

Simulating planetary systems

Newton's law of gravity, machine learning,
and everything in-between



Hanno Rein, University of Toronto

Image credit: NASA/SOFIA/Lynette Cook

Agenda

Early work on Solar System Dynamics

Chaos

Modern numerical methods

Machine learning

REBOUND, REBOUNDx, ASSIST

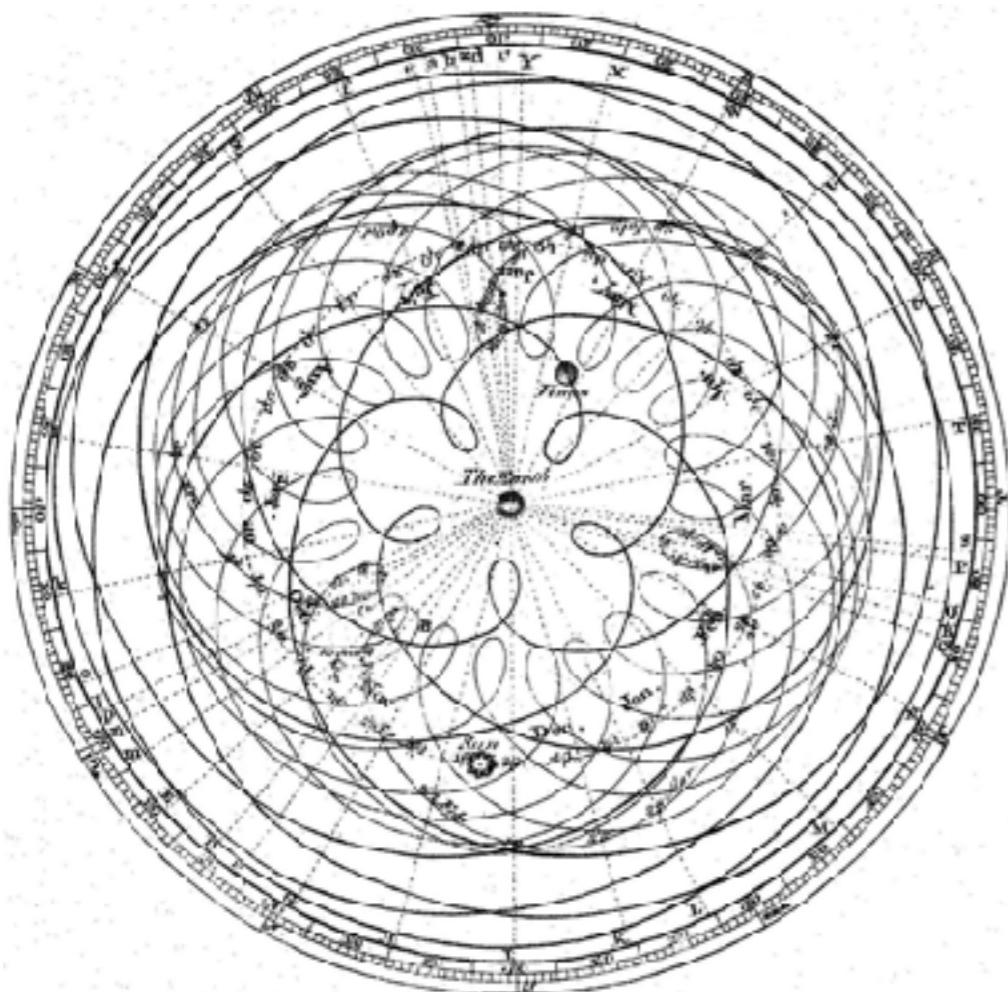
Light pollution

Early work on Solar System Dynamics

Newton (1687)

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j \neq i}}^N m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}$$

Newton (Opticks 1717, 1730)



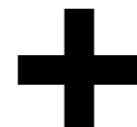
For while comets move in very excentrick orbs in all manner of positions, blind fate could never make all the planets move one and the same way in orbs concentrick, some inconsiderable irregularities excepted, which may have risen from the **mutual actions** of comets and planets upon one another, and which will be apt to increase, **till this system wants a reformation.**

Evidence for irregularity/instability



Ptolemy

On March 1st, 228 BC, at 4:23 am, mean Paris time, Saturn was observed two fingers under Gamma in Virgo.



Observations from 1590 and 1650.



Six million years ago Jupiter and Saturn were at the same distance from the Sun.

Demo with REBOUND

Laplace-Lagrange Secular Dynamics



Average over
short time scales



Perturbation theory

Explanations for the irregularities?



Euler was twice awarded a prize in 1748 and 1752 related to this problem by the Paris Academy of Sciences.

Lagrange thought that Euler's calculations were wrong and did his own.

Laplace (1776)



Mr. Euler, in his second piece on the irregularities of Jupiter and Saturn, find it **equal for both these planets**. According to Mr. de Lagrange, on the contrary, [...] it is very **different for these two bodies**. [...] I have some reasons to believe, however, that the formula is still not accurate. The one which I obtain is quite different. [...] by substituting these values in the formula of the secular equation, **I found absolutely zero**, from which I conclude the alteration of the mean motion of Jupiter, if it exists, does not result from the action of Saturn.

Lagrange (in a letter to d'Alembert, 1775)



I am ready to give a **complete theory for the variations of the elements of the planets** under their mutual action. That Mr. de la Place did on this subject I liked, and I flatter myself that he will not be offended if I do not hold the kind of promise that I made to completely abandon this subject to him; I could not resist to the desire to look into it again, but I am no less charmed that he is also working on it on his side; I am even very eager to read his subsequent research on this topic, but I do ask him not to send me any manuscript and send them to me only in printed form.

Fundamental modes, eigenfrequencies

	S₁	S₂	S₃	S₄	S₅	S₆	S₇	S₈
Lagrange (1774)	5.98	6.31	19.80	18.31	0	25.34	-	-
Brown & Rein (2019)	5.59	7.05	18.84	17.74	0	26.35	2.99	0.69

Note: No semi-major axis changes to first and also second order (Poisson, Haretu and Poincaré) in the expansion.

This still contradicts Ptolemy's observations from antiquity.

Demo with REBOUND

Laplace (1785)

Simple energy argument implies:

$$\frac{m_J}{a_J} + \frac{m_S}{a_S} = \text{const}$$

Thus, can be confident that the change in orbits must be due to mutual interactions.

He's also shown, no secular terms. Hence must be short period.

Near 5:2 mean motion resonance. Period of 900 years.

Secular Dynamics



Average over
short time scales



Perturbation theory



5:2 near MMR

Secular Dynamics



Average over
short time scales

Accurate over
> 1 million years



5:2 near MMR

Chaos

Le Verrier (1840, 1841)



Follow up on the work of Lagrange and Laplace but to higher order.

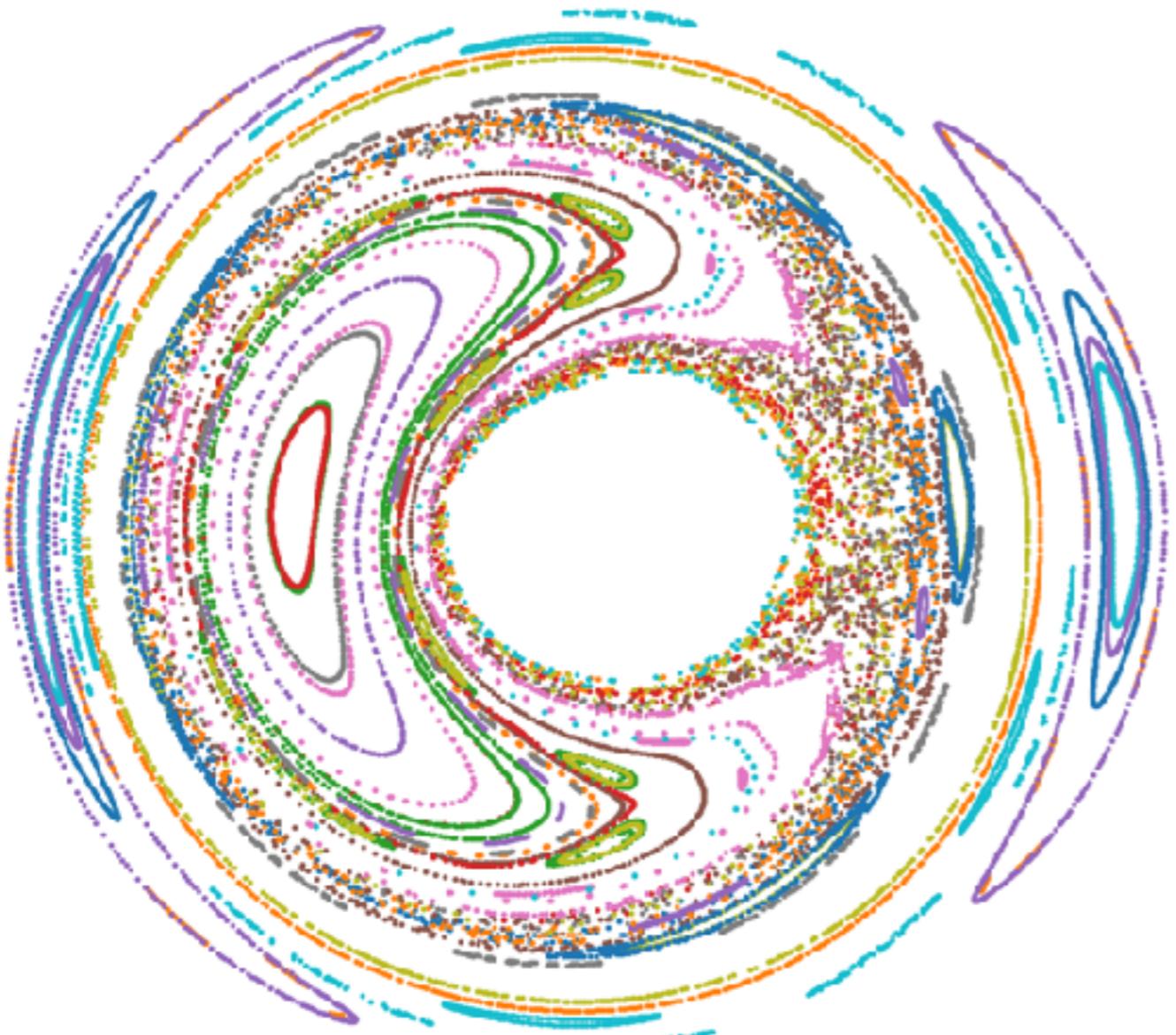
Small divisor problem:
third order could be larger than second order terms

Poincaré (1897)



The terms of these series, in fact, decrease first very quickly and then begin to grow, **but as the Astronomers' stop after the first terms of the series**, and well before these terms have stopped to decrease, the approximation is sufficient for the practical use. The divergence of these expansions would have some disadvantages only if one wanted to use them to rigorously establish some specific results, as the **stability of the Solar System**.

Kolmogorov (1954), Arnold (1963), Moser (1962)

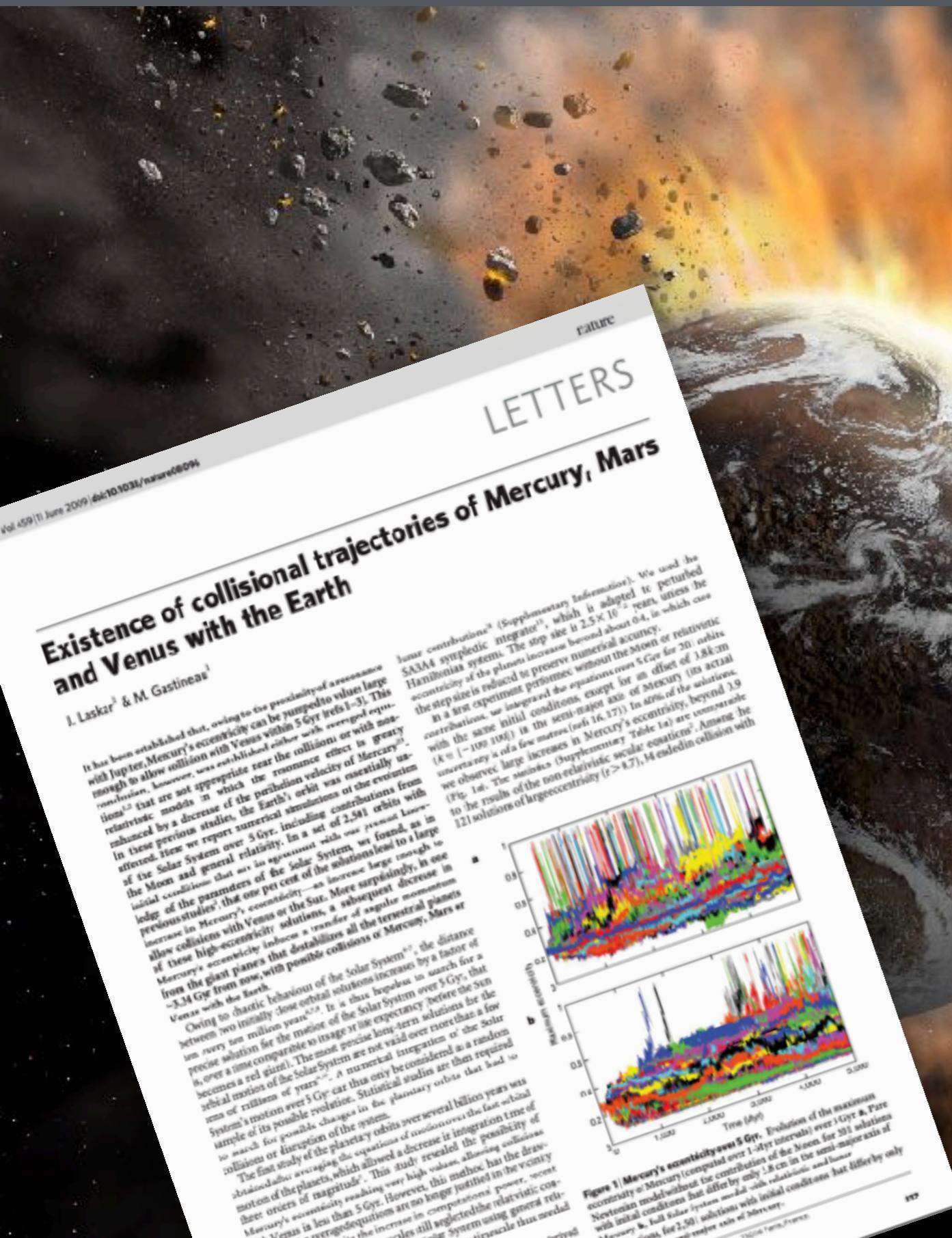


Kolmogorov showed that convergent perturbation series can exist.

