Structural Econometrics in Labor and IO
Week 2 - Static Discrete Choice in IO

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5 May 2022

Organisation

- Overall grade determined by
  - 2 problem sets (to be completed in groups of max. 2 participants), weighted 1/3 each, and
  - a final exam, weighted 1/3.

Organisation

- Next class, May 12: recap demand estimation, discuss details, practical session (BLP)
- Read (again) Berry et al. (1995) and Nevo (2000)
- Hand in problem set

Advertisement

- Every other Thursday, 17:00: Virtual Digital Economy Seminar
  - https://www.digitalecon.org/seminar

Plan for today

- Structural econometrics in Industrial Organization
- Demand for differentiated products: Berry (1994), Berry et al. (1995)
- Practical session: logit demand with aggregate data

Structural econometrics in Industrial Organization

- Highly recommended slides to read: `http://www.econ.yale.edu/~pah29/intro.pdf`
- "Structural: estimate features of a data generating process (i.e., a model) that are (assumed to be) invariant to the policy changes or other counterfactuals of interest"

Structural econometrics in Industrial Organization

- IO applications: study the behavior of firms in markets, their strategic interactions, and the implications on profits and consumer welfare.
- Why use economic theory for structural assumptions in IO (and elsewhere)?
  E.g. why not simply regress prices on number of firms, or market shares on prices?
- Counterfactuals for ex-ante evaluation of merger policy, antitrust, regulation (e.g. price or entry barriers), subsidies, etc.

Typical IO Models

How is economic theory used as <u>structure</u> in IO applications?

1. Model of consumer behavior (demand), product differentiation
2. Model for firms costs, economies of scale, economies of scope, entry costs, investment costs
3. Equilibrium model of static competition, price (Bertrand), quantity (Cournot), bargaining, etc.
4. Equilibrium model of market entry-exit
5. Equilibrium model of dynamic competition, investment, advertising, quality, product characteristics, stores, etc.

Industrial organization: applications

- Price and quantity choice,
- Investment in capacity, inventories, physical capital, R&D, patents,
- Advertising,
- Product characteristics,
- Externalities in two-sided markets,
- Geographic location of plants and stores,
- Entry in new markets

Industrial organization: applications continued

- Adoption of new technologies,
- Contracts with suppliers, and customers,
- Market power along vertical supply chain,
- Assess effects of price, entry, cost-side, and other regulation on industry, demand, and welfare.

$\rightarrow$ Require a well-specified demand system, in particular for (consumer) welfare assessment.

Plan for today

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- Practical session: logit demand with aggregate data

Typical Structure of IO Models...again

1. Model of consumer behavior (demand)
2. Model for firms costs
3. Equilibrium model of static competition

Demand for differentiated products: discrete choice models

- Empirical observation:
  - Products mostly not homogenous
  - Firms typically set prices strategically
- Then, how to model and estimate demand for differentiated products
- Aggregation to market-level important. In 1990s, move from
  - representative consumer to
  - micro-modelling of consumer preferences.

Demand for differentiated products: discrete choice models

- One way: system of linear demand equations

$$q = D(p, x, \theta), \text{ with } D(p) = Ap$$

- J products
- Flexible demand system but number of parameters to be estimated: $J \times J$ matrix of price coefficients

Demand for differentiated products: discrete choice models

- Discrete choice models as models of product demand:
  - Characteristics space to reduce dimensionality (McFadden, 1974)
- Berry (1994): foundation for discrete choice demand estimation using market-level data, single-product firm oligopoly pricing
- Berry et al. (1995): first application of the proposed framework, multi-product firm oligopoly pricing, proof of contraction mapping

Demand for differentiated products: discrete choice models

- In markets with imperfect competition, primitives of the model
  - Product characteristics
  - Consumer preferences
  - Equilibrium notion (price setting, quantity setting, bargaining, etc.)

Demand for differentiated products: Berry (1994)

- Assume consumer i's linear utility from buying product j:

$$u_{ij} = \delta_j + \varepsilon_{ij}$$

  where $\delta_j = x_j\beta - \alpha p_j + \xi_j$.

- Assumption: each consumer buys one unit of utility-maximizing good.

- If ε_{ij} is extreme value distributed, the probability that consumer i chooses product j is:

$$s_j = \frac{exp(\delta_j)}{1 + \sum_{m=1}^{J} exp(\delta_m)}$$

Demand for differentiated products: Berry (1994)

- Consumers homogenous w.r.t. α, β.
- Structural product-specific error ξ_j: unobserved product characteristic, typically correlated with price

Demand for differentiated products: Berry (1994)

- Market share inversion to allow for linear IV estimation
- s_j can be inverted such that

$$\ln(s_j) - \ln(s_0) = \delta_j = x_j\beta - \alpha p_j + \xi_j$$

- Role of the outside good. What is a reasonable market definition?
- With strategic price setting:
  p_j correlated with unobserved characteristic $\rightarrow$ endogeneity problem

Demand for differentiated products: Berry (1994)

- Fixed effects
- Instruments - do we have good ones (for price)?
- What about endogeneity of other product characteristics?

