Object Recognition

Chapter 2: Image Processing Fundamentals

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Document History

Version Nr.	Date	Changes
1.0		Initial Version
1.1	26.03.2013	Added test images for comparing average- and gaussian filter
1.2	20.03.2017	Adaptations for SS 17
1.3	03.04.2017	Added slides for Non-local-Means Filter
1.4	19.04.2019	Adaptations for SS 19

Chapter 2: Image Processing

Motivation

- Object Recognition integrates image processing techniques
- Image Processing is realized by applying filters on the image.
- Examples for filtering in Object Recognition:
  - Preprocessing of images to enhance quality, e.g. noise surpression
  - Local feature based recognition techniques apply filter methods for finding keypoints
  - Template Matching is a simple object recognition method which just correlates a filter with the image.
  - In order to achieve scale invariance in recognition, usually Gaussian Pyramids are applied to generate different scale levels.

1-Dimensional Filtering

Notation:

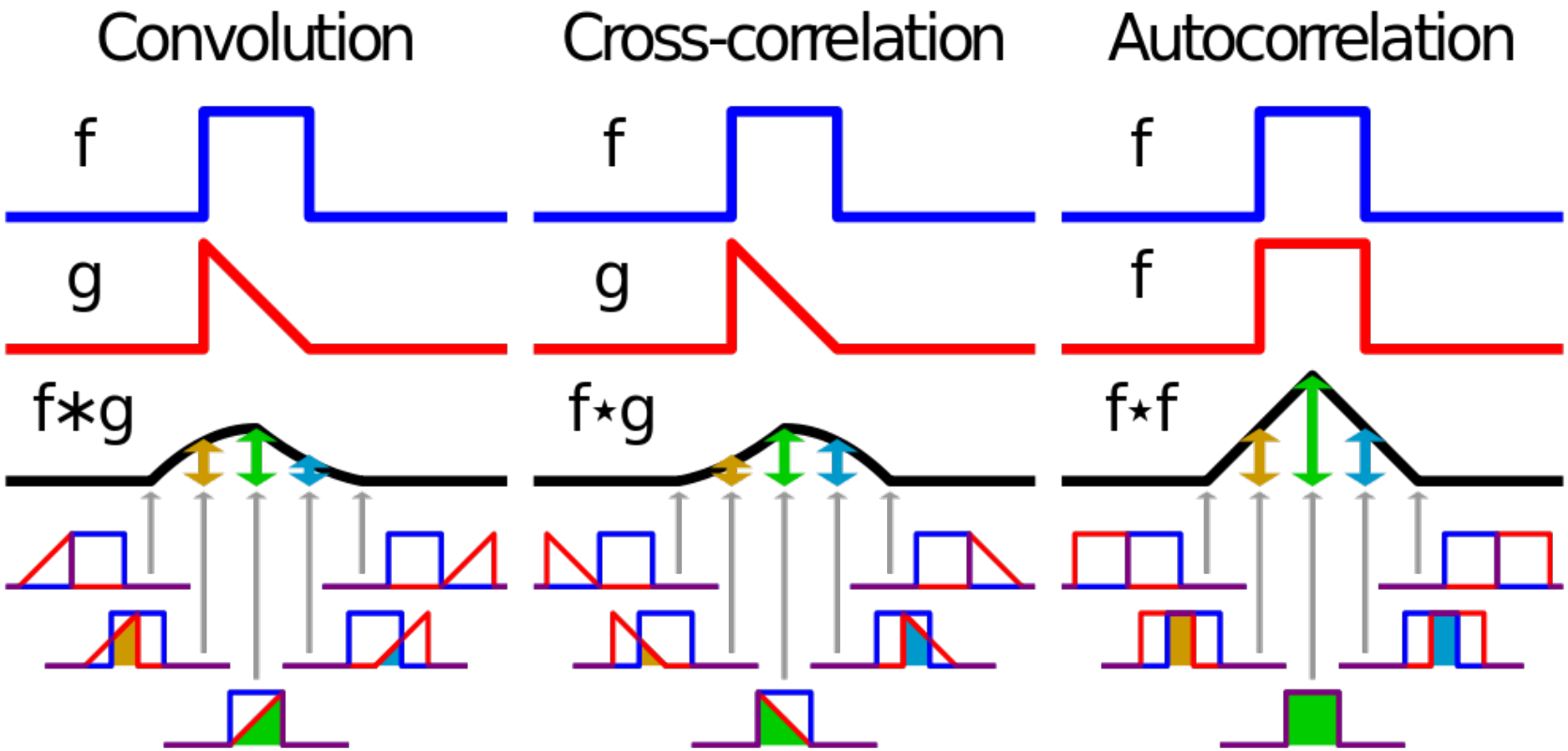
- $f(i)$ is the $i.th$ component of the 1-D diskret input signal
- $h(u)$ is the $u.th$ component of the 1-D diskret filter kernel (or the $u.th$ filter weight). Total filter length is $2k + 1$
- $g(i)$ is the $i.th$ component of the 1-D diskret output signal
- Correlation Filtering $g = h \circledast f$

$$g(i) = \sum_{u=-k}^{u=k} h(u) f(i+u) \tag{1}$$

- Convolution Filtering $g = h * f$

$$g(i) = \sum_{u=-k}^{u=k} h(u) f(i-u) \tag{2}$$

Comparision: Convolution and Correlation



Quelle: http://en.wikipedia.org/wiki/Convolution

Options for Border Extensions

Since the length of the filter h is > 1 the input signal f must be extended at the borders in order to calculate the filter response g at the first and last coordinate(s). Extension options are: [1] .

- nearest Use the value at the boundary

$$[1, 2, 3] \Rightarrow [1, 1, 2, 3, 3]$$

- wrap Periodically replicate the array

$$[1, 2, 3] \Rightarrow [3, 1, 2, 3, 1]$$

- reflect Reflect the array at the boundary

$$[1, 2, 3] \Rightarrow [1, 1, 2, 3, 3]$$

- constant Use a constant value, e.g. 0.0

$$[1, 2, 3] \Rightarrow [0, 1, 2, 3, 0]$$

[1] http://docs.scipy.org/doc/scipy/reference/tutorial/ndimage.html

1-Dimensional Convolution Filtering Examples

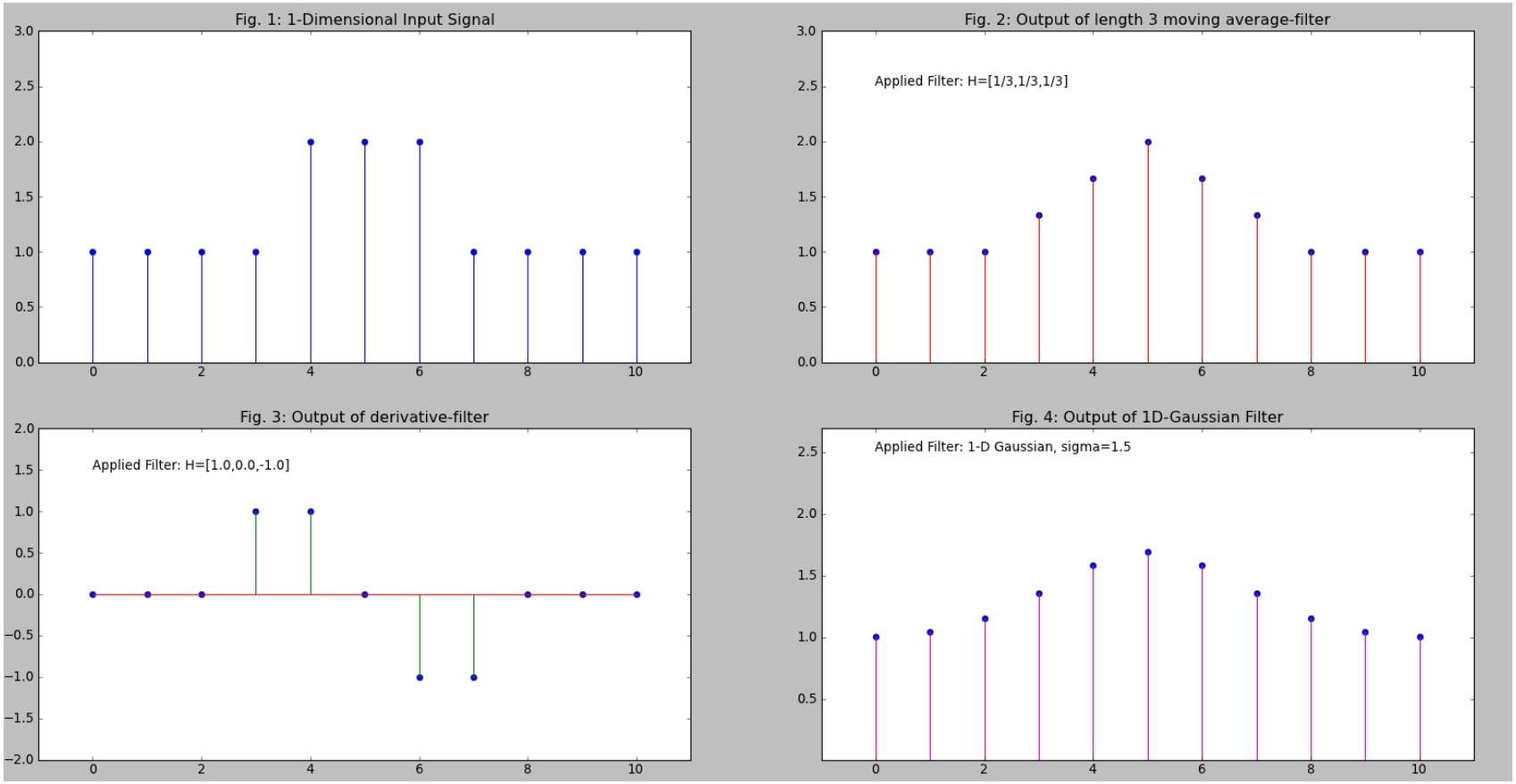
- Input Signal (Fig. 1 in next slide):

$$f = [1, 1, 1, 1, 2, 2, 2, 1, 1, 1, 1]$$

- Output of convolutional filtering with length-3 moving average filter (Fig. 2)

$$\begin{aligned} g &= h * f \\ &= \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right] * [1, 1, 1, 1, 2, 2, 2, 1, 1, 1, 1] \\ &= \left[1, 1, 1, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{5}{3}, \frac{4}{3}, 1, 1, 1\right] \end{aligned}$$

