Structural Econometrics in Labour and IO
Partial Job Search

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Preliminaries: Presentation

Presentation

- MCC Berlin & DIW Berlin, LHaywood@diw.de
- Why am I here?
  Search models as valuable & useful.
  Some numerical methods.
- And you?

Preliminaries: Organisation

- **Next class** on Thursday **7th July** from 2 - 5pm back in Karl Popper
- Hand in 2nd problem set on 6th July by 2pm (all info online)
- **Exam** on 14th July from 2 - 4pm (details next week)

What are job search models trying to do?

What is the **motivation for** job search models?

Motivation of Job Search Models

Labour market inequality in imperfect market with

- (ex ante) homogeneous workers
- (ex ante) homogeneous firms

Motivation of Job Search Models

Stochastic labour market: search explains

- means people are **happy to get a job**.
- (some) **unemployment is involuntary** and random.

Cause of **labour market inequality** in standard models?

Contrast: neoclassical models

Labour market inequality in stylized standard models

"Neoclassical" causes of **unemployment**

- low productivity
- better outside options
- minimum wage legislation
  $\Rightarrow$ whose responsibility ?

What about **wage inequality**?

Contrast: neoclassical models II

Standard **causes of wage inequality**

- productivity differentials
- compensating differentials
  $\Rightarrow$ whose responsibility ?

Job search models

- **also** informational & matching frictions

The McCall Model

1. Motivation & McCall
   - Motivation
   - McCall Model
2. Problem Set Part 1: Values & Strategies
   - Focus on alternative models of labour market
3. Practical Session
   - PS Part II: Simulation
   - Numerical integration
   - Inverse Probability Sampling

McCall (1970)

Research Questions

McCall (1970): Research Questions

- Is searching for jobs a source of unemployment?
- How to distinguish **frictional vs structural unemployment**?
- What role do **expectations** play for **unemployment**?
- How do **labour market policies** interact with frictions?

What does McCall find?

McCall (1970): Research Findings

- Is **searching** for jobs a source of unemployment?
  $\Rightarrow$ Yes, frictions create economic rationale for searching.
  Is searching compatible with working? (λ_0 vs. λ_1)

- **Frictional vs structural unemployment**?
  $\Rightarrow$ Job-seeking $\neq$ **non-participation**.

- **Expectations & unemployment**?
  $\Rightarrow$ Optimal search requires **knowing what you can find**.

- How do **labour market policies** interact with search unemployment?

What does the model predict on **minimum wages**?

McCall on minimum wages

Minimum wages: What does the model predict?

- job-seekers accept fewer jobs
- **unemp duration rises**
- **unemployment level increases**

Comments?

McCall on minimum wages

Analysis of min-wages here

- **fewer offers** (offer distribution truncated)
  $\Rightarrow$ leads to **longer unemp duration**
  $\Rightarrow$ more unemp, less employment
- fewer offers $\Leftrightarrow$ fewer jobs (petitio principii?)

Other modelling strategies?

Other search models on minimum wages

Min-w implications in **search models with endogenous wage**

- raise some wages (K to L redistribution or L to L)
- make other jobs non-competitive (if min-w $>$ p)

Effects of min-wage then...

- some redistribution
- raise unemployment (why?)
- participation effect ambiguous (why?)
- raise aggregate productivity

Other search models on Minimum wages

More likely **effects of min-wage**

- some redistribution
  across workers (e.g. via prices), from capital
- raise unemployment (why?)
  if low-paid jobs paid more, value of search increases
- participation effect ambiguous (why?)
  $\Rightarrow$ low-earning workers may find fewer jobs - discourage
  $\Rightarrow$ low-earning workers may find better-paid jobs - encourage
- raise aggregate productivity
  $\Rightarrow$ low-productivity jobs removed from market

Problem Set Part 1: Values & Strategies

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Model set-up

What is basic model set-up?

Model set-up

Jobseeking **unemployed**

- unemployment **benefit** b
- receive **wage offer** w with probability λ
- wage offer **distribution** $F(.)$ ... if offer accepted...

Employed individuals

- w paid until...
- job destroyed (at rate δ)

In your view: **most important restrictions**?

Mechanics of our job search model

What are the **primitives** of the model?

What are the **state variables**?

