



# Formation of resonant multi- planetary systems

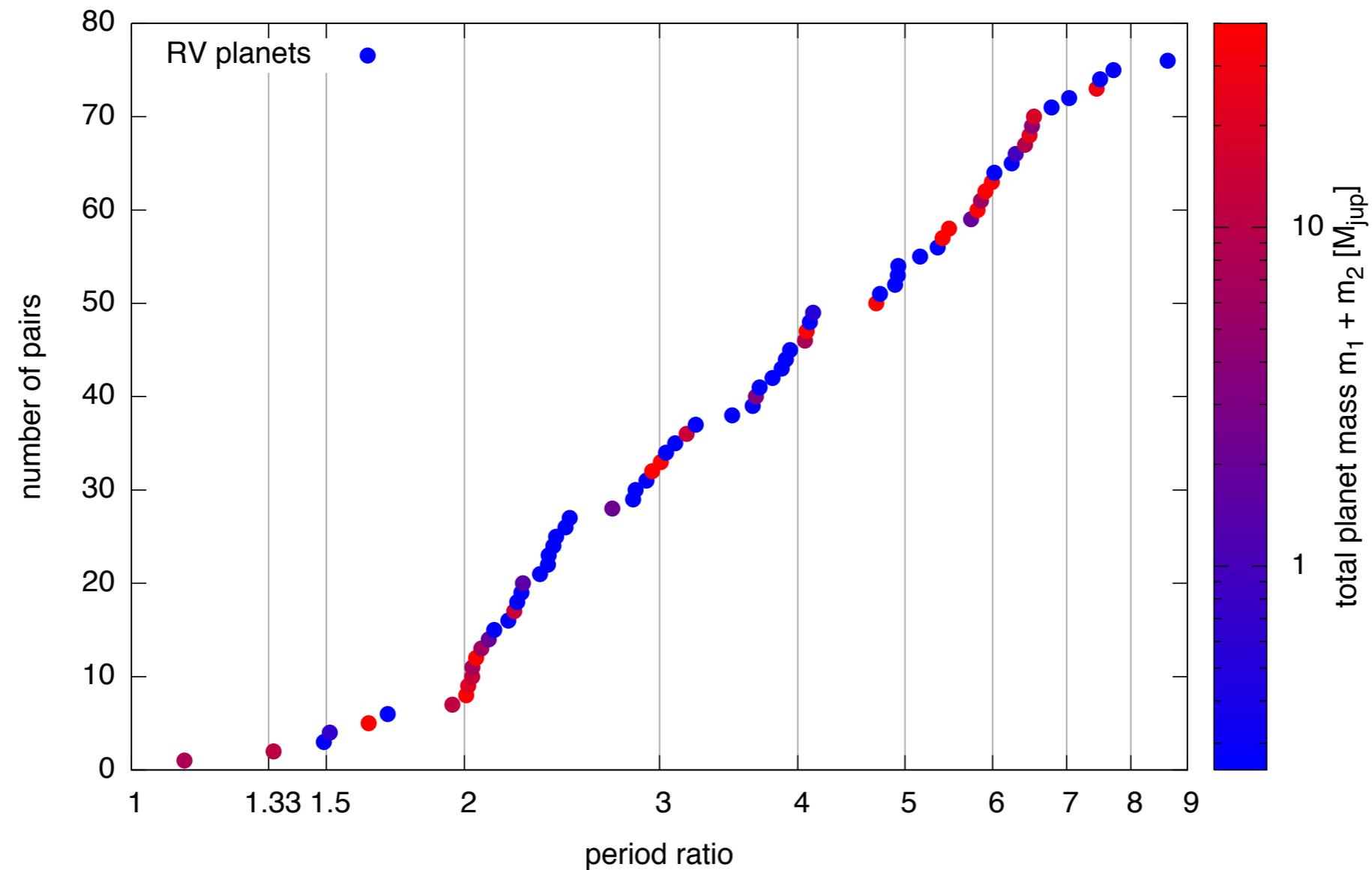
Hanno Rein @ NAOJ, Tokyo, March 2012

# Statistics of multiple planets (using iPhone App)

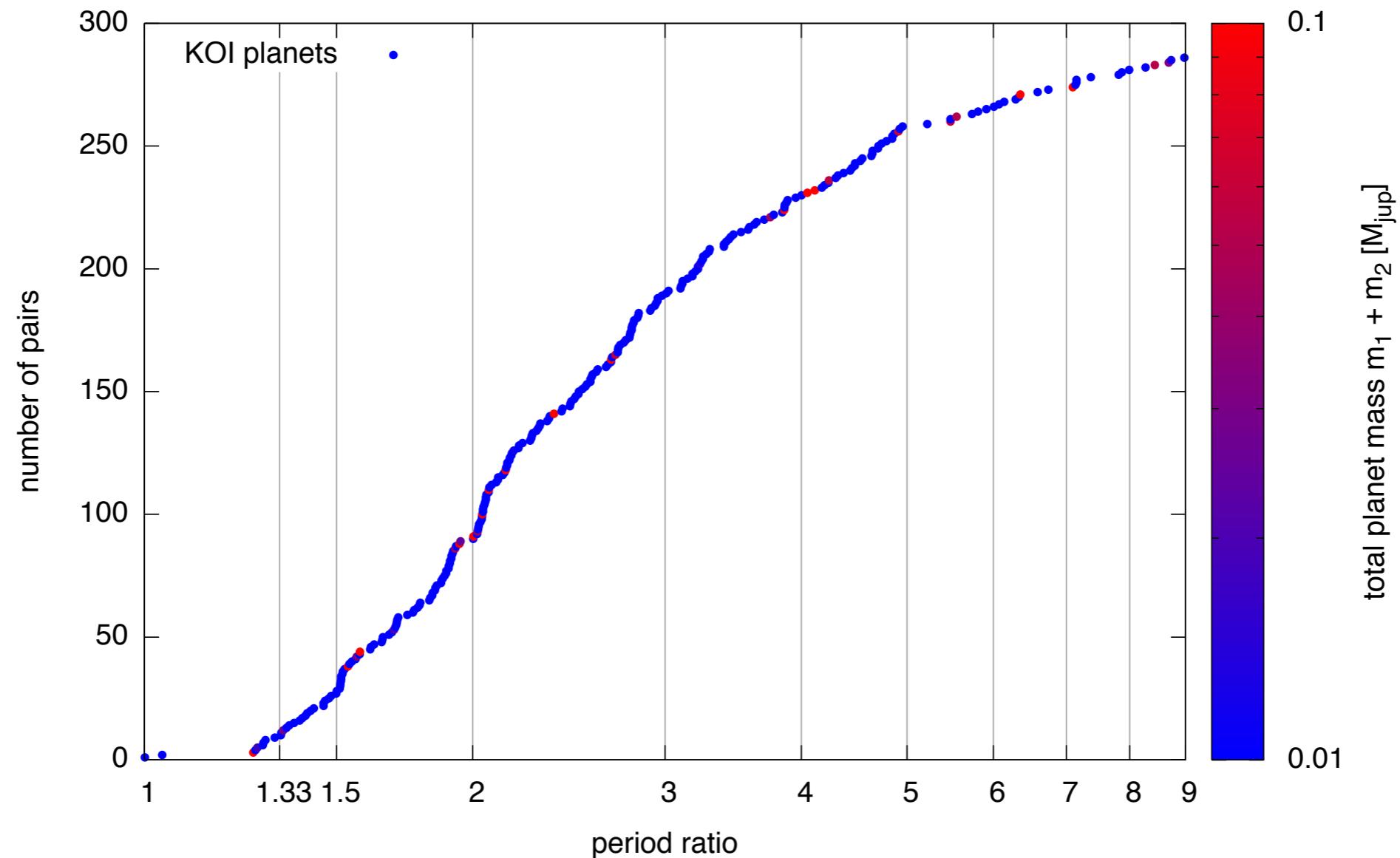


Available for free on the Apple AppStore.