1-Dimensional Filtering Examples



1-dimensional Gaussian Filter

- 1-dimensional Gaussian Kernel:

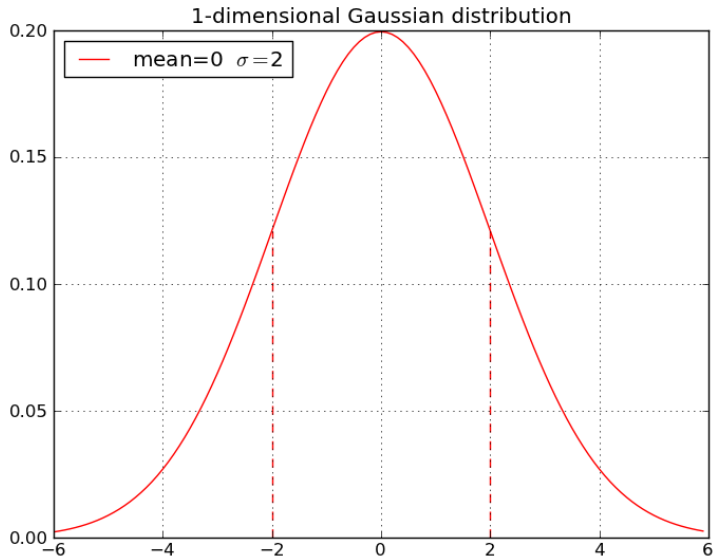
$$G_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

- Convolution of discrete signal f with Gaussian kernel (Fig 4):

$$g(i) = \sum_{u=-k}^{k} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{u^2}{2\sigma^2}\right) f(i-u)$$

  i.e. the Filter is cut to the finite range $[-k, \ldots, 0, \ldots, k]$.

- Standard deviation σ defines the spread of the filter
- Choose finite filter range $k \approx 3\sigma$.



2-Dimensional Filtering

Notation:

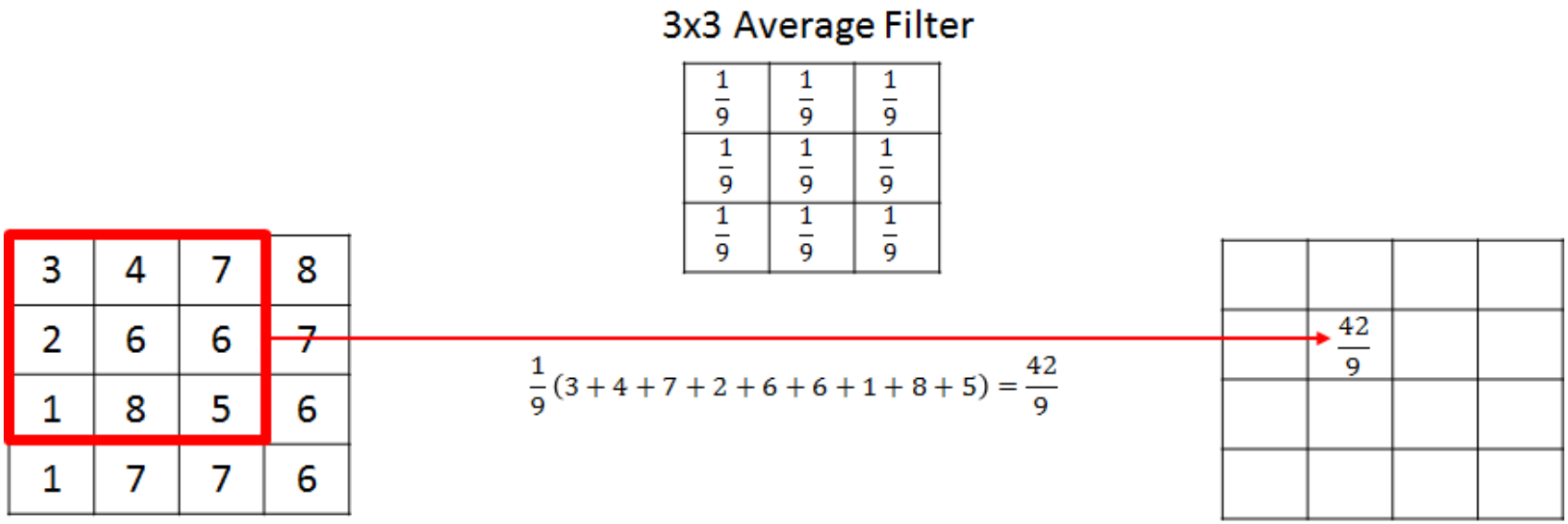
- $F(i,j)$ is the component in row i, column j of the 2-dimensional diskret input signal
- $H(u,v)$ is the component in row u, column v of the 2-dimensional diskret filter kernel
- $G(i,j)$ is the component in row i, column j of the 2-dimensional diskret output signal
- Correlation Filtering $G = H \circledast F$

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H(u,v)F(i+u,j+v) \tag{3}$$

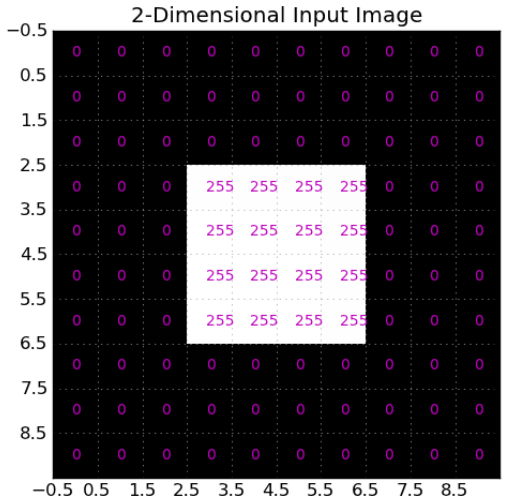
- Convolution Filtering $G = H * F$

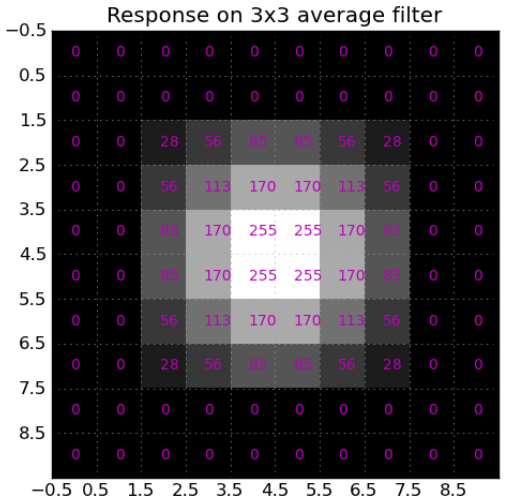
$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H(u,v)F(i-u,j-v) \tag{4}$$

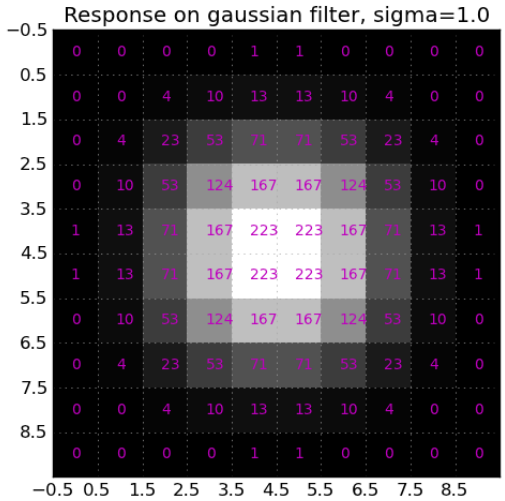
2-Dimensional Filtering: Example Average Filter

