Labor Problem Set 1
Example Solutions

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Advanced applied econometrics

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Robinson has to ensure his survival for the next 3 days. He has to work the land in order to produce food. Each day he can choose to work for 8, 12, or 16 hours. He can enjoy the remaining part of the day, such that his leisure time is given by $l_t = 24 - h_t$, where h_t denotes his choice of hours of work.
On Day 1 Robinson is completely unskilled and no matter how many hours he works, he gets 1 unit of produce. Working more than 8 hours a day allows Robinson to learn how to take better care of the crops, such that he is able to produce more. In particular, for every additional 4 hours of work on either day Robinson levels up his farming skills by 1 unit. Produce c_t is a function of experience e_t:

$$c_t = (1 + 0.25e_t)^2$$

Robinson gains utility from consumption and leisure:

$$u(c_t; l_t) = {c_t}^{\alpha} {l_t}^{\beta}$$

with $\alpha = \beta = 0.5$

1. If Robinson had to survive only for 1 day, how many hours would he work? How much utility would he get? What incentive does Robinson have to make a different choice on Day 1 if he has to survive for 3 days?

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$$max\, u(c_t, h_t) = c_t^{\alpha}(T - h_t)^{\beta}$$

$$max\, u(h_t) = 1(24 - h_t)^{0.5}$$

For c_t fixed to 1 unit, utility is maximal when hours of work are minimized. If Robinson had to survive only for 1 day, he would choose to work 8 hours and have 16 hours of leisure time with $u = 16^{0.5}$.

When Robinson has to survive for 3 days, and consumption depends on experience, dynamic incentives come into play. Depending on the parameters of the model, it may well be optimal to work longer hours on the first day(s) and reap the returns to experience gathered in the following day(s).

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The curvature of the utility function measures the consumers attitude towards risk. The Cobb-Douglas utility is a concave function (can be verified by showing that the Hessian is negative semidefinite), implying that the utility of the expected consumption is higher than the expected utility of consumption. Therefore Robinson is risk averse.

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We can check for substitutability by verifying that the marginal rate of substitution between the goods is diminishing.

$$MRS = \frac{c_t}{l_t}$$

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One obvious state variable in the model is experience. Given that this is a finite discrete time model, there is a second state variable: period.
There are 9 states.

period	experience
1	0
2	0
2	1
2	2
3	0
3	1
3	2
3	3
3	4

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State specific value function on the last day is given by

$$V(h_t, x) = c_t(x)^\alpha (T - h_t)^\beta + \delta V(t = 3 + 1, x)$$

$$V(h_t, x) = [(1 + 0.25x)^2]^\alpha (T - h_t)^\beta + \delta V(x + 1)$$

where $V(x+1)$ is the maximum value function next period and δ is the discount factor that is assumed 1 in this case.

Choice

$$\underset{h_t}{\operatorname{argmax}}\, V(h_t, x)$$

Generally, the optimal choice is state specific.

5. What would be optimal number of hours of work for Robinson on Day 3?

period	experience	$h_t = 8$	$h_t = 12$	$h_t = 16$	choice
3	0	4	3.4641	2.8284	8h
3	1	5	4.3301	3.5355	8h
3	1	6	5.1962	4.2426	8h
3	3	7	6.0621	4.9497	8h
3	4	8	6.9282	5.6569	8h

Upon calculating the choice specific value functions in this state it becomes clear that no matter what choices were made on day 1 and day 2, on day 3 it is always optimal for Robinson to work for 8 hours only.

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The table above summarises the choice specific value functions on Day 1 and Day 2. Comparing the values one arrives at the *state specific* optimal number of hours to work. One gets to these values by *solving the model backwards*.

period	experience	$h_t = 8$	$h_t = 12$	$h_t = 16$	choice
1	0	12.8284	13.9996	15.0711	
2	0	8	8.4641	8.8284	
2	1	10	10.3301	10.5355	
2	2	12	12.1962	12.2426	

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period	experience	$h_t = 8$	$h_t = 12$	$h_t = 16$	choice
1	0	12.8284	13.9996	15.0711	16
2	0	8	8.4641	8.8284	16
2	1	10	10.3301	10.5355	16
2	2	12	12.1962	12.2426	16

Finally, moving forward through the model, we can establish that Robinson will choose to work for 16h on Day 1 and 2 and 8 hours on Day 3.

See code for Question B.