In prep. for JPO

Dynamical insights from Lagrangian-filtered structure functions

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TRR 181 Eddy-Wave Meeting, .2023

Gulf of Mexico surface drifter

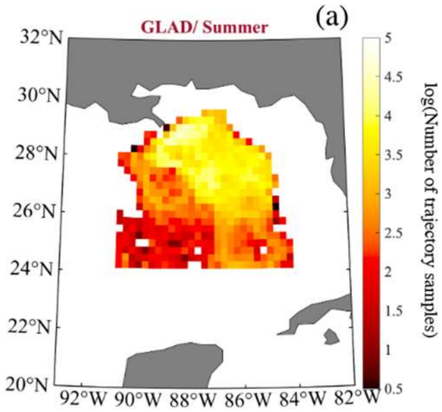
A Waterlust & CARTHE film

Similar spatial region, drogued on the surface.
Velocities at 15 min interval. Spans 100 m – 100 km.

Lagrangian filtering   2nd order structure functions

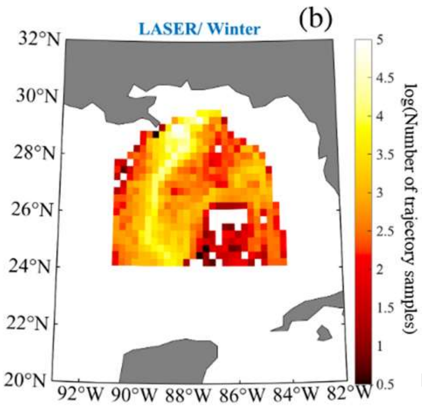
Grand LAgrangian Deployment (GLAD)

- Summer 2012



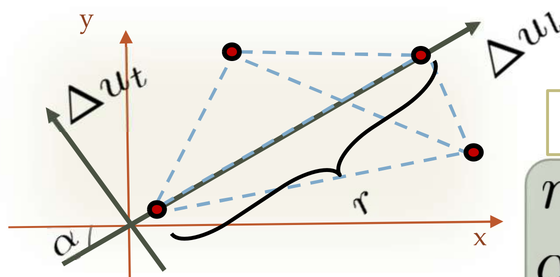
Lagrangian Submesoscale Experiment (LASER)

- Winter 2016



Balwada, Xie, Marino and Feraco 2022

2nd order structure functions



Separation vector

r Separation distance

α Separation angle

- At each snapshot, $\Delta \mathbf{u}$ projected into directions along/across the separation vector (r, α)
- Under horizontal homogeneity and stationarity,
  $\overline{\left(u_l\left(\mathbf{r_0}; t_o\right) - u_l\left(\mathbf{r_0} + \mathbf{r}; t_o\right)\right)^2}$ is a function of $\mathbf{r}$ only

→second order structure functions :

$$D_{ll}(r,\alpha) = \overline{\Delta {u_l}^2(r,\alpha)}$$

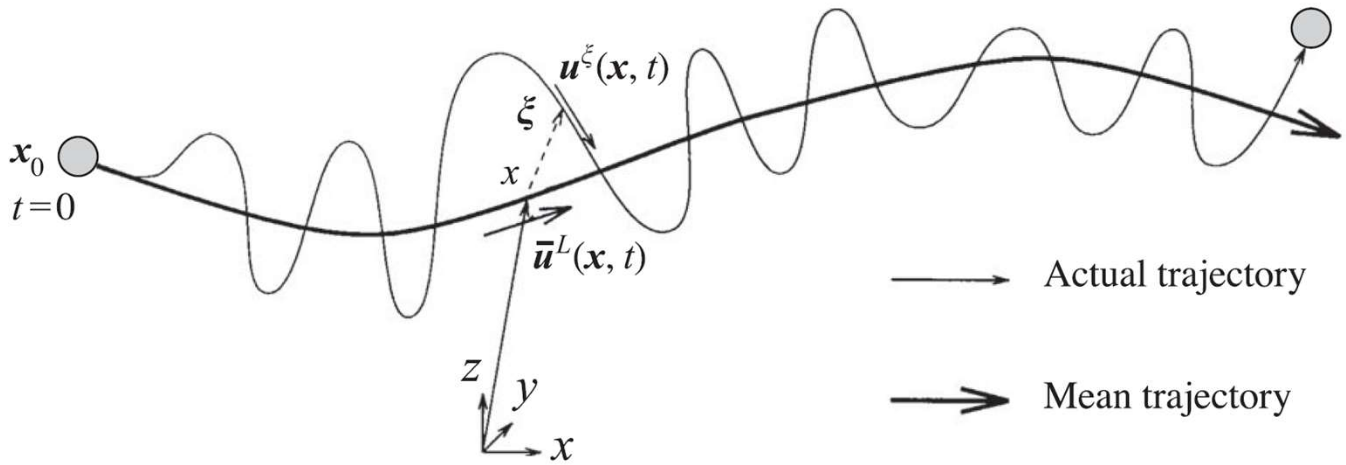
$$D_{tt} = \overline{\Delta {u_t}^2(r,\alpha)}, D_{lt} = \overline{\Delta u_l(r,\alpha)\Delta u_t(r,\alpha)}$$

Lagrangian filtering ("LF")

