

# Advanced Applied Econometrics

## Week 9 - Static discrete choice with market-level data

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# Organisation

- ▶ My part has three problem sets (to be completed in groups of max. 2 participants)
- ▶ The third problem set is graded.
- ▶ The 3 best of 4 graded problem sets will count in the course.
- ▶ Felix's first problem set counts as one of the 4.

# Organisation

- ▶ June 21: recap demand estimation, discuss details, practical session (BLP)
- ▶ Read (again) Berry et al. (1995) and Conlon and Gortemaker (2020)
- ▶ Hand in second problem set
- ▶ June 26 (Wednesday, another change in date): dynamic discrete choice

# Plan for today

- ▶ Structural econometrics in Industrial Organization
- ▶ Demand for differentiated products: Berry (1994), Berry et al. (1995)
- ▶ Practical session: logit demand with aggregate data

# Structural econometrics in Industrial Organization

- ▶ Highly recommended slides by Phil Haile on “Models, Measurement, and the Language of Empirical Economics” to read at:  
<https://sites.google.com/view/philhaile/home/teaching>
- ▶ “Structural: estimate features of a data generating process (i.e., a model) that are (assumed to be) invariant to the policy changes or other counterfactuals of interest”
- ▶ Fun fact: they are generative models because structural models specify and estimate the data-generating process, thus can generate  $p(X, Y)$ .

# Structural econometrics in Industrial Organization

- ▶ IO applications: study the behavior of firms in markets, their strategic interactions, and the implications on profits and consumer welfare.

# Recent policy discussions on inflation and markups

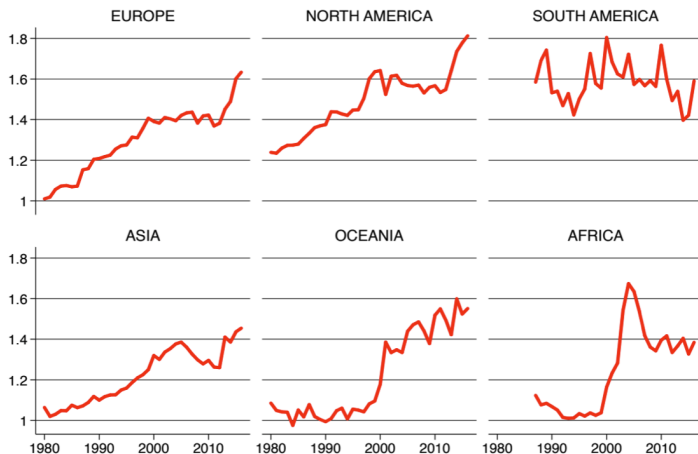


Figure 5: Global Regions

De Loecker, Jan and Jan Eeckhout (2021), "Global Market Power", working paper

# Recent policy discussions on inflation and markups

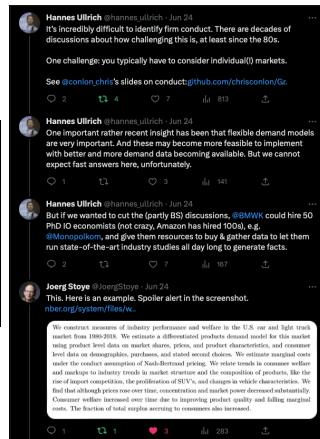
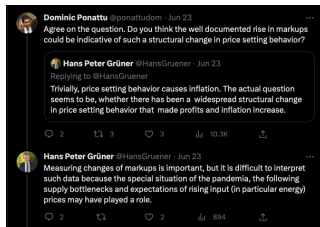




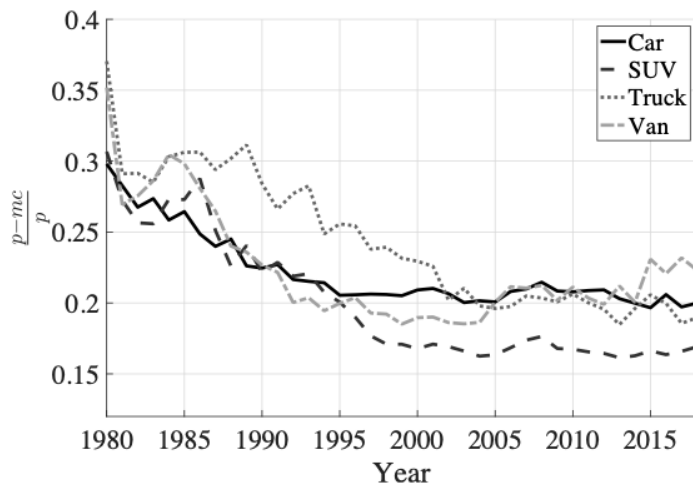
# Recent policy discussions on inflation and markups