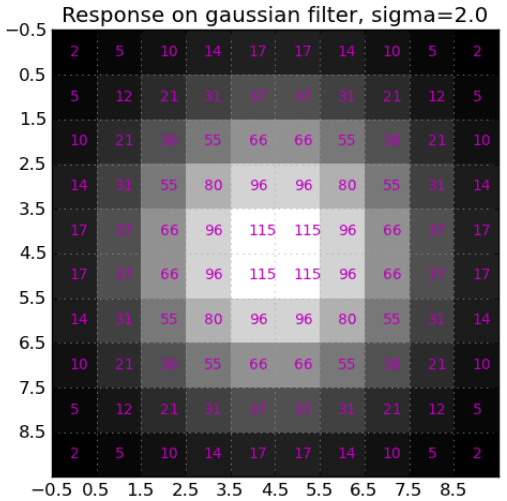


2-Dimensional Filtering: Examples Low Pass Filter

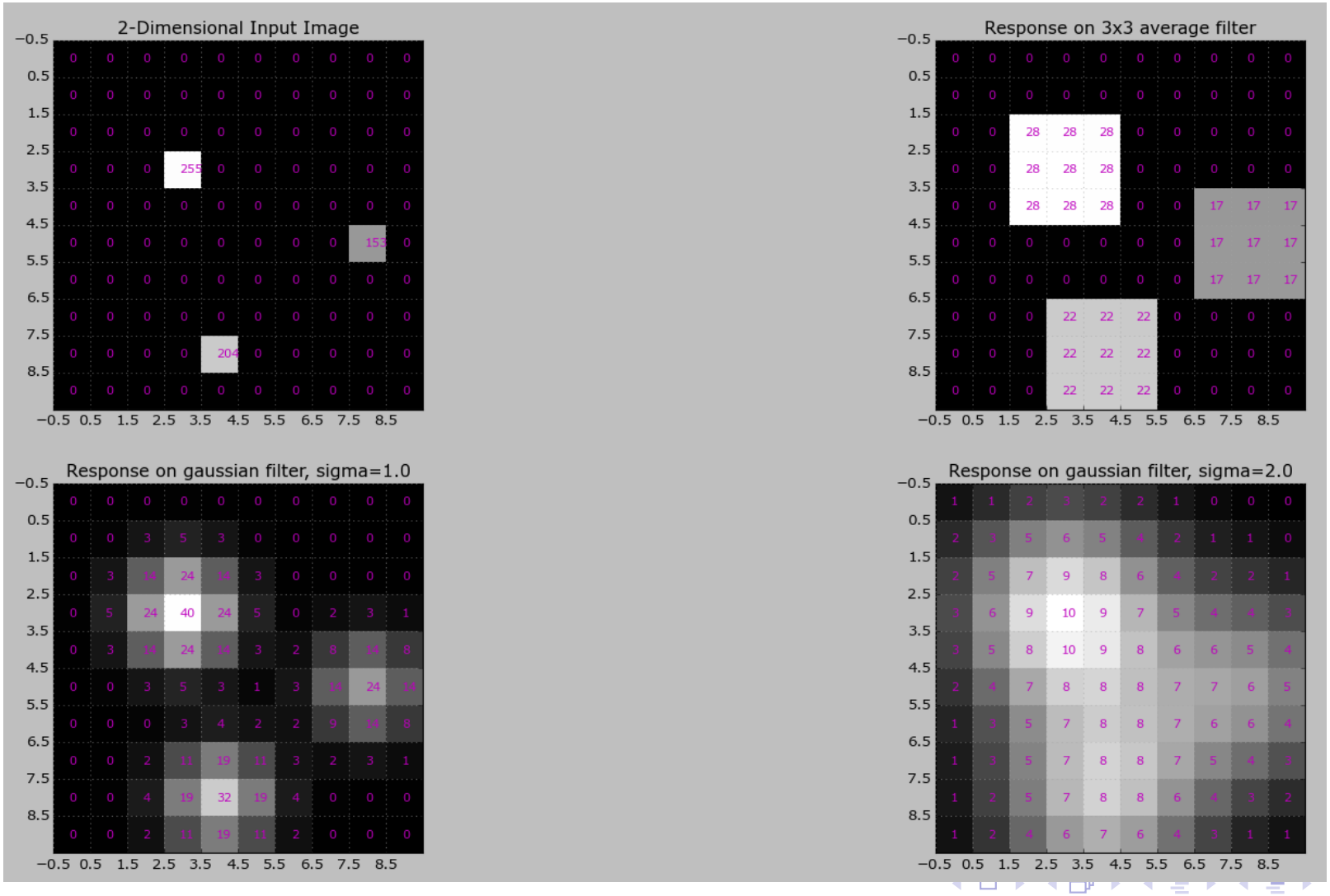








2-Dimensional Filtering: Examples Low Pass Filter



Symmetric 2-dimensional Gaussian Filter

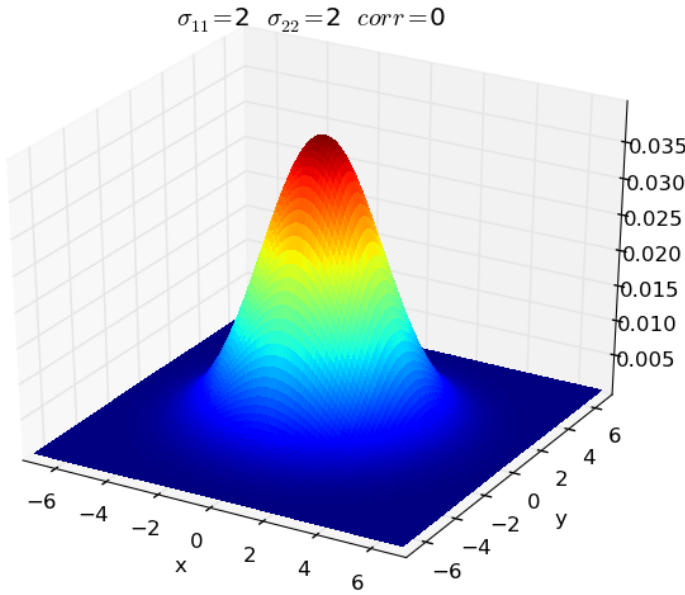
- 2-dimensional Gaussian Kernel

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \qquad (5)$$

- Convolution of discrete signal f with Gaussian kernel:

$$g(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{u^2+v^2}{2\sigma^2}\right) f(i-u, j-v).$$

- Standard deviation σ defines the spread of the filter
- Choose finite filter range $k \approx 3\sigma$.



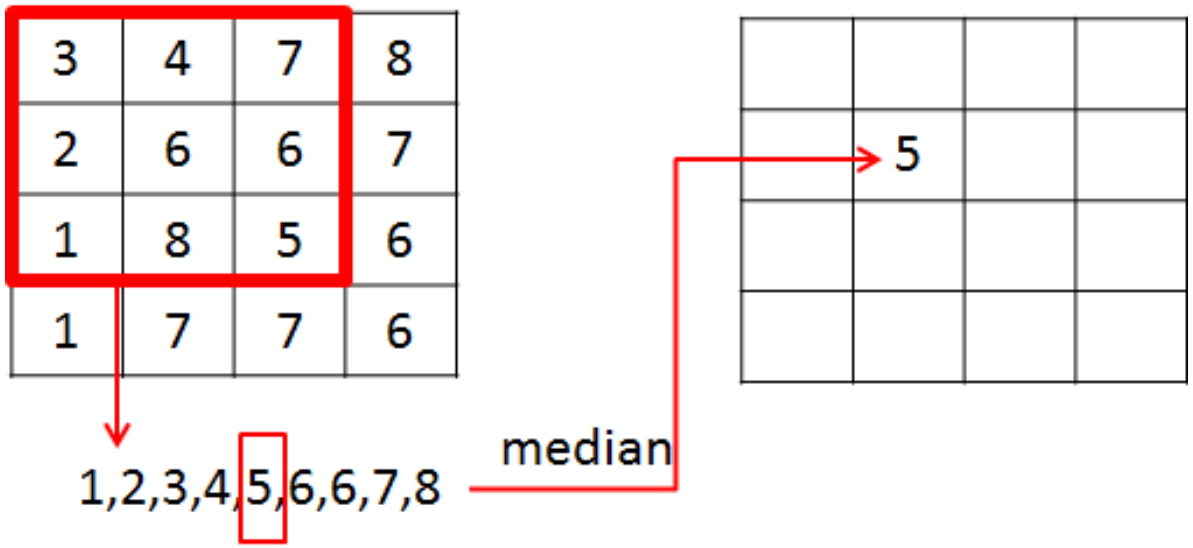
Median Filter

Calculation of output $G(i,j)$ if Median Filter H_k of half-width k is applied to Input F:

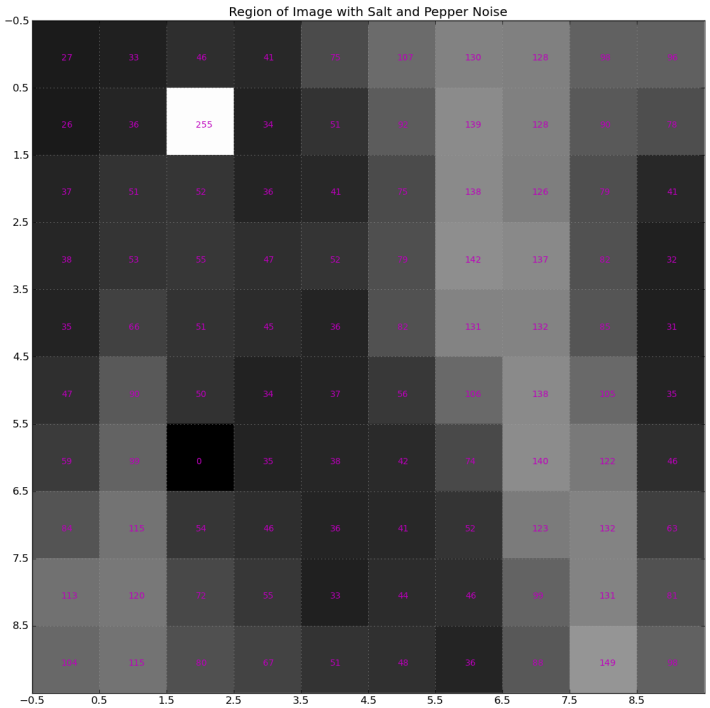
- Order all pixel values in the region

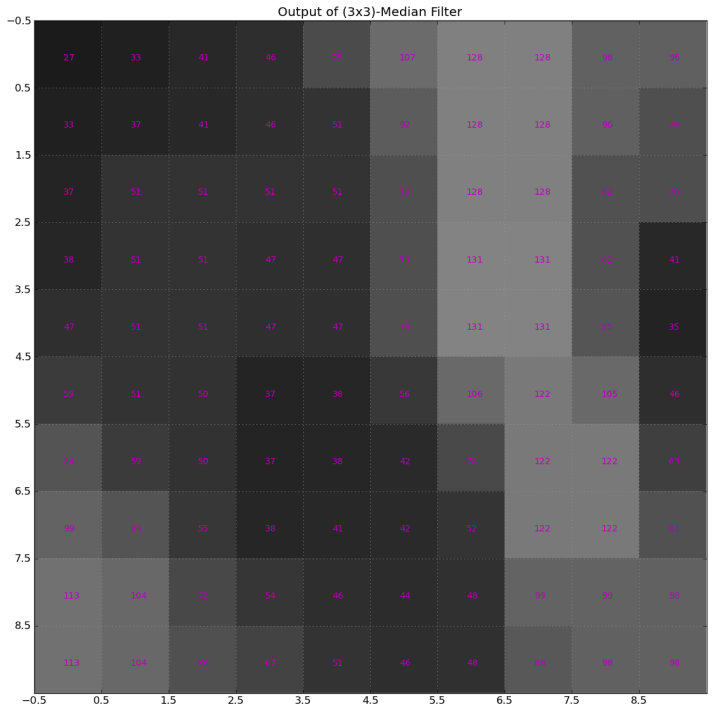
$$F[i-k:i+k, j-k:j+k]$$

- Then the filter output $G(i,j)$ is the value, which is in the center of the ordered list.



2-Dimensional Median Filter applied to Image with Salt and Pepper Noise





Features of Median Filter:

- No new pixel values
- Keeps edges sharp
- Removes spikes (Salt and Pepper Noise)

Properties of Filtering

- Shift Invariance: Filter output depends only on the values in the relevant region, not on the position of the region.
- Linear Filter, iff
  - Superposition
  $$H \odot (F_1 + F_2) = (H \odot F_1) + (H \odot F_2) \tag{6}$$
  - and Scaling
  $$H \odot (kF) = k(H \odot F) \tag{7}$$

  are fulfilled[2].

[2] Here $\odot$ denotes an arbitrary filter operation, not necessarily correlation or convolution

Properties of convolution filtering

- The convolution operation $*$ is shift invariant and linear.
- Commutative: $H * G = G * H$
- Associative: $F * (G * H) = (F * G) * H$
- Identity: $H * \delta = H$, where δ is the unit impulse $\delta = [\ldots, 0, 0, 1, 0, 0, \ldots]$
- Differentiation:

$$\frac{\partial}{\partial x}(H * G) = \frac{\partial H}{\partial x} * G$$

- Fourier Transform

$$\mathcal{F}(H * G) = \mathcal{F}(H) \cdot \mathcal{F}(G)$$

Examples:

- Averaging Filter (=Uniform Filter) and Gaussian Filter can be described as convolution. Thus they are linear and shift invariant.
- The Median Filter can not be described as convolution and is not linear.

Comparison: Frequency Filtering of Average- and Gaussian-Filter

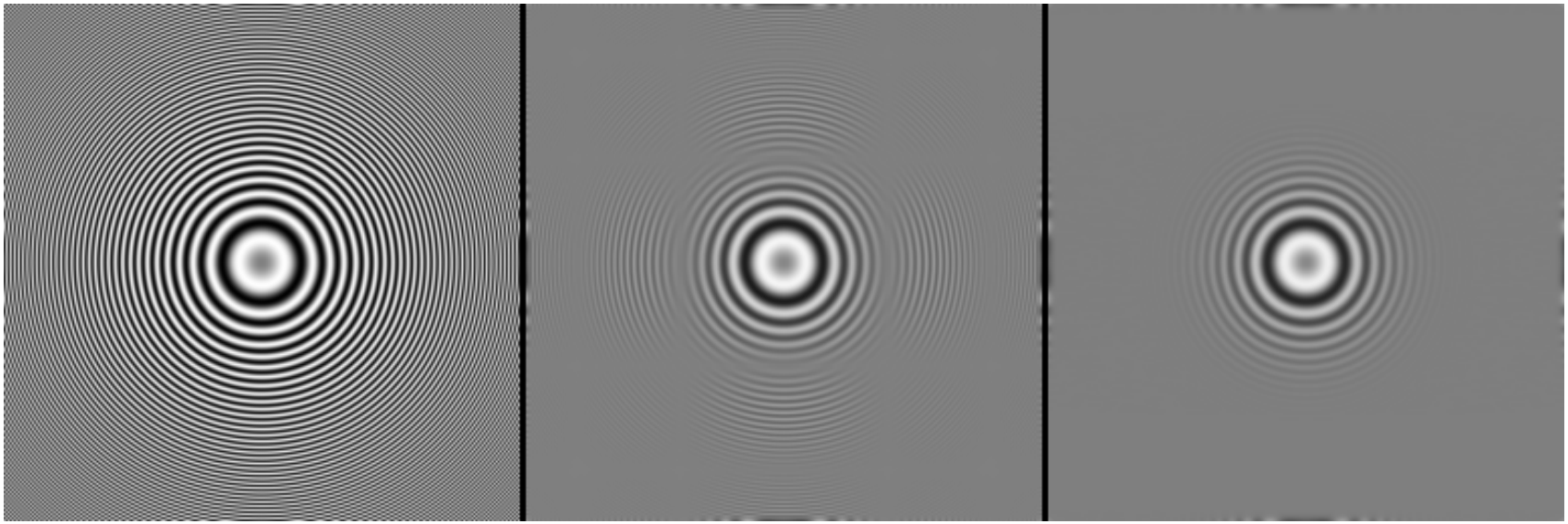


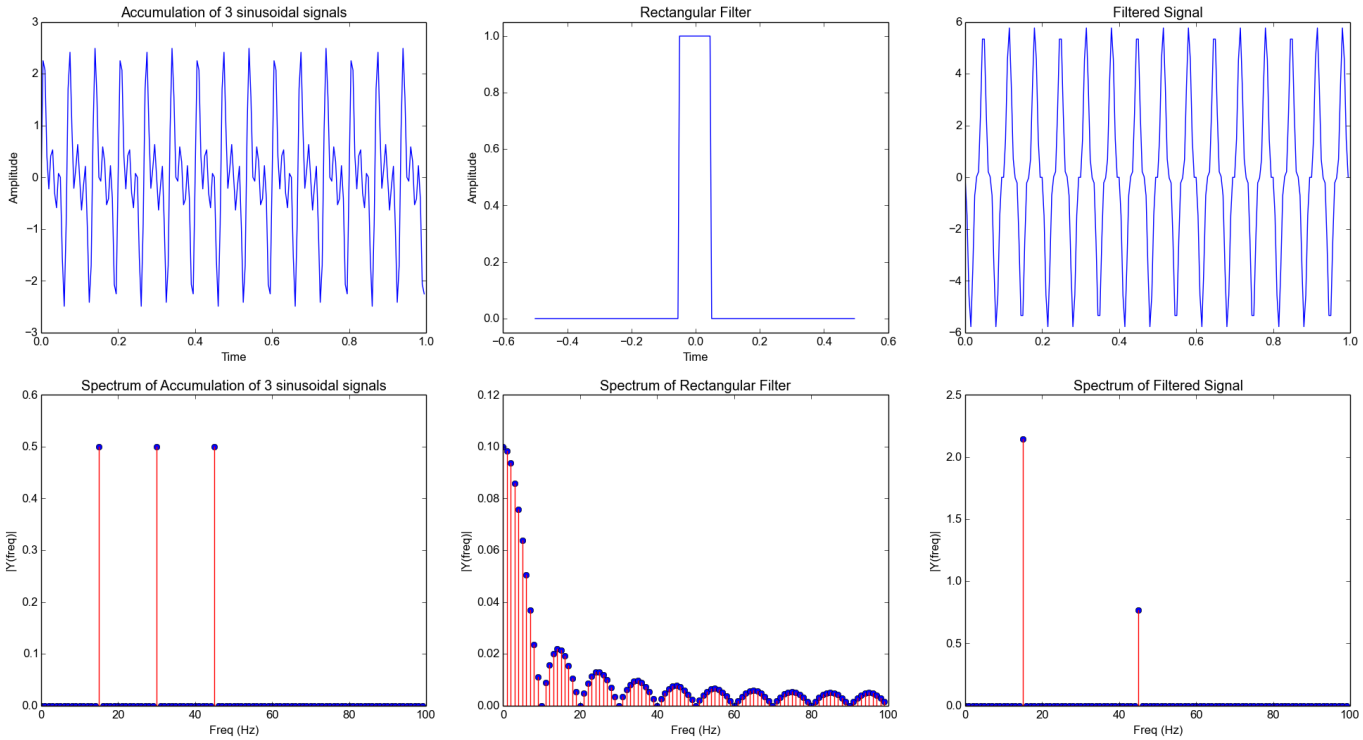
Abbildung: Original image (left), output of Average filter (center) and output of Gaussian filter (right)

What makes the difference?

