Delta_t(p_t - c_t) = 0$, where $\Delta_t(j, k)$ denotes a diagonal matrix of own-price derivatives and off-diagonal elements according to market structure (equal to zero in the case of single-product firms). If marginal cost are known, we obtain the supply side by solving the system for c_t:

$$p_t + \Delta_t^{-1} s_t = c_t \tag{5}$$

When changes in market structure are of interest (e.g. merger analysis), it is useful to generate a full $\tilde{\Delta}_t$ derivative matrix, an equally-sized matrix O_t designating firms' product ownership, and element-wise multiply $\tilde{\Delta}_t$ and O_t. $\partial s_{kt}/\partial p_{jt}$ is given by $\alpha s_{jt}(1 - s_{jt})$ if $j = k$ and $-\alpha s_{jt} s_{kt}$ otherwise.

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[1]The crucial assumption in Berry (1994) and virtually all subsequent work is that market shares are generated by a large number (technically, infinity) of consumers in the market and, hence, can be used as a measure of individual choice probabilities.

Problems

Together with your answers, please submit your computer code for the questions below.

1. Simulate data based on the described model assuming the following

   - $J = 10$ products are sold in $T = 250$ markets (size $L_t = 1$) by single-product firms.
   - Two observable product characteristics $x_{jt} = (1, x_{jt}^1)$, with $x_{jt}^1 \sim U(1, 2)$.
   - Marginal cost $c_{jt} = x_{jt}\gamma_1 + w_{jt}\gamma_2 + \omega_{jt}$.
   - Three observable cost shifters $w_{jt} = (w_{jt}^1, w_{jt}^2, w_{jt}^3)$, all i.i.d. $U(0, 1)$.
   - Marginal cost parameters: $\gamma_1 = (0.7, 0.7)$ and $\gamma_2 = (1, 1, 1)$.
   - Unobserved demand and cost characteristic $(\xi_{jt}, \omega_{jt}) \sim N(0, \sigma_c)$ with $\sigma_c = \begin{bmatrix} 1 & 0.7 \\ 0.7 & 1 \end{bmatrix}$.
   - Preference parameters $\beta = (2, 2), \alpha = -2, \sigma = (0, 1)$.
   - $\nu \sim N(0, 1)$.

   To simulate the data, you must code functions computing

   - a $(T \times J) \times 1$ vector containing market shares using equation (2) - integration needs to be done numerically, either using a Monte Carlo method or a quadrature rule (`https://en.wikipedia.org/wiki/Gauss-Hermite_quadrature` and the appropriate scipy function is *scipy.special.roots_hermite*),
   - a $(T \times J) \times J$ matrix containing market share derivatives with respect to price (own-price on the diagonal and cross-price off the diagonal),
   - a $(T \times J) \times 1$ vector containing (Bertrand-Nash) equilibrium prices, using equation (5) and python's root-finding function *scipy.optimize.fsolve* or *scipy.optimize.root.*

2. The simulated data at hand, forget the parameters specified in Problem 1 and

   - assume that $\sigma = (0, 0)$. Estimate $\{\alpha, \beta\}$ using OLS and report the results.
   - still assume that $\sigma = (0, 0)$. Estimate $\{\alpha, \beta\}$ using linear IV and report the results for two alternative sets of instrumental variables:

     (a) x_{jt}, the squared term $(x_{jt}^1)^2$, and BLP instruments (sums of other firms' x_{jt})
     (b) all variables in a) plus cost shifters (w_{jt}).

3. Code the BLP contraction mapping to obtain δ for a given set of parameters (requires observed market shares and the market share function as input): $\delta_t = s_t^{-1}(s_t, \sigma) \equiv \delta_t(s_t, \sigma)$. The algorithm should iterate as follows: $\delta^{h+1} = \delta^h + \ln(s) - \ln(s(\delta^h, \sigma))$.

4. Build the python function that nests the δ contraction mapping and the GMM objective function. Then, find the parameter values that minimize this function using *scipy.optimize.minimize.*