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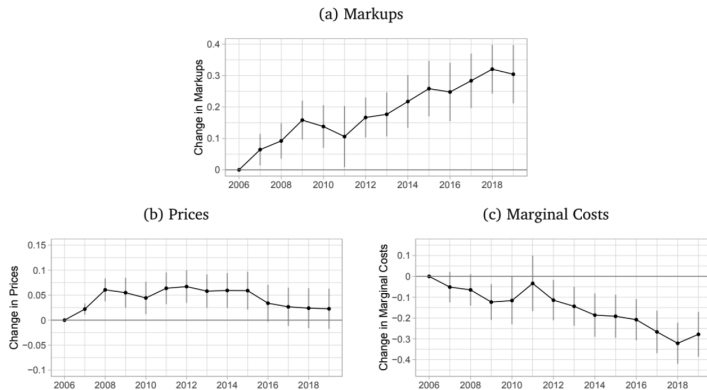
## Recent policy discussions on inflation and markups



(a) Markups by Vehicle Style

Grieco, Paul, Charles Murry, and Ali Yurukoglu (2024), "The Evolution of Market Power in the US Auto Industry", Quarterly Journal of Economics

# Recent policy discussions on inflation and markups

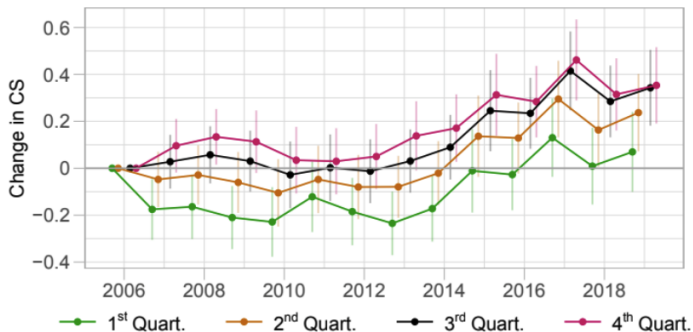


Notes: This figure shows coefficients and 95 percent confidence intervals of a regressions of the log of the Lerner index, real prices, and real marginal costs at the product-chain-DMA-quarter-year level on year dummies controlling for product-chain-DMA and quarter fixed effects. The year 2006 is the base category.

Döpfer, Hendrik, Alexander MacKay, Nathan H. Miller, and Joel Stiebale (2023), “Rising Markups and the Role of Consumer Preferences”, R&R Journal of Political Economy

# Recent policy discussions on inflation and markups

Figure 10: Consumer Surplus Over Time By Income Group



Döpper, Hendrik, Alexander MacKay, Nathan H. Miller, and Joel Stiebale (2023), “Rising Markups and the Role of Consumer Preferences”, R&R Journal of Political Economy

# Structural econometrics in Industrial Organization

- ▶ IO applications: study the behavior of firms in markets, their strategic interactions, and the implications on profits and consumer welfare.
  - ▶ Why use economic theory for structural assumptions in IO (and elsewhere)?  
E.g. why not simply regress prices on number of firms, or market shares on prices?
  - ▶ Counterfactuals for ex-ante (welfare) evaluation of mergers, antitrust cases, regulation (e.g. price or entry barriers), subsidies, etc.
- Require a flexible, well-specified demand system

# Typical IO Models

How is economic theory used as structure in IO applications?

1. Model of consumer behavior (demand), product differentiation
2. Model for firms costs, economies of scale, economies of scope, entry costs, investment costs

## Conduct

3. Equilibrium model of static competition, price (Bertrand), quantity (Cournot), bargaining, collusion, etc.
4. Equilibrium model of market entry-exit
5. Equilibrium model of dynamic competition, investment, advertising, quality, product characteristics, stores, etc.

# Plan for today

- ▶ Structural econometrics in Industrial Organization
- ▶ Demand for differentiated products: Berry (1994), Berry et al. (1995)
- ▶ Practical session: logit demand with aggregate data



# Typical Structure of IO Models

For this course:

1. Model of consumer behavior (demand)
2. Model for firms costs
3. Equilibrium model of static competition

# Demand for differentiated products: discrete choice models

- ▶ Empirical observation:
  - ▶ Products mostly not homogenous
  - ▶ Firms typically set prices strategically
- ▶ Then, how to model and estimate demand for differentiated products
- ▶ Aggregation to market-level important. In 1990s, move from
  - ▶ representative consumer to
  - ▶ micro-modelling of consumer preferences.