Many subtleties (degeneracy, small masses, slow Arnold diffusion)

In short: expansions are not useful for determining the stability of our Solar System.

Laskar (2009)



Existence of collisional trajectories of Mercury, Mars and Venus with the Earth

I. Laskar¹ & M. Gastineau¹

It has been established that, owing to the proximity of resonance with Jupiter, Mercury's eccentricity can be pumped to values large enough to allow collision with Venus within 5 Gyr (refs 1–3). This conclusion, however, was established either with averaged equations² that are not appropriate near the millions or with general relativistic models³ in which the resonance effect is greatly enhanced by a decrease of the perihelion velocity of Mercury⁴. In these previous studies, the Earth's orbit was essentially unaffected. Here we report numerical simulations of the evolution of the Solar System over 5 Gyr, including contributions from the Moon and general relativity. In a set of 2,581 orbits with initial conditions that are in agreement with our present knowledge of the parameters of the Solar System, we found, as in previous studies⁴, that one per cent of the solutions lead to a large increase in Mercury's eccentricity—an increase large enough to allow collisions with Venus or the Sun. More surprisingly, in one of these high-eccentricity solutions, a subsequent decrease in Mercury's eccentricity induces a transfer of angular momentum from the giant planets that destabilizes all the terrestrial planets \sim 3.34 Gyr from now, with possible collisions of Mercury, Mars or Venus with the Earth.

Owing to chaotic behaviour of the Solar System^{5,6}, the distance between two initially close orbital solutions increases by a factor of ten every few million years^{7,8}. It is thus hopeless to search for a precise solution for the motion of the Solar System over 5 Gyr, that is, over a time comparable to its age. To search for long-term solutions for the orbital motion of the Solar System is not valid over more than a few tens of millions of years^{9,10}. A numerical integration of the Solar System's motion over 5 Gyr can thus only be considered as a random sample of its possible evolution. Statistical studies are then required to search for possible changes in the planetary orbits that had no influence on the evolution of the system.

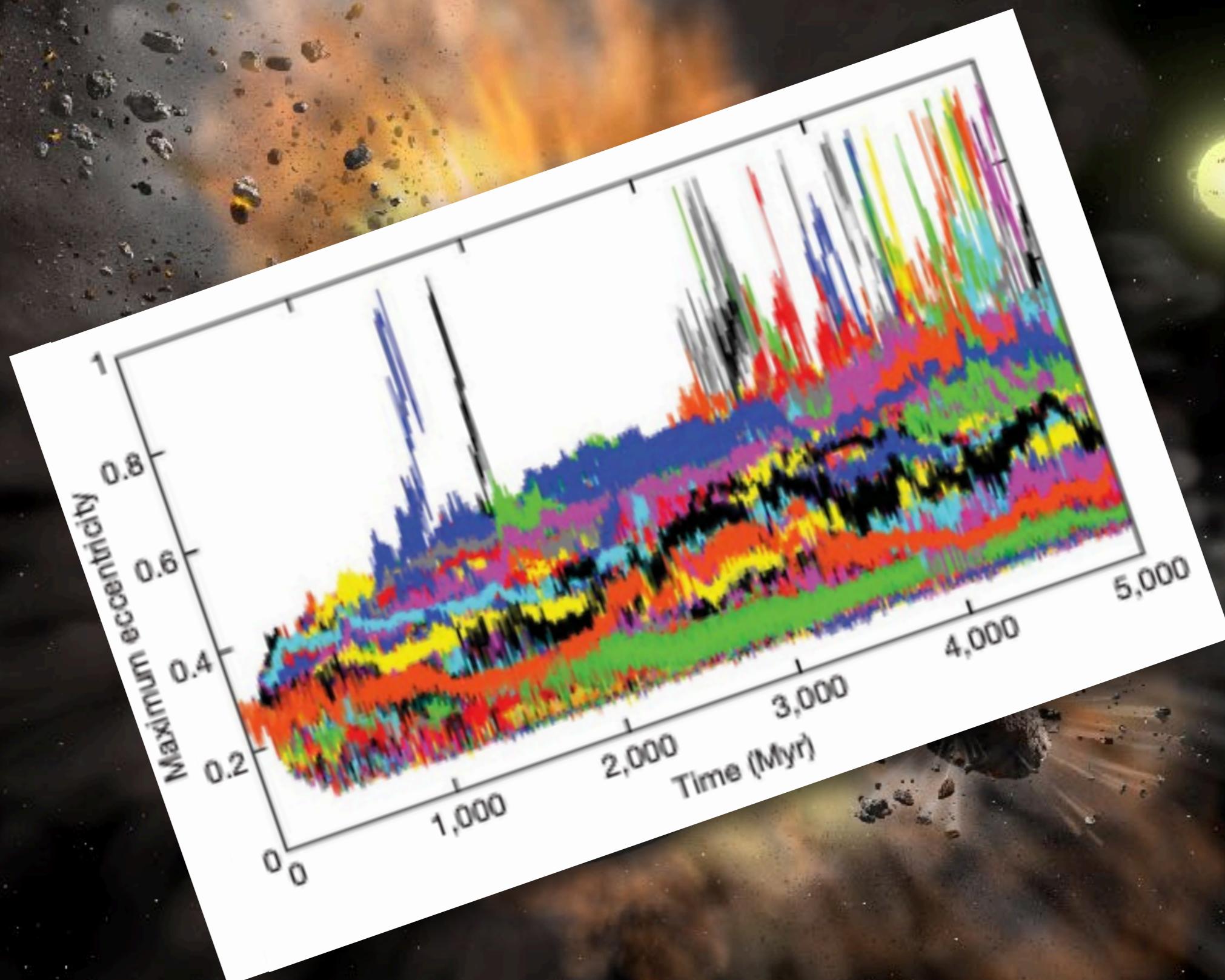
The first study of the planetary orbits over several billion years was obtained after averaging the equations of motion over the fast orbital motion of the planets, which allowed a decrease in integration time of three orders of magnitude¹¹. This study revealed the possibility of Mercury's eccentricity reaching very high values, allowing collisions of Venus with the Sun. However, this method has the disadvantage of averaging the equations of motion over a long time interval. The increase in computational power, recent developments in supercomputers, and the use of general relativistic models still neglected the relativistic corrections to the equations of motion of the Solar System using general relativity have made it possible to simulate the evolution of the Solar System over billions of years¹².

Jupiter contributes¹³ (Supplementary Information). We used the SABA4 symplectic integrator¹⁴, which is adapted to perturbed Hamiltonian systems. The step size is 2.5×10^{-2} years, unless the eccentricities of the planets increase beyond about 0.4, in which case the step size is reduced to preserve numerical accuracy.

In a first experiment performed without the Moon or relativistic contributions, we integrated the equations over 5 Gyr for 201 orbits with the same initial conditions except for an offset of 3.8 km ($k = 1\text{--}100$) in the semi-major axis of Mercury (its actual uncertainty is of a few meters (ref. 16, 17)). In 40% of the solutions, we observe large increases in Mercury's eccentricity, beyond 1.9 (Fig. 1a). The statistics (Supplementary Table 1a) are consonant to the results of the non-relativistic secular equations². Among the 121 solutions of large eccentricity ($e > 0.7$), 44 ended in collision with

Figure 1 | Mercury's eccentricity over 5 Gyr. Evolution of the maximum eccentricity of Mercury (computed over 1-Myr intervals) over 1-Gyr a. Pareto distribution without the contribution of the Moon for 201 solutions with initial conditions that differ by only 3.8 cm in the semi-major axis of Mercury b. full Solar system model with relativistic and lunar corrections for 2,581 solutions with initial conditions that differ by only 3.8 cm in the semi-major axis of Mercury.

Laskar (2009)



Modern numerical methods

Newton (1687)

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j \neq i}}^N m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}$$

How to solve N-body ODE

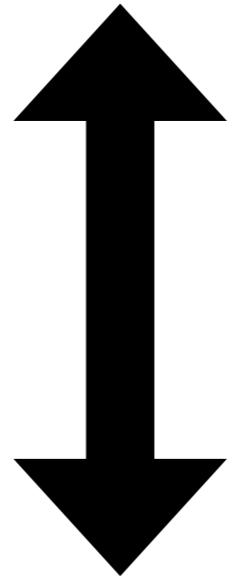
Brute force approach
(IAS15)

Physical approach
(WHFast/EOS)

Fundamentals

Equations of motions

$$\ddot{\mathbf{r}}_i = \sum_{\substack{j=1 \\ j \neq i}}^N m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}$$



Hamiltonian

$$H = T + U$$

kinetic term potential term

Two arrows originate from the text labels "kinetic term" and "potential term". The arrow from "kinetic term" points to the term T in the equation. The arrow from "potential term" points to the term U in the equation.

Leap frog integrator

$$H = T + U$$

A

$$T = \sum_{i=0}^{N-1} T_i$$

B

$$U = \sum_{i=0}^{N-1} \sum_{j=i+1}^{N-1} U_{ij}$$

Both solutions are trivial!

$$\dot{r}_i = v_i$$

$$\dot{v}_i = \sum_{j \neq i} a_{ij}$$

Wisdom-Holman integrator

$$H = A + B$$

A

$$\sum_{i=0}^{N-1} T_i + \sum_{i=1}^{N-1} U_{i0}$$

Dominant part of motion

B

$$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N-1} U_{ij}$$

Perturbation

Solution for B is still trivial.

Solution for A is more complicated. We need a “Kepler solver”.

Embedded Operator Splitting Method (EOS)

