Euler angles

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1 Solve for α, β, γ

The rotation matrix of spin 1/2 by euler angles is shown in Equation. 1

$$D(\alpha, \beta, \gamma) = \begin{bmatrix} e^{-i(\alpha+\gamma)/2} cos(\beta/2) & -e^{-i(\alpha-\gamma)/2} sin(\beta/2) \\ e^{i(\alpha-\gamma)/2} sin(\beta/2) & e^{i(\alpha+\gamma)/2} cos(\beta/2) \end{bmatrix} \tag{1}$$

Compare this to the matrix

$$D(\alpha, \beta, \gamma) = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix} \tag{2}$$

We find

$$|a| = cos(\beta/2) \tag{3}$$
$$|b| = sin(\beta/2) \tag{4}$$
$$a \cdot b = -e^{-i\alpha} sin(\beta/2) cos(\beta/2) \tag{5}$$
$$a/b = -e^{-i\gamma} cot(\beta/2) \tag{6}$$

From which we can solve for

$$\beta = 2arccos(|a|) \tag{7}$$

$$\alpha = ilog\left(-\frac{a \cdot b}{|a| \cdot |b|}\right) \tag{8}$$

$$\gamma = ilog\left(-\frac{a/b}{|a|/|b|}\right) \tag{9}$$

$$\tag{10}$$

2 Singularities

In case of singularities, there is little I can do in terms of stability, but I attempt to crudely handle them so that they do not blow up.

2.1 $a = 0$

Since this correponds to a $D^y(\pi)$ rotation, which commutes arbitrary z rotation, α and γ are not fully constrained. One can also see this from equation 11, where $(\alpha + \gamma)$ can be anything. We choose $\alpha = 0$ for an opaque reason that is consistent with the theory.

$$a = e^{-i(\alpha+\gamma)/2} cos(\beta/2) = 0 \tag{11}$$

implies that

$$\beta = \pi \tag{12}$$

$$\gamma = -2ilog(-b) \tag{13}$$

2.2 $b = 0$

The analysis is similar to the case of $a = 0$. This case corresponds to a free precession. Taking $\alpha = 0$ as the choice.

$$\beta = 0 \tag{14}$$

$$\gamma = 2ilog(a) \tag{15}$$