# Radial velocity planets



# Kepler's transiting planet candidates



# Recipe

Migration

Resonances

# Migration in a non-turbulent disc

# Planet formation

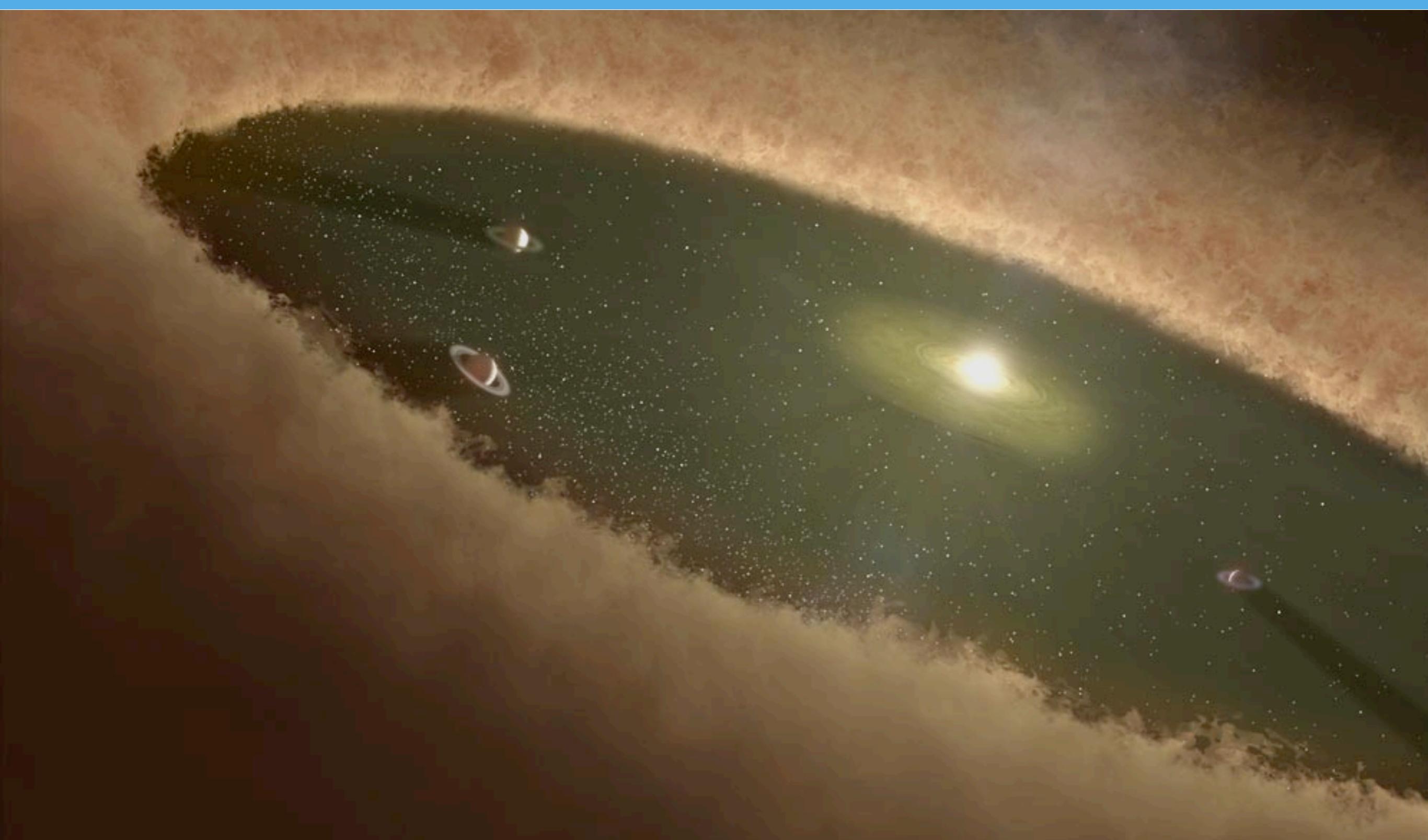
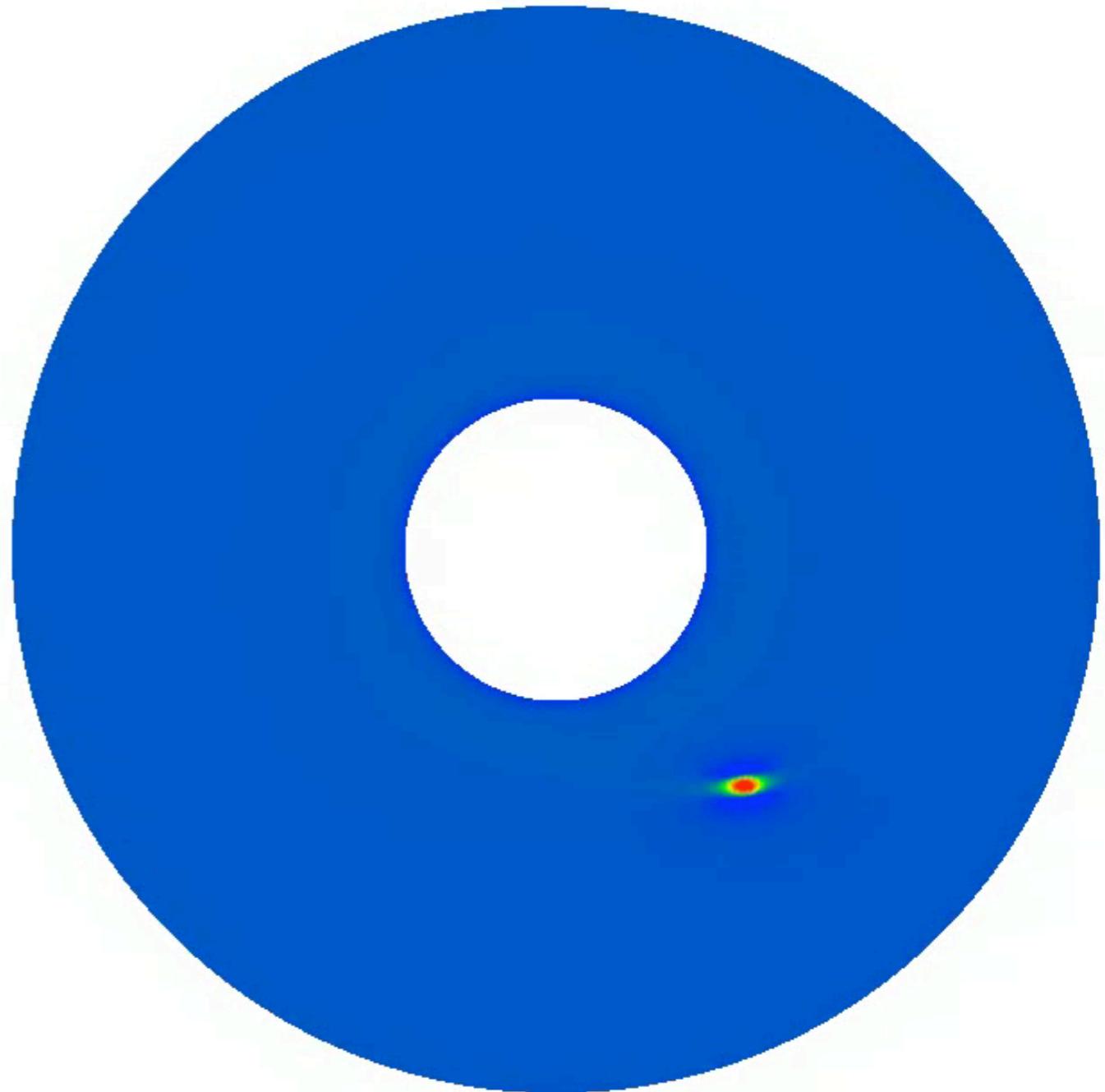


Image credit: NASA/JPL-Caltech

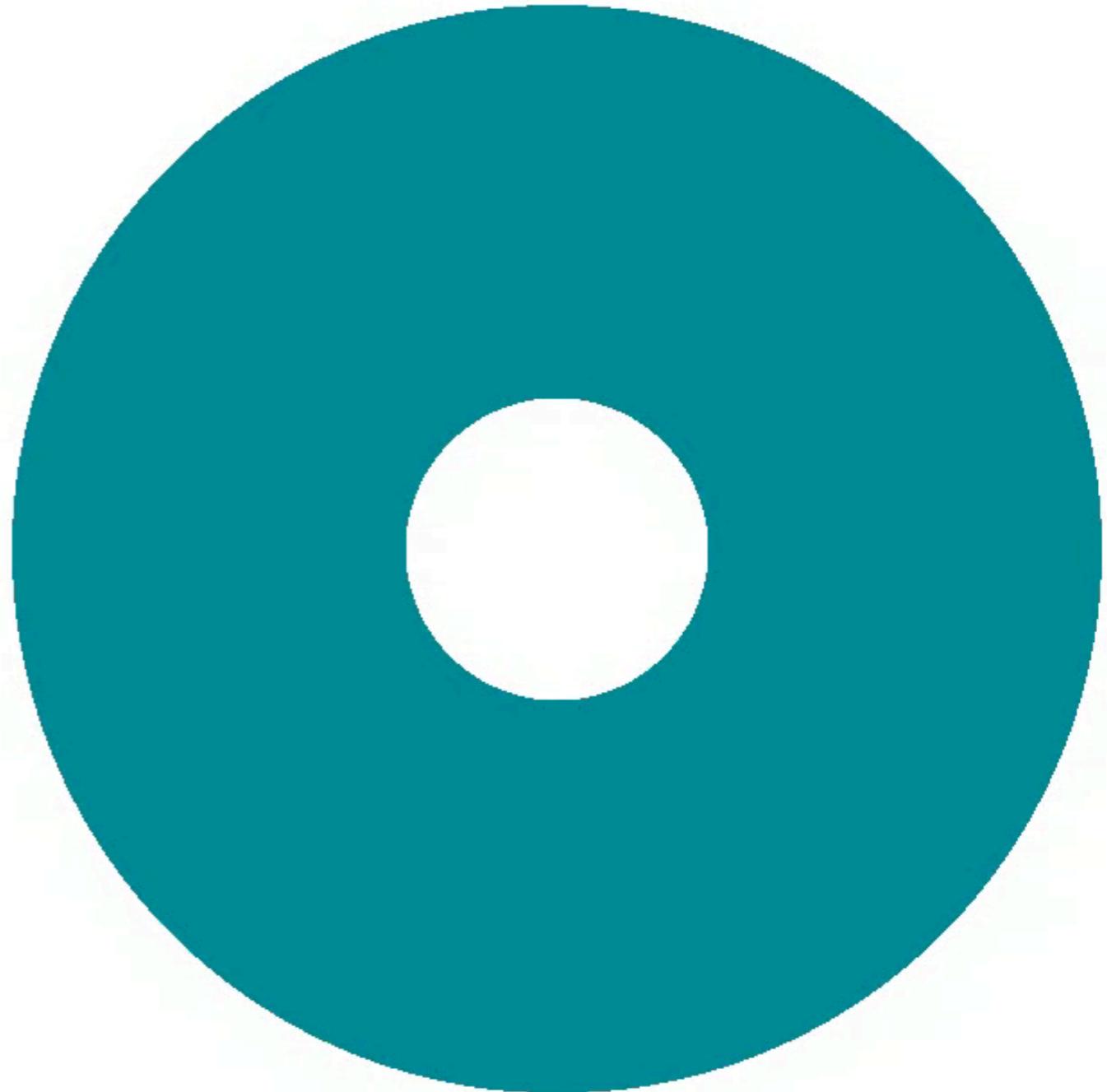
# Migration - Type I

- Low mass planets
- No gap opening in disc
- Migration rate is fast
- Depends strongly on thermodynamics of the disc