$$\boldsymbol{x} + \boldsymbol{\xi}(\boldsymbol{x}, t)$$

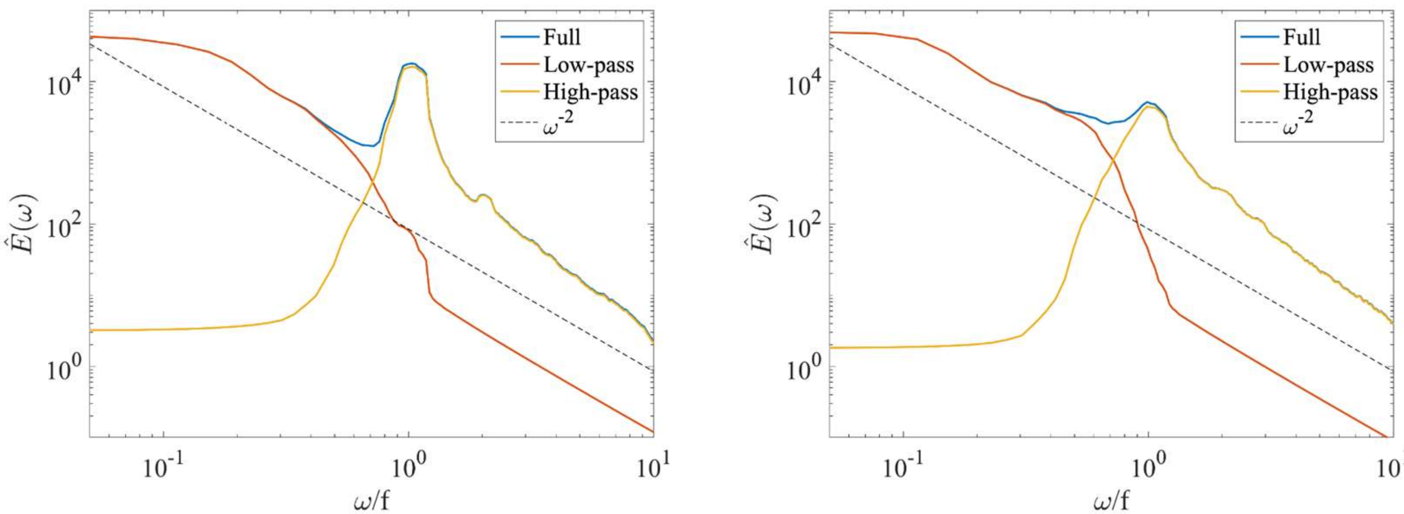
slow fast

$$\overline{\boldsymbol{u}}^L + \boldsymbol{u}^\xi$$



Bühler 2009

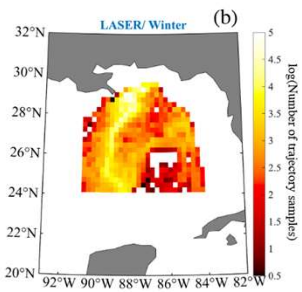
LF-ed velocities

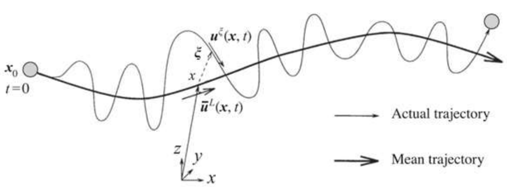


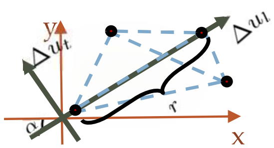
(a) *GLAD* (b) *LASER*

- 1.5 days
- High-pass includes f

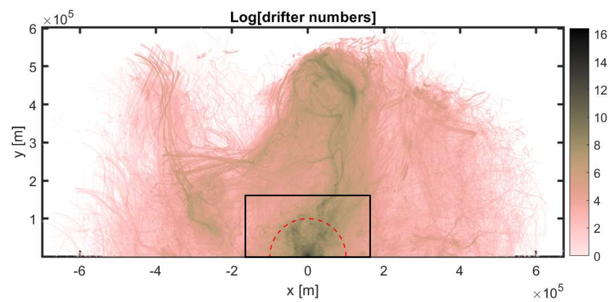
Mean (low-pass) trajectories

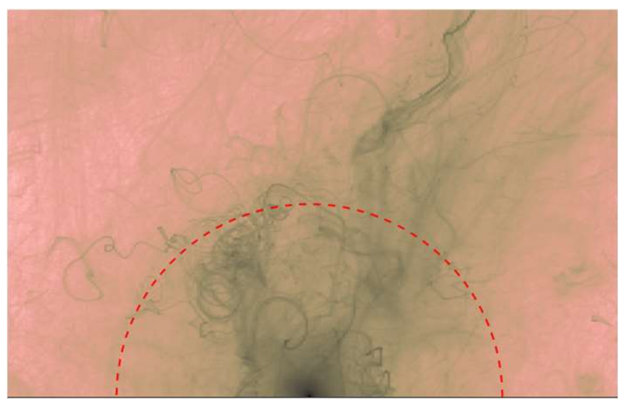




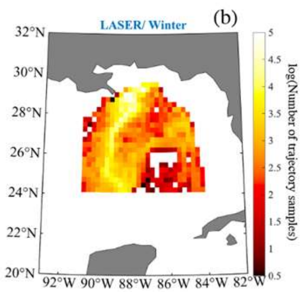


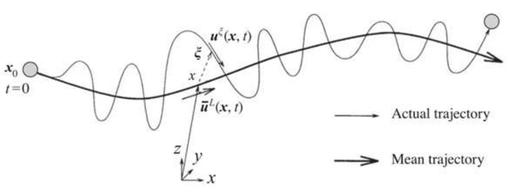
Separation vector from mean trajectories

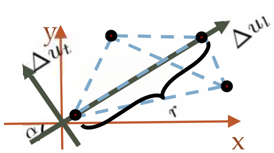




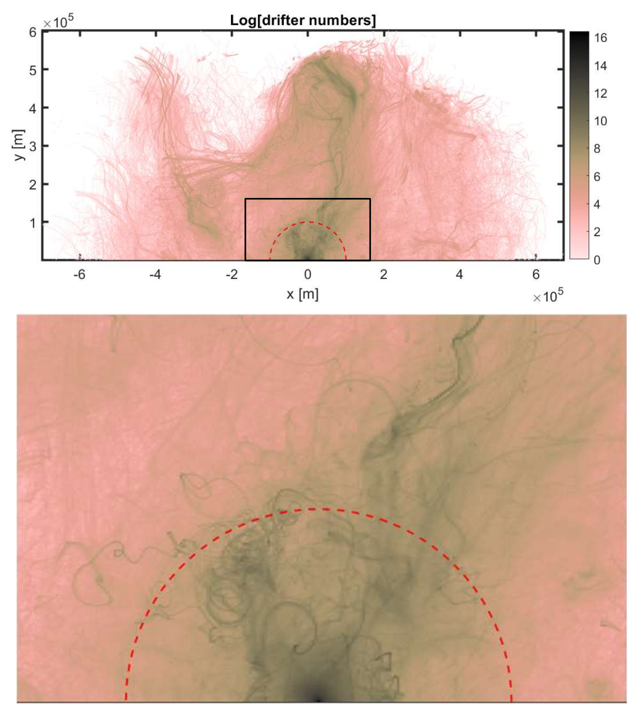
Effects of LF



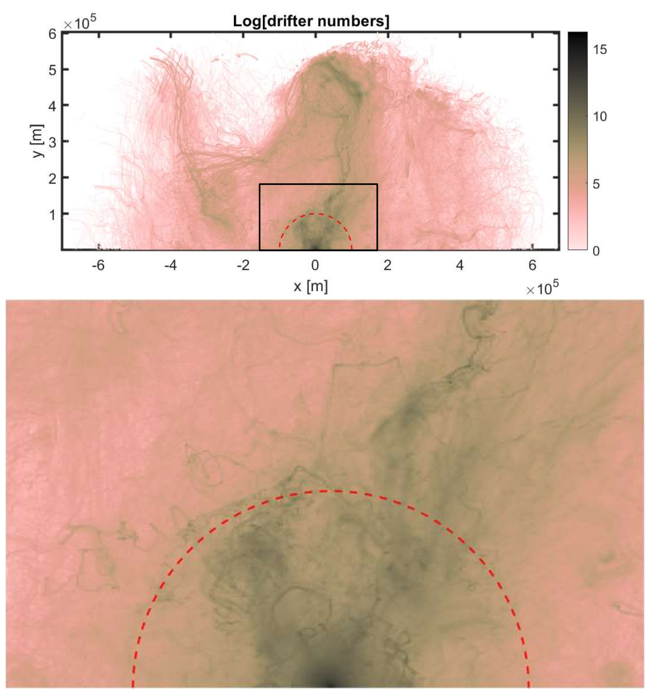




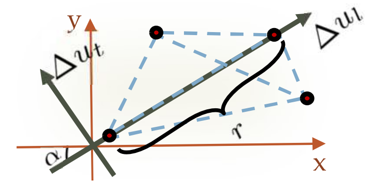
Separation vector from mean trajectories