Average Low Pass Filter

Input Signal to be filtered:

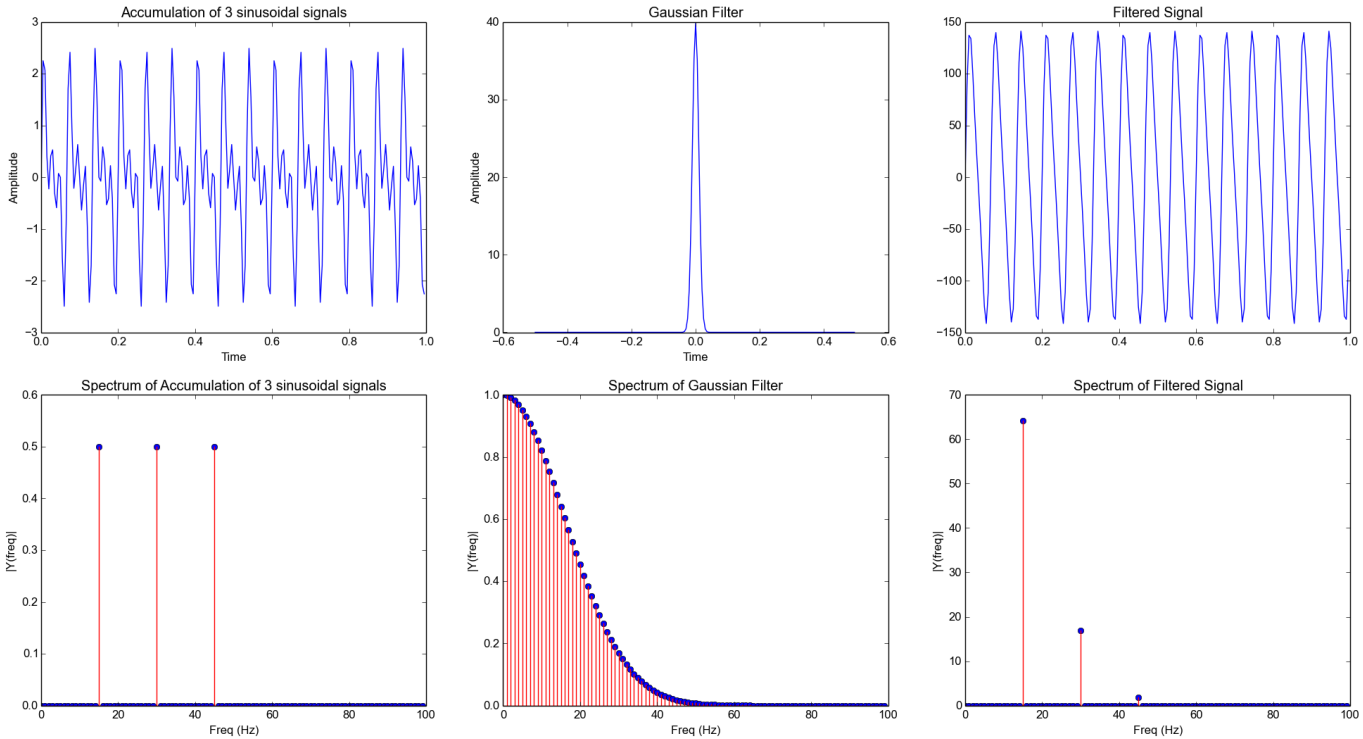
$$y(t) = sin(2\pi \cdot 15 \cdot t) + sin(2\pi \cdot 30 \cdot t) + sin(2\pi \cdot 45 \cdot t)$$



Gaussian Low Pass Filter

Input Signal to be filtered:

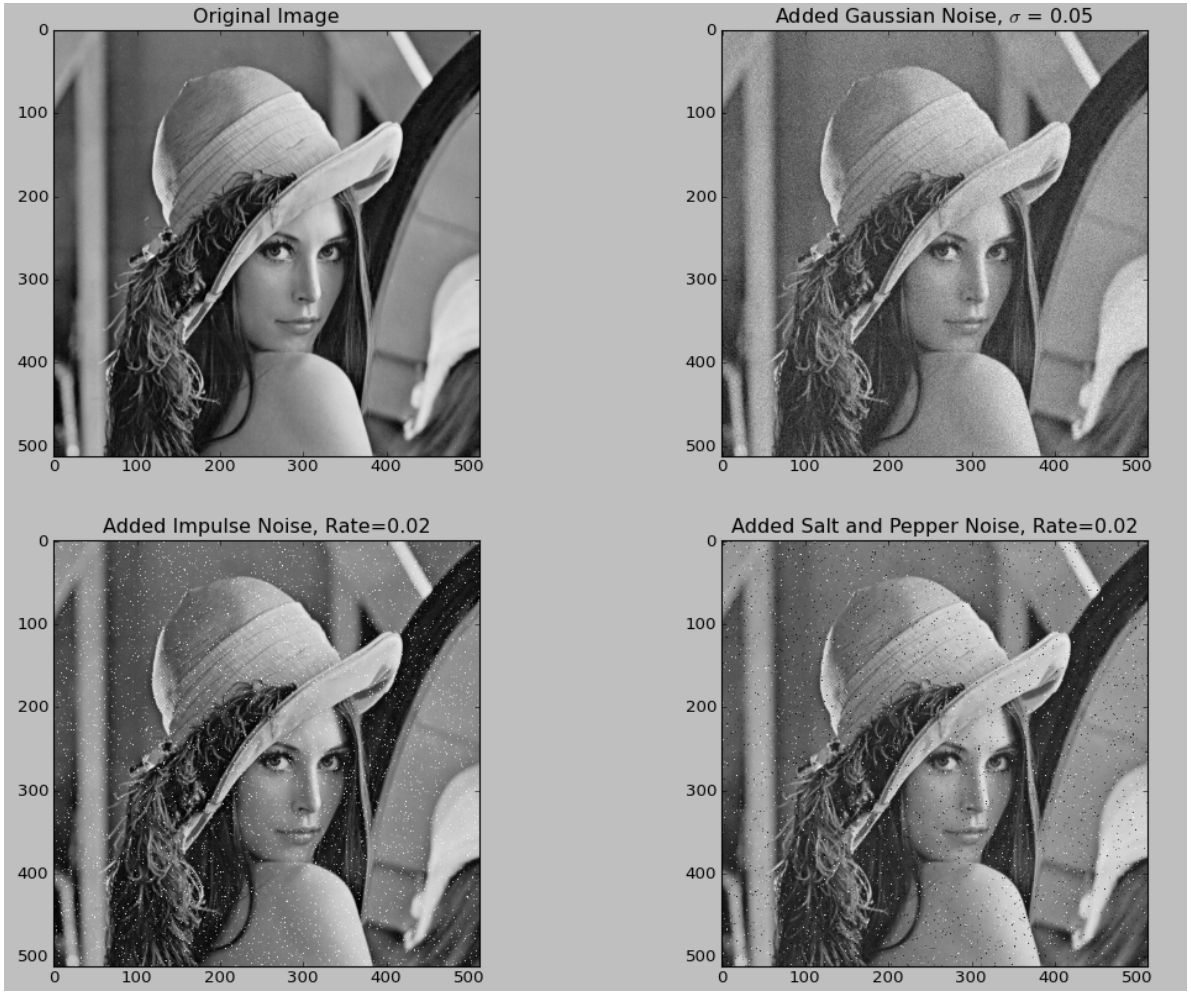
$$y(t) = sin(2\pi \cdot 15 \cdot t) + sin(2\pi \cdot 30 \cdot t) + sin(2\pi \cdot 45 \cdot t)$$



Different Types of Noise

- General assumption: Noise is independent and indentically distributed over entire image
- Types:
  - Gaussian Noise: Additive Noise sampled from a Gaussian distribution.
  - Impulse Noise: White pixels, that are randomly distributed over the image.
  - Salt and Pepper Noise: White and black pixels, that are randomly distributed over the image.

Examples for different types of noise



Principles of noise reduction

- It is assumed, that in the noise-free picture nearby pixels have similar values
- Since noise is assumed to be independent noisy pixels may vary effectively from their neighbours.
- The assumed noise is high-frequent
- Surpress high frequent noise by applying a low pass filter.
- Calculation of (weighted) averages corresponds to low pass filtering.
- The Fourier Transform (spectrum) of a Gaussian is again a Gaussian with inverse variance. I.e. a smaller variance in time domain implies a large bandwidth in the frequency domain and vice versa.