# Migration - Type II

- Massive planets (typically bigger than Saturn)
- Opens a (clear) gap
- Migration rate is slow
- Follows viscous evolution of the disc



# Gap opening criteria

$$\frac{3}{4} \frac{H}{R_{\text{Hill}}} + \frac{50M_*}{M_p \mathcal{R}} \leq 1$$

Disc scale height →

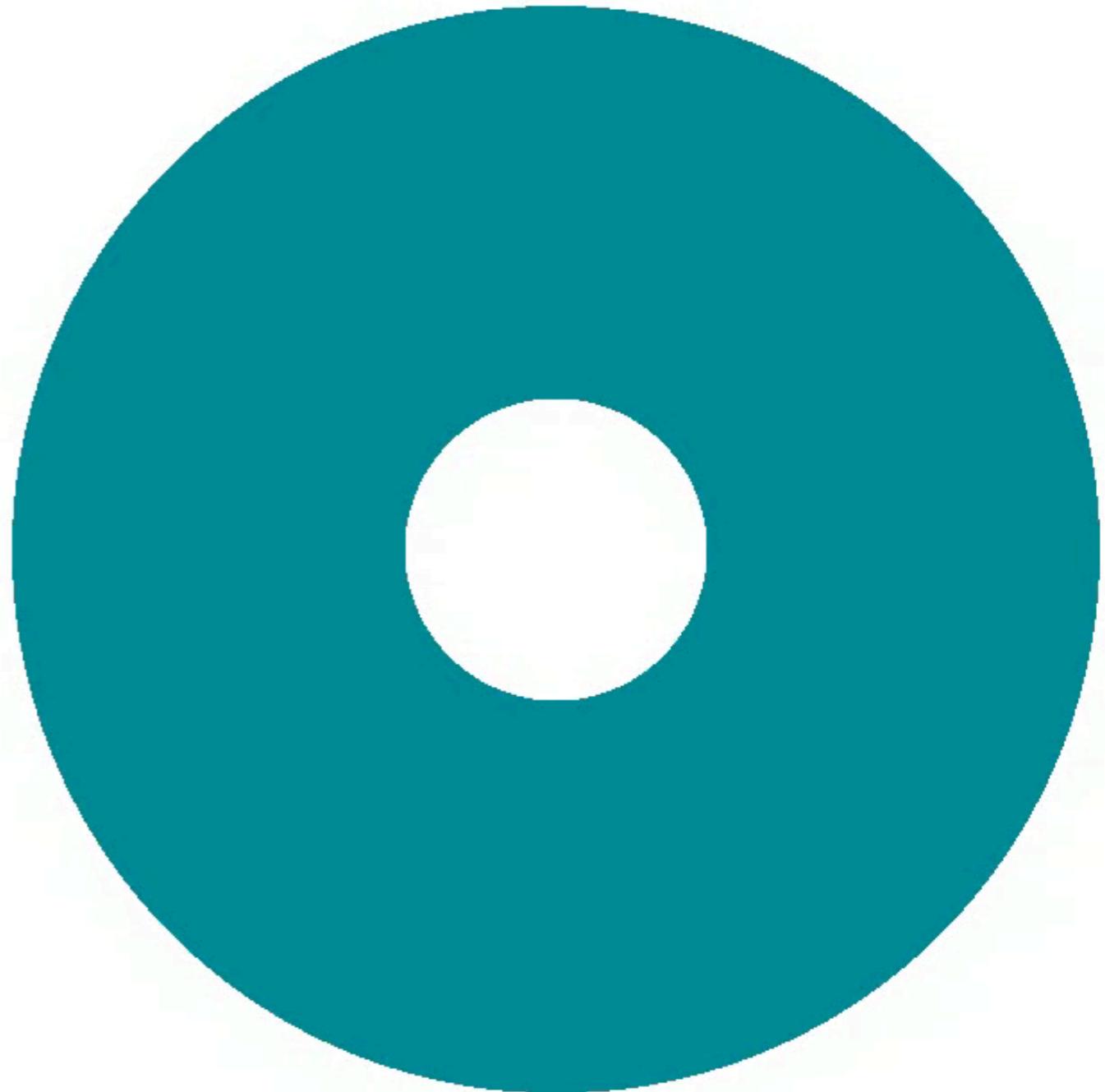
Stellar mass →

Planet mass ↗

Viscosity  $^{-1}$  ↗

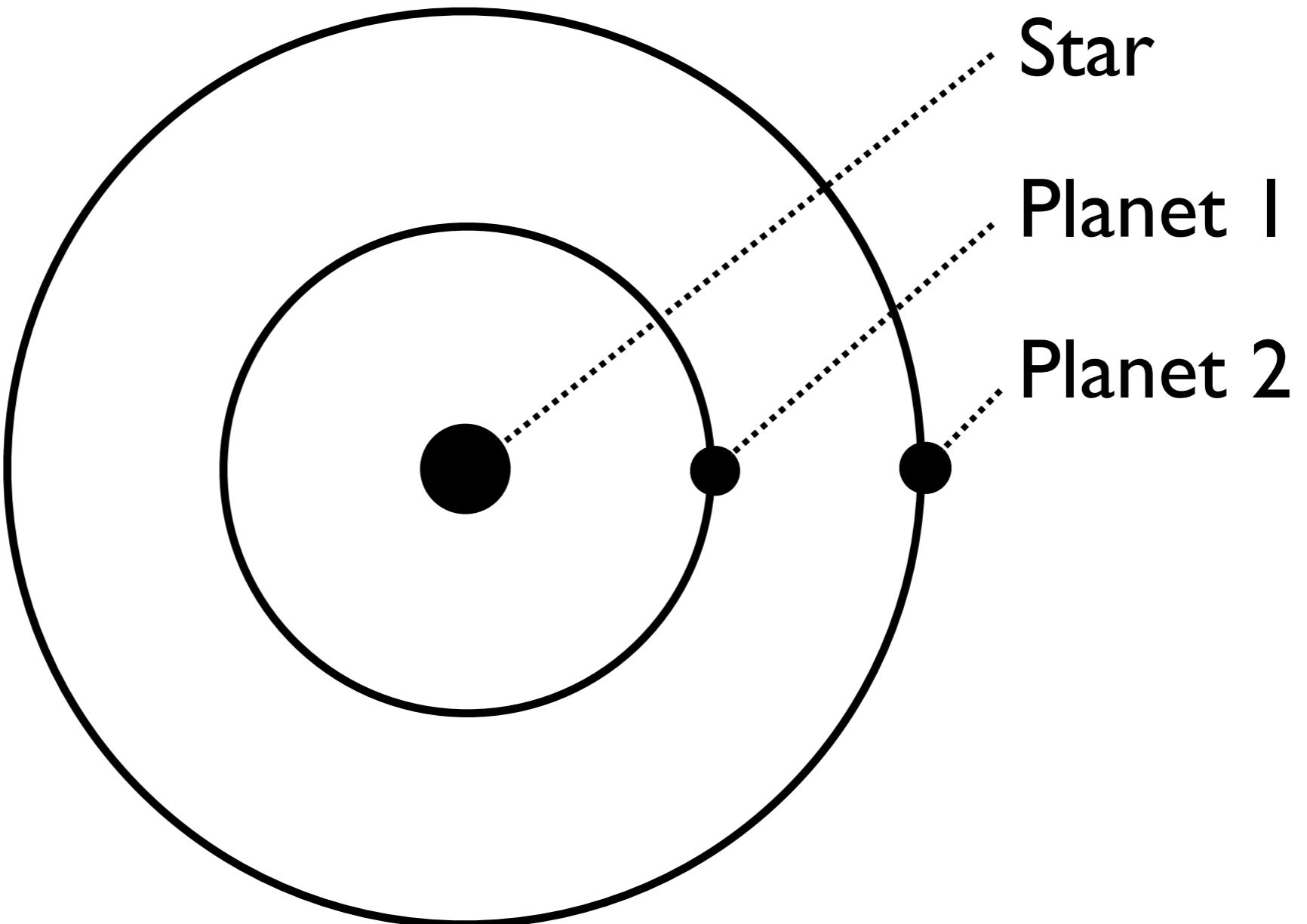
# Migration - Type III

- Massive disc
- Intermediate planet mass
- Tries to open gap
- Very fast, few orbital timescales