$$H = A + B$$

A

$$\sum_{i=0}^{N-1} T_i + \sum_{i=1}^{N-1} U_{i0}$$

B

$$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N-1} U_{ij}$$

$$A = A_1 + A_2$$

A₁

$$\sum_{i=0}^{N-1} T_i$$

A₂

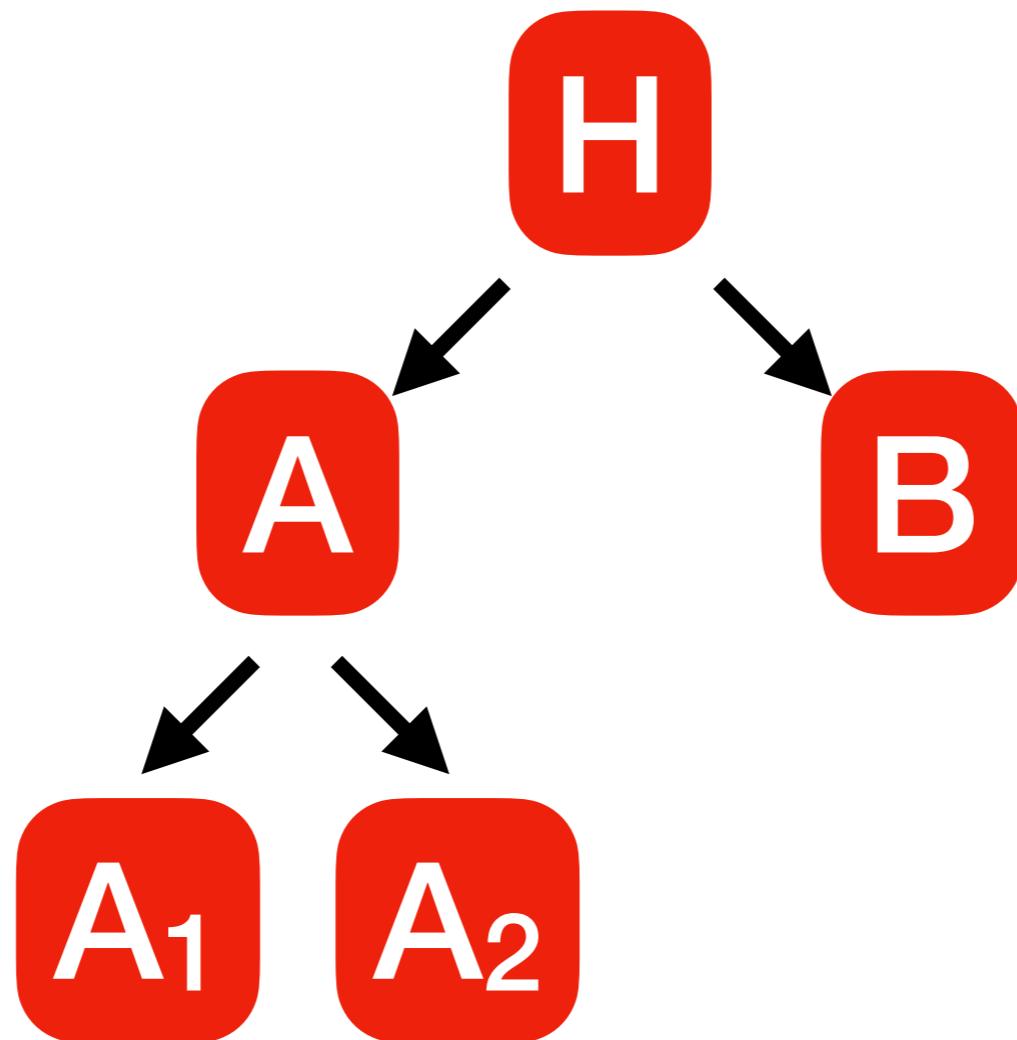
$$\sum_{i=1}^{N-1} U_{i0}$$

A lot of choice

Full Hamiltonian

1st splitting

2nd splitting

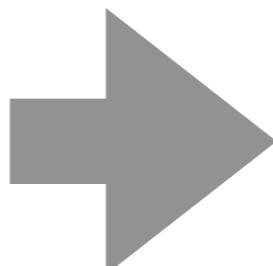


At each splitting, can choose:

- How to split Hamiltonian into two parts
- Which splitting method to use
- Timestep

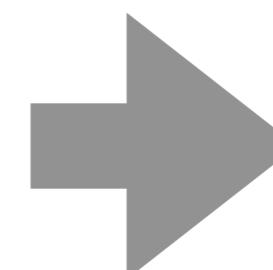
EOS methods are extremely flexible

splitting into $T + U$



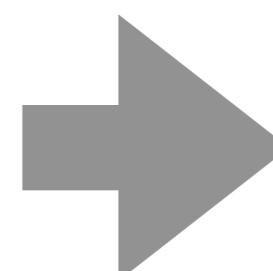
leap frog,
higher order leap frog

splitting into Keplerian
motion + perturbations



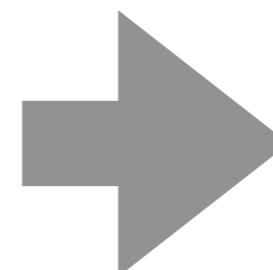
Wisdom Holman integrator,
higher order generalizations

splitting into near and far
interactions



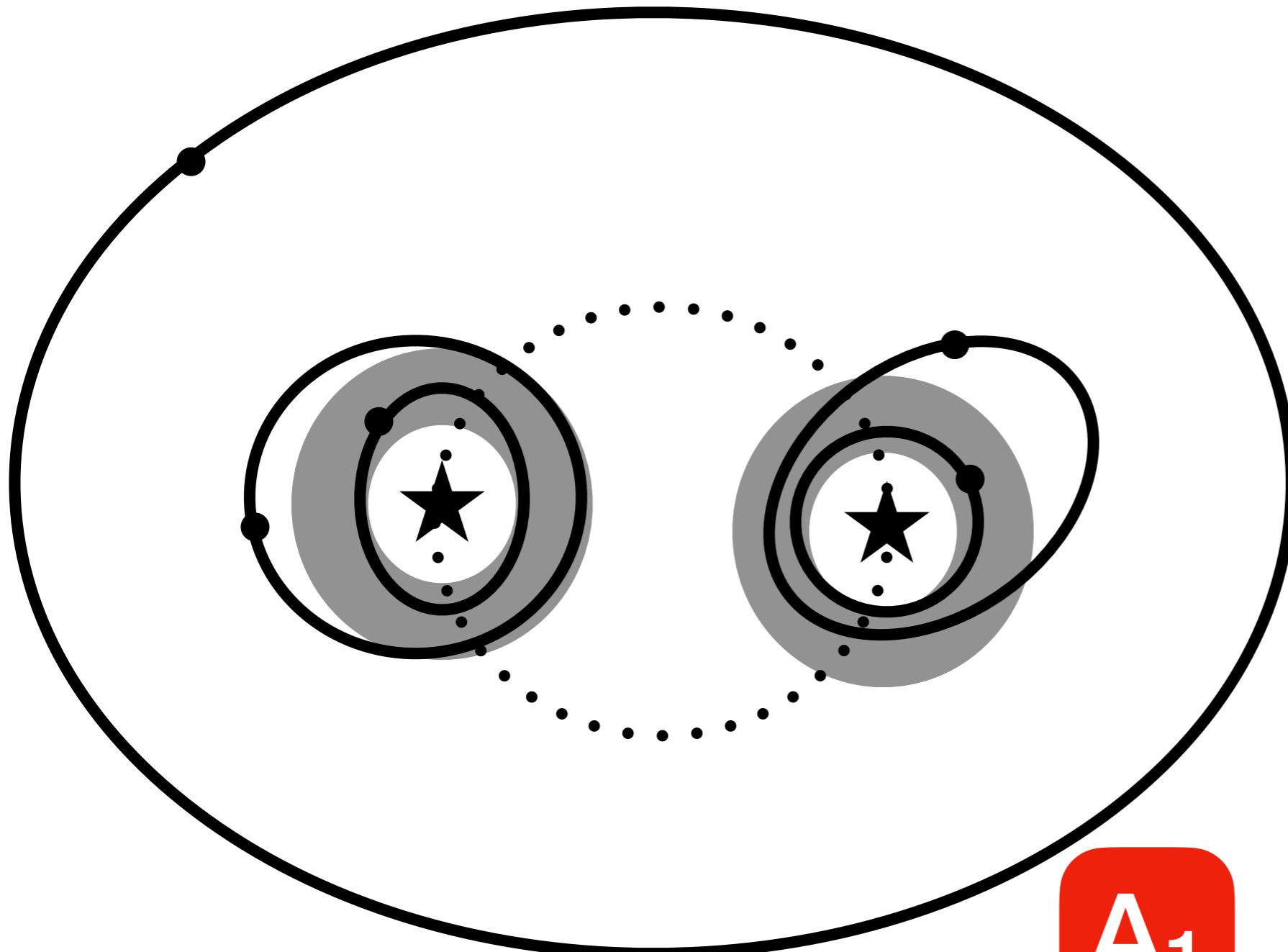
Hybrid symplectic
integrators, Mercury

splitting into many different
“shells”



SYMBA

Example: complicated hierarchical systems



B

planet-planet interactions
(kick)

A₁

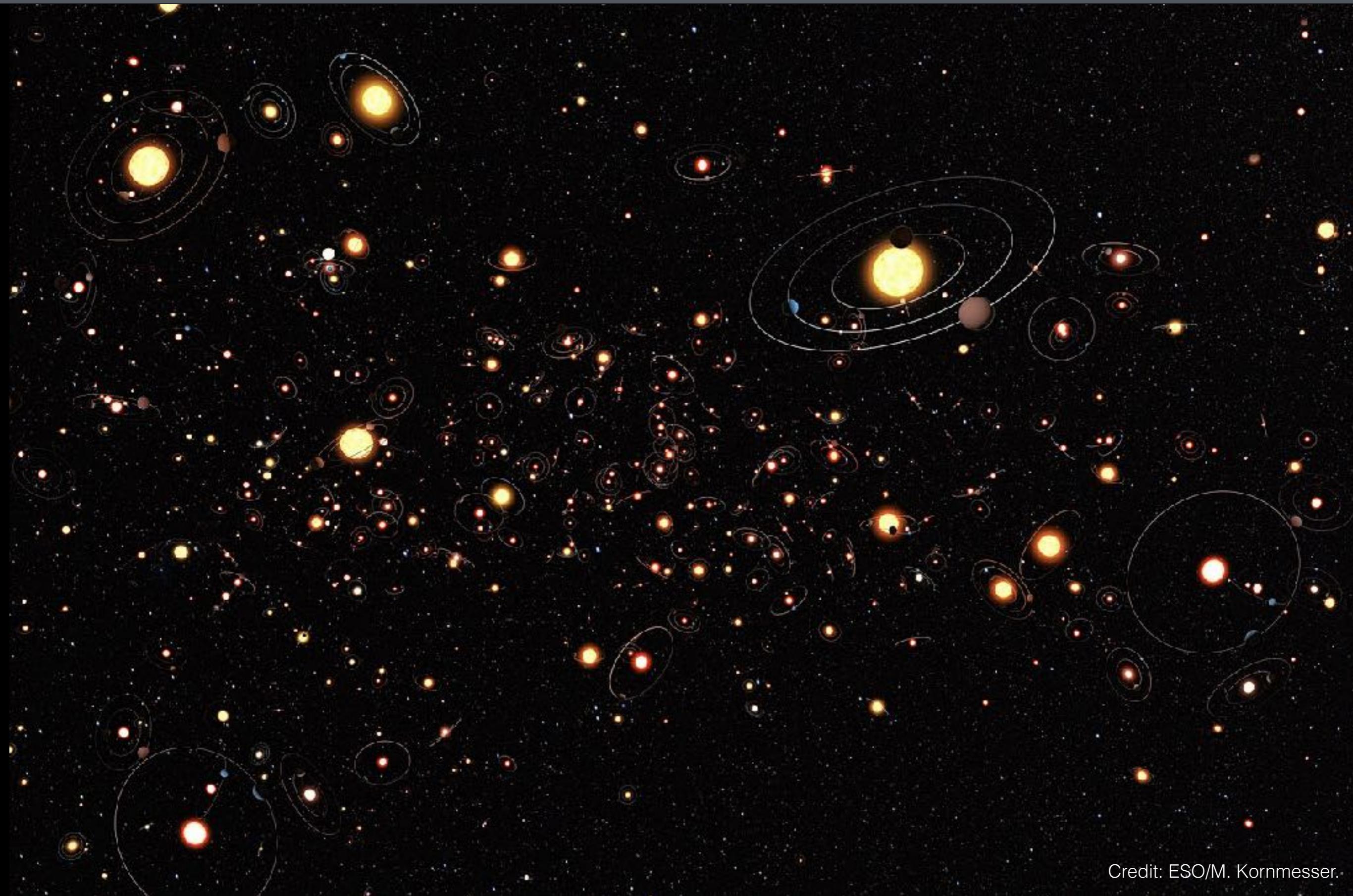
kinetic term (drift)

A₂

planet-star and star-
star interactions (kick)