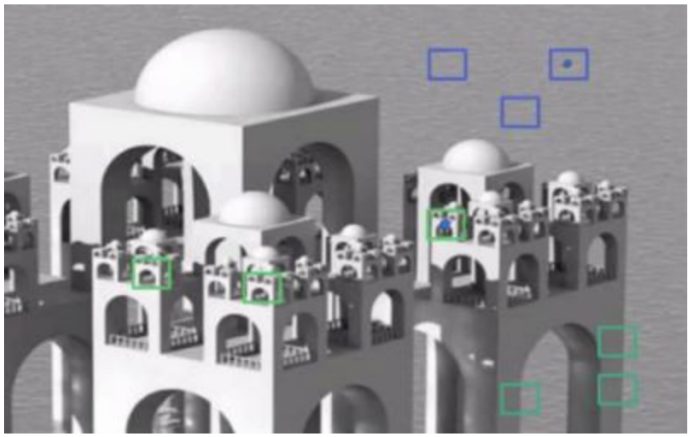
Non-Local-Means Noise Reduction

- Average Filter: Replace each pixel value by average of neighbouring pixel values.
- Gauss-Filter: Replace each pixel value by weighted average of neighbouring pixel values.
- Non-Local-Means-Filter: Replace each pixel value by weighted average of pixel values in similar image regions.
- Idea behind NLM: Noisy Picture:

$$P_{noise} = P_{true} + N_0,$$

  where N_0 is Gaussian distributed noise.
- By adding many noisy versions of the same image-region, the noise-term will be averaged out and the result is the true image-region.
- Link to NLM demo.
- html-version of paper on NLM Denoising.

Non-Local-Means Noise Reduction



image

The blue patches in the image looks the similar. Green patches looks similar. So we take a pixel, take small window around it, search for similar windows in the image, average all the windows and replace the pixel with the result we got. This method is Non-Local Means Denoising. It takes more time compared to

Source: http://docs.opencv.org/trunk/d5/d69/tutorial_py_non_local_means.html

Reduce Gaussian Noise

Reduce Salt and Pepper Noise

Gaussian Pyramide

Generate Gaussian Pyramide:

1. Set level index $l = 0$; Set $G_{l=0}$ to be the original image
2. Smooth G_l by applying a Gaussian filter
3. Downsample the smoothed G_l by a factor of 2. The result is G_{l+1}
4. Set $l = l + 1$ and continue with step 2 until G_l contains only a single pixel

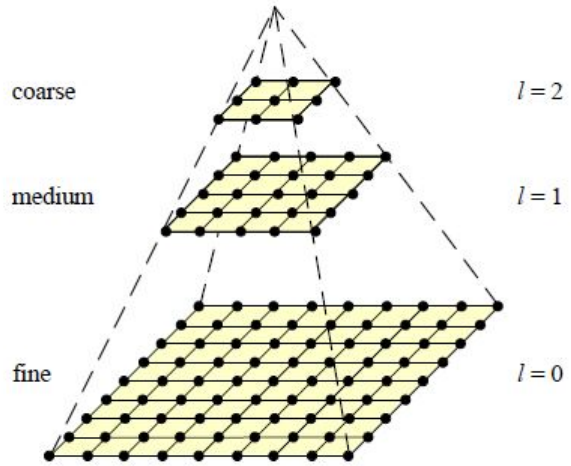


Figure 3.32 A traditional image pyramid: each level has half the resolution (width and height), and hence a quarter of the pixels, of its parent level.

Quelle: [1]

Gaussian and Laplacian Pyramide

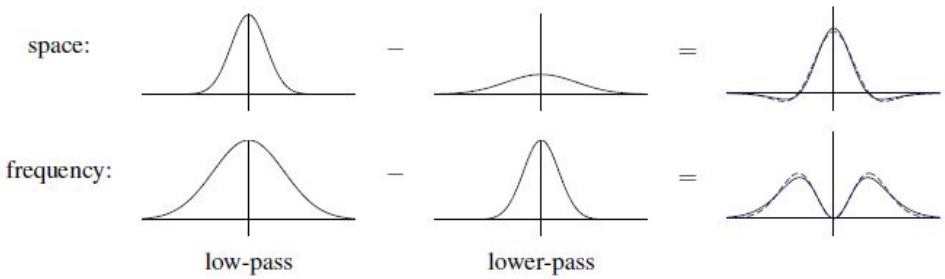


Figure 3.35 The difference of two low-pass filters results in a band-pass filter. The dashed blue lines show the close fit to a half-octave Laplacian of Gaussian.

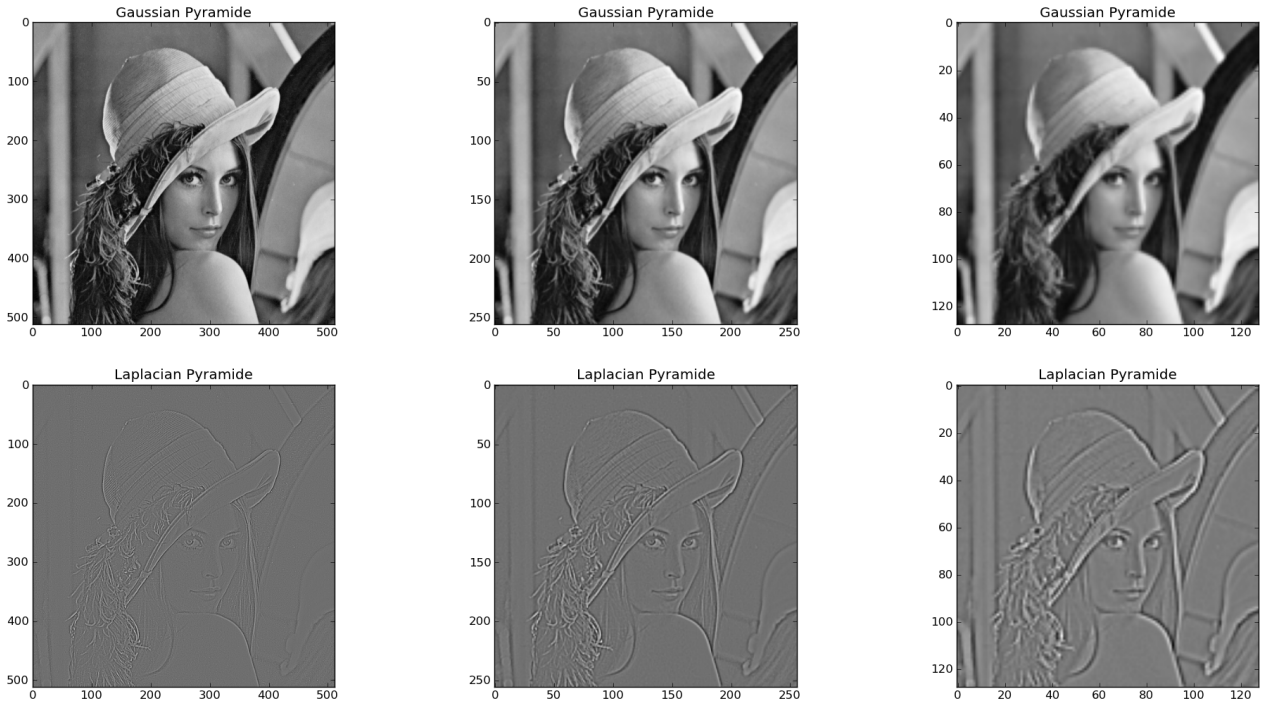
Quelle: [1]

- In each level the input image is low pass filtered
- Thus in each level finer structures (higher frequencies) are filtered out.
- At high levels l only coarse structures (low frequencies) are represented in G_l.
- The Difference

$$D_l = G_l - G_{l+1}^{\uparrow}$$

  represents the high frequencies which are filtered out in level l, where $G_{l+1}^{\uparrow}$ is the upsampled (interpolated) version of G_{l+1}.
- The pyramide of all D_l constitute the Laplacian Pyramide.

Gaussian and Laplacian Pyramide (first 3 levels)



Applications for Gaussian and Laplacian Pyramide

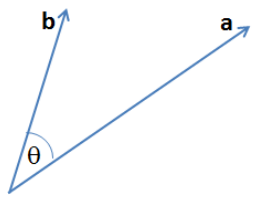
- Generate scale space: For local image descriptors (see corresponding lecture) scale space extrema constitute the keypoints.
- Image Retrieval: Find images, which contain the same objects as a *query-Image*.
  - Efficient search for matching objects: First search high level representation G_l, which contain much less pixels than the original image, only in the case of a coarse match continue with lower levels.
  - The object in the database can be represented with another scale than the object in the query-image. Matching at different scales eliminates this problem.

Finding patterns in images

- Correlation Filtering at a given position (i, j) according to equation (3) is the same as calculation of the dot product of two vectors.
- The dot product normed by the length of the 2 vectors defines the cosine between these two vectors.

$$\cos(\Theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|}$$

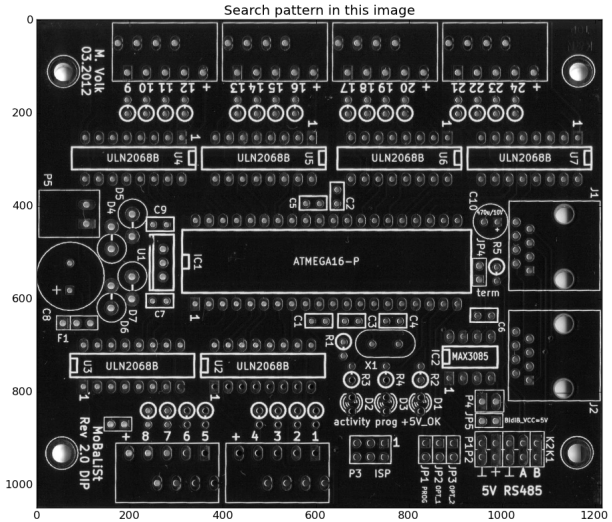
- The cosine is a similarity measure, with a maximum value of 1 if the vectors point in the same direction.
- $\Rightarrow$ At the positions (i, j), where the normed correlation (Equation (3)) is ≈ 1 the image region looks similar as the filter.
- $\Rightarrow$ In order to find patterns in an image one can apply a filter, which looks like the pattern.