... from raw trajectories



Helmholtz decomposition



$$\boxed{\begin{aligned} u &= \phi_x - \psi_y \\ v &= \phi_y + \psi_x \end{aligned}}$$

$$\Delta \mathbf{u} = \Delta \mathbf{u}_{\mathrm{rot}} + \Delta \mathbf{u}_{\mathrm{div}}$$

due to rotational

Consider isotropy

To relax later

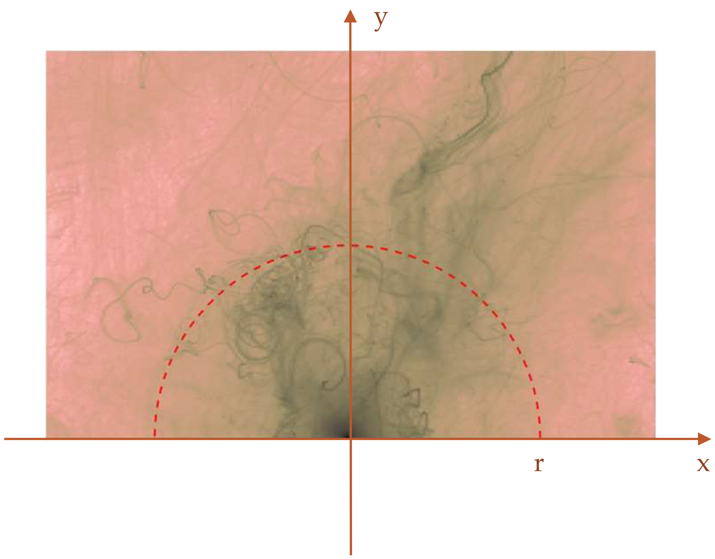
$$D_{ll}(r, \cancel{\alpha}) = \overline{\Delta {u_l}^2 (r, \cancel{\alpha})}$$

$$D_{tt} = \overline{\Delta {u_t}^2 (r, \cancel{\alpha})}, D_{lt} = \overline{\Delta u_l (r, \cancel{\alpha}) \Delta u_t (r, \cancel{\alpha})}$$

$$\boxed{\begin{aligned} D_{\mathrm{rot}}(r) &= \int_0^r \left(\frac{-D_{ll}(\tau) + D_{tt}(\tau)}{\tau} \right) \mathrm{d}\tau + D_{tt}(r), \\ D_{\mathrm{div}}(r) &= -\int_0^r \left(\frac{-D_{ll}(\tau) + D_{tt}(\tau)}{\tau} \right) \mathrm{d}\tau + D_{ll}(r) \end{aligned}}$$

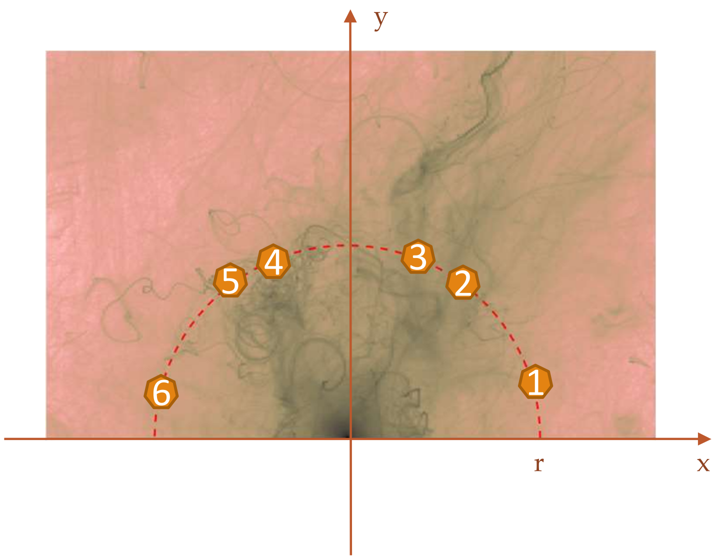
Bühler, Callies and Ferrari 2014
Lindborg 2015

Angle weighting



- Anisotropic distribution of separation vector

Angle weighting



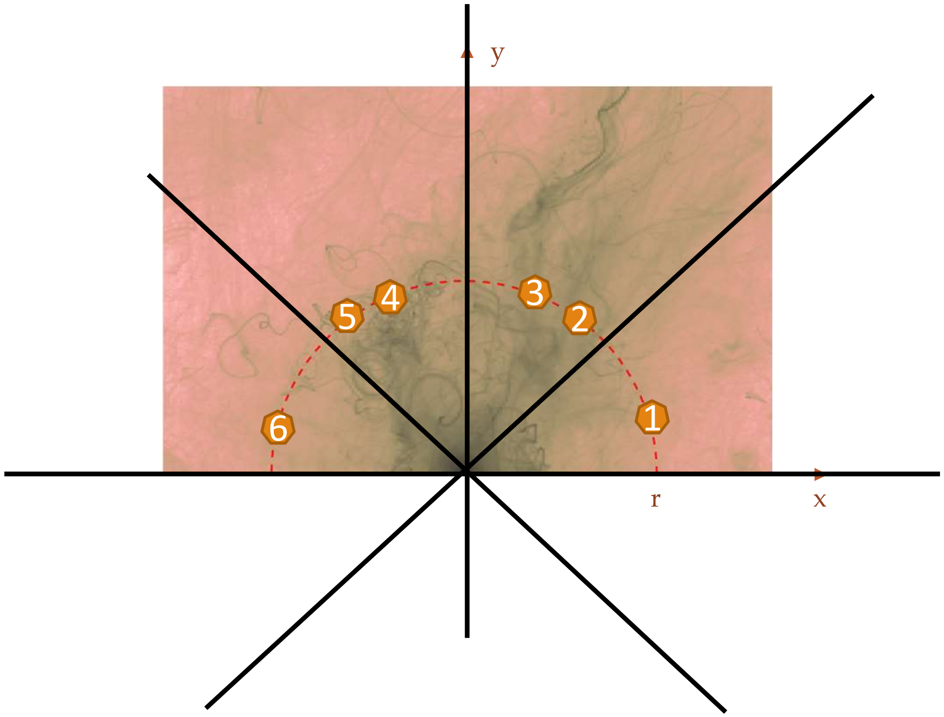
- Anisotropic distribution of separation vector

To compute $D_{ll}(r) = \overline{\Delta u_l{}^2(r)}$

- Traditional: unweighted average over all observations at each r
  (1 + 2 + 3 + 4 + 5 + 6)/6