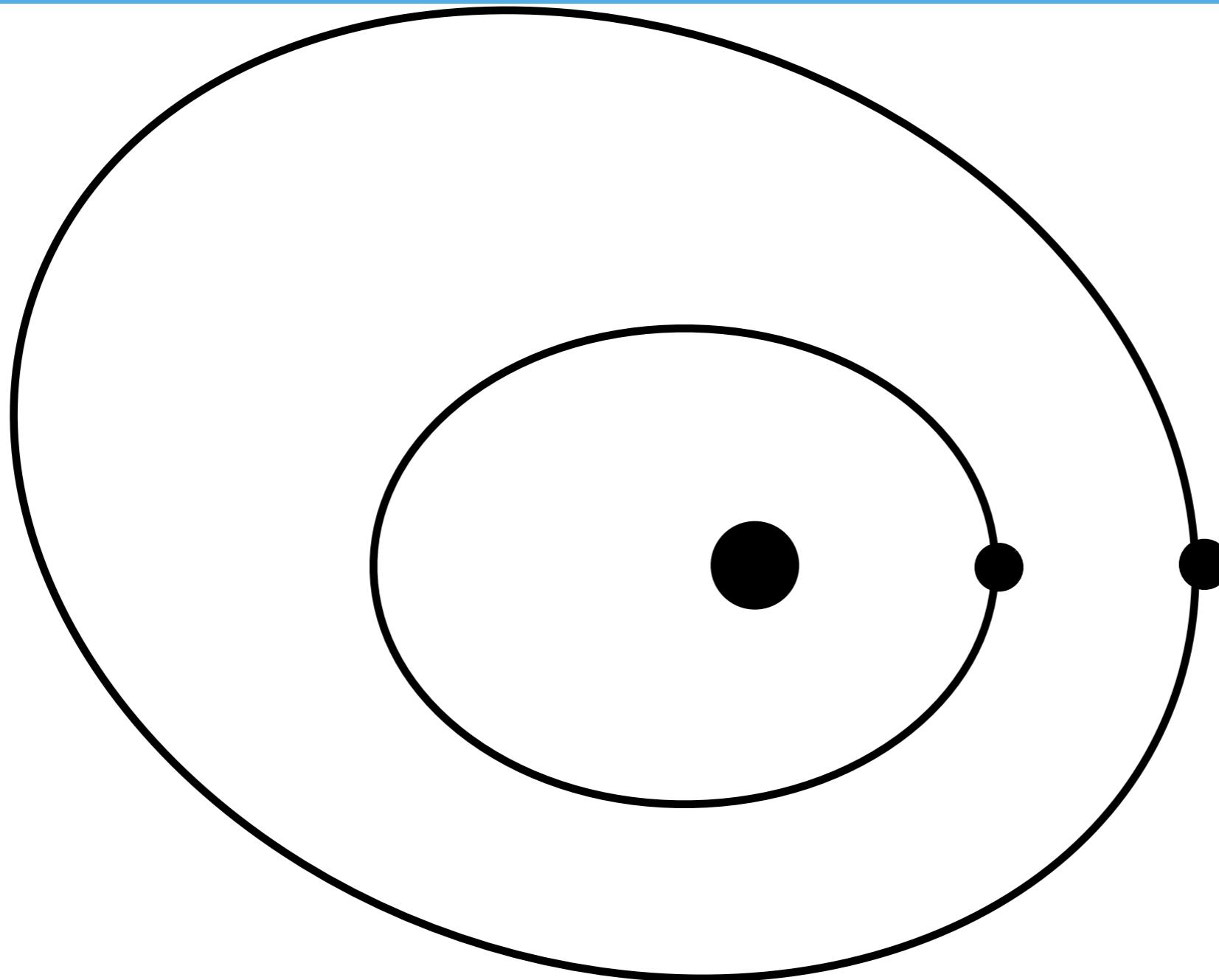


# Resonance capture

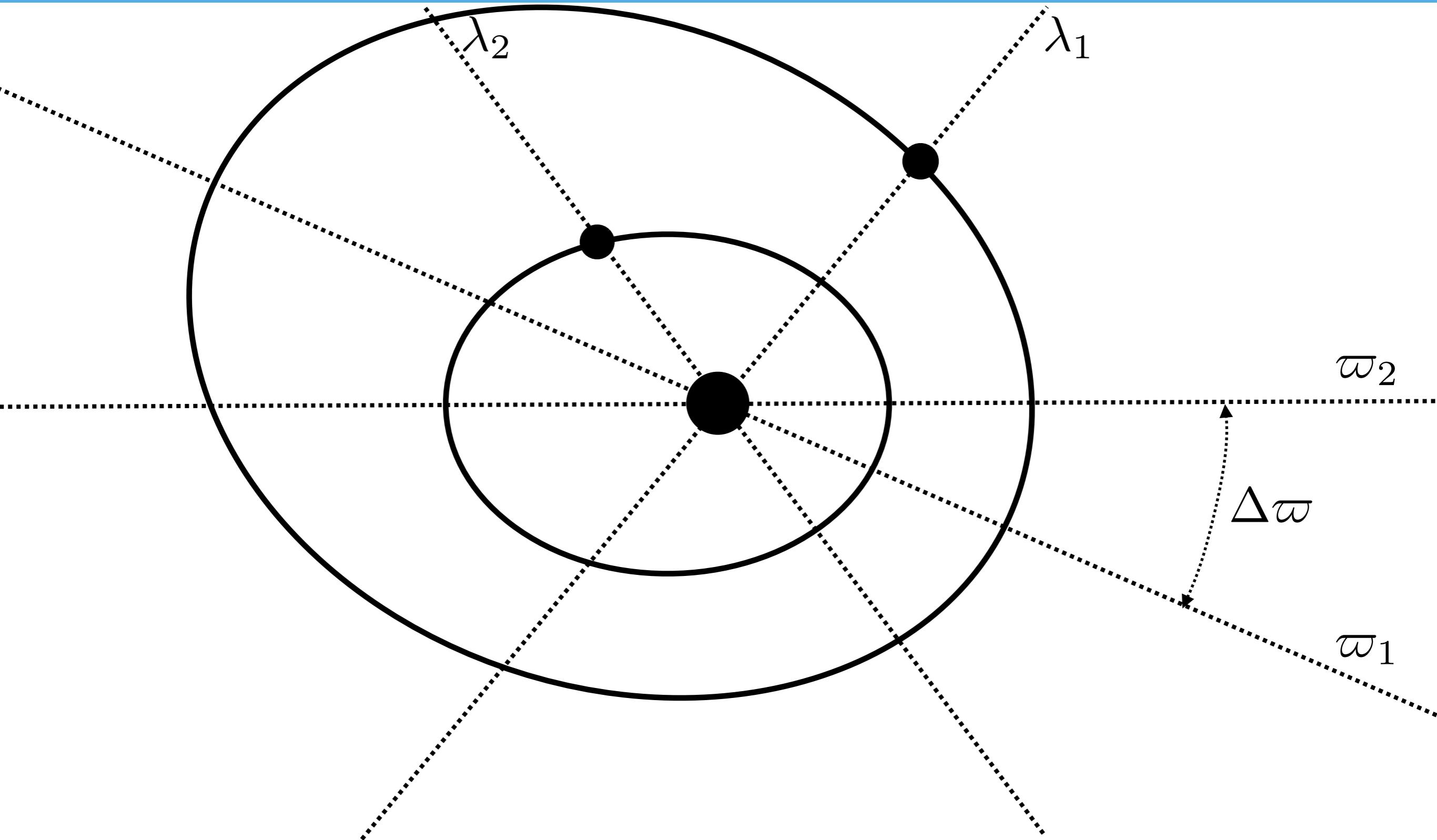
# 2:1 Mean Motion Resonance



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# 2:1 Mean Motion Resonance



# Resonant angles

- Fast varying angles

$$\lambda_1 - \varpi_1$$

$$\lambda_2 - \varpi_2$$

- Slowly varying combinations