# Demand for differentiated products: discrete choice models

- ▶ One way: system of linear demand equations

$$q = D(p, x, \theta), \text{ with } D(p) = Ap$$

- ▶  $J$  products
- ▶ Flexible demand system but number of parameters to be estimated:  $J \times J$  matrix of price coefficients

# Demand for differentiated products: discrete choice models

- ▶ Discrete choice models as models of product demand:
  - ▶ Characteristics space to reduce dimensionality (McFadden, 1974)
- ▶ Berry (1994): foundation for discrete choice demand estimation using market-level data, single-product firm oligopoly pricing
- ▶ Berry et al. (1995): first application of the proposed framework, multi-product firm oligopoly pricing, proof of contraction mapping

# Demand for differentiated products: discrete choice models

- ▶ In markets with imperfect competition, primitives of the model
  - ▶ Product characteristics
  - ▶ Consumer preferences
  - ▶ Equilibrium notion (price setting, quantity setting, bargaining, etc.)

## Demand for differentiated products: Berry (1994)

- ▶ Assume consumer  $i$ 's linear utility from buying product  $j$ :

$$u_{ij} = \delta_j + \varepsilon_{ij}$$

where  $\delta_j = x_j\beta - \alpha p_j + \xi_j$ .

- ▶ Assumption: each consumer buys one unit of utility-maximizing good.
- ▶ If  $\varepsilon_{ij}$  is extreme value distributed, the probability that consumer  $i$  chooses product  $j$  is:

$$s_j = \frac{\exp(\delta_j)}{1 + \sum_{m=1}^J \exp(\delta_m)}$$

- ▶ Role of the outside good. What is a reasonable market definition?

## Demand for differentiated products: Berry (1994)

- ▶ With strategic price setting:  
 $p_j$  correlated with unobserved characteristic  $\xi_j \rightarrow$  endogeneity problem
- ▶ Idea:  $s_j$  can be inverted such that

$$\ln(s_j) - \ln(s_0) = \delta_j = x_j\beta - \alpha p_j + \xi_j$$

- ▶ Market share inversion allows for linear IV estimation

# Demand for differentiated products: Berry (1994)

- ▶ Fixed effects
- ▶ Instruments - do we have good ones (for price)?
- ▶ What about endogeneity of other product characteristics?



# Instruments

- ▶ Good instruments might be the regressors  
(derivative of the moment function with respect to the parameters)
- ▶ Cost shifters uncorrelated with the demand shock (rarely observed)...
- ▶ Bresnahan (1987), BLP (1995): Assume product characteristics exogenous. Use
  - ▶ observed product characteristics; sums of the same characteristics of other products offered by that firm; sums of the same characteristics of products offered by other firms
  - ▶ Intuition: closeness in characteristics space, in oligopoly markups decreases with the presence of good substitutes
- ▶ Gandhi and Houde (2019): Differentiation IVs
  - ▶ Polynomial of differences in observed product characteristics

## Demand for differentiated products: Berry (1994)

- ▶ Homogenous consumers  $(\alpha, \beta)$  strong assumption  
→ representative consumer up to i.i.d.  $\varepsilon_{ij}$
- ▶ Leads to substitution patterns depending only on market shares

# Unobserved preference heterogeneity

- ▶ Random coefficients: allow consumer to have heterogeneous preferences over certain characteristics, i.e.  $\beta_i$

$$u_{ij} = x_j \beta_i - \alpha p_j + \xi_j + \varepsilon_{ij}$$

- ▶ Berry (1994) inversion to obtain linear equation no longer possible
- ▶ Market shares, as we did not integrate out all individual errors:

$$s_j = \int \dots \int (s_{ij} dF_1(\alpha|\theta) \dots dF_{K+1}(\beta_k|\theta))$$

- ▶ Now can estimate completely flexible substitution patterns (market share derivatives depend on all characteristics in  $u_{ij}$ )

# Unobserved preference heterogeneity

- ▶ Assume that individual  $i$  can be described by a vector of unobserved characteristics  $(\nu_i, \varepsilon_{i0}, \dots, \varepsilon_{iJ})$ .
- ▶ We can re-write consumer  $i$ 's utility:

$$u_{ij} = \delta_j(x_j, p_j, \xi_j; \theta_1) + \mu_{ij}(x_j, p_j, \nu_i, \theta_2) + \varepsilon_{ij}$$

→ Partition utility into mean utility  $\delta_j$  and consumer-specific  $\mu_{ij}$

- ▶ Vector  $\theta_1 = (\alpha, \beta)$  contains linear and  $\theta_2 = (\sigma)$  nonlinear parameters

# Unobserved preference heterogeneity

- ▶ For simplicity, assume that consumers only have heterogeneous preferences over one characteristic in  $x_j$ , so that  $\theta_2 = (\sigma)$  is a scalar.
- ▶ Nonlinear part of consumer utility:

$$\mu_{ij}(x_j, p_j, v_i, \theta_2) = x_j v_i \sigma$$

where the distribution of  $v_i$  is assumed. Typically,  $N(0, 1)$ .