Instruments

- Good instruments might be the regressors
  (derivative of the moment function with respect to the parameters)
- Cost shifters uncorrelated with the demand shock (rarely observed)...
- Bresnahan (1987), BLP (1995): Assume product characteristics exogenous. Use
  - observed product characteristics; sums of the same characteristics of other products offered by that firm; sums of the same characteristics of products offered by other firms
  - Intuition: closeness in characteristics space, in oligopoly markups decreases with the presence of good substitutes
- Gandhi and Houde (2019): Differentiation IVs
  - Polynomial of differences in observed product characteristics

Demand for differentiated products: Berry (1994)

- Homogenous consumers strong assumption
  $\rightarrow$ representative consumer up to ε_{ij}

- Leads to substitution patterns depending only on market shares

Unobserved preference heterogeneity

- Random coefficients: allow consumer to have heterogenous preferences over certain characteristics, i.e. β_i
$$u_{ij} = x_j \beta_i - \alpha p_j + \xi_j + \varepsilon_{ij}$$

- Berry (1994) inversion to obtain linear equation no longer possible

- Market shares, as we did not integrate out all individual errors:
$$s_j = \int ... \int (s_{ij} dF_1(\alpha|\theta) ... dF_{K+1}(\beta_k|\theta))$$

- Now can estimate completely flexible substitution patterns (market share derivatives depend on all characteristics in u_{ij})

Unobserved preference heterogeneity

- Assume that individual i can be described by a vector of unobserved characteristics $(\nu_i, \varepsilon_{i0}, ..., \varepsilon_{iJ})$.
- Can/Should also include observed characteristics (age, income, ...)
- We can re-write consumer i's utility:

$$u_{ij} = \delta_j(x_j, p_j, \xi_j; \theta_1) + \mu_{ij}(x_j, p_j, \nu_i, \theta_2) + \varepsilon_{ij}$$

$\rightarrow$ Partition utility into mean utility δ_j and consumer-specific μ_{ij}

- Vector $\theta_1 = (\alpha, \beta)$ contains linear and $\theta_2 = (\sigma)$ nonlinear parameters

Unobserved preference heterogeneity

- For simplicity, assume that consumers only have heterogenous preferences over one characteristic in x_j, so that $\theta_2 = (\sigma)$ is a scalar.
- Nonlinear part of consumer utility:

$$\mu_{ij}(x_j, p_j, \nu_i, \theta_2) = x_j \nu_i \sigma$$

  where the distribution of ν_i is assumed. Typically, $N(0,1)$.
- If observe consumer demographics, empirical or fitted distributions

Simulation

- Idiosyncratic terms ε_{ij} integrated out (EV distribution assumption)
- But due to ν_i no closed form solution for the integral
- Invert the system of equation $s_j = s_j(x, p, \delta_j, \theta_2)$ numerically

→ Numerical integration / simulation to compute market shares

→ Contraction mapping to compute δ_j

Monte Carlo Simulation

- Take s draws from the distribution of ν for each observation
- Given s draws of ν, σ, and δ, compute the market shares:

$$s_j = \frac{1}{ns}\sum_{i=1}^{ns} s_{ij} = \frac{1}{ns}\sum_{1}^{ns}\frac{exp(\delta_j + x_j \nu_i \sigma)}{1 + \sum_{m=1}^{J} exp(\delta_m + x_m \nu_i \sigma)}$$

Intermission: Newton's Method

- Idea: Approximate (univariate) nonlinear function $f(x)$ by its tangent at x_n, where $x_{n+1} = x_n + \epsilon$:

$$y = f'(x_n)(x_{n+1} - x_n) + f(x_n)$$

- Finding the root of this tangent is easy, set $y = 0$ and solve for x_{n+1}:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Iterating until $x_{n+1} \approx x_n$ yields the (local) root of f(x)

- Optimization: Apply Newton's method to $f'(x)$, then check $f''(x)$

Intermission: Newton's Method

```
% Newton's root-finding method
function xt1 = newt_sqrt(a,xt)

if a<0
    disp('No negative numbers.');
    xt1=0;
    return
else
tol = 10e-12;
i = 0;
xdiff = 100;
while xdiff > tol
    fx = a-xt^2;
    dfdx = -2*xt;
    xt1 = xt - fx/dfdx;
    i = i + 1;
    fprintf('Value at iteration...
     %d: %16.14f \n', i, xt1);
    xdiff = abs(xt1-xt);
    xt = xt1;
end
end
```

Find $\sqrt{a}$.