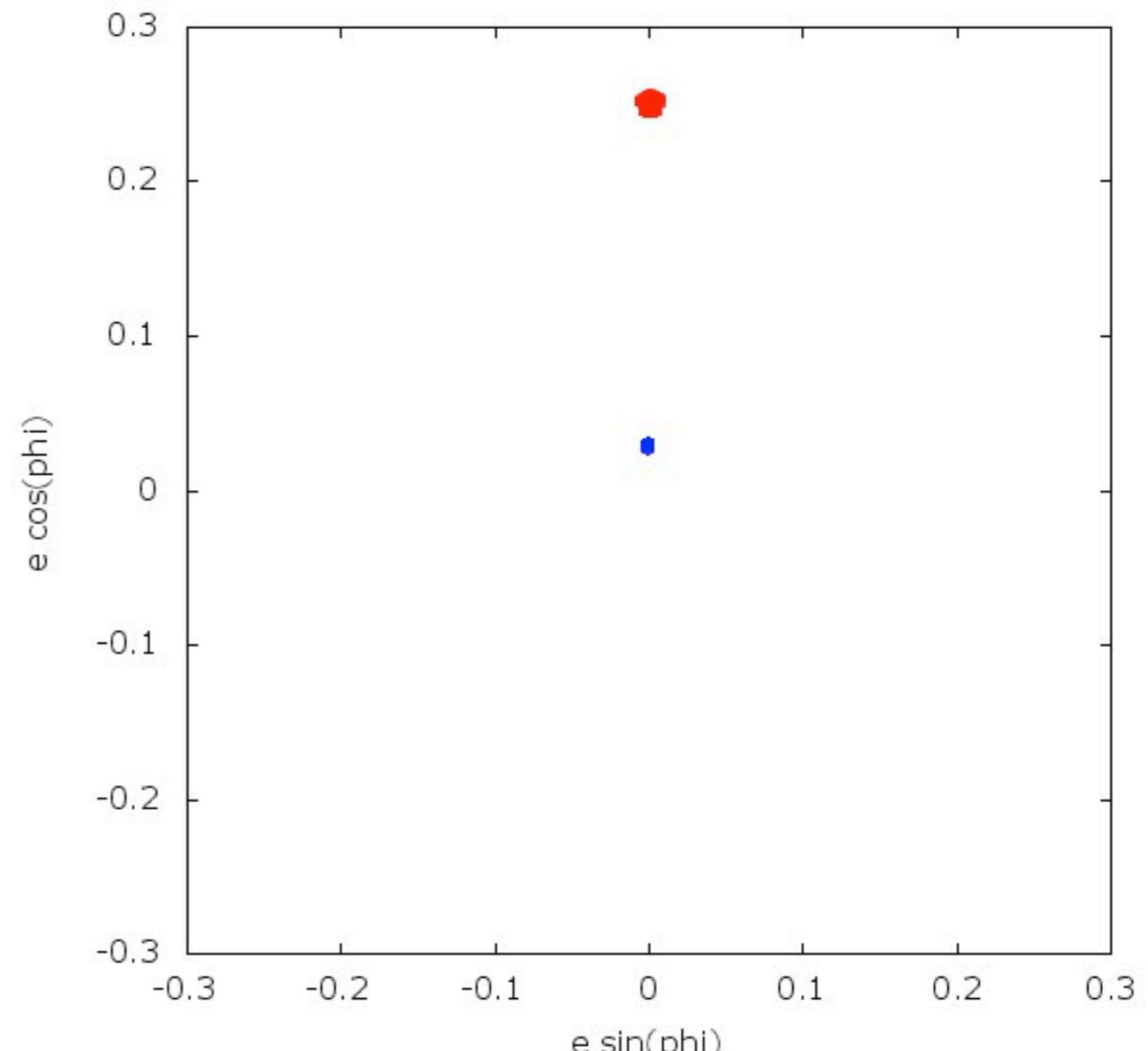
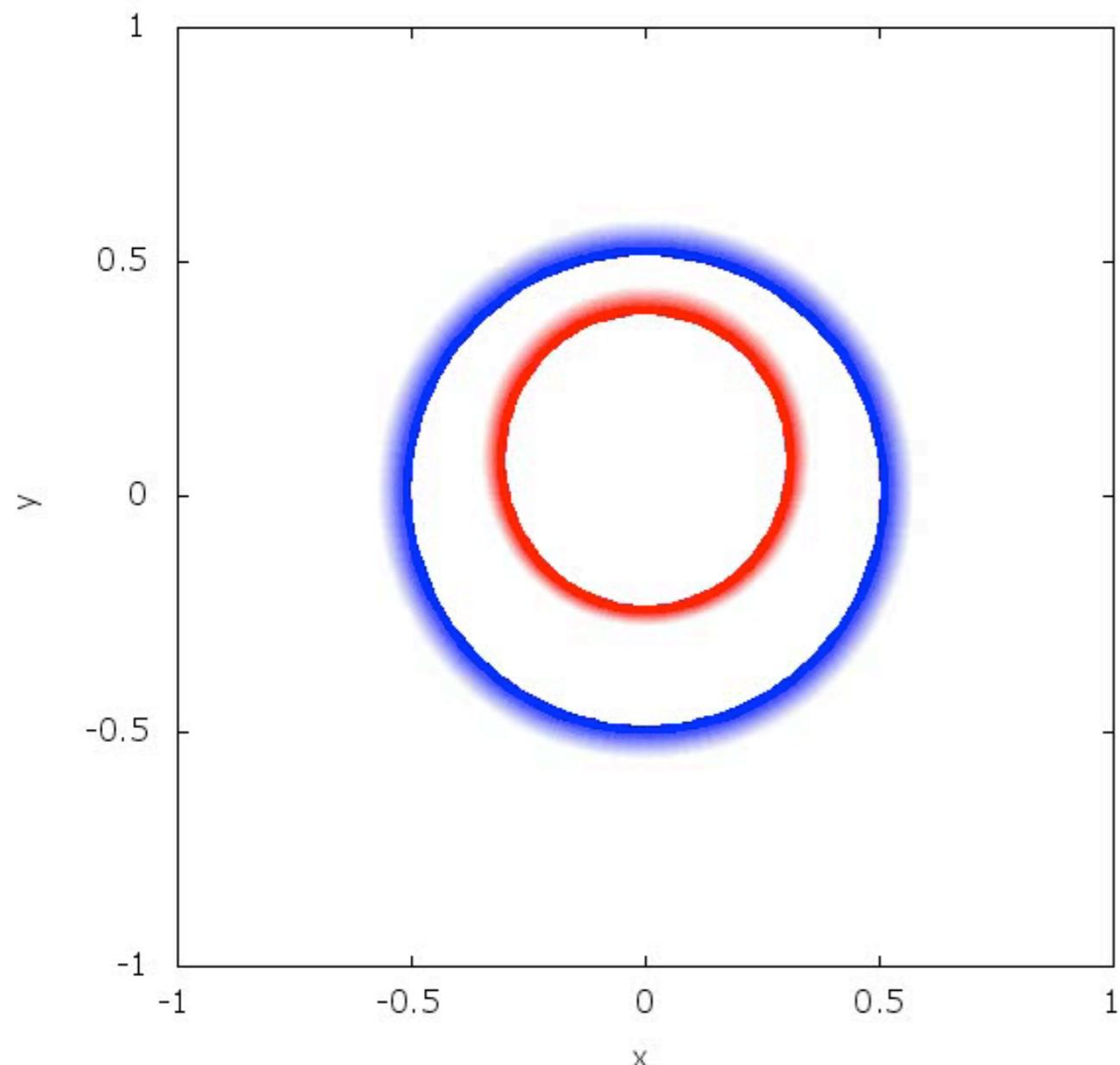
$$\phi_1 = \lambda_2 - 2\lambda_1 + \varpi_2$$

$$\phi_2 = \lambda_2 - 2\lambda_1 + \varpi_1$$

$$\Delta\varpi = \varpi_1 - \varpi_2$$

- Two are linear independent

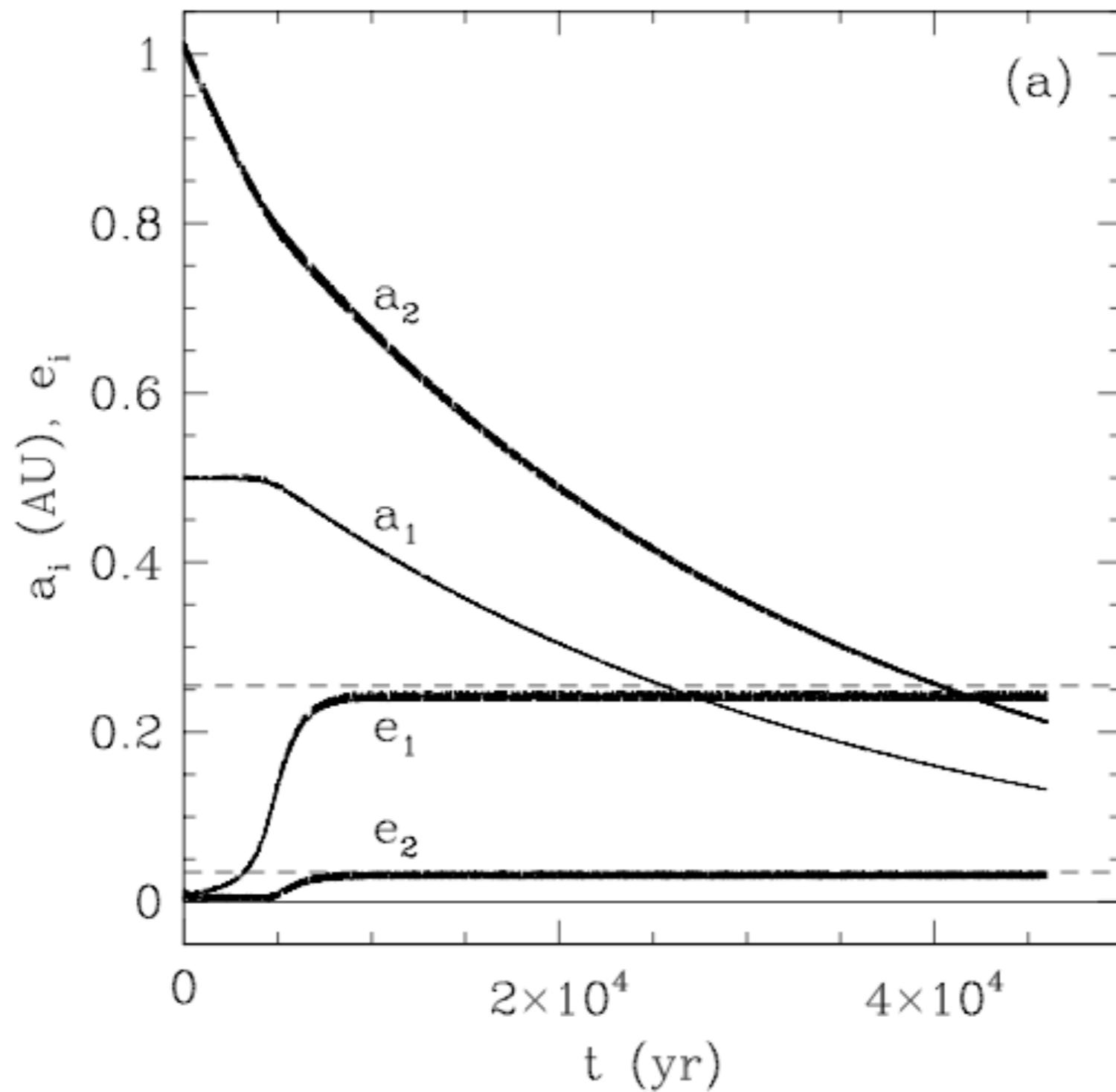
# Non-turbulent resonance capture: two planets