- Angle-weighted (Wang and Bühler 2021): Weight by angles to account for nonuniformity

Angle weighting



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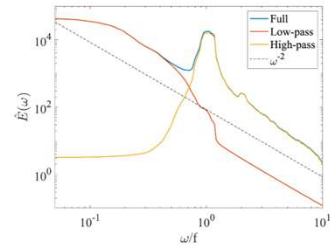
- Traditional: unweighted average over all observations at each r
  (1 + 2 + 3 + 4 + 5 + 6)/6

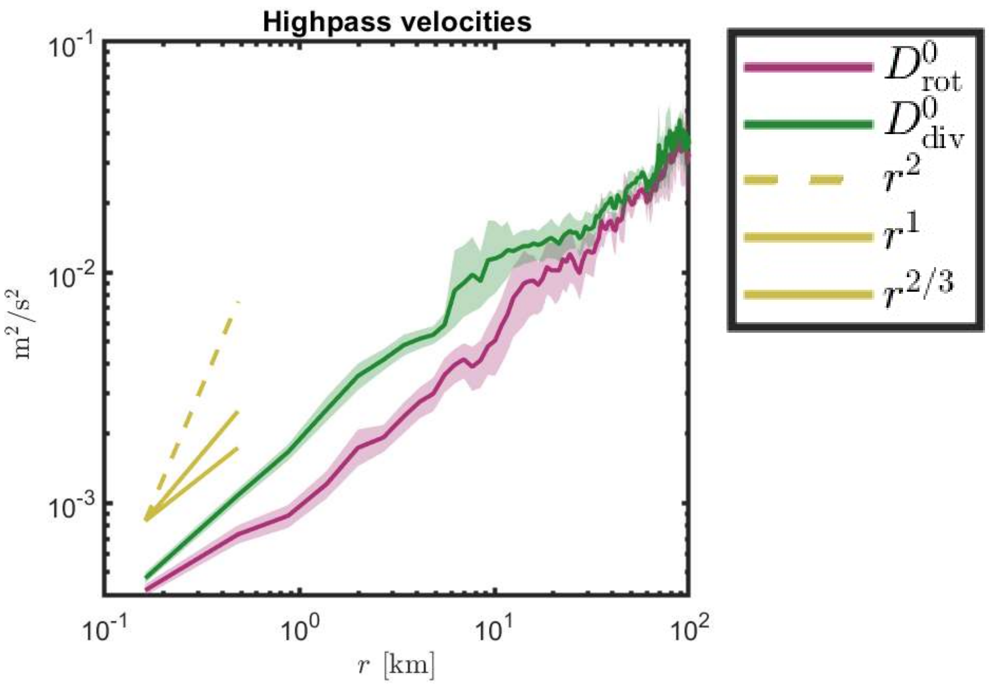
- Angle-weighted (Wang and Bühler 2021): Weight by angles to account for nonuniformity
  [1 +(2 + 3)/2 +(4 + 5)/2 + 6]/4



(a) GLAD

$$\boxed{\begin{aligned} u &= \phi_x - \psi_y \\ v &= \phi_y + \psi_x \end{aligned}}$$

Helmholtz: GLAD, highpass



- div >= rot, consistent with linear theory

$$\boldsymbol{u}_t + \cancel{\boldsymbol{u}\cdot\nabla\boldsymbol{u}} + \boldsymbol{f}\times\boldsymbol{u} + g\nabla\eta = 0$$

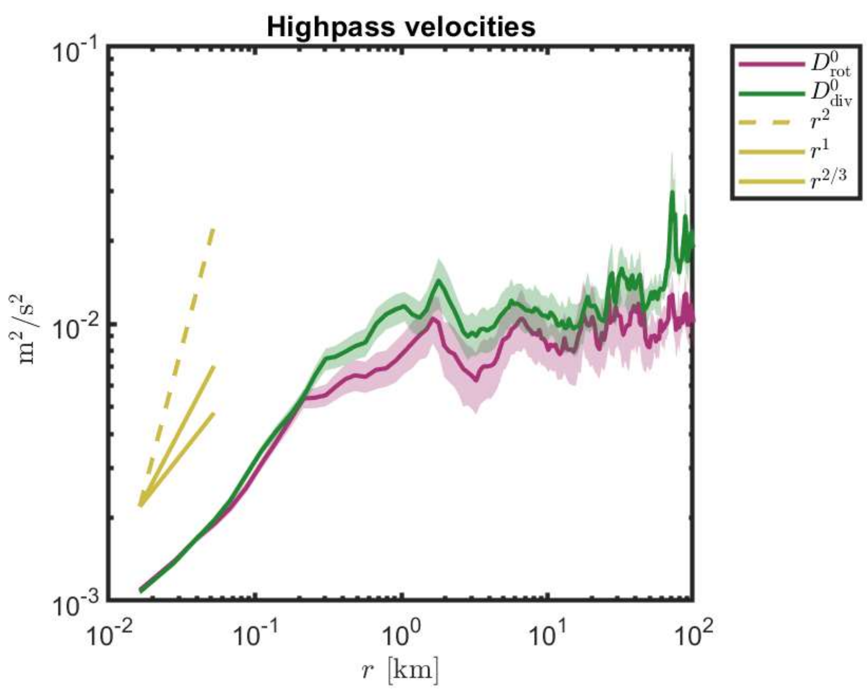
$$\Rightarrow \quad \frac{\partial\psi}{\partial t} + f\phi = 0$$

$$\Rightarrow \quad \frac{D_{\mathrm{div}}(r)}{D_{\mathrm{rot}}(r)} \approx \left|\frac{\omega(k)}{f}\right|^2, \quad k = 2\pi/r$$