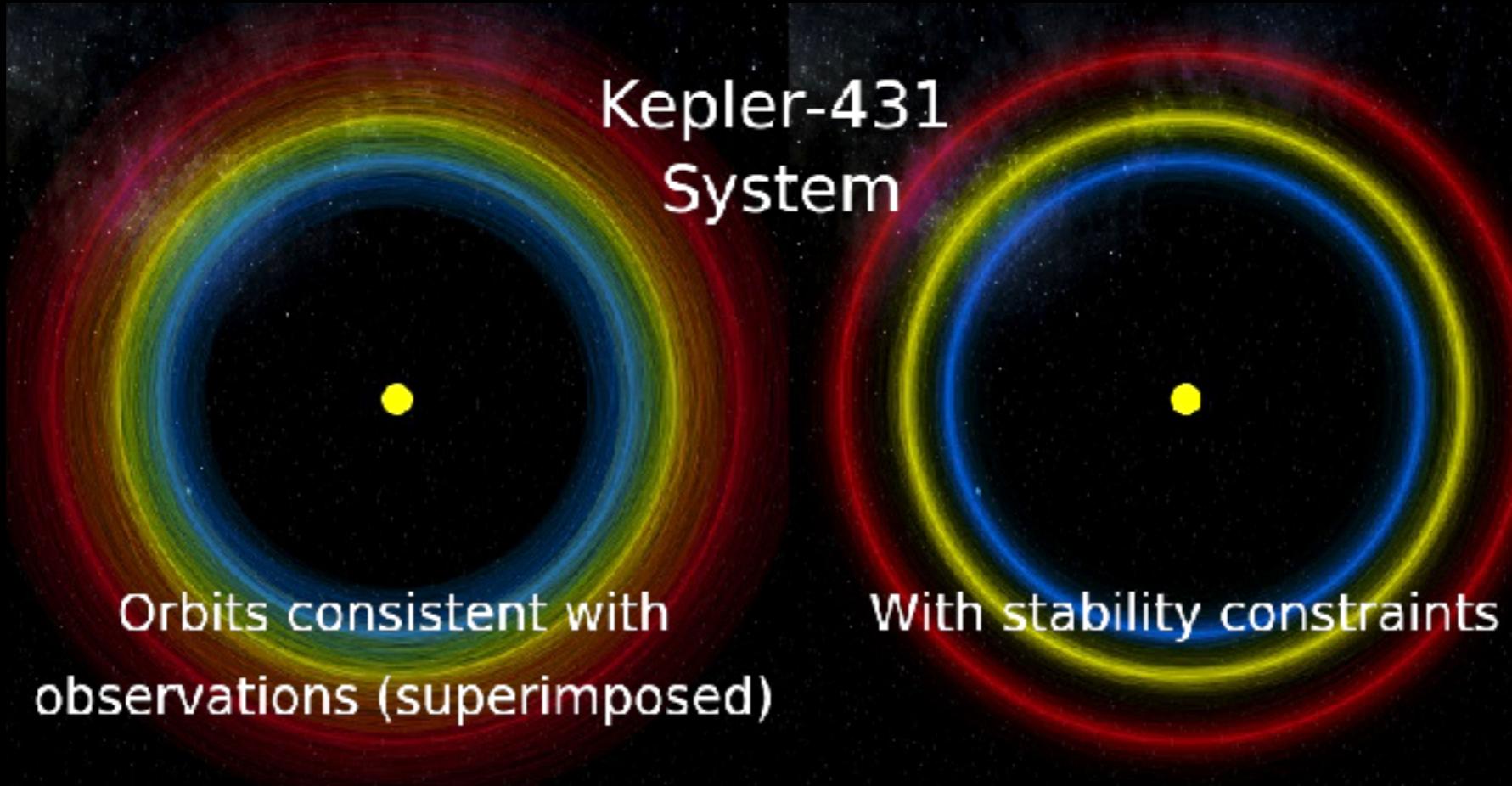
Predicting the stability of planetary systems with machine learning

Planets everywhere



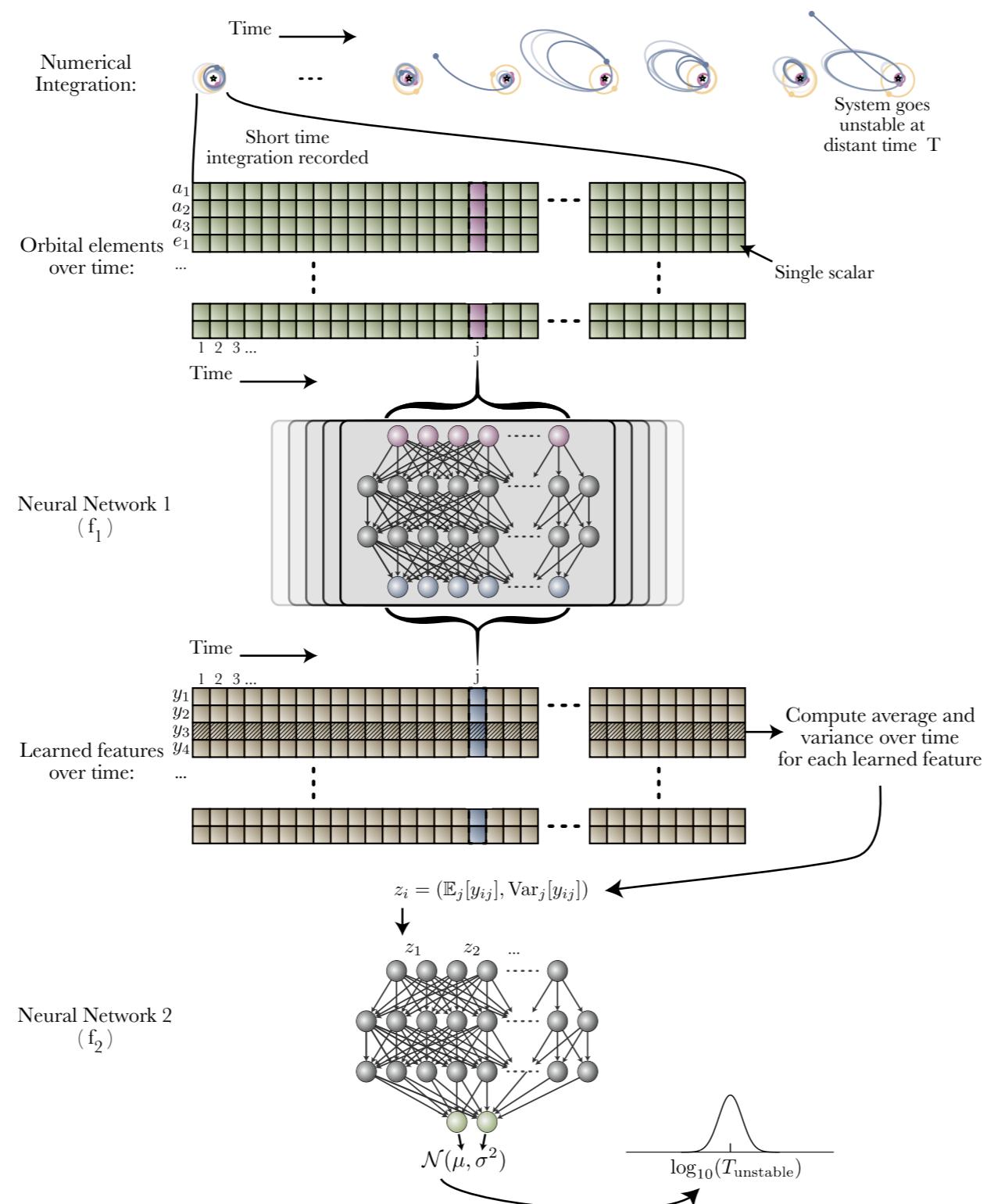
Credit: ESO/M. Kornmesser.

Constrain orbital parameters

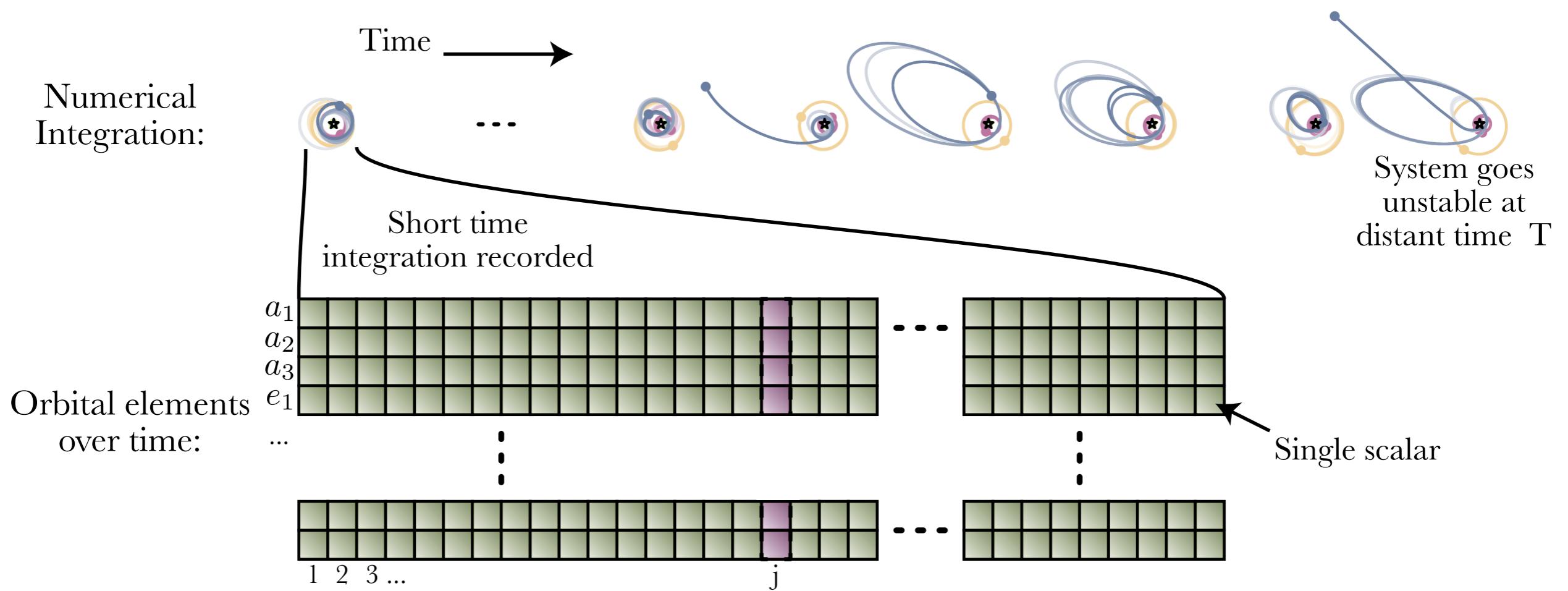


- ▶ This is a hard problem!
- ▶ Machine learning models can help solve this problem.
- ▶ Doesn't have to be a black box! Can be part of a Bayesian analysis.