Newton's method is a contraction mapping if $f''(x)$ is continuous, $f'(x) \neq 0 \; \forall \; x \in \mathbb{R}$, and $q \in (0,1)$ such that $|f(x)f''(x)| \leq q|f'(x)|^2 \; \forall x \in \mathbb{R}$

Contraction mapping for δ_j

- Given σ, compute vector δ that equates model predicted to observed market shares
- Contraction mapping

  $$\delta^{h+1} = \delta^h + ln(s) - ln(s(p, x, \delta^h, \sigma)), h = 0, ..., H,$$

  where H is the smallest integer such that $\|\delta^H - \delta^{H-1}\|$is smaller than some tolerance level, and δ^H is the approximation to δ.
- Compute vector $s(x, p, \delta, \sigma)$ at each iteration

Estimation

- Nested simulated GMM procedure
- IV estimation. Population moment conditions $E[Z\xi(\theta^*)] = 0$
- Using δ_j, computed at each parameter iteration in the GMM objective function, obtain: $\xi_j = \delta_j - (x_j\beta - \alpha p_j)$

Estimation

- GMM objective function:
$$f = \xi(\theta)' Z \Phi^{-1} Z' \xi(\theta),$$
  where Φ^{-1} is a consistent estimate of $E[Z'\xi\xi' Z]$
- Search $\theta = (\alpha, \beta, \sigma)$ that minimizes the objective function
- Note: finding the nonlinear parameter σ requires a nonlinear search.

Equilibrium Model

- How are prices determined in oligopolistic markets with differentiated products?
- Increased efficiency of demand estimates by adding structure on the supply side
- Assume prices are result of firms' optimization problem
- Use information from first order conditions for estimation
- Single-product vs. multi-product firms
- Require cost function and equilibrium notion

Equilibrium Model

Berry (1994) single-product firms

- Profits of firm j are given by $\Pi_j(\mathrm{p}) = p_j q_j(\mathrm{p}) - C_j(q_j(\mathrm{p}))$
- Assumption: compete à la Nash in prices
- The first-order condition is:
$$q_j + (p_j - mc_j)\frac{\partial q_j}{\partial p_j} = 0$$
- which can be rewritten as
$$p_j = mc_j + b_j(\mathrm{p})$$
where $b_j(\mathrm{p}) = \frac{-q_j}{\partial q_j \backslash \partial p_j}$ is the *markup*

Multiproduct Firms

BLP (1995) look at the more general case of a multiproduct firm producing a subset $\Im_f$ of the J products

- Profits of firm f are given by:

$$\Pi_f = \sum_{j \in \Im_f} (p_j - mc_j) M s_j(p, x, \theta_1, \theta_2)$$

- The usual assumption is that firms compete à la Nash in prices

Multiproduct Firms

- The first-order condition is:

$$s_j(p,x,\theta_1,\theta_2) + \sum_{r \in \Im_f} (p_r - mc_r) \frac{\partial s_j(p,x,\theta_1,\theta_2)}{\partial p_j} = 0$$

- which can be rewritten in vector notation as:

$$p_j = mc_j + \Delta(p,x,\theta_1,\theta_2)^{-1} s(p,x,\theta_1,\theta_2)$$

where the (j,r) element of the J by J matrix $\Delta(p,x,\theta_1,\theta_2)$ is

$$\Delta_{jr} \begin{cases} \frac{-\partial s_r}{\partial p_j} & \text{if r and j are produced by the same firm} \\ 0 & \text{otherwise} \end{cases}$$

Multiproduct Firms

- For simplicity, assume that marginal costs are constant and take the form:

$$ln(mc) = w_j\gamma + \omega_j \tag{1}$$

- The equation to be estimated is therefore

$$ln(p - b(p, x, \theta_1, \theta_2)) = w_j\gamma + \omega_j$$

  where w_j are observed and ω_j unobserved product characteristics

- The markup $b(p, x, \theta_1, \theta_2) = \Delta(p, x, \theta_1, \theta_2)^{-1}s(p, x, \theta_1, \theta_2)$ depends on demand parameter and, through the equilibrium price, on the unobserved cost component ω

- We therefore need some instruments, which are uncorrelated with ω as well as with the structural demand error ξ

GMM estimation

- Estimate demand and supply simultaneously. Define additional instruments $z_j = (x_j, w_j)$.
- Additional moment conditions

$$E[\xi_j|z] = E[\omega_j|z] = 0$$

where $\omega = mc - [x, w]\gamma$

Problem set

Struggles?