$$|\omega| \geq f \Rightarrow D_{\mathrm{div}}(r) \geq D_{\mathrm{rot}}(r)$$

- div ~ rot at larger scales: NIWs! $|\omega| \approx f, \quad k << m$
- Slope

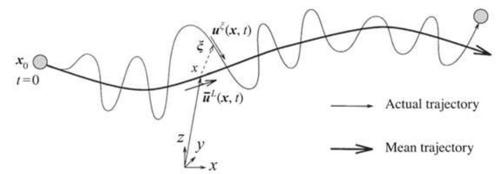
Helmholtz: LASER, highpass



- div >= rot, consistent with linear theory

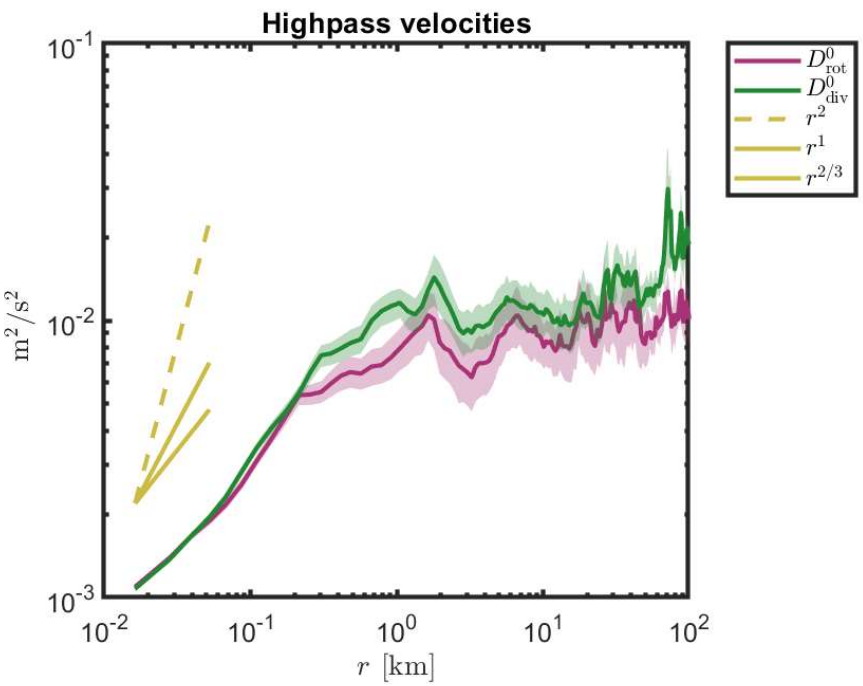
$$\frac{D_{\mathrm{div}}(r)}{D_{\mathrm{rot}}(r)} \approx \left|\frac{\omega(k)}{f}\right|^2, \quad k = 2\pi/r$$

- Decorrelation; may be impacts from balanced flows?

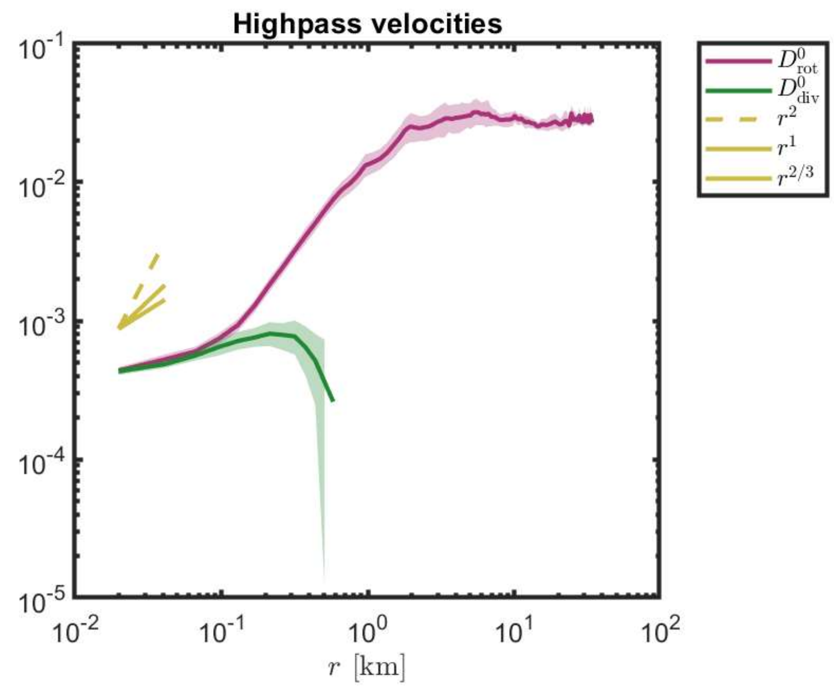


Mean trajectories make more sense!