- ▶ If consumer demographics are observed (e.g. age, income), empirical or fitted distributions

# Simulation

- ▶ Idiosyncratic terms  $\varepsilon_{ij}$  integrated out (EV distribution assumption)
- ▶ But due to  $v_i$  no closed form solution for the integral
- ▶ Numerically invert the system of equation  $s_j = s_j(x, p, \delta_j, \theta_2)$  to obtain  $\delta_j$  given  $\theta_2$
- ▶ Two numerical ingredients needed:
  - Numerical integration / simulation to compute market shares
  - Contraction mapping to compute  $\delta_j$

## Integration to compute market shares

- ▶ Take  $s$  draws from the distribution of  $\nu$  for each observation
- ▶ Given  $s$  draws of  $\nu$ ,  $\sigma$ , and  $\delta$ , compute the market shares:

$$s_j = \frac{1}{ns} \sum_{i=1}^{ns} s_{ij} = \frac{1}{ns} \sum_1^{ns} \frac{\exp(\delta_j + x_j \nu_i \sigma)}{1 + \sum_{m=1}^J \exp(\delta_m + x_m \nu_i \sigma)}$$

# Integration to compute market shares

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- ▶ Monte Carlo simulation
  - ▶ Pseudo-random draws
  - ▶ Quasi-random - e.g. Halton, modified Latin hypercube sampling
- ▶ Quadrature: Gauss-Hermite for (univariate) normal distribution, unbounded support



## Intermission: Newton's Method

- ▶ Idea: Approximate (univariate) nonlinear function  $f(x)$  by its tangent at  $x_n$ , where  $x_{n+1} = x_n + \epsilon$ :

$$y = f'(x_n)(x_{n+1} - x_n) + f(x_n)$$

- ▶ Finding the root of this tangent is easy, set  $y = 0$  and solve for  $x_{n+1}$ :

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- ▶ Iterating until  $x_{n+1} \approx x_n$  yields the (local) root of  $f(x)$
- ▶ Optimization: Apply Newton's method to  $f'(x)$ , then check  $f''(x)$

## Intermission: Newton's Method

```
# Newton's root-finding method
def newton_sqrt(a, tol=1e-12):

    if a < 0:
        raise ValueError("Number a should ...
                           be positive.")

    x_t = a / 2
    x_t1 = a / 2 - 1
    i = 0

    while abs(x_t1 - x_t) > tol:
        x_t = x_t1
        f_t = a - x_t**2
        f_t_derivative = -2 * x_t
        x_t1 = x_t - f_t / f_t_derivative
        i += 1
        print(f"Value at iteration {i} is ...
              {x_t1:.10f}.")

    return x_t1
```

Find  $\sqrt{a}$ ,  $\rightarrow$  find  $x = \sqrt{a}$ .

$$\rightarrow x^2 = a$$

$$\rightarrow f(x) = a - x^2 = 0$$

Newton's method is a contraction mapping if  $f''(x)$  is continuous,  $f'(x) \neq 0 \forall x \in \mathbb{R}$ , and  $q \in (0, 1)$  such that  $|f(x)f''(x)| \leq q|f'(x)|^2 \forall x \in \mathbb{R}$

## Contraction mapping for $\delta_j$

- ▶ Given  $\sigma$ , compute vector  $\delta$  that equates model predicted to observed market shares
- ▶ Contraction mapping

$$\delta^{h+1} = \delta^h + \ln(s) - \ln(s(p, x, \delta^h, \sigma)), h = 0, \dots, H,$$

where  $H$  is the smallest integer such that  $\|\delta^H - \delta^{H-1}\|$  is smaller than some tolerance level, and  $\delta^H$  is the approximation to  $\delta$ .

- ▶ Compute vector  $s(x, p, \delta, \sigma)$  at each iteration

# Estimation

- ▶ Nested simulated GMM procedure
- ▶ IV estimation. Population moment conditions  $E[Z\xi(\theta^*)] = 0$
- ▶ Using  $\delta_j$ , computed at each parameter iteration in the GMM objective function, obtain:  
$$\xi_j = \delta_j - (x_j\beta - \alpha p_j)$$

# Estimation

- ▶ GMM objective function:

$$f = \zeta(\theta)' Z \Phi^{-1} Z' \zeta(\theta),$$

where  $\Phi^{-1}$  is a consistent estimate of  $E[Z' \zeta \zeta' Z]$

- ▶ Search  $\theta = (\alpha, \beta, \sigma)$  that minimizes the objective function
- ▶ Note: finding the nonlinear parameter  $\sigma$  requires a nonlinear search.

# Problem set