Quantum Mechanics I
Problem Set 9

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Due: Monday, March 28th 2016

Problem 9.1

We combine a particle with spin $s_1 = \frac{1}{2}$ and a particle with arbitrary spin s_2. Consider the state

$$|s,m\rangle = A\left[\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \otimes \left|s_2,\left(m-\tfrac{1}{2}\right)\right\rangle\right] + B\left[\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \otimes \left|s_2,\left(m+\tfrac{1}{2}\right)\right\rangle\right].$$

(a) What are the possible values for s and m?

(b) Find the coefficients A and B for arbitrary values of s_2, s and m.

This question is about Clebsch-Gordan coefficients. You should calculate the coefficients by hand for this question. You may check your answers by looking up the coefficients in a table.

Hints for part (b):

- You know that $|s,m\rangle$ is an eigenstate of S^2.
- $S^2 = (S^{(1)})^2 + (S^{(2)})^2 + 2\,\mathbf{S}^1 \cdot \mathbf{S}^{(2)}$
- $S_\pm|s,m\rangle = \hbar\sqrt{s(s+1) - m(m\pm 1)}|s,(m\pm 1)\rangle$ and $S_\pm = S_x \pm iS_y$.
- You can check your result using the Glebsch-Gordon coefficient table.

Problem 9.2

In this problem, we are looking at the Helium atom. We ignore the interaction term. Thus the Hamiltonian is

$$H = \frac{\hbar^2}{2m}\nabla_1^2 + \frac{\hbar^2}{2m}\nabla_2^2 - \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_1} - \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_2}.$$

- Write down the solutions to the Schroedinger equation $H\psi(r_1, r_2) = E\psi(r_1, r_2)$ in terms of Hydrogen wavefunctions.
- If we put both electrons in the $n = 2$ state, one electron will fall back to the groundstate and the other electron will get ejected. What is the energy of the ejected electron?

Problem 9.3

In the above problem, we ignored the interaction term

$$H_{\text{int}} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}.$$

We now try to estimate the order of magnitude of this term. To do that, we assume that both electrons are in the groundstate for the Hamiltonian given in Problem 8.1. Calculate the expectation value

$$\left\langle \frac{1}{4\pi\epsilon_0}\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} \right\rangle$$

for that state and use your result to predict a better estimate of the total energy of the electrons in the groundstate. Our naiive estimate without the interaction term was -109 eV, the experimental value is -78.975 eV. Comment on the level of agreement of your result.

Hint: Use Wolfram Alpha to solve the integrals or look for tipps in Griffiths, Problem 5.11.