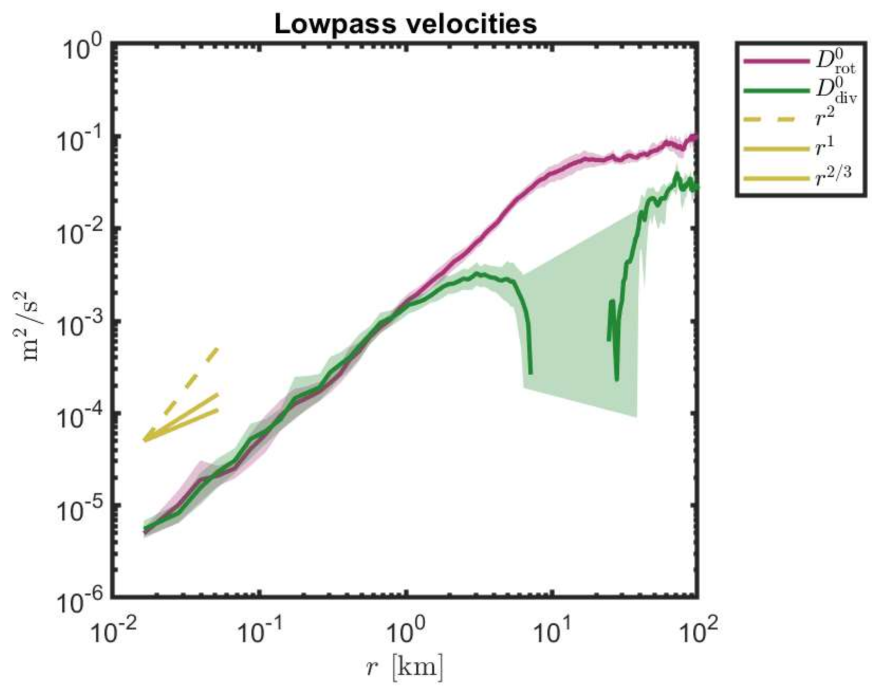
On mean trajectories, LASER



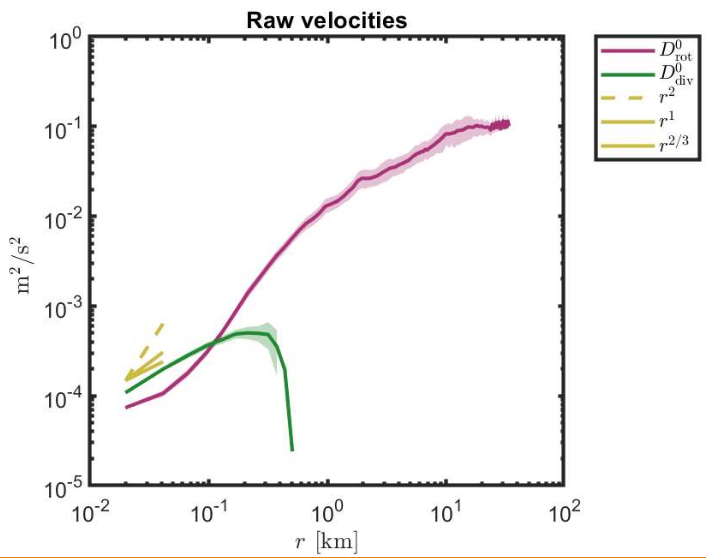
On raw trajectories, LASER



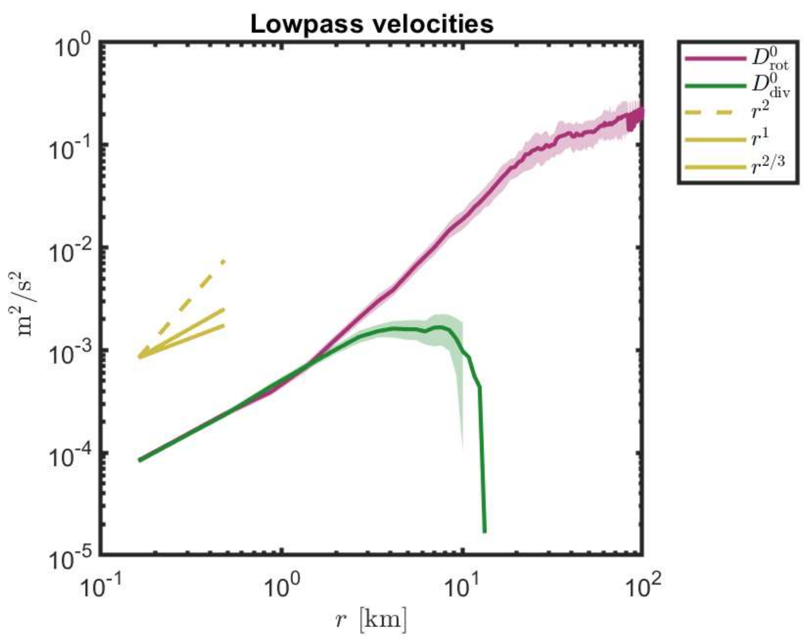
Helmholtz: LASER, lowpass



- Dominance of rot at > 10 km
- At <1 km, equipartition
- Compare with raw:



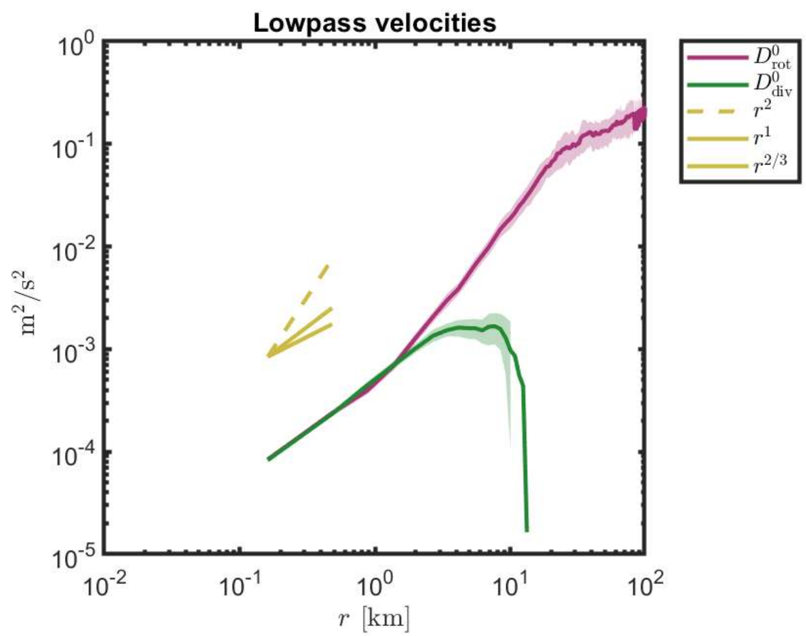
Helmholtz: GLAD, lowpass



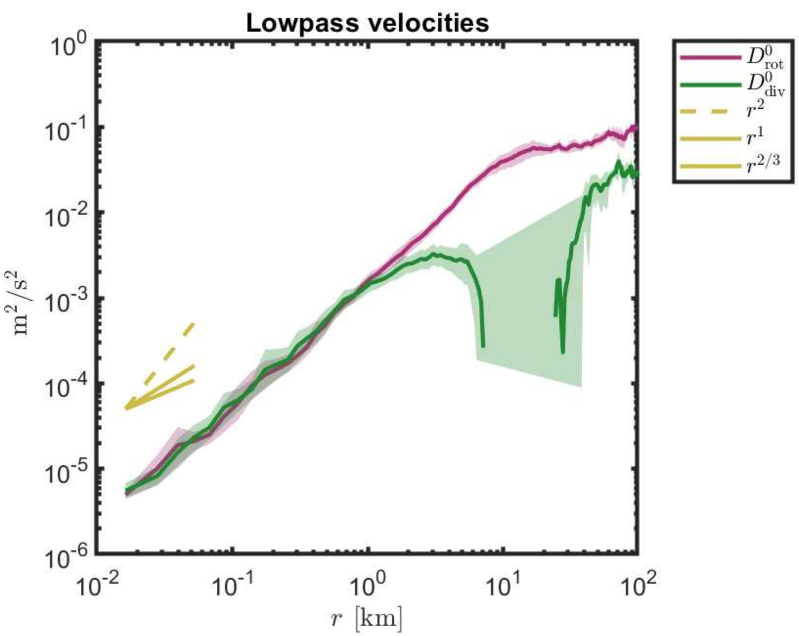
- Dominance of rot at > 10 km
- At <1 km, equipartition

Transitional scales insensitive to mixed layer thickness

GLAD (summer)

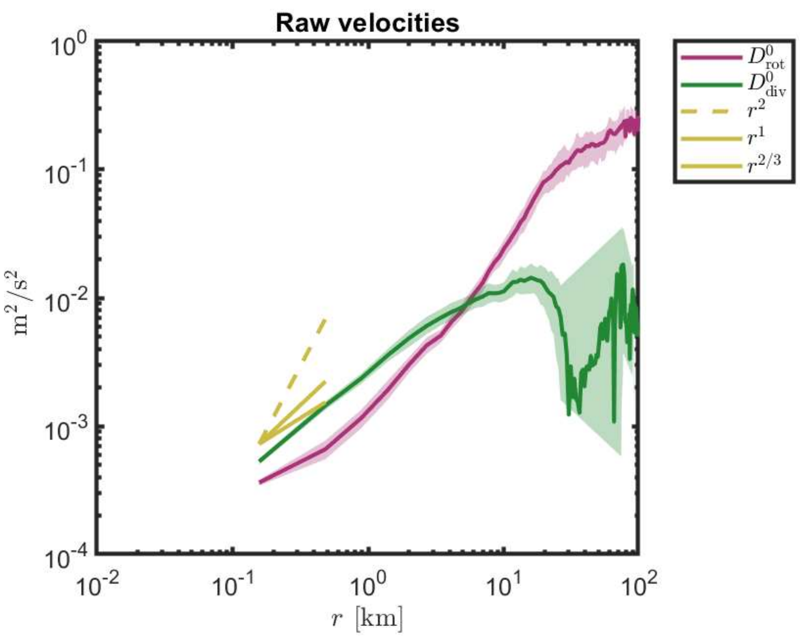


LASER (winter)