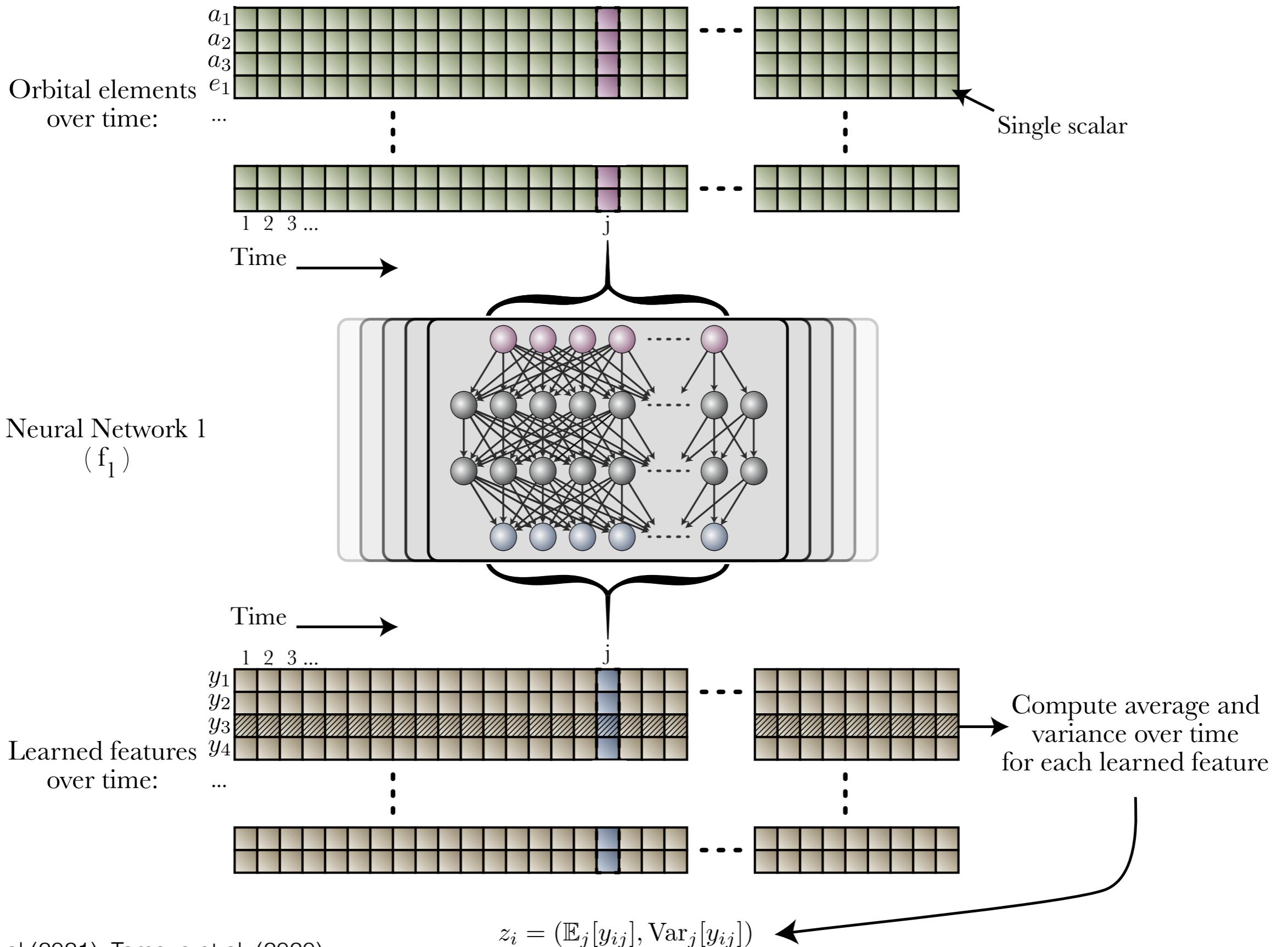
Bayesian Neural Network



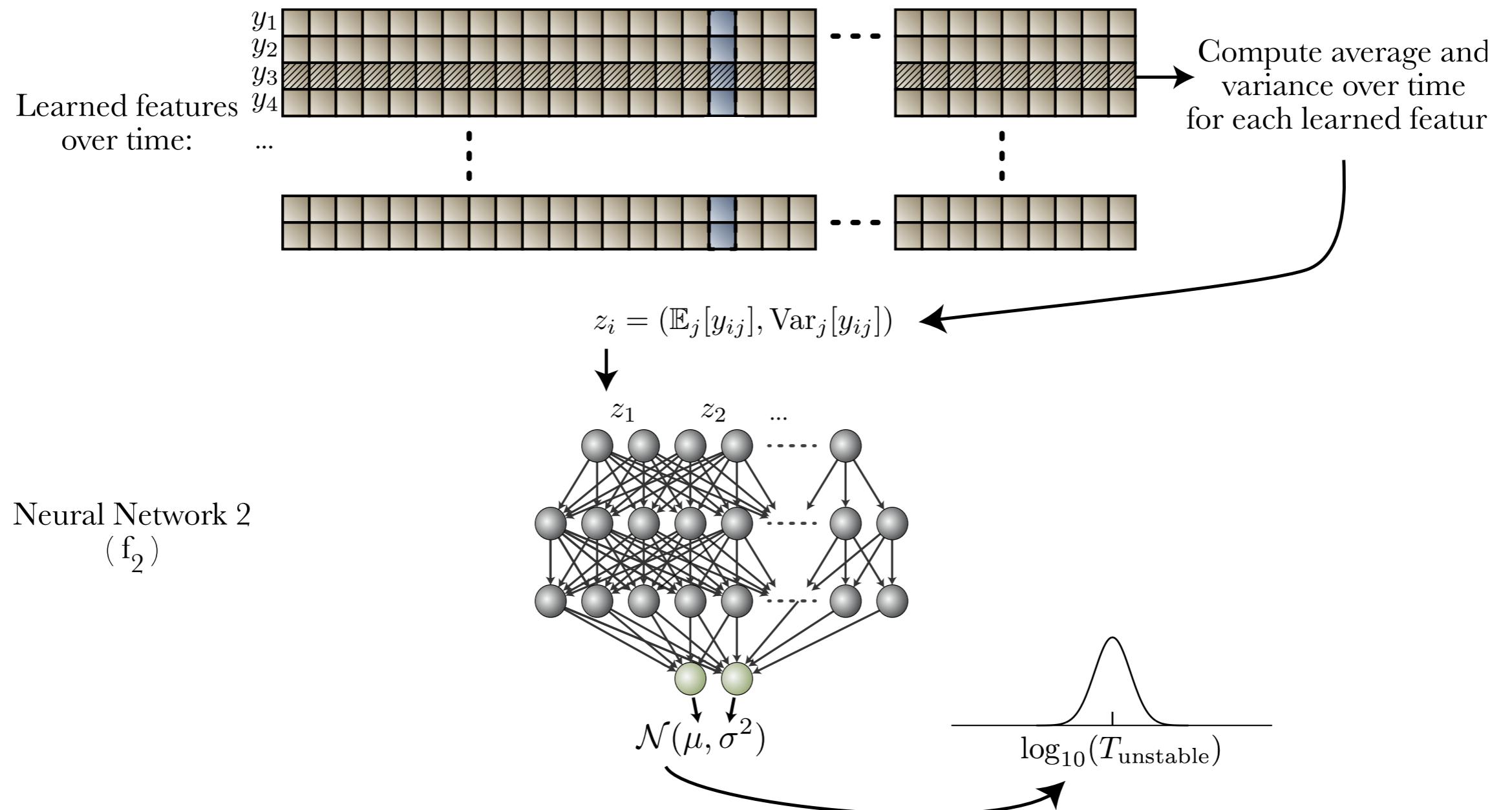
Step 1: Short integration



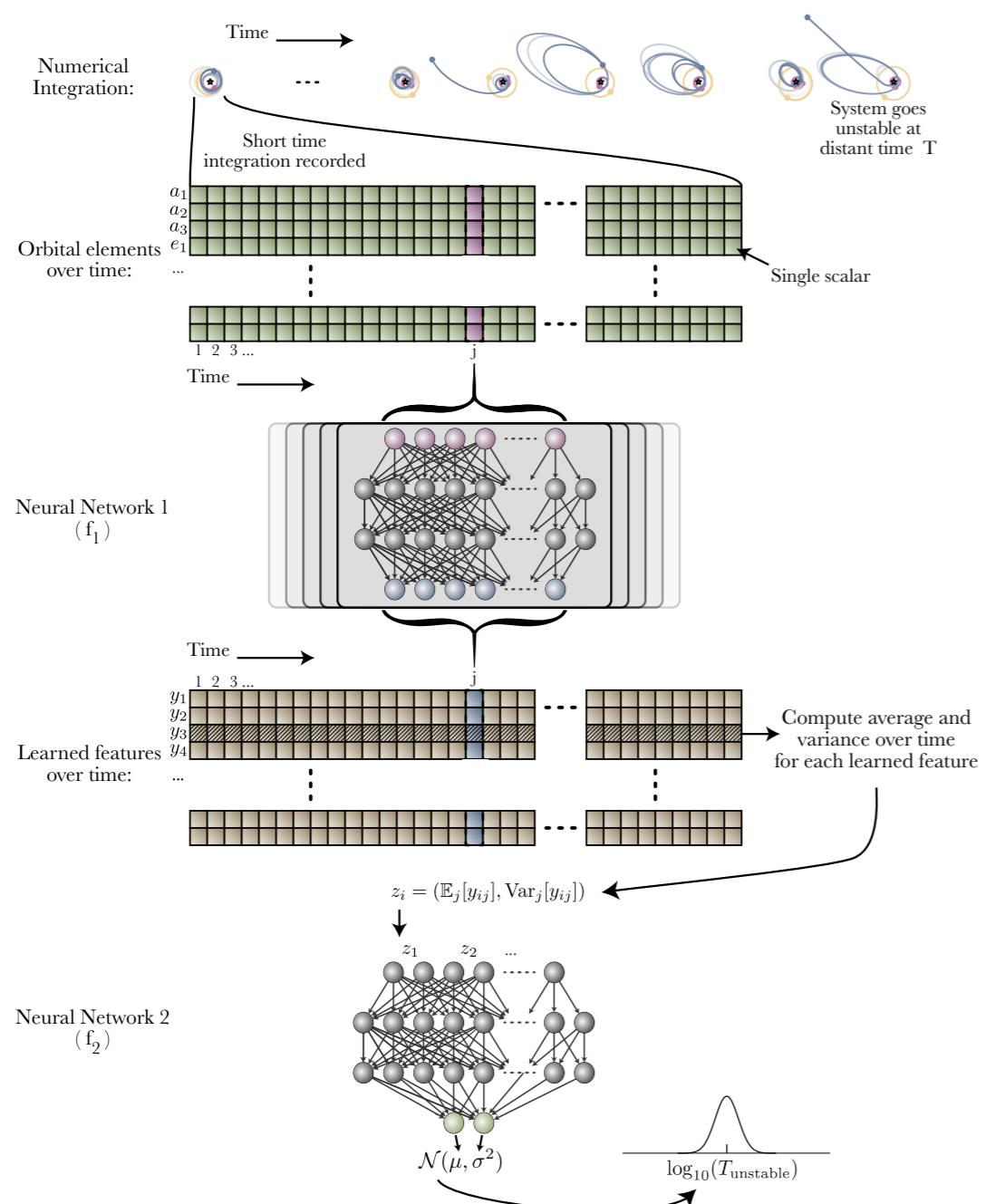
Step 2: Learning features



Step 3: Predicting time of instability

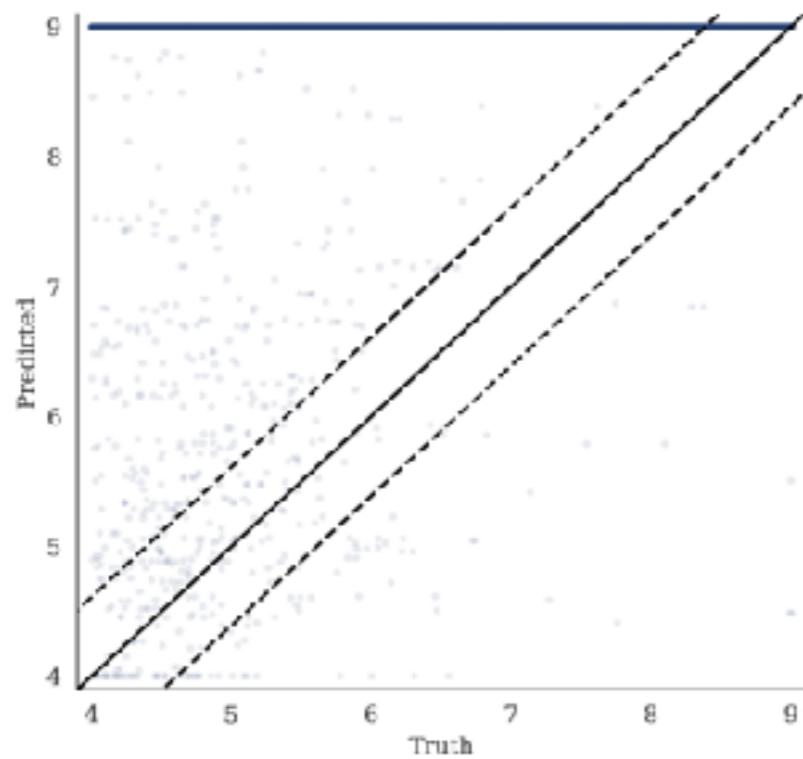


Bayesian Neural Network

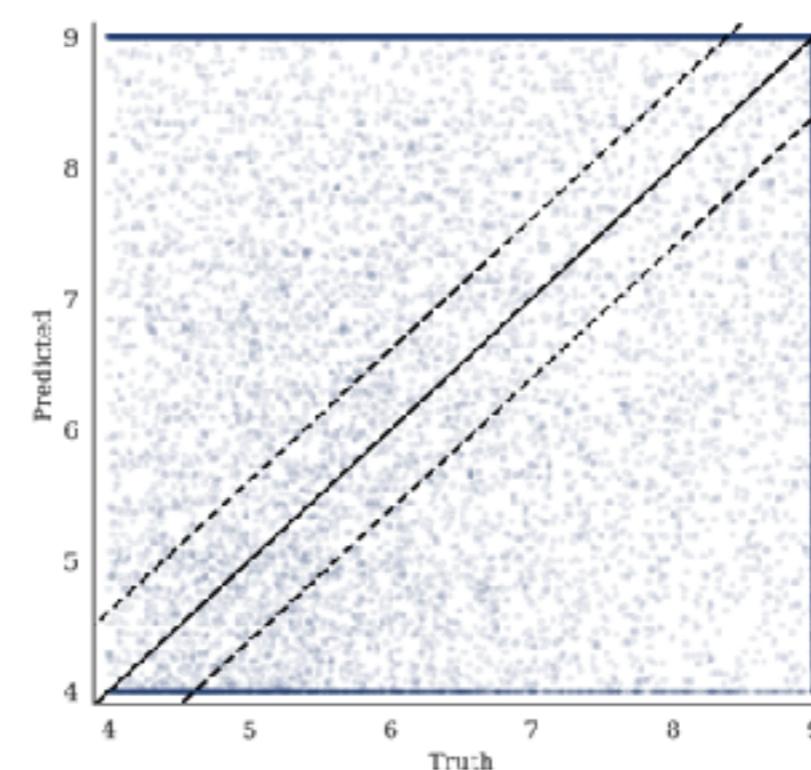


- ▶ Large training dataset: 113,543 simulation
- ▶ Generating training dataset is tricky
- ▶ Sampling a highly complex parameter space
- ▶ Computationally very expensive, billion orbits