$$\phi_1 = \lambda_2 - 2\lambda_1 + \varpi_2$$

parameters of GJ 876

# GJ 876



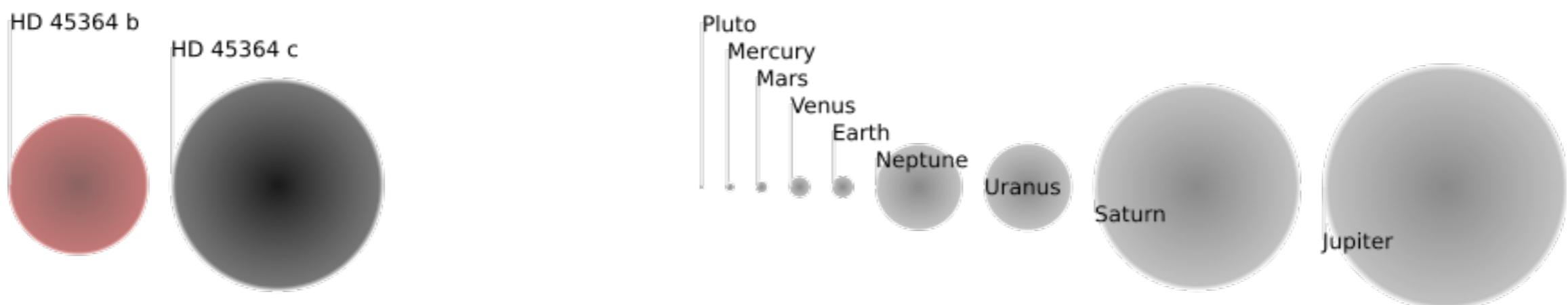
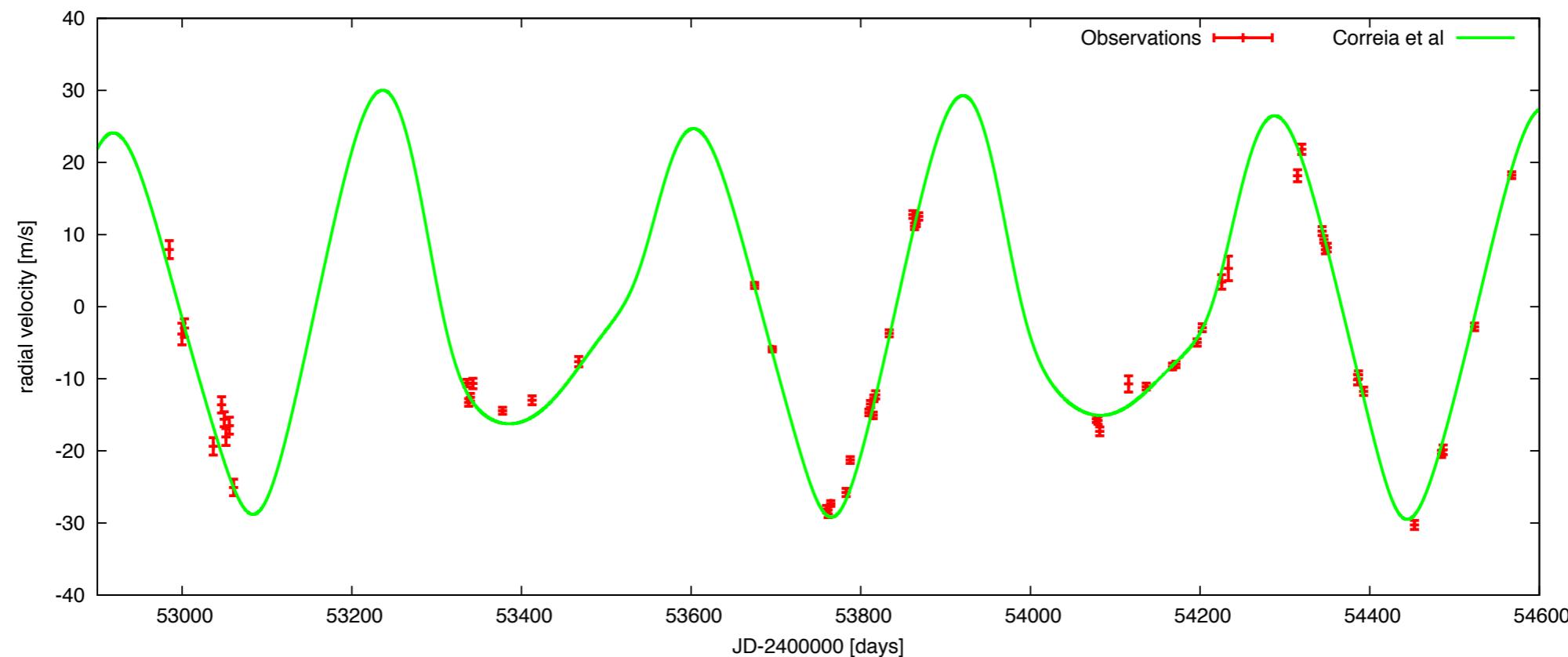
# Take home message I