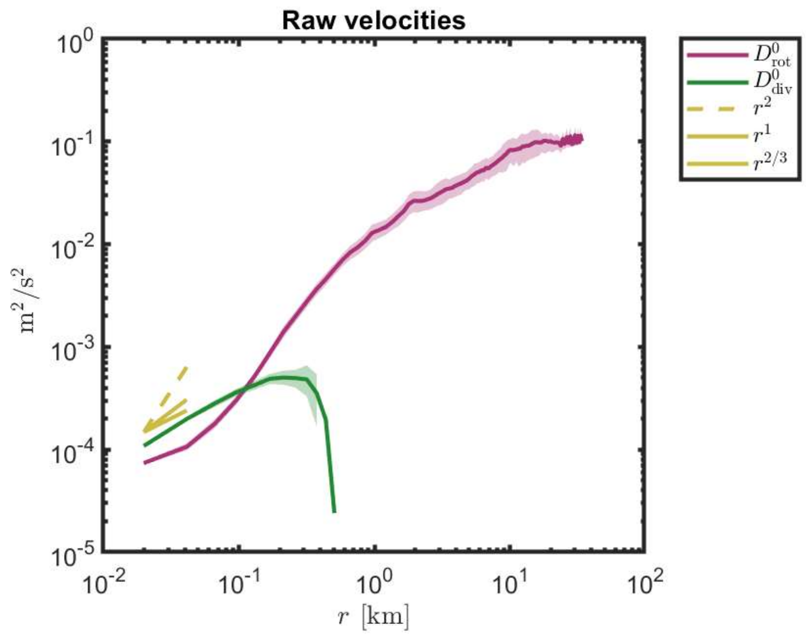


… which can't be seen in unfiltered data

GLAD



LASER



Anisotropic Helmholtz decomp.

$$f(r,\alpha) = \sum_{n=0}^{\infty} f^{cn}(r)\cos(n\alpha) + f^{sn}(r)\sin(n\alpha),$$

→ Evaluate $D^{c0}_{\mathrm{rot}}(r), D^{c2}_{\mathrm{rot}}(r), D^{s2}_{\mathrm{rot}}(r), \ldots$

→ Compare

$$D^{0}_{\mathrm{rot}} = D^{c0}_{\mathrm{rot}}$$

with

$$D^{2}_{\mathrm{rot}} = \sqrt{\left(D^{c2}_{\mathrm{rot}}\right)^2 + \left(D^{s2}_{\mathrm{rot}}\right)^2}$$

2: "2nd Fourier mode"

$$D^{cn}_{\mathrm{rot}}(r) = \int_0^r \left(\frac{-D^{cn}_{ll}(\tau) + D^{cn}_{tt}(\tau)}{\tau} - n\frac{D^{sn}_{lt}(\tau)}{\tau} \right) \mathrm{d}\tau + D^{cn}_{tt}(r),$$

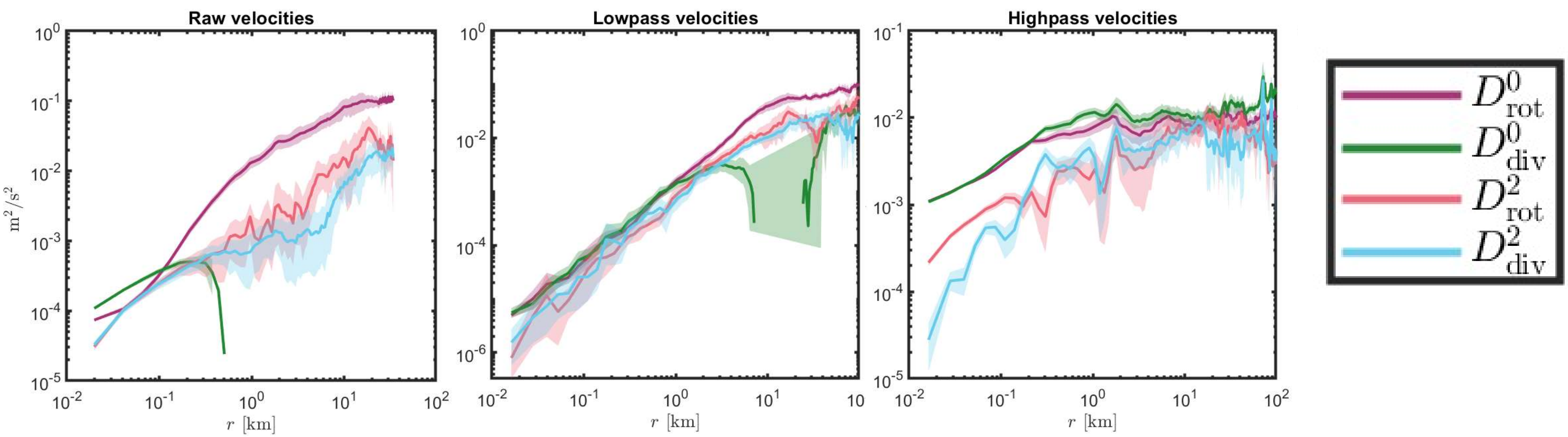
$$D^{sn}_{\mathrm{rot}}(r) = \int_0^r \left(\frac{-D^{sn}_{ll}(\tau) + D^{sn}_{tt}(\tau)}{\tau} + n\frac{D^{cn}_{lt}(\tau)}{\tau} \right) \mathrm{d}\tau + D^{sn}_{tt}(r),$$

$$D^{cn}_{\mathrm{div}}(r) = -\int_0^r \left(\frac{-D^{cn}_{ll}(\tau) + D^{cn}_{tt}(\tau)}{\tau} - n\frac{D^{sn}_{lt}(\tau)}{\tau} \right) \mathrm{d}\tau + D^{cn}_{ll}(r),$$

$$D^{sn}_{\mathrm{div}}(r) = -\int_0^r \left(\frac{-D^{sn}_{ll}(\tau) + D^{sn}_{tt}(\tau)}{\tau} + n\frac{D^{cn}_{lt}(\tau)}{\tau} \right) \mathrm{d}\tau + D^{sn}_{ll}(r).$$

