

Structural Econometrics in Labor and IO.

Problem Set: Dynamic Discrete Choice Estimation.

Description

Superintendent Harry Zurcher manages the bus fleet at the Madison Metropolitan Bus Company. Notably, at every point in time t , he decides whether to replace an old bus engine with a new one or to keep operating the old one.

We are interested in estimating the cost of engine maintenance and replacement, using observed data on: (1) mileage x_t at period t , (2) observed replacement decision i_t at period t , and (3) mileage x_{t+1} next period.

The data generating process is as follows:

- x_t can take on 11 values, $x_t \in \{0, 1, 2, \dots, 10\}$ (i.e. mileage discretized into 11 categories).
- At any time period,

$$x_{t+1} = \begin{cases} \min\{x_t + 1, 10\}, & \text{with probability } \lambda, \\ x_t, & \text{with probability } 1 - \lambda \end{cases}$$

Note that λ does not depend on the replacement decision i_t .

- The agent's replacement decision is made at the beginning of each period and is effective immediately (i.e. if $i_t = 1$, the agent uses machine with mileage 0 and the next period state x_{t+1} is 1 with probability λ and 0 with probability $1 - \lambda$).
- The per-period maintenance cost for a bus with mileage x is $C(x, \theta) = \theta_1 x + \theta_2 x^2$.
- The cost of replacement is θ_3 .
- The per-period utility function is given by

$$u(x_t, i_t, \epsilon_{1t}, \epsilon_{2t}; \theta) = \begin{cases} -\theta_1 x_t - \theta_2 x_t^2 + \epsilon_{0t}, & \text{if } i_t = 0, \\ -\theta_3 + \epsilon_{1t}, & \text{if } i_t = 1 \end{cases}$$

where $(\epsilon_{0t}, \epsilon_{1t}) \stackrel{iid}{\sim}$ Type 1 Extreme Value.

- Assume that the decision-maker's discount factor is known, $\beta = 0.95$.

Problems

Together with your answers, please submit your well-commented computer code for the questions below before next session. If possible, please send all files compressed into one single zip-file.

1. Simulate a fake data set with 1000 observations of x_t , x_{t+1} , and i_t . For this, set $\theta_1 = 0.13$, $\theta_2 = -0.004$, $\theta_3 = 3.10$, and $\lambda = 0.82$. Hint: All busses are assumed to be operated and maintained independently.
2. Now, forget the chosen parameter values and use the simulated data to:
 - report an estimate for λ (this does not require any nonlinear optimization).
 - report the parameters of the model estimated using MLE with a nested fixed point algorithm:
 - (a) Guess initial parameter values;
 - (b) Solve dynamic programming problem;
 - (c) Calculate the probability of replacement at each state;
 - (d) Use model predictions (the probability calculated in the previous step) and data to compute the (log-)likelihood function;
 - (e) Search over parameter values (e.g. using *fminsearch*) by repeating steps (b) – (d).
 - Report the probability of replacement for every state: $P(1|x_t, \theta)$.