**planet + disc = migration**

**2 planets + migration = resonance**

# HD 45364

# HD45364

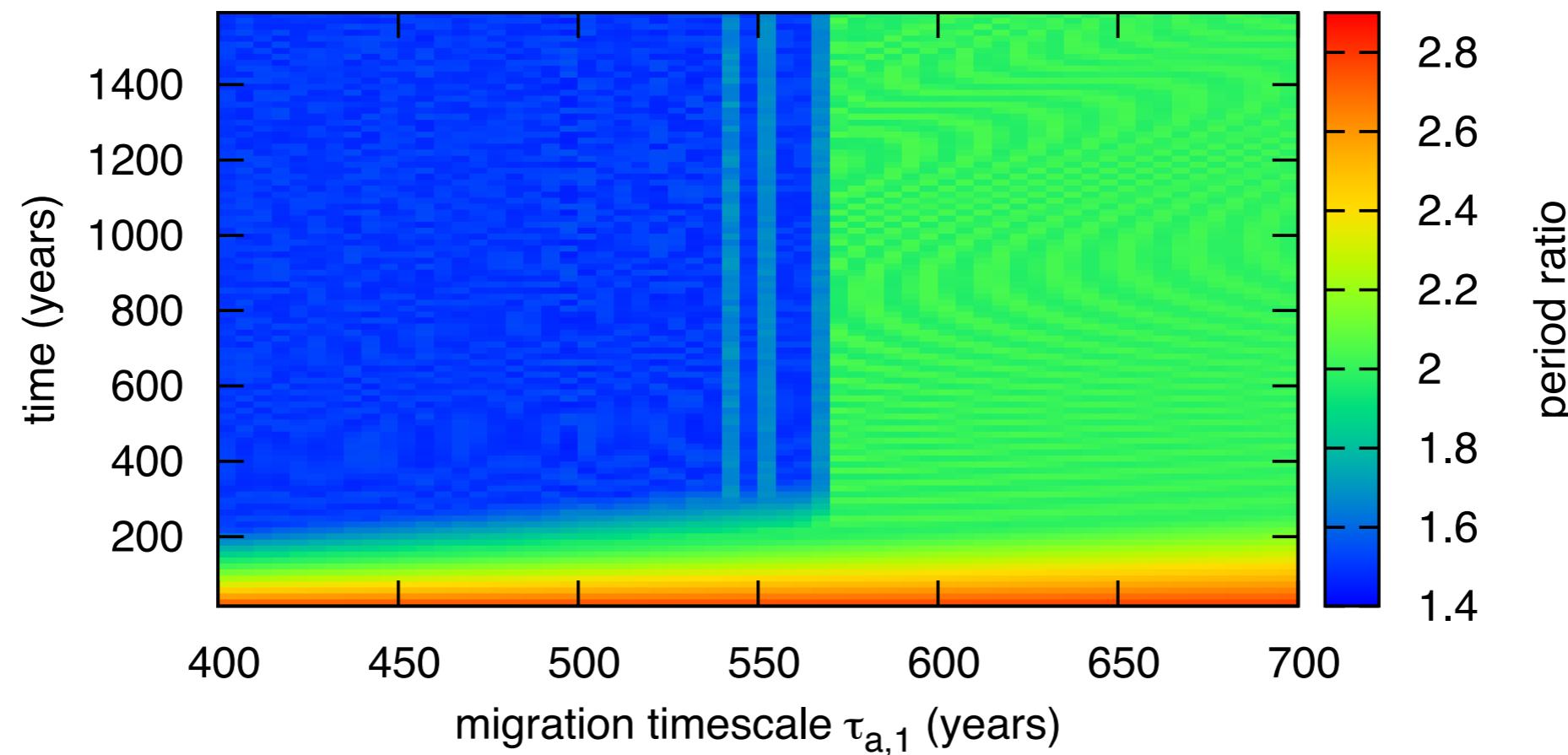


# Formation scenario for HD45364

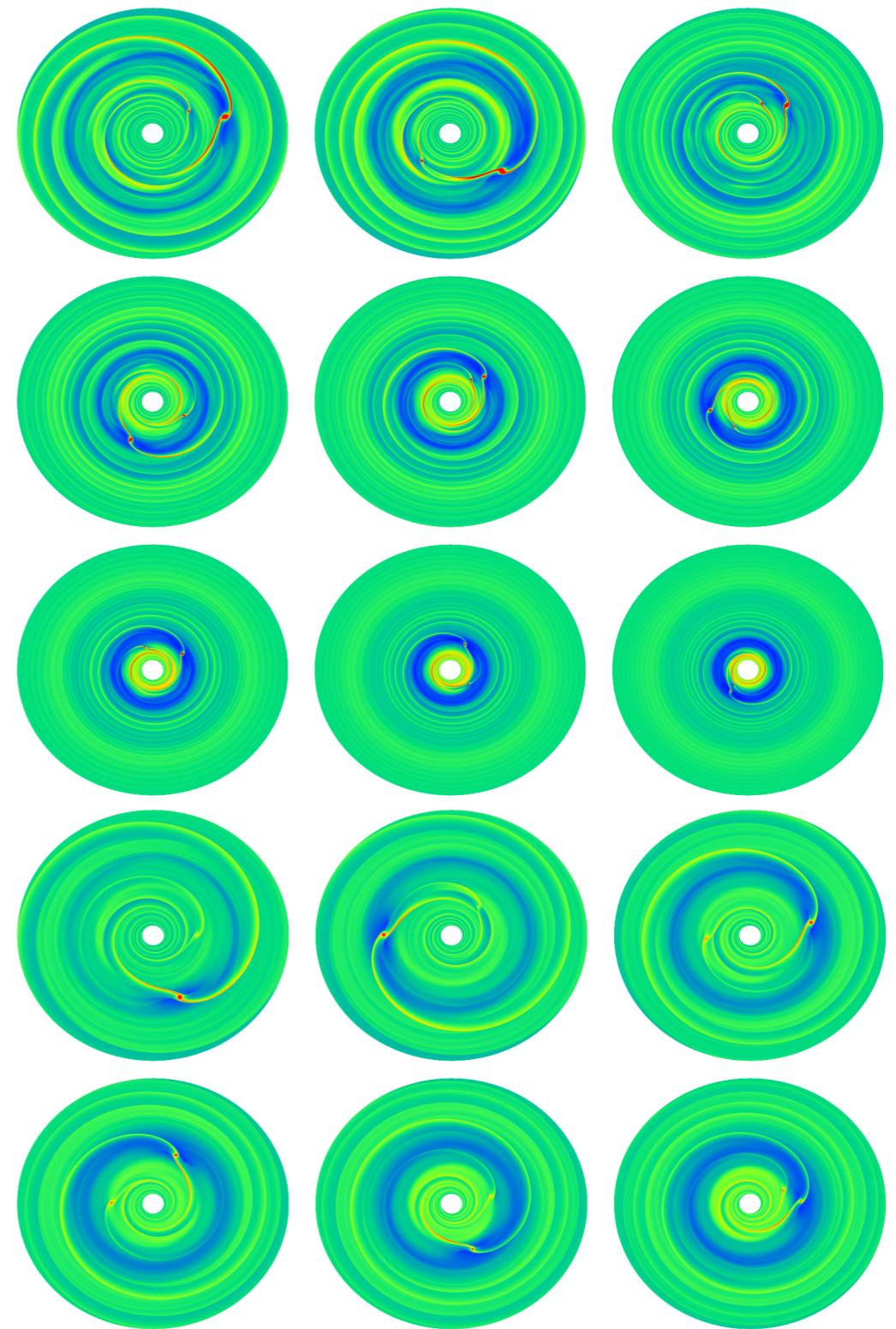
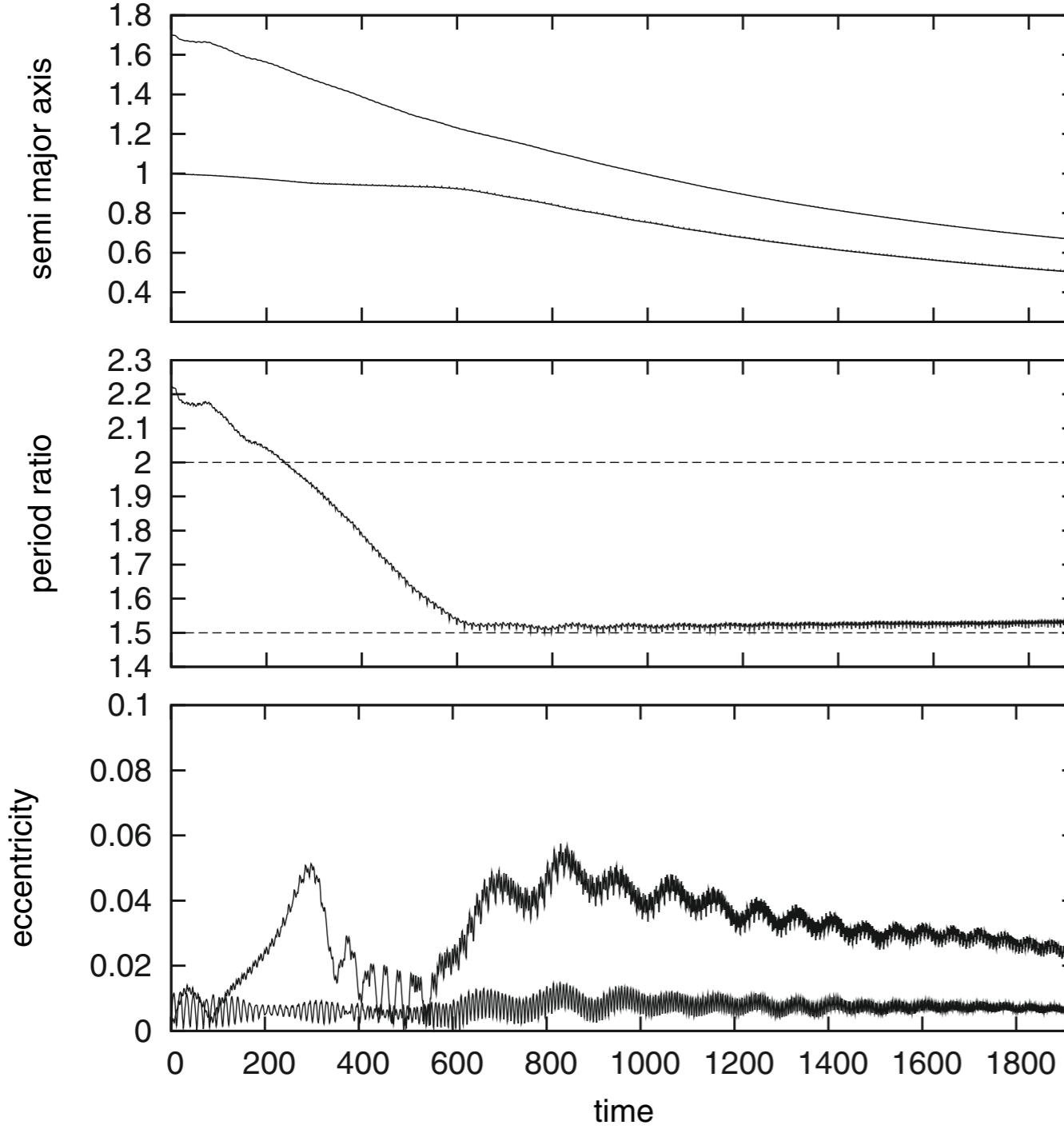
- Two migrating planets
- Infinite number of resonances

1:2    7:8    3:2    1:3    3:4

- Migration speed is crucial
- Resonance width and libration period define critical migration rate