Wang and Bühler, 2021

Anisotropic modes in LASER



Not so significant

More significant at smaller scales

More significant at larger scales

Take home message

Methods

- Combine spatial and temporal info

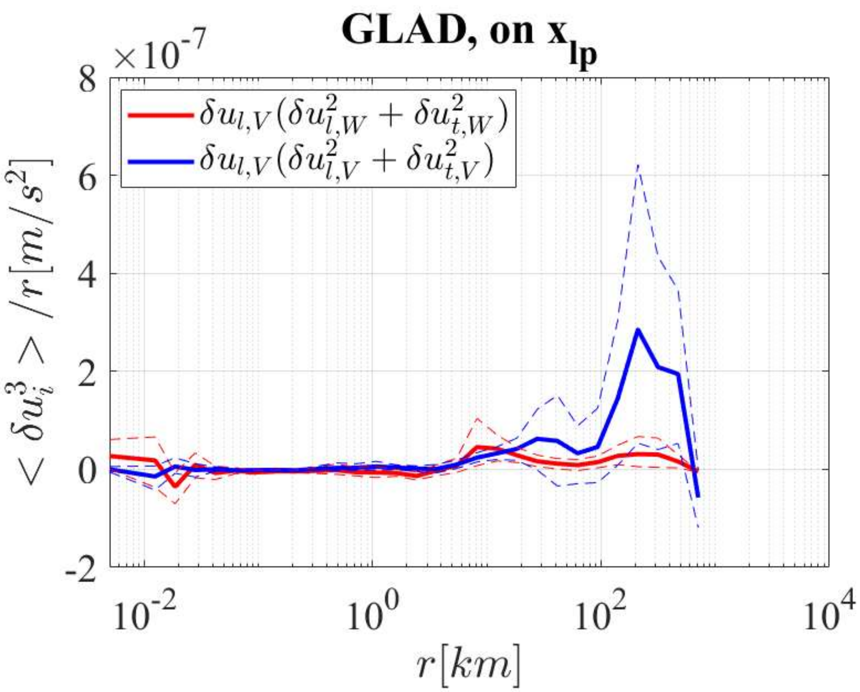
  Helmholtz   LF

- Angle weighting: simple idea to avoid bias from nonuniform angle distribution. Useful for isotropic modes too
- LF: High/low-pass velocities on mean trajectories

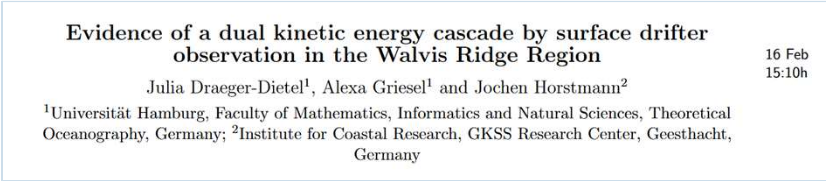
Observations (LASER and GLAD)

- High-pass component consistent with linear wave theory, esp. near-inertial waves
- Low-pass component: transitional scale insensitive to mixed layer thickness
- See anisotropy more in frequency-filtered fields

Third-order structure functions



- Reflects energy transfer rate
- Recent work on unfiltered data:
  Balwada, Xie, Marino and Feraco 2022



Evidence of a dual kinetic energy cascade by surface drifter observation in the Walvis Ridge Region

16 Feb
15:10h

Julia Draeger-Dietel[1], Alexa Griesel[1] and Jochen Horstmann[2]

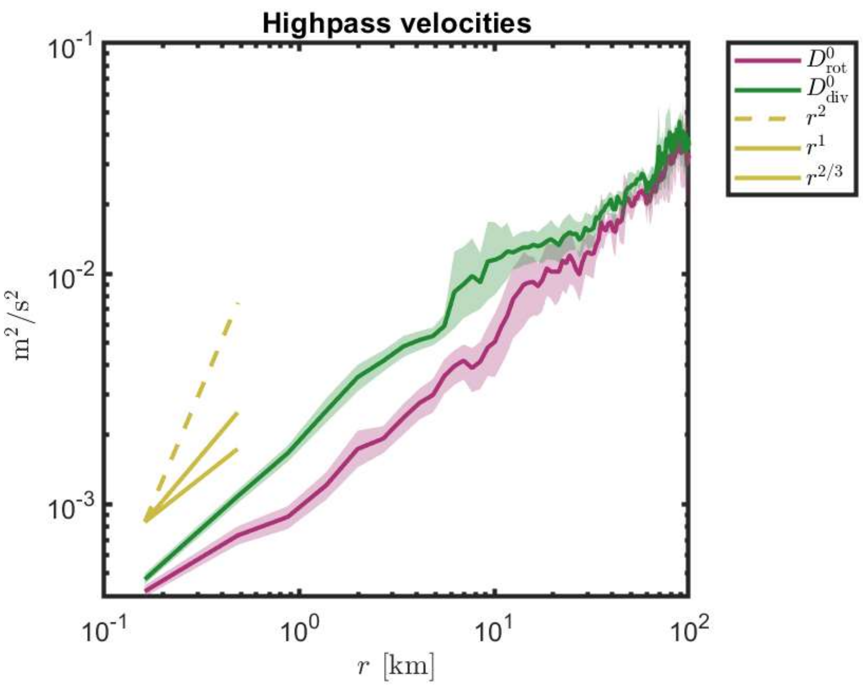
[1]Universität Hamburg, Faculty of Mathematics, Informatics and Natural Sciences, Theoretical Oceanography, Germany; [2]Institute for Coastal Research, GKSS Research Center, Geesthacht, Germany

- Under some theoretical constraints, we can decompose slow-fast and slow-slow transfer rates
- Collaboration welcome

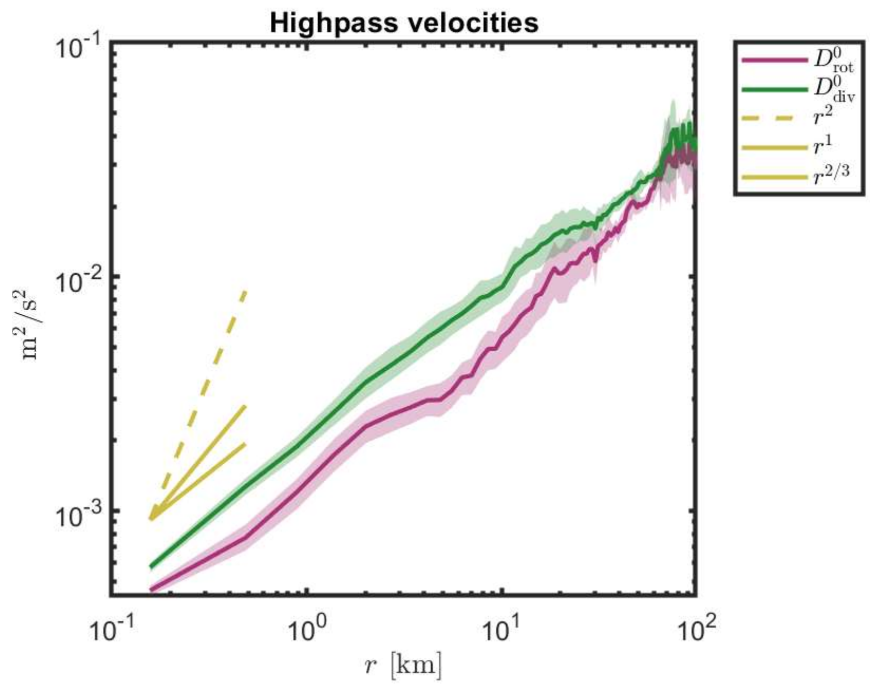
Backup slides

…and it's not coding error

On LF-ed locations, GLAD



On raw locations, GLAD



GLAD