Mechanics of our job search model

Model primitives

- λ, δ, β
- $F(\cdot)$
- b

State variables

- employment status
- wages

Model assumptions

Assumptions of model in problem set

- homogeneous workers
- wage posting - no bargaining
- random search - job offers arrive at Poisson rate λ
- exogenous quits - jobs destroyed at Poisson rate δ

What are **alternatives** to these assumptions?

Mechanics of our job search model

What about...

- discounting $\beta \equiv \frac{1}{1+r}$?
- retirement?
- search cost c?

Job search model without search

No **explicit search cost**... (cf. McCall)

- "Waiting for jobs" model
- cannot distinguish **non-participation from job-seeking**

...but is search free?

Mechanics of our job search model

How does this **job search problem** compare to Rust and Keane & Wolpin?

Job search

Dynamic sequential decisions in uncertain world

Dynamic Bellman equation (Rust, Keane & Wolpin):

$$\begin{aligned} V_t(S_{i,t}, \theta) =& max_{d(S_{i,t})} E\left[\sum \beta^t u_t(d_t, X_{i,t}, \theta) + \varepsilon\right] \\ =& max_{d(S_{i,t})}\left[u_t(d_t, X_{i,t}, \theta) + \beta\ E\left[V_{t+1}(S_{i,t+1}, \theta | d_{i,t})\right]\right] \end{aligned}$$

Interpret job search problem in this way.

Discrete choice dynamic programming

Optimal decision rule depends on **expected maximum**.

What is **decision rule** in the model & why ?

What is **option value**? How is model solved?

(1.1) Present values

Question (1.1)
What are the values of unemployment U and employment $W(w')$ in this model?

(1.1) Present values

Value of employment

$$r\ W(w) = w + \delta\ (U - W(w)) \tag{1}$$

$$W(w) = \frac{w}{\delta + r} + \frac{\delta\ U}{\delta + r} \tag{2}$$

Value of unemployment

$$r\ U = b + \lambda_0 \int_0^{\infty} max\,(0, W(w) - U)\, dF(w) \tag{3}$$

Using $W(w^r) = U$ the expressions can be further simplified

(see Q(1.3))

(1.2) Workers' strategy

Question (1.2) Explain why the reservation wage satisfies $W(w^r) = U$.

Stationary infinite horizon

Strategy is reservation wage - an optimal stopping strategy. Why?

Conversely: Why would you accept a higher or lower wage offer?

Stationary infinite horizon

Stationary setting. What does that mean?

Stationary infinite horizon

Stationary setting

- processes return to zero (conditional on L mkt status)
- problem has infinite horizon
- What is option value in *Emax* here?

How to calculate optimal decision rule in this case?

(1.3) Derive the reservation wage

Derive a useful expression for the reservation wage: Using $W(w^r) = U$, we get $W(w^r) = \frac{w^r}{r}$ from (2).

This can be used to show

$$W(w) - U = \frac{w}{\delta + r} - \frac{\delta}{\delta + r} U \tag{4}$$

$$= \frac{w - w^r}{\delta + r} \tag{5}$$

Use this in expression (3) to get an analytic expression for w^r...

(1.3) Derive the reservation wage II

$$w^r = b + \lambda_0 \int_{w^r}^{\infty} \frac{w' - w^r}{\delta + r} dF(w') \tag{6}$$

a little algebra and noting that $r = \frac{1-\beta}{\beta}$ we get the desired expression

$$[1 - \beta(1 - \delta)](w^R - b) = \beta\lambda \int_{w^R}^{\infty} (w - w^R) dF(w) \tag{7}$$

(1.4) Alternatives to optimal stopping

When is it not a good idea to adopt a reservation wage strategy?

Stationary infinite horizon

For example, whenever the environment is not stationary or infinite horizon.

Why might this be the case?

Alternative models of the labor market?

Alternative models of L-mkt

More sophisticated models

- **Firms set wages** & search for workers
- workers may have different **outside options**
- unemployment **benefits fall** over time
- job offers not only arrive, workers choose **search intensity**
- wages evolve: not wage but wage profile
- **wages may depend** on productivity
- workers may **search on the job**

Alternative models of L-mkt II

Consider three alternative labour market models

1. workers have **different outside options**
2. with **firm side** (firms set wages)
3. where **dynamic discrete choice** governs labour supply

Alternative 1: Models with diff outside options

Workers may receive **different levels of benefits** b

Why might this be?