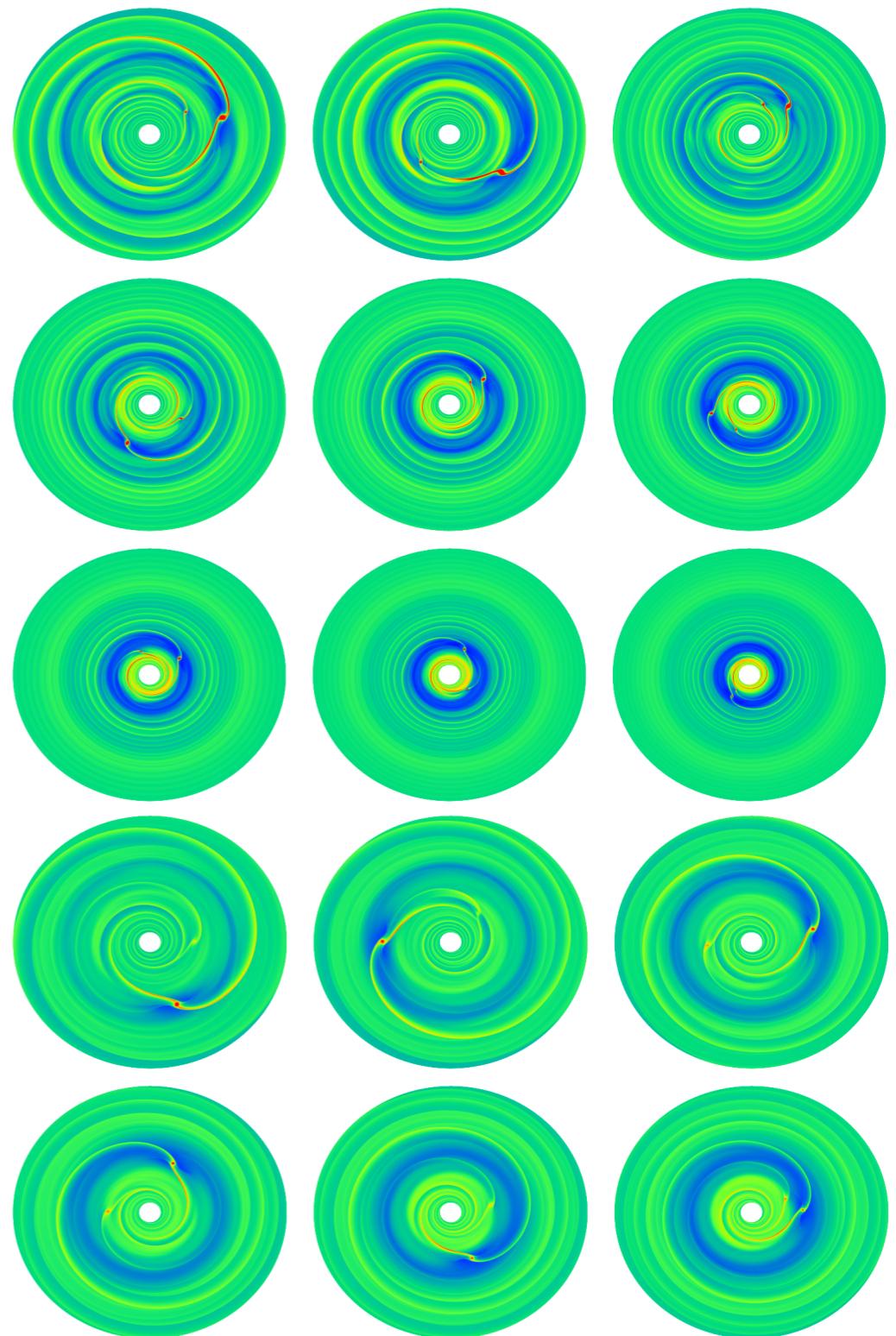
# Formation scenario for HD45364



# Formation scenario for HD45364

## Massive disc (5 times MMSN)

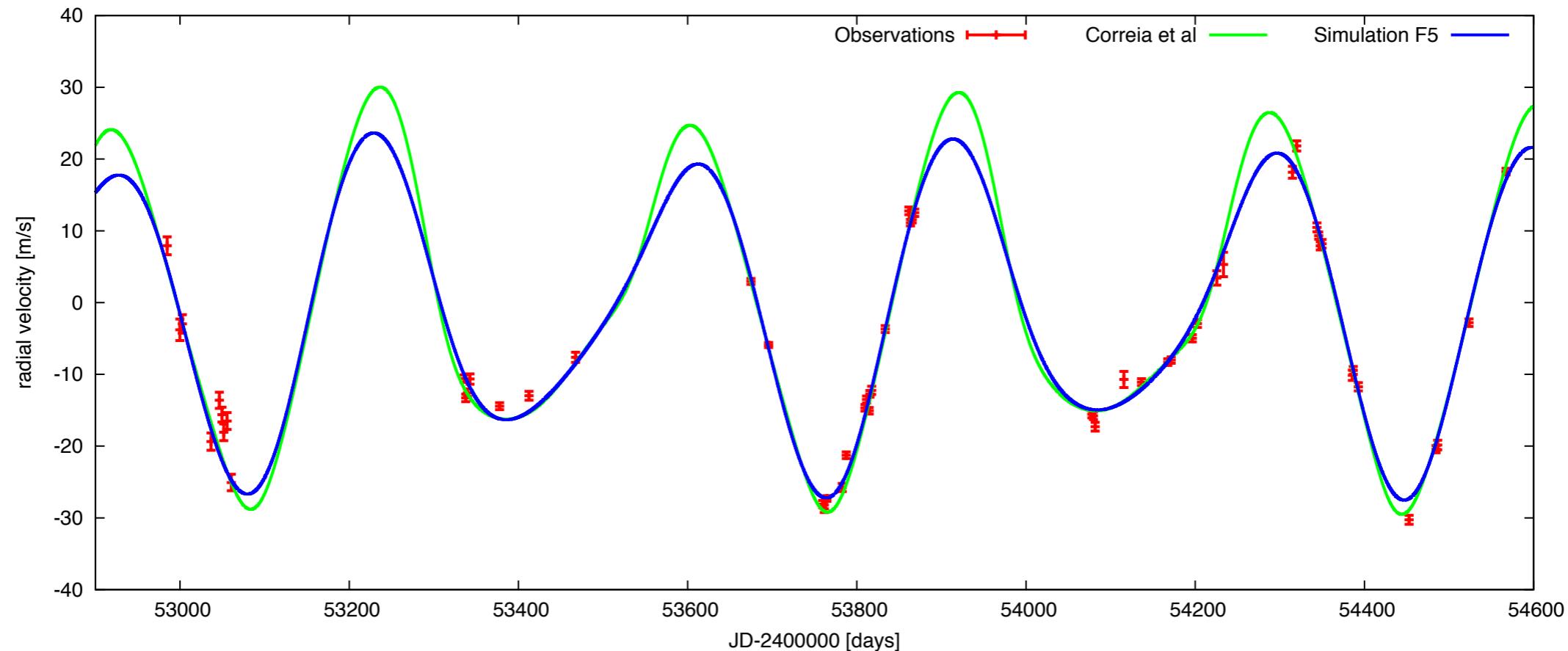
- Short, rapid Type III migration
- Passage of 2:1 resonance
- Capture into 3:2 resonance



## Large scale-height (0.07)

- Slow Type I migration once in resonance
- Resonance is stable
- Consistent with radiation hydrodynamics

# Formation scenario leads to a better ‘fit’

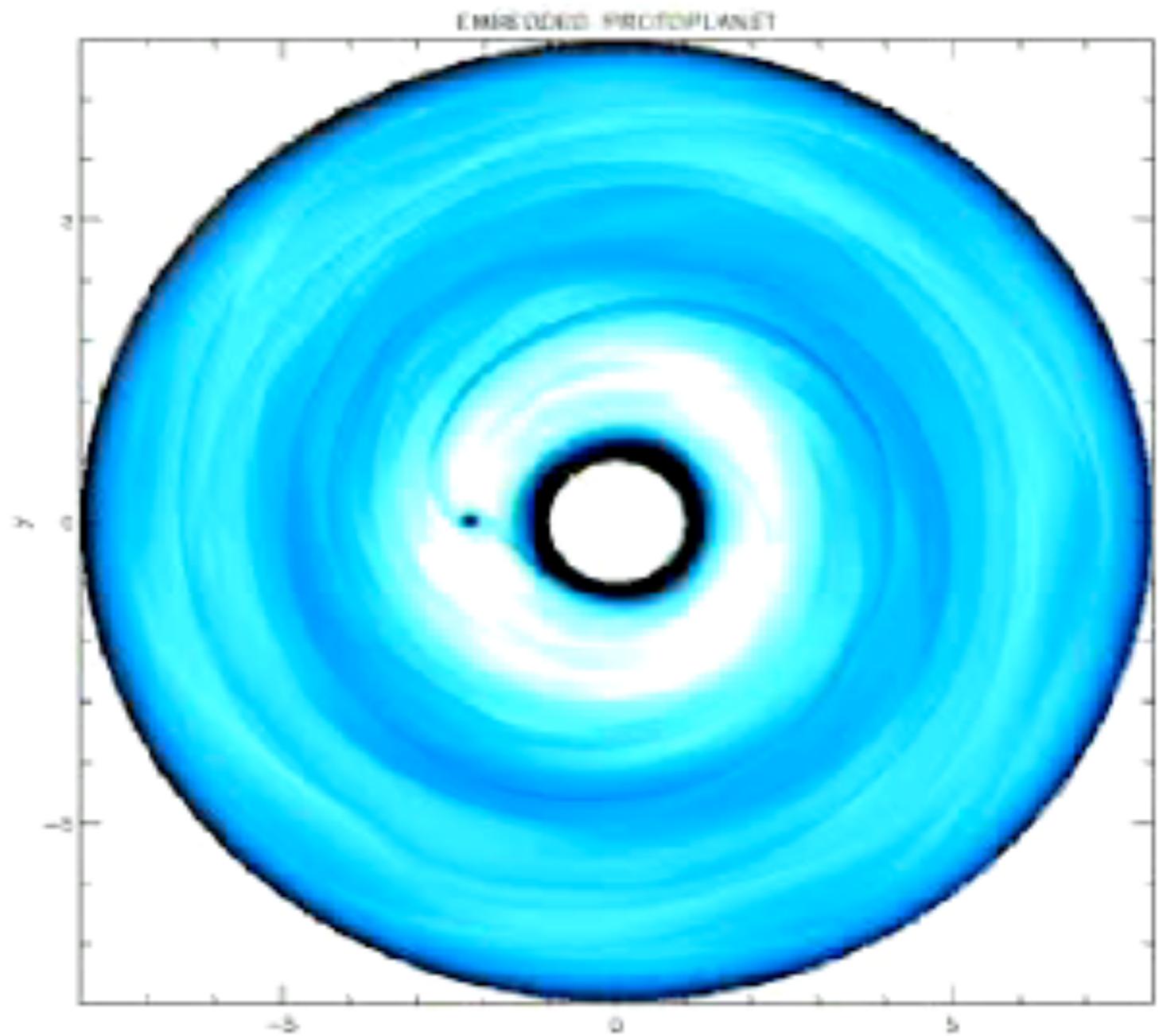


Parameter	Unit	Correia et al. (2009)		Simulation F5	
		b	c	b	c
$M \sin i$	[ $M_{\text{Jup}}$ ]	0.1872	0.6579	0.1872	0.6579
$M_*$	[ $M_{\odot}$ ]		0.82		0.82
$a$	[AU]	0.6813	0.8972	0.6804	0.8994
$e$		$0.17 \pm 0.02$	$0.097 \pm 0.012$	0.036	0.017
$\lambda$	[deg]	$105.8 \pm 1.4$	$269.5 \pm 0.6$	352.5	153.9
$\varpi^a$	[deg]	$162.6 \pm 6.3$	$7.4 \pm 4.3$	87.9	292.2
$\sqrt{\chi^2}$			2.79	$2.76^b$ (3.51)	
Date	[JD]		2453500	2453500	

# Migration in a turbulent disc

