

$$\begin{aligned}3 &= a_0 + (-2)a_1 + 4a_2 + e_0 \\1 &= a_0 + (-1)a_1 + 1a_2 + e_1 \\1 &= a_0 + (1)a_1 + 1a_2 + e_2 \\2 &= a_0 + (2)a_1 + 4a_2 + e_3\end{aligned}$$

$$e^2 = e_0^2 + e_1^2 + e_2^2 + e_3^2$$

$$\begin{aligned}&= (3 - a_0 + 2a_1 - 4a_2)^2 + (1 - a_0 + a_1 - a_2)^2 \\&\quad + (1 - a_0 - a_1 - a_2)^2 + (2 - a_0 - 2a_1 - 4a_2)^2\end{aligned}$$

$$\begin{aligned}\frac{\partial e^2}{\partial a_0} &= -1 \cdot 2 \cdot (3 - a_0 + 2a_1 - 4a_2) - 1 \cdot 2 \cdot (1 - a_0 + a_1 - a_2) \\&\quad - 1 \cdot 2 \cdot (1 - a_0 - a_1 - a_2) - 1 \cdot 2 \cdot (2 - a_0 - 2a_1 - 4a_2) = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e^2}{\partial a_1} &= 2 \cdot 2 \cdot (3 - a_0 + 2a_1 - 4a_2) + 1 \cdot 2 \cdot (1 - a_0 + a_1 - a_2) \\&\quad - 1 \cdot 2 \cdot (1 - a_0 - a_1 - a_2) - 2 \cdot 2 \cdot (2 - a_0 - 2a_1 - 4a_2) = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e^2}{\partial a_2} &= -4 \cdot 2 \cdot (3 - a_0 + 2a_1 - 4a_2) - 1 \cdot 2 \cdot (1 - a_0 + a_1 - a_2) \\&\quad - 1 \cdot 2 \cdot (1 - a_0 - a_1 - a_2) - 4 \cdot 2 \cdot (2 - a_0 - 2a_1 - 4a_2) = 0\end{aligned}$$

$$-14 + 8a_0 + 20a_2 = 0$$

$$4 + 20a_1 = 0$$

$$-44 + 20a_0 + 68a_2 = 0$$

$$a_1 = -\frac{4}{20} = -0.2$$

$$a_0 = 0.5$$

$$a_2 = 0.5$$

$$\begin{pmatrix} 3 \\ 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

$$e^2 = \left\| \begin{pmatrix} 3 \\ 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} \right\|^2$$

$$\frac{\partial e^2}{\partial a} = -2 \cdot \begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}^T \cdot \begin{pmatrix} 3 \\ 1 \\ 1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}^T \begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

$$= -2 \begin{pmatrix} 7 \\ -2 \\ 22 \end{pmatrix} + 2 \begin{pmatrix} 4 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 34 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

$$= \begin{pmatrix} -14 \\ 4 \\ -44 \end{pmatrix} + \begin{pmatrix} 8 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 68 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \mathbf{0}$$

$$\Rightarrow \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0.5 \\ -0.2 \\ 0.5 \end{pmatrix}$$