Alternative 1: Diff outside options II

Causes of different *home production*

- **family care** obligations - kids or parents need help
- family transfers (**rich spouse** pays)
- non-labour income (**profits** from your hotel chain)
- unemployment **benefits vary** (ALG 1 $>$ ALG 2)

What impacts compared to current model?

Alternative 1: Diff outside options III

What **effects of different home production**?

- some unemployed **reject offers** that others do not
- negative **duration dependency** of unemp exit hazard - why?
- How might **firms react** to this ?

Alternative 1: Diff outside options IV

What **effects of different home production**?

- some unemployed **reject offers** that others do not
  - Leads to heterogeneity in w^R
  - Unless firms can condition offers on w^R - w sometimes too low.
- negative **duration dependency** of unemp exit hazard - why?
  - individuals with lowest w^R leave unemployment first
  - can model using individual effects ("frailty")
- How might **firms react** to this ?
  - Advantage of low-wage firms?
  - Advantage of high-wage firms?

Next week: Alternative form of reservation wage heterogeneity

Alternative 2: Equilibrium Models

Equilibrium between workers' and firms' strategies

- workers **choose** w^R given wage distribution
- **firm competition** over workers
- **firms maximize profits given workers' strategy**

Next week: An equilibrium model of the labour market

Alternative 3: Dynamic discrete choice models

What differences compared to DDC?

Alternative 3: Dynamic discrete choice II

Compare to dynamic **discrete choice models** of labour supply.

Typically, these have

- **intensive** margin
- **voluntary** transitions
- What about the wage distribution?

(1.5) Job-finding over time

Question (1.5) A mass layoff occurs somewhere. In which period after becoming unemployed do the maximum number of layed-off workers find a new job?

(1.5) Job-finding over time

Job finding **rate (hazard) constant** over duration ("memoryless")

but number of unemployed decreases each round... so max in first period.

(1.6) Steady State

Question (1.6) What defines the “steady state equilibrium” of this model?

(1.6) Steady State II

Steady state equilibrium

- Constant **distribution of state variables**

What are the state variables in our model?

(1.6) Steady State III

Distribution of state variables in our model

- distribution of **labour market status**
- **wage distribution**

When are these distributions in equilibrium?

(1.6) Steady State IV

Distributions will be in equilibrium iff

1. Workers **choose** w^R **optimally**
2. u and e are **stationary**

Pausa

10 minutes break

Then: problem set part 2 - practical session

Practical session

See code.

Important methods in this problem set

- **Numerical integration**
- **Inverse probability sampling**

Integrals are your friends

Reservation wage contains integral

$$\int_{w^R}^{\infty} (w' - w^R) dF(w')$$

$$= \int_{w^R}^{\infty} (w' - w^R)\ f(w') dw'$$

How do we go about this?

Integrals are your friends

Assume you want to **integrate some function** $f(.)$ between w_{min} and w.

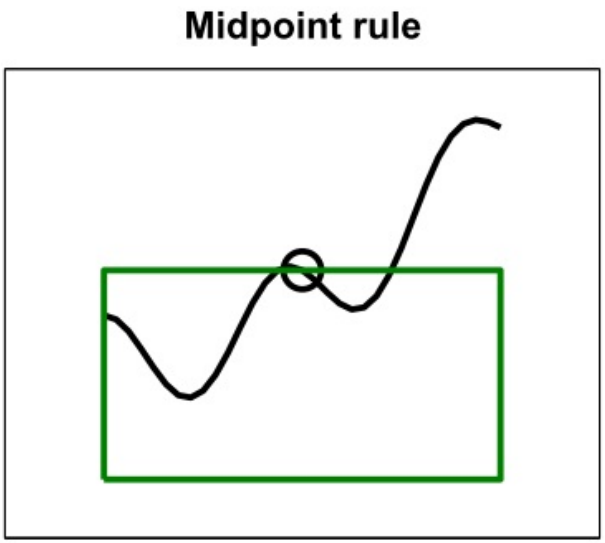
Most basically, what is integration ?

Most simple way to integrate ?

Integration - Midpoint

Assume value of $f(.)$ is $\approx$ constant between w_{min} and w. Evaluate function in the middle and multiply by distance $w - w_{min}$.

