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109-1 Statistics
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Homework #1

Due by Sunday 10/11 23:55

Question #A

Please derive the score function as a function of β for the Rayleigh distribution.

Solution:

The Rayleigh distribution is defined as:

$$f(x;\beta) = \frac{x}{\beta^2} e^{-x^2/2\beta^2} \tag{1}$$

And the score function (log-likelihood) is:

$$l(\theta) = logL(\theta) = \sum_{i=1}^{n} log f_\theta(x_i) \tag{2}$$

Therefore, the score function of the Rayleigh distribution is:

$$l(\beta) = \sum_{i=1}^{n} log(\frac{x_i}{\beta^2} e^{-x_i^2/2\beta^2}) = -nlog\beta^2 + \sum_{i=1}^{n}(logx_i - \frac{x_i^2}{2\beta^2}) \tag{3}$$

Question #B

Please find the fitted value of β for the Rayleigh distribution. Also draw the score function.

Solution:

From last section, we have

$$l(\beta) = -nlog\beta^2 + \sum_{i=1}^{n}(logx_i - \frac{x_i^2}{2\beta^2}) \tag{4}$$

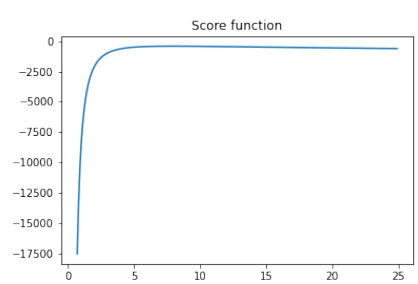
To find the fitted value of β, we have to maximize $l(\beta)$, that is, solve $\frac{\partial l(\beta)}{\partial \beta} = 0$. Then we have:

$$\frac{\partial l(\beta)}{\partial \beta} = \frac{2n}{\beta} - \sum_{i=1}^{n} \frac{x_i^2}{4\beta^3} = 0 \tag{5}$$

Thus,

$$\beta_{MLE} = \sqrt{\frac{1}{2n}\sum_{i=1}^{n} x_i^2} = 7.872894308240928 \tag{6}$$

The following is the figure of the score function.



Question #C

Please draw the empirical function and the fitted Rayleigh distribution on the same plot. Comment on whether the Rayleigh assumption is plausible.

Solution:

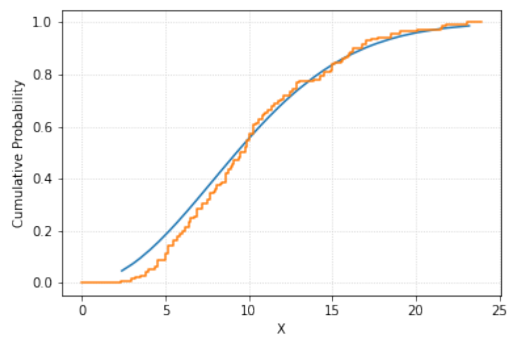
The cumulative distribution function of Rayleigh distribution is:

$$F(x; \beta_{MLE}) = 1 - exp(\frac{-x^2}{2\beta_{MLE}^2}) \tag{7}$$

And the empirical function is:

$$\overline{F}(x) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq x) \tag{8}$$

Thus, the following is the figure of the empirical function and the fitted Rayleigh distribution on the same plot.



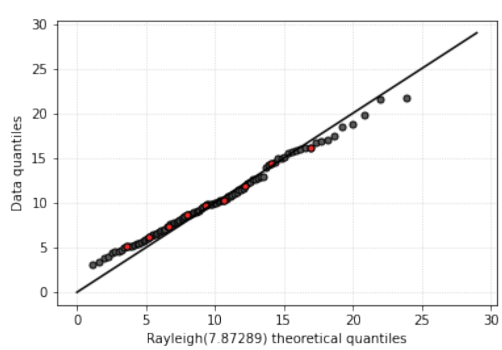
Since the two curves are very close, the parametric assumption is plausible.□

Question #D

Please draw a Q-Q plot. Comment on whether the Rayleigh assumption is plausible.

Solution:

The following is the figure of the Q-Q plot.



Since the points are located along 45 degree line, the model assumption is plausible.□