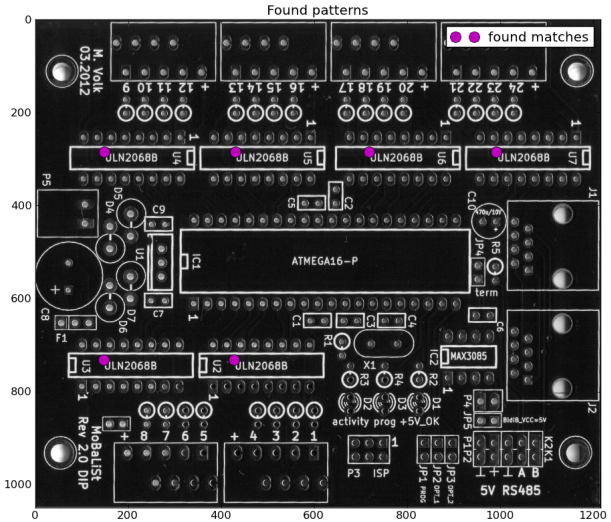


Example: Find module on a platine

Pattern:

Image:





Criteria for Edge Detection

- Edges: Distinctive change of pixel intensities
- Change of values: is measured by 1.st order derivative.
  - 1-dimensional continuous:
$$f'(x) = \frac{\partial f(x)}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$$
  - 1-dimensional discrete:
$$f'(x) = \frac{\partial f(x)}{\partial x} \approx \frac{f(x+1) - f(x)}{1}$$
  - 2-dimensional continuous:
$$\frac{\partial f(x,y)}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon, y) - f(x,y)}{\epsilon}$$
$$\frac{\partial f(x,y)}{\partial y} = \lim_{\epsilon \to 0} \frac{f(x, y+\epsilon) - f(x,y)}{\epsilon}$$
  - 2-dimensional discrete:
$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$
$$\frac{\partial f(x,y)}{\partial y} \approx \frac{f(x, y+1) - f(x,y)}{1}$$

Determining derivatives by convolution filtering (1-dimensional signals)

- For discrete 1-dimensional signals the derivative is just the difference between two successive values.
- Calculating the difference of successive values in signal f can be realized by convolution of the signal with the filter

$$h = (1, -1) \tag{8}$$

or better (due to symmetry)

$$h = (1, 0, -1) \tag{9}$$

- The discrete derivative of f is

$$\frac{\partial f}{\partial x} = h * f$$

Prewitt Edge Detector Filters

- For 2-dimensional discrete signals F the partial derivative of F in the direction of x is calculated as convolution

$$\frac{\partial F}{\partial x} = H_x * F$$

with the derivative filter

$$H_x = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \tag{10}$$

- The partial derivative of F in the direction of y is calculated as convolution

$$\frac{\partial F}{\partial y} = H_y * F$$

with the derivative filter

$$H_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \tag{11}$$

Sobel Edge Detector Filters

- The filter pair in (10) and (11) is called Prewitt Filter
- Another edge detector is the Sobel Filter, defined by

$$H_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and } H_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \qquad (12)$$

1st order derivative of Gaussian edge detector

- In the case of noisy (Gaussian Noise) images the 1st order derivatives of the 2-dimensional Gaussian are suitable edge detector filters.
- For the 2-dimensional Gaussian $G_\sigma(x, y)$ (Equation (5)) the partial derivatives are

$$G_{\sigma,x}(x, y) = \frac{\partial G_\sigma(x, y)}{\partial x} = -\frac{x}{\sigma^2} G_\sigma(x, y) \quad (13)$$

$$G_{\sigma,y}(x, y) = \frac{\partial G_\sigma(x, y)}{\partial y} = -\frac{y}{\sigma^2} G_\sigma(x, y) \quad (14)$$

- Convolution of 1st order derivative Gaussian with the Image is the same as derivation of the convolution of Gaussian and Image (see slide properties of convolution):

$$\frac{\partial \left(G_\sigma(x, y) * F\right)}{\partial x} = \frac{\partial G_\sigma(x, y)}{\partial x} * F$$

- The edge detectors H_x, H_y are the discretization of (13) and (14).

Magnitude and Direction of Gradient

- For all of the edge detectors

$$\frac{\partial F}{\partial x} = H_x * F \quad \text{and} \quad \frac{\partial F}{\partial y} = H_y * F$$

constitute the gradient of image F:

$$\nabla F = \begin{pmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{pmatrix} \tag{15}$$

- The magnitude of the gradient is

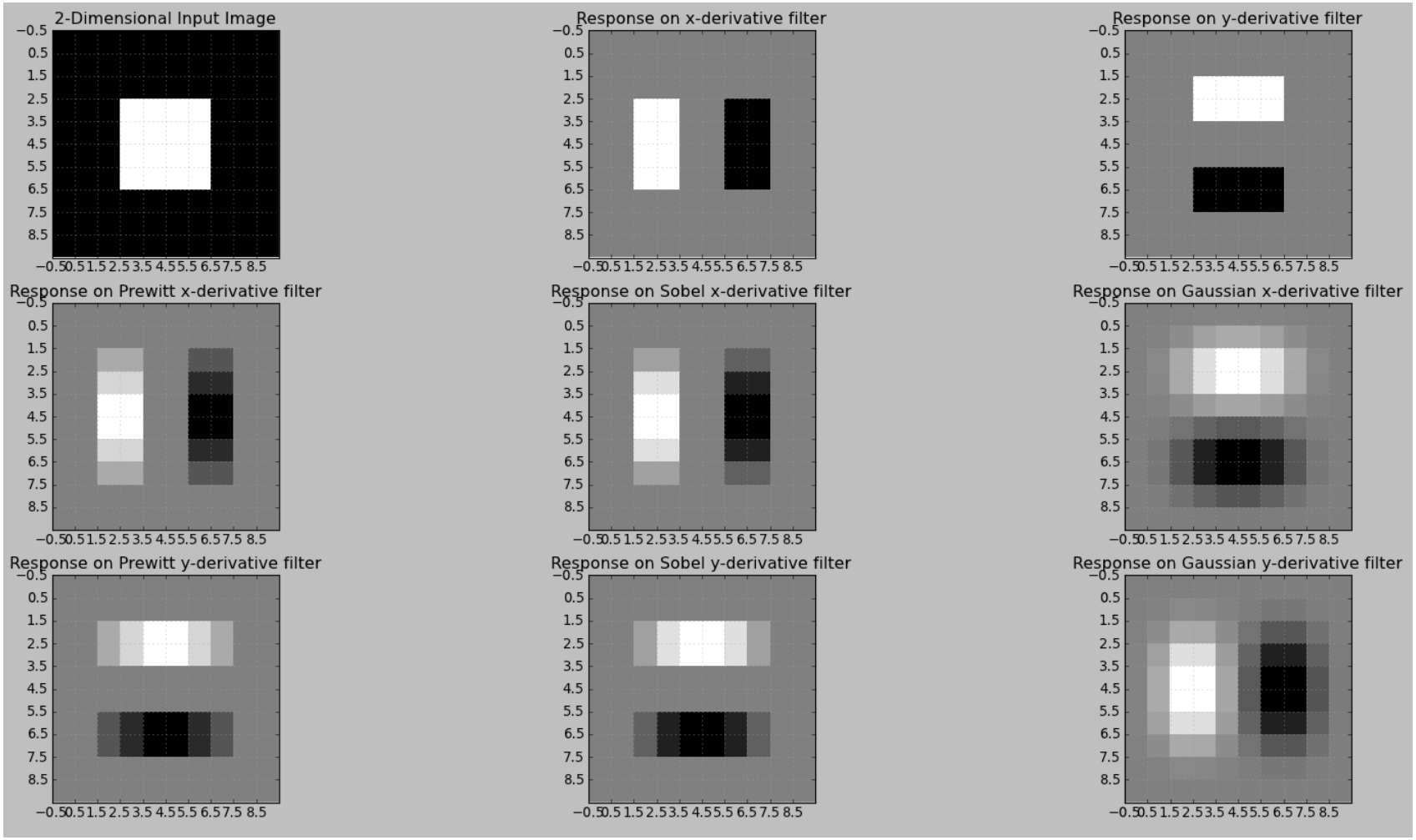
$$\|\nabla F\| = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2} \tag{16}$$