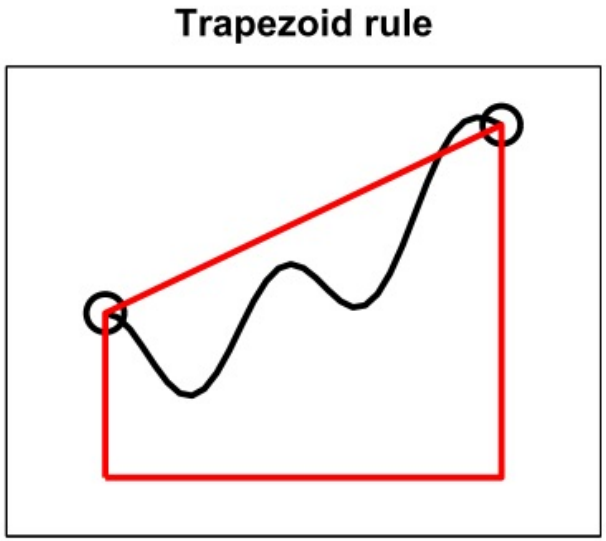
$$(w - w_{min}) \left[f\left(\frac{w + w_{min}}{2} \right) \right]$$



Integration - Trapezoidal

Evaluate function at both ends, weight evaluations equally and multiply by distance $w - w_{min}$.

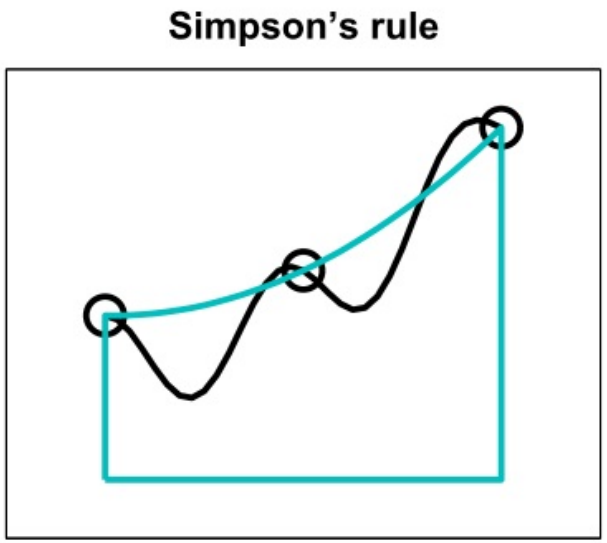
$$(w - w_{min}) \left[\frac{1}{2} f(w_{min}) + \frac{1}{2} f(w) \right]$$



Integration - Simpsons

Best combination of both:

$$(w - w_{min}) \left[\frac{1}{6} f(w_{min}) + \frac{4}{6} f\left(\frac{w_{min} + w}{2} \right) + \frac{1}{6} f(w) \right]$$



Numerical integration in practice

1. n_r set of points (**nodes**) at which to evaluate
2. ω_r set of **weights** with which evaluations enter integral
3. What should determine **number of nodes**?

Numerous Matlab scripts generate $[n_r, \omega_r]$.

What do these scripts need as input?

Numerical integration in practice

What they do not need

- Information about the function...
- ...rarely important

- but... number of points to evaluate can be.

How to integrate anything

Get vectors of **weights & nodes**...

Change of variables & use interval -1 to 1

$$\int_a^b f(w)dw = \frac{b-a}{2} \int_{-1}^{1} f\left(\frac{a+b}{2} + \frac{b-a}{2} * x\right) dx$$

$$= \frac{b-a}{2} \sum_{x_r=-1}^{x_r=1} w_r \, f\left(\frac{a+b}{2} + \frac{b-a}{2} * x_r\right)$$

(see Wikipedia article on Gaussian Quadrature)

Inverse Probability Sampling

Inverse Probability Sampling

Sample from any distribution... if you know inverse.

Example: Duration in unemployment

$$Pr(X < t) = 1 - [1 - \delta]^t$$

Inverse Probability Sampling

Sample from any distribution... if you know inverse.

Example: Duration in unemployment governed by exponential

$$
\begin{aligned}
Pr(X < t) &= 1 - [1 - \delta]^t \\
F &= 1 - [1 - \delta]^t \\
1 - F &= [1 - \delta]^t \\
log(1 - F) &= t * log[1 - \delta] \\
t &= log(1 - F)./[log(1 - \delta)]
\end{aligned}
$$

If I draw from unemp duration randomly, how will F-values be distributed?

Next class on **Thursday 7th July at 2pm**

Please:
(1) read **Burdett & Mortensen (1998)**
(2) **hand in a problem set** by 6th July, 2pm