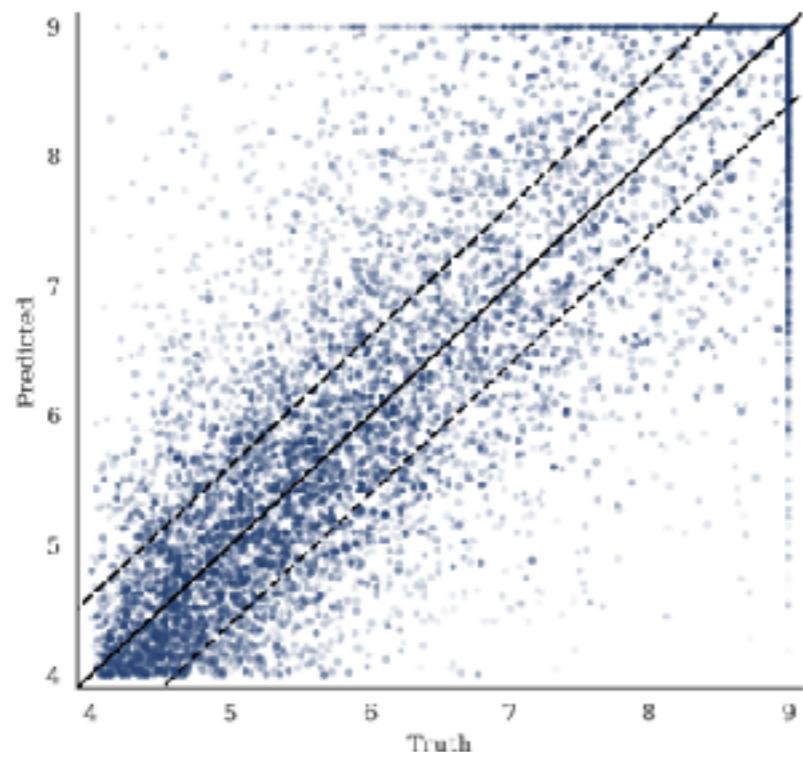
Results



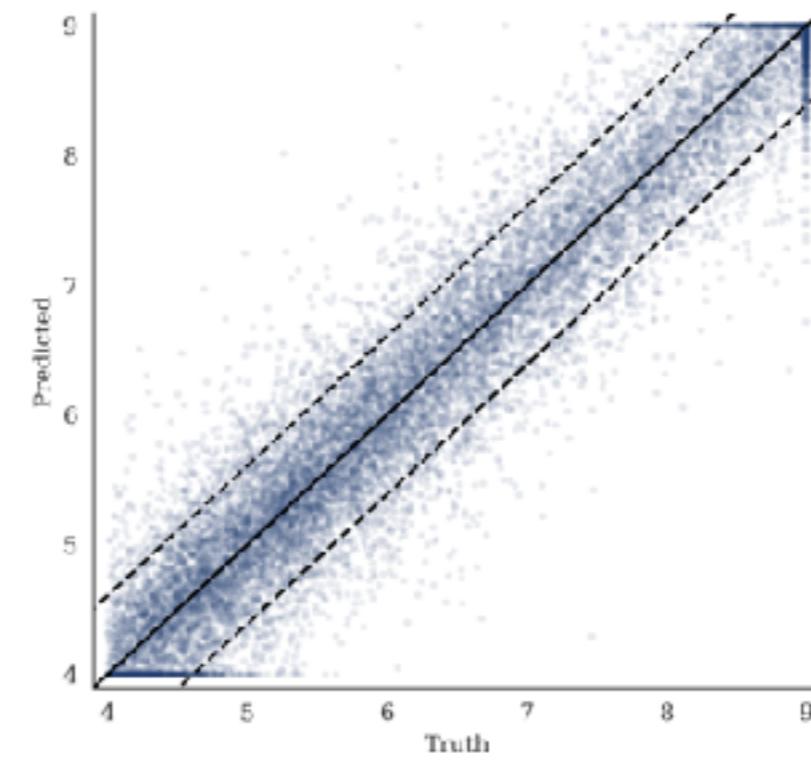
Petit et al. (2020)



Obertas et al. (2017)

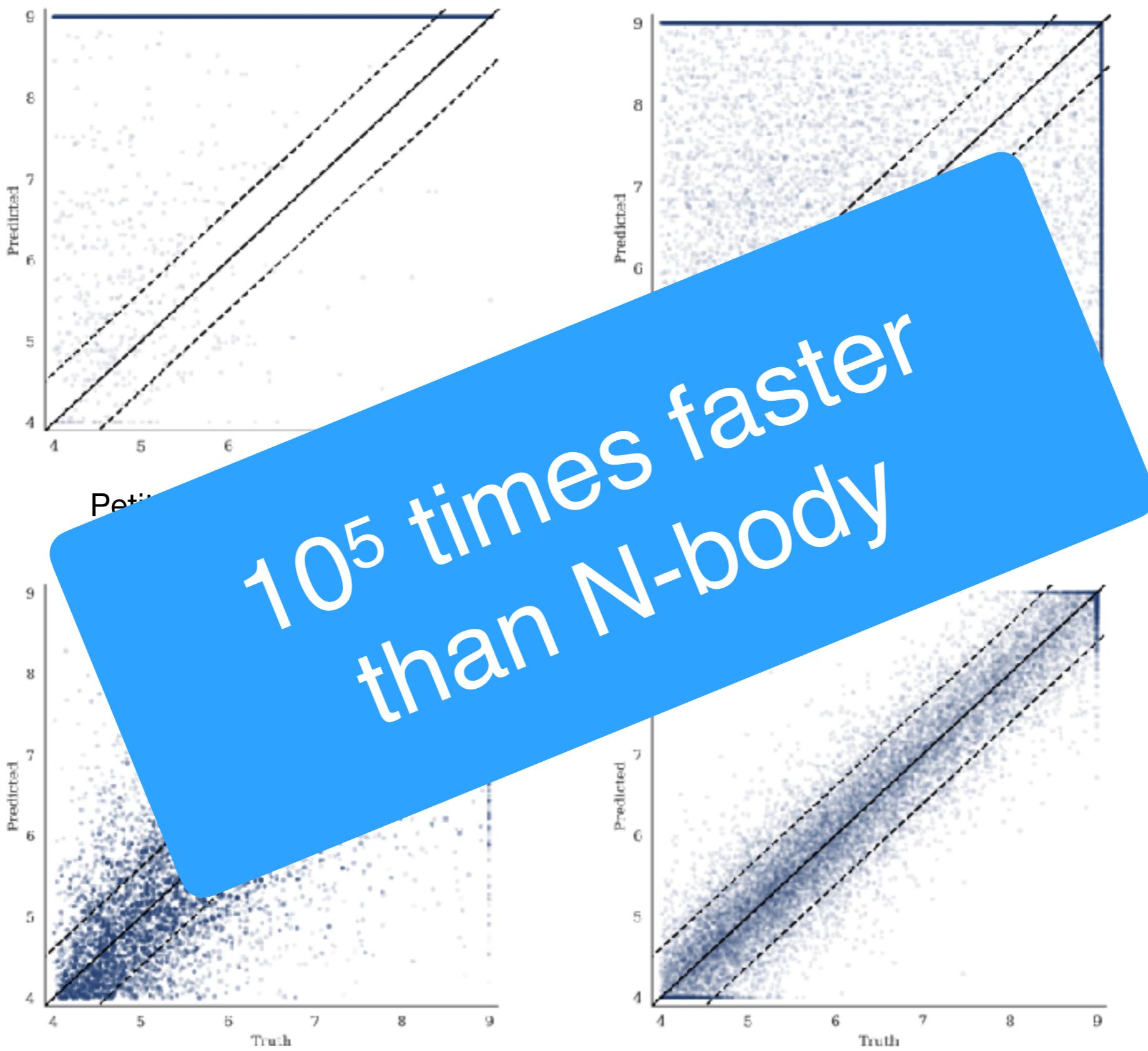


Our ML model, Cranmer et al (2021)



Theoretical limit

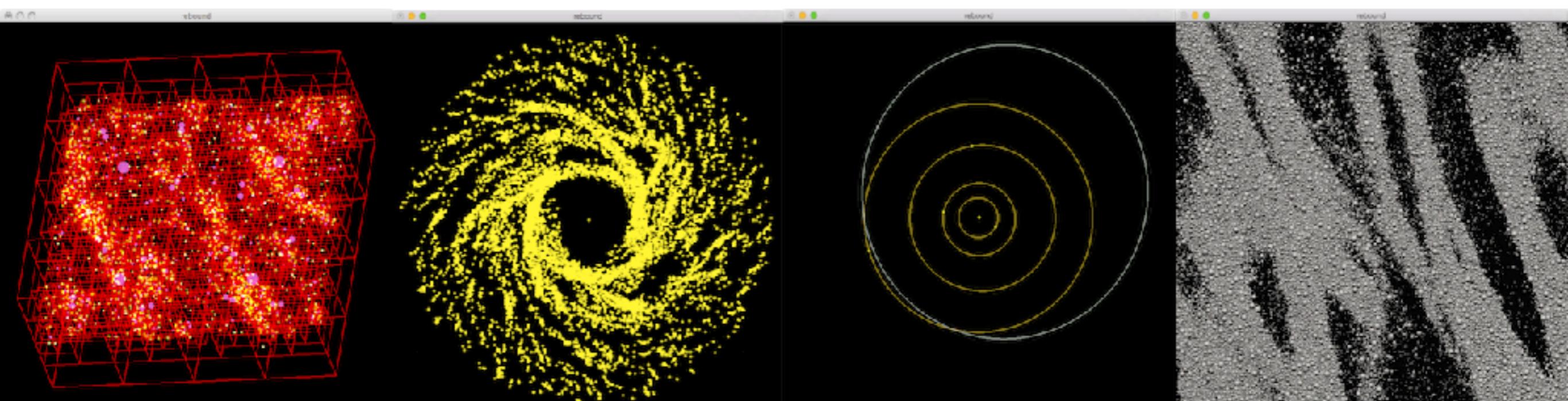
Results



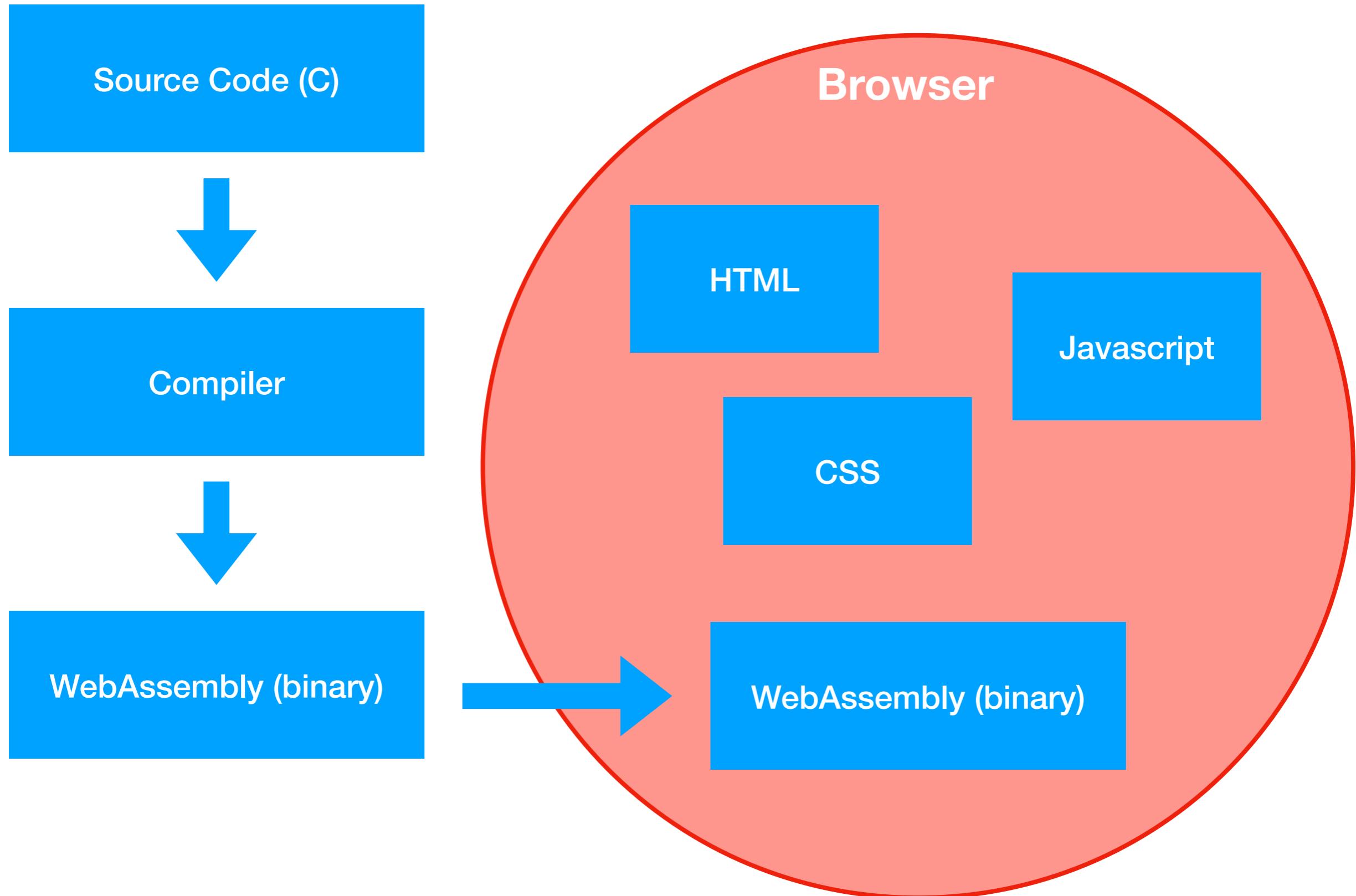
REBOUND, REBOUNDx, ASSIST

REBOUND

- ▶ N-body integrator package
- ▶ Many different built-in integrators
- ▶ Planetary systems
- ▶ Collisional simulations of planetary rings
- ▶ Written in C with an easy to use python interface
- ▶ No dependencies