- and its direction

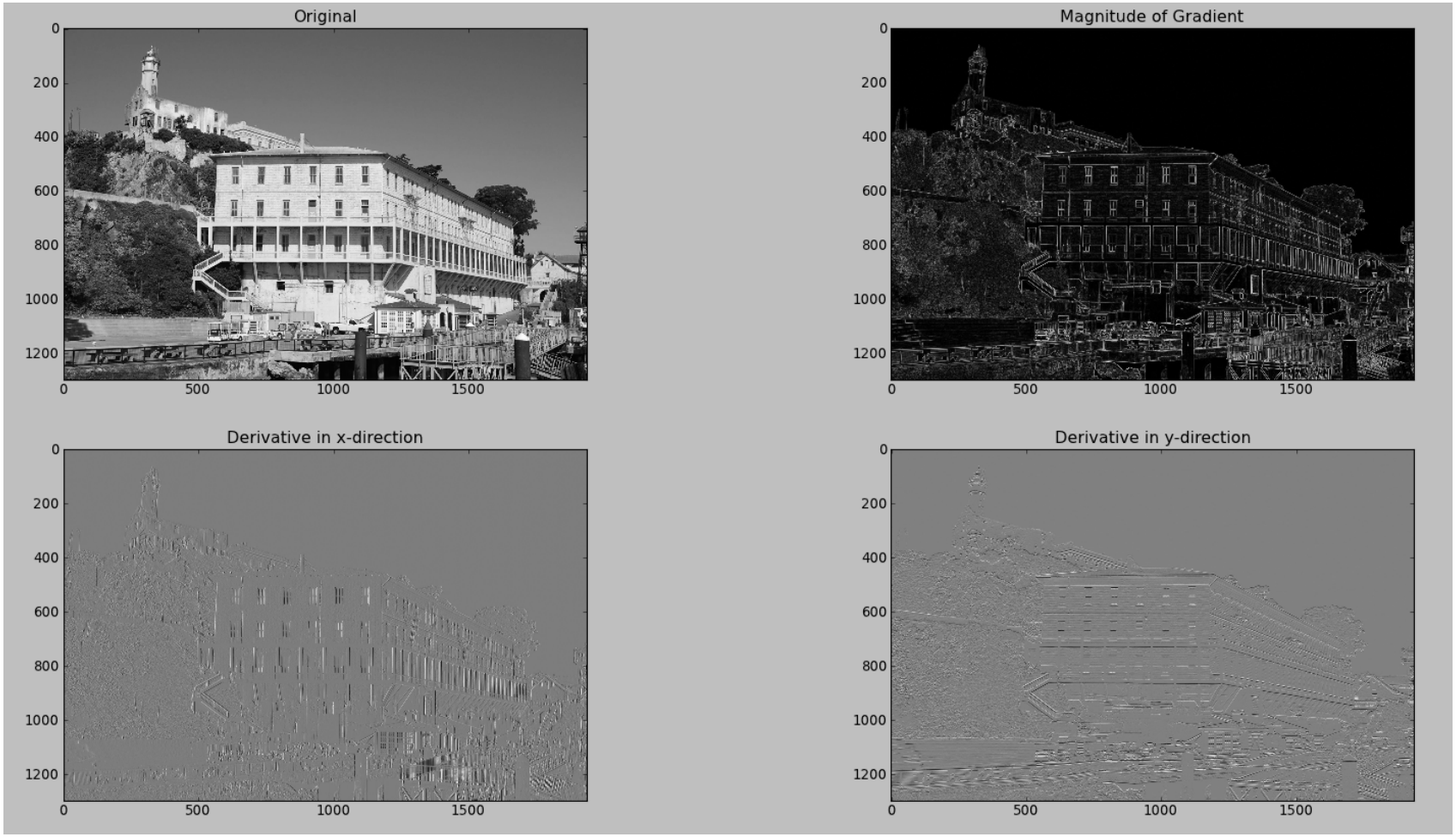
$$\theta = \arctan\left(\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}\right) \tag{17}$$

- Edges are at the extremas of $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$.

Application of edge detectors to simple image



Application of Sobel edge detectors to real image



Laplacian Operator, Laplacian of Gaussians

- The Laplacian Operator $\nabla^2 F$ is the undirected 2nd derivative of a 2-dimensional image F:

$$\nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} \tag{18}$$

- Further derivation of the 1st order derivatives of the Gaussian from equations (13) and (14) yields:

$$G_{\sigma,xx} = \frac{\partial^2(G_\sigma(x,y))}{\partial x^2} = \frac{\partial(G_{\sigma,x}(x,y))}{\partial x} = \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right) G_\sigma(x,y) \tag{19}$$

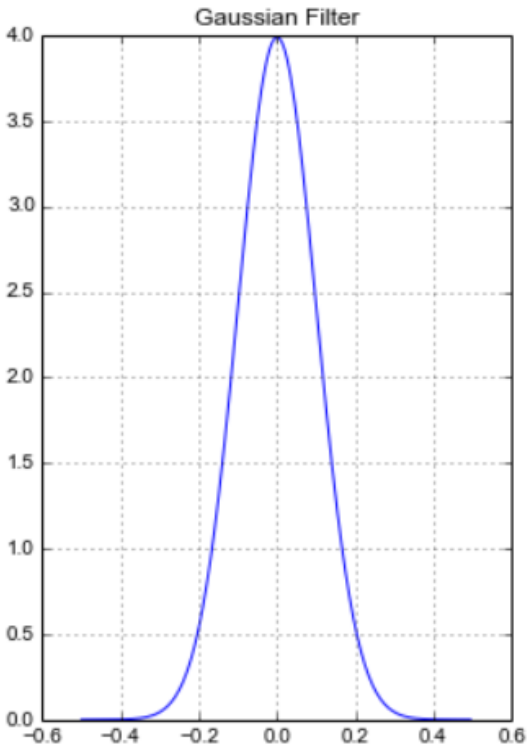
$$G_{\sigma,yy} = \frac{\partial^2(G_\sigma(x,y))}{\partial y^2} = \frac{\partial(G_{\sigma,y}(x,y))}{\partial y} = \left(\frac{y^2}{\sigma^4} - \frac{1}{\sigma^2}\right) G_\sigma(x,y) \tag{20}$$

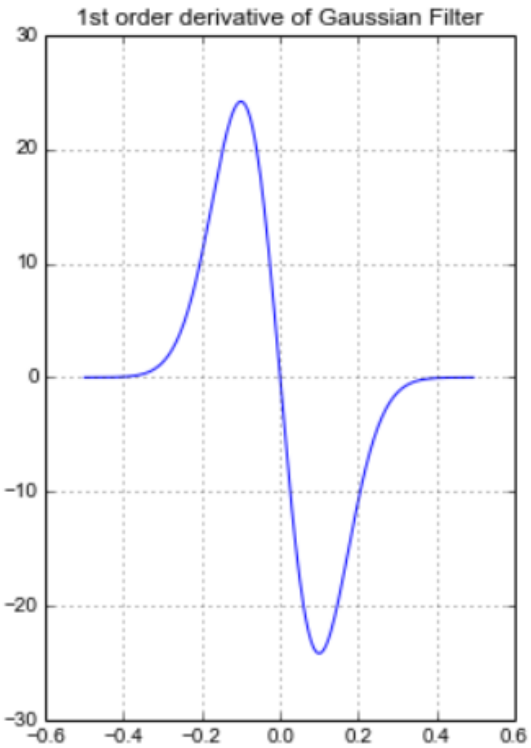
- Smoothing an image F with a Gaussian and then taking its Laplacian operator is equivalent to convolving the image F with the Laplacian of Gaussian (LoG) Filter:

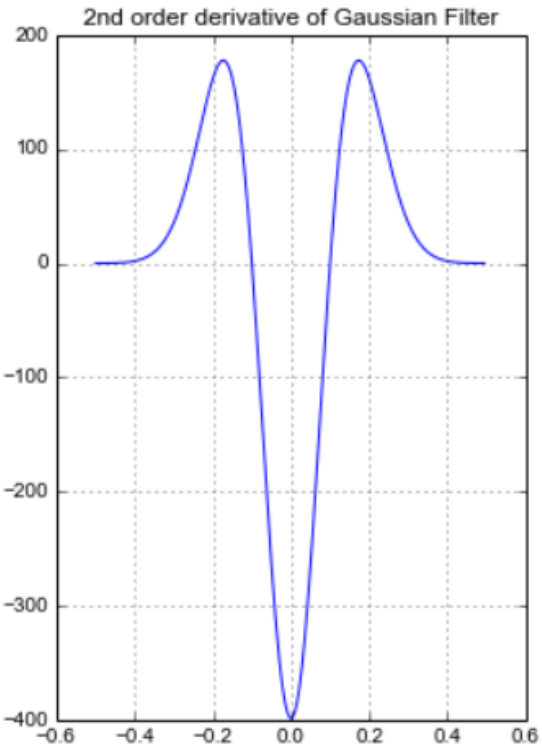
$$\nabla^2 G_\sigma(x,y) = \left(\frac{x^2+y^2}{\sigma^4} - \frac{2}{\sigma^2}\right) G_\sigma(x,y) \tag{21}$$

- Edges in F are zero crossings of the Laplacian operator applied to F

Derivatives of Gaussian Filter







Remarks and Outlook

- Much more edge detection techniques exist, e.g. Canny
- Edge detection and corner detection is an essential step in local feature extraction (chapter 5). Corner detection is introduced there.

References I

[1] R. Szeliski. *Computer Vision: Algorithms and Applications*. Springer-Verlag New York, Inc., New York, NY, USA, 1st edition, 2010.