WebAssembly

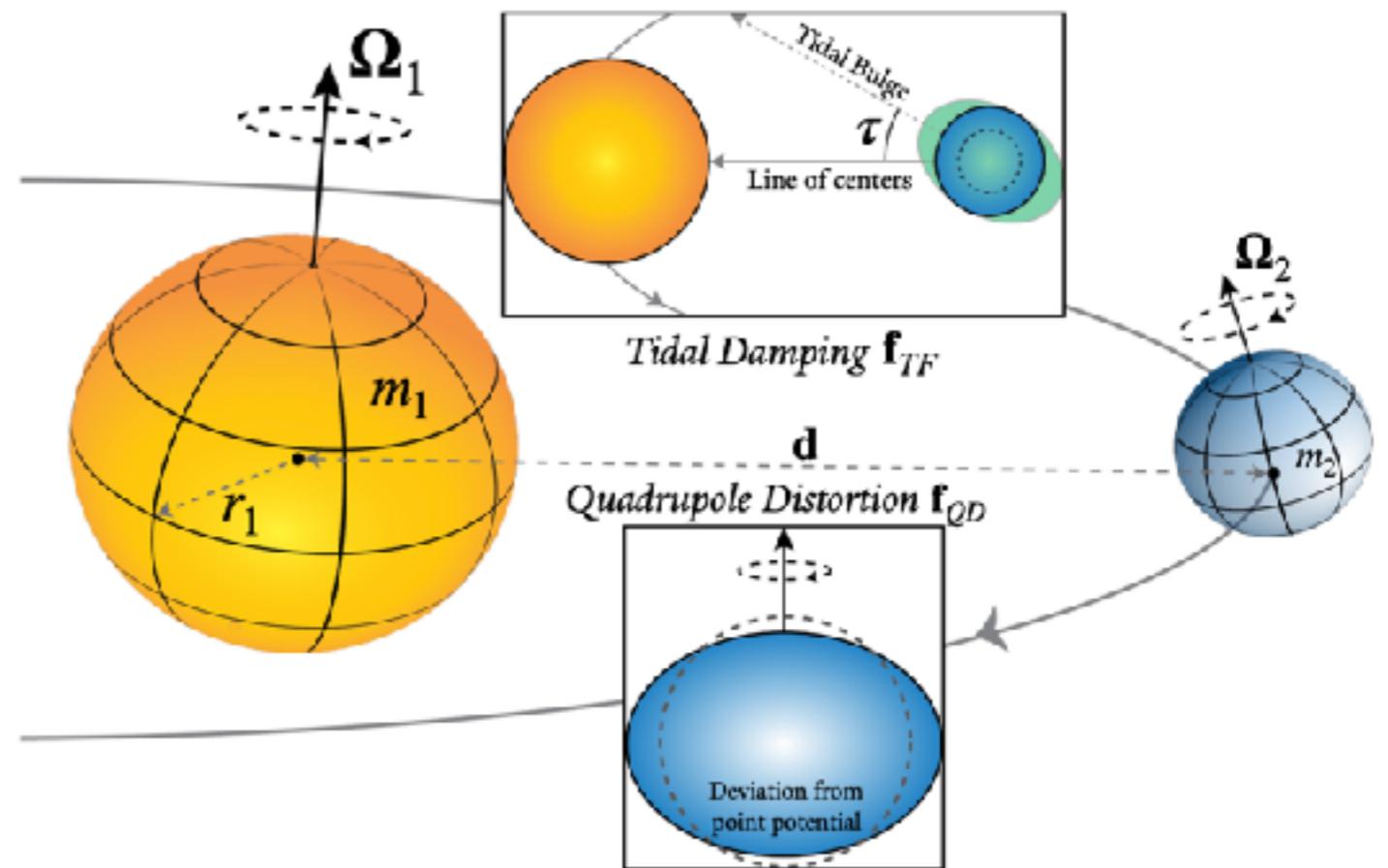


REBOUNDx Add-on

- ▶ Development led by Dan Tamayo (Harvey Mudd)
- ▶ Incorporate additional physics into N-body simulations:
 - ▶ Orbit modifications/migration
 - ▶ General relativity
 - ▶ Radiation pressure, Yarkovsky effect
 - ▶ Gravitational harmonics
 - ▶ Tides
- ▶ Very easy to use
- ▶ Does a lot of smart things behind the scenes!

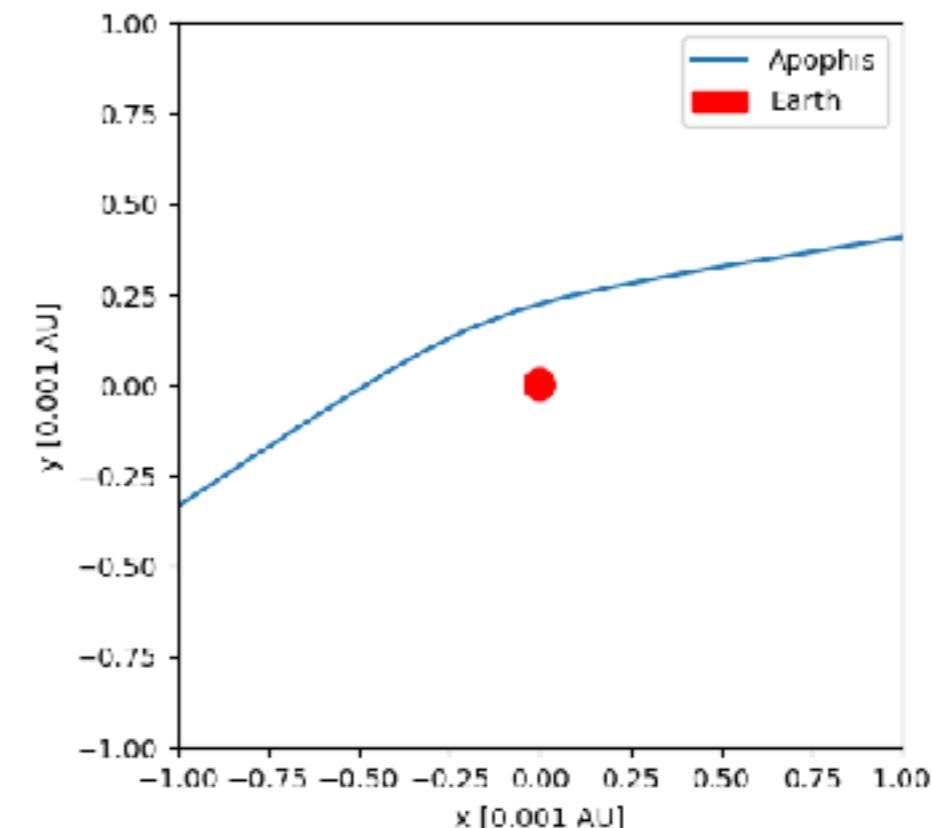
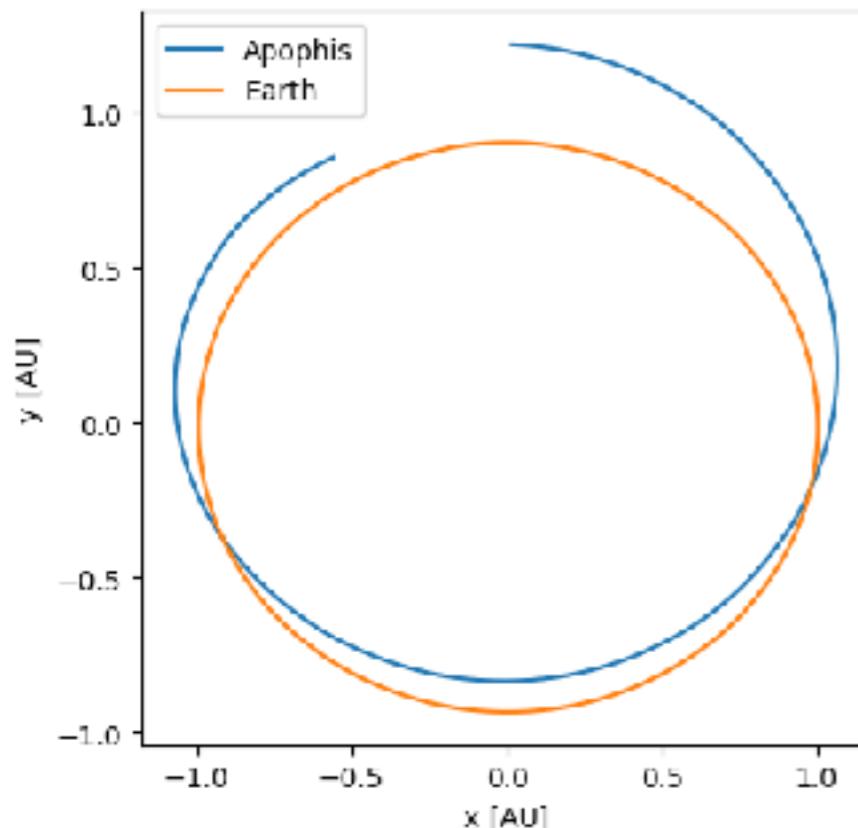
REBOUNDx Tides

- ▶ Self-consistent spin, tidal and dynamical equations of motion
- ▶ Constant time lag approximation
- ▶ Part of REBOUNDx



ASSIST Add-on

- ▶ Development led by Matt Holman (Minor Planet Center)
- ▶ Integration of asteroids, spacecrafts, artificial satellites in gravitational field of sun + planets (DE440 ephemeris)
- ▶ GR, radiation forces, higher order harmonics
- ▶ **Very** high accuracy, ~cm



Conclusions

Determining the stability of planetary systems is a very old problem. Analytic solutions cannot answer all questions.

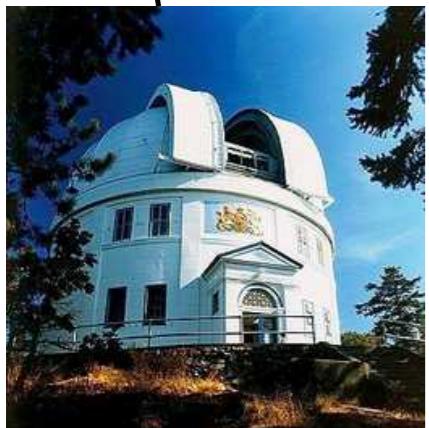
Chaos leads to collisional trajectories in the Solar System.

Embedded Operator Splitting methods (EOS) are very easy to implement. Can be configured to be equivalent to: leap-frog, Wisdom-Holman, Mercury, SYMBA, and many new methods.

Our machine-learning classifier can predict the stability of planetary systems 10^5 times faster.

Use the REBOUND/REBOUNDx/ASSIST ecosystem for all your small N dynamics needs.

Light pollution and mega constellations



Plaskett Telescope

Aaron Boley,
UBC



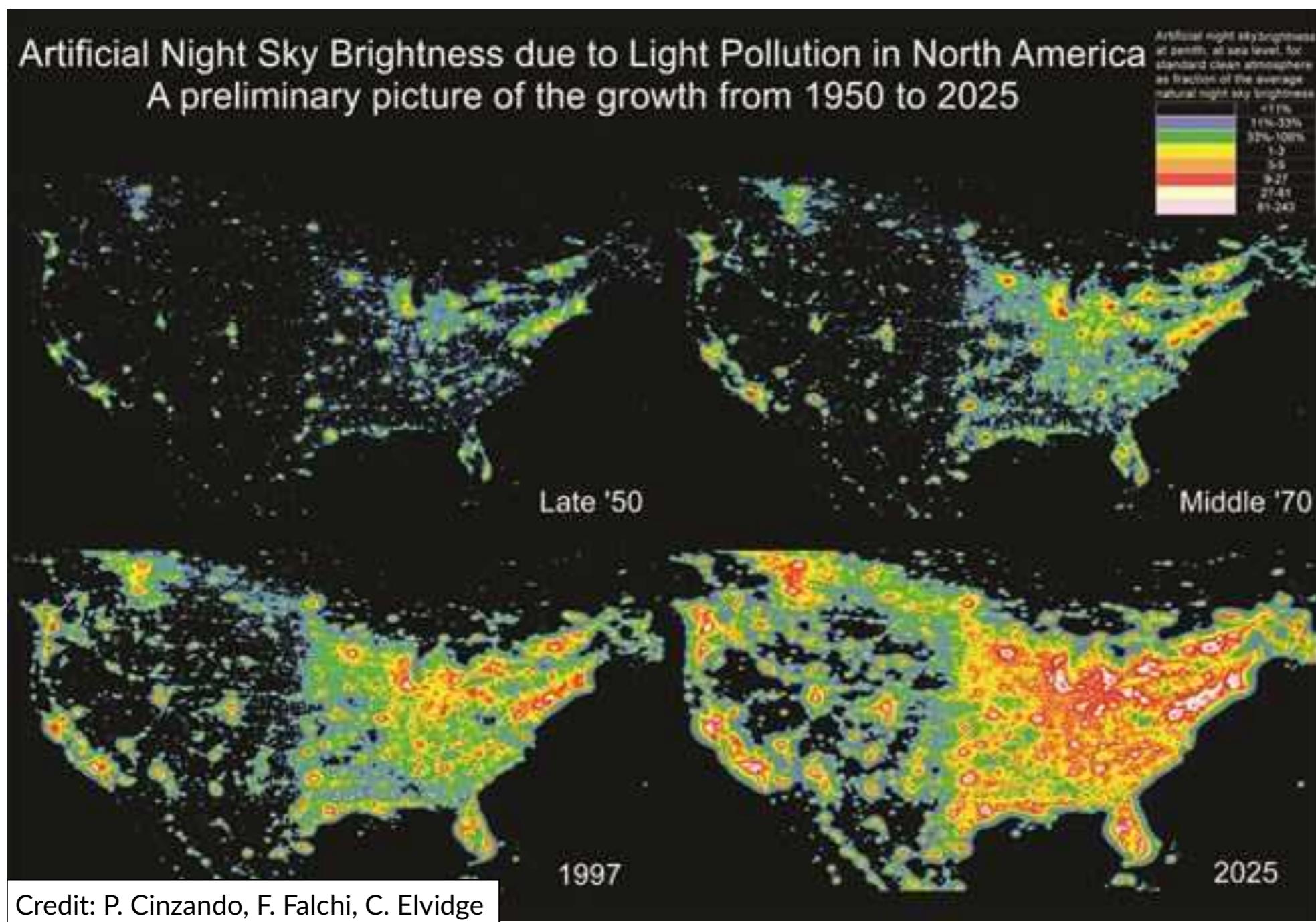
Samantha Lawler,
U. of Regina



Hanno Rein,
U. of Toronto



Access to the night sky is restricted by urban light pollution



But there are many groups fighting!
(ex: International Dark Sky Association)

LEDs (a sudden leap in technology access) took these groups by surprise

LEDs are good – use less energy for more light. BUT are massively over-used because they're cheap.

A new source of globally visible light pollution



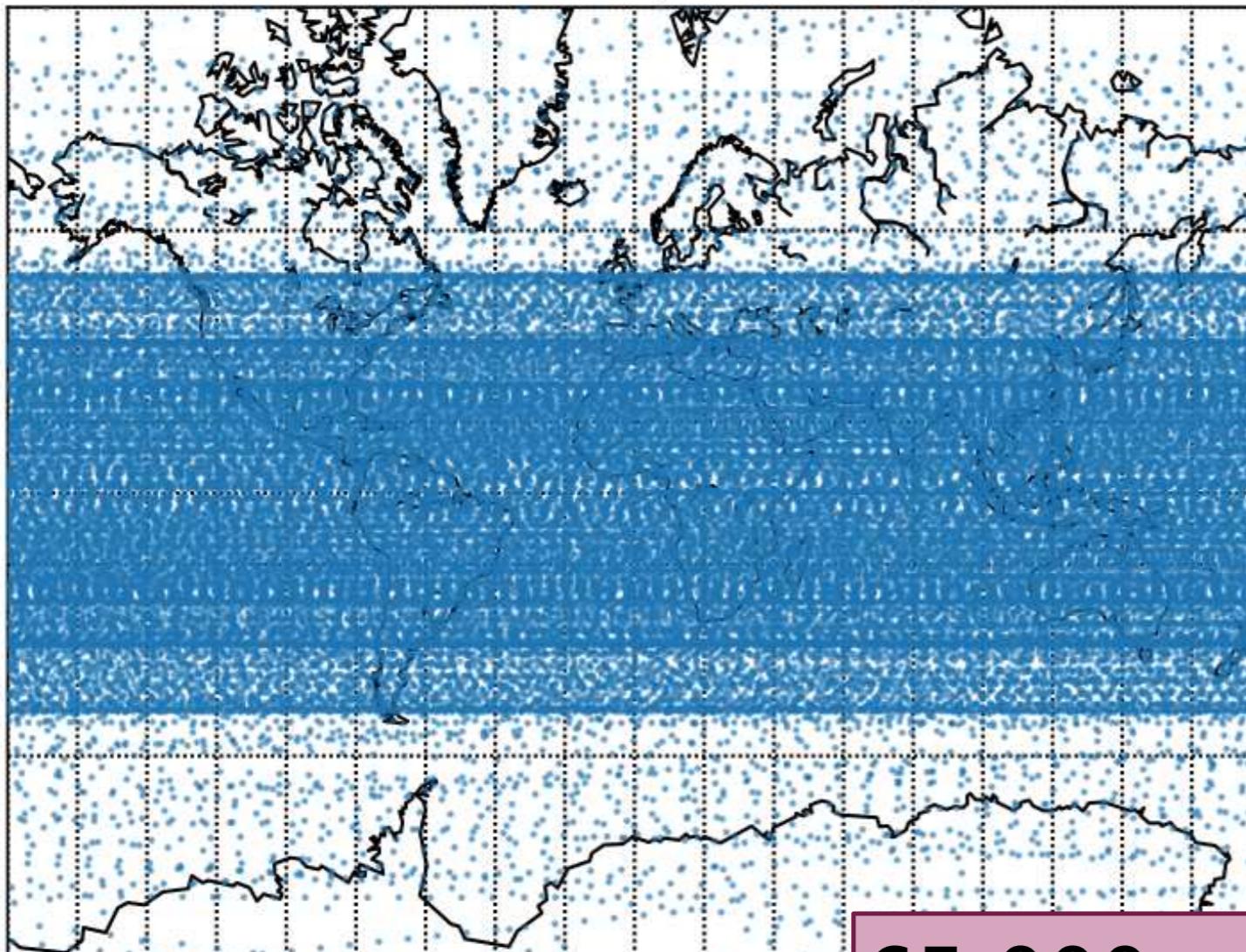
American private company SpaceX is launching batches of 60 satellites into low-Earth orbit **every 2-3 weeks**.

There are currently 3,633 Starlink satellites in orbit (out of 3,930 so far launched)

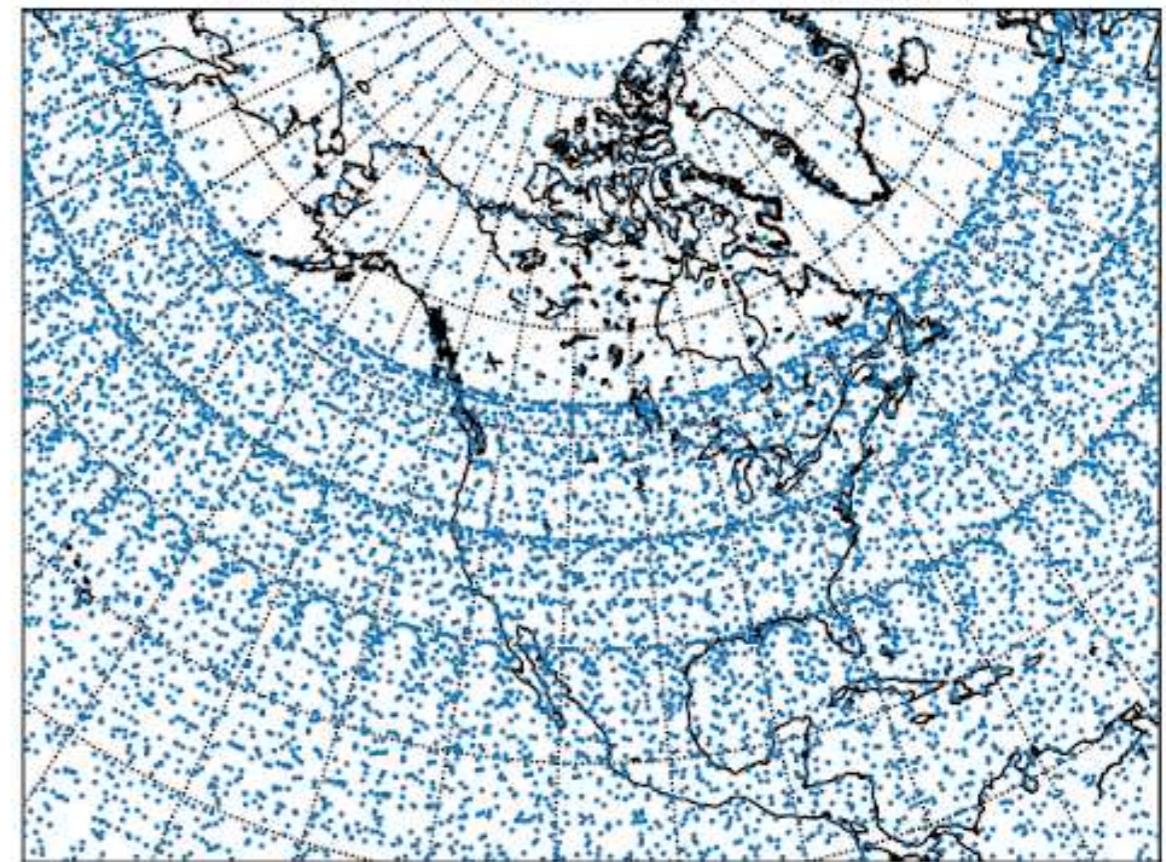
[Numbers from Jonathan McDowell's Starlink Statistics Page up to date as of 16 Feb 2023]

How bad could it get?

Satellite Distribution (Lat-Lon Projection)



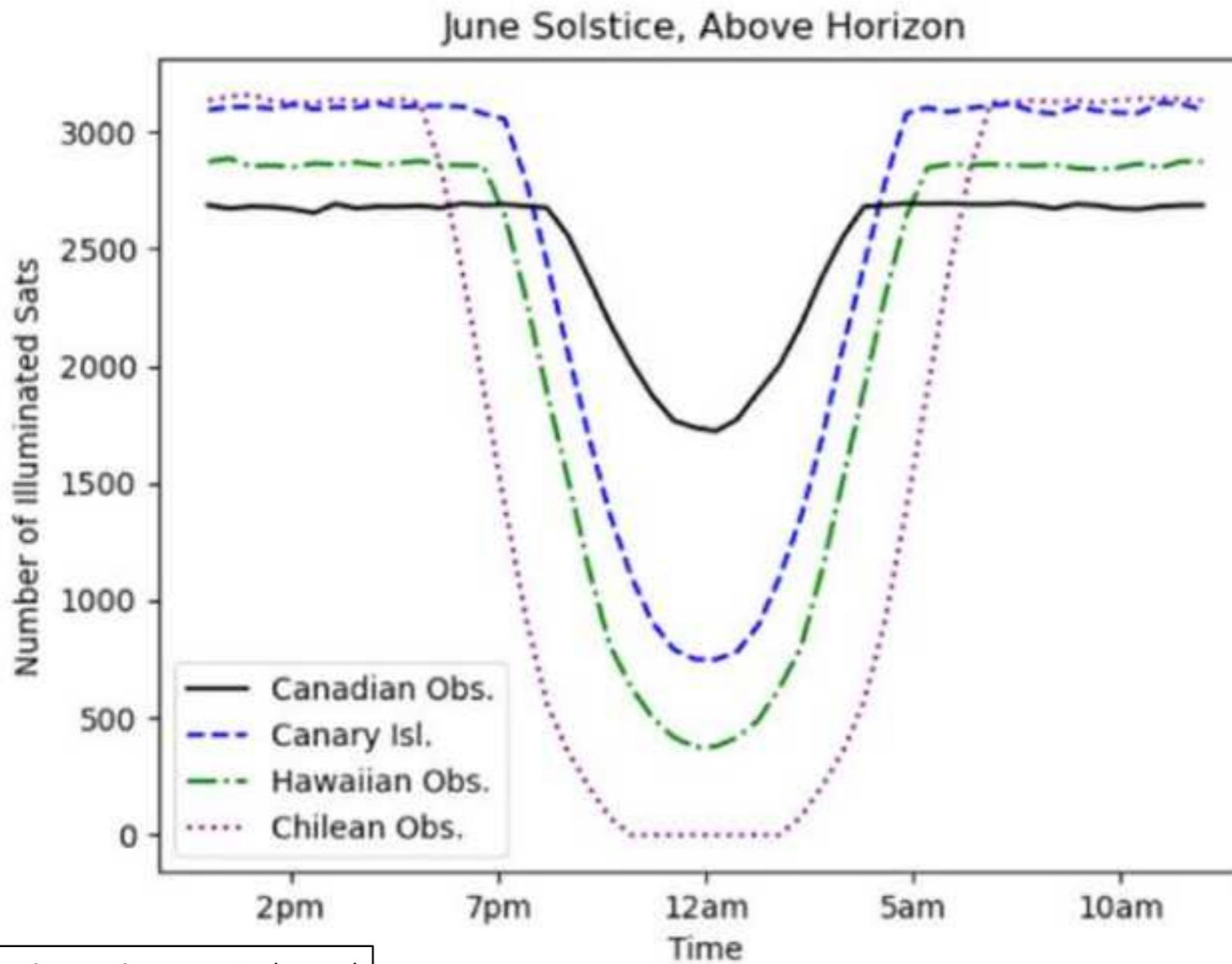
Satellite Distribution (Lat-Lon Projection)



65,000 satellites

Lawler, Boley & Rein (2022)

How bad could it get?



Straightforward to calculate how many satellites are above the horizon and illuminated by sunlight.

... But how bright the satellites are when illuminated in orbit depends entirely on unknown engineering.



Megaconstellations
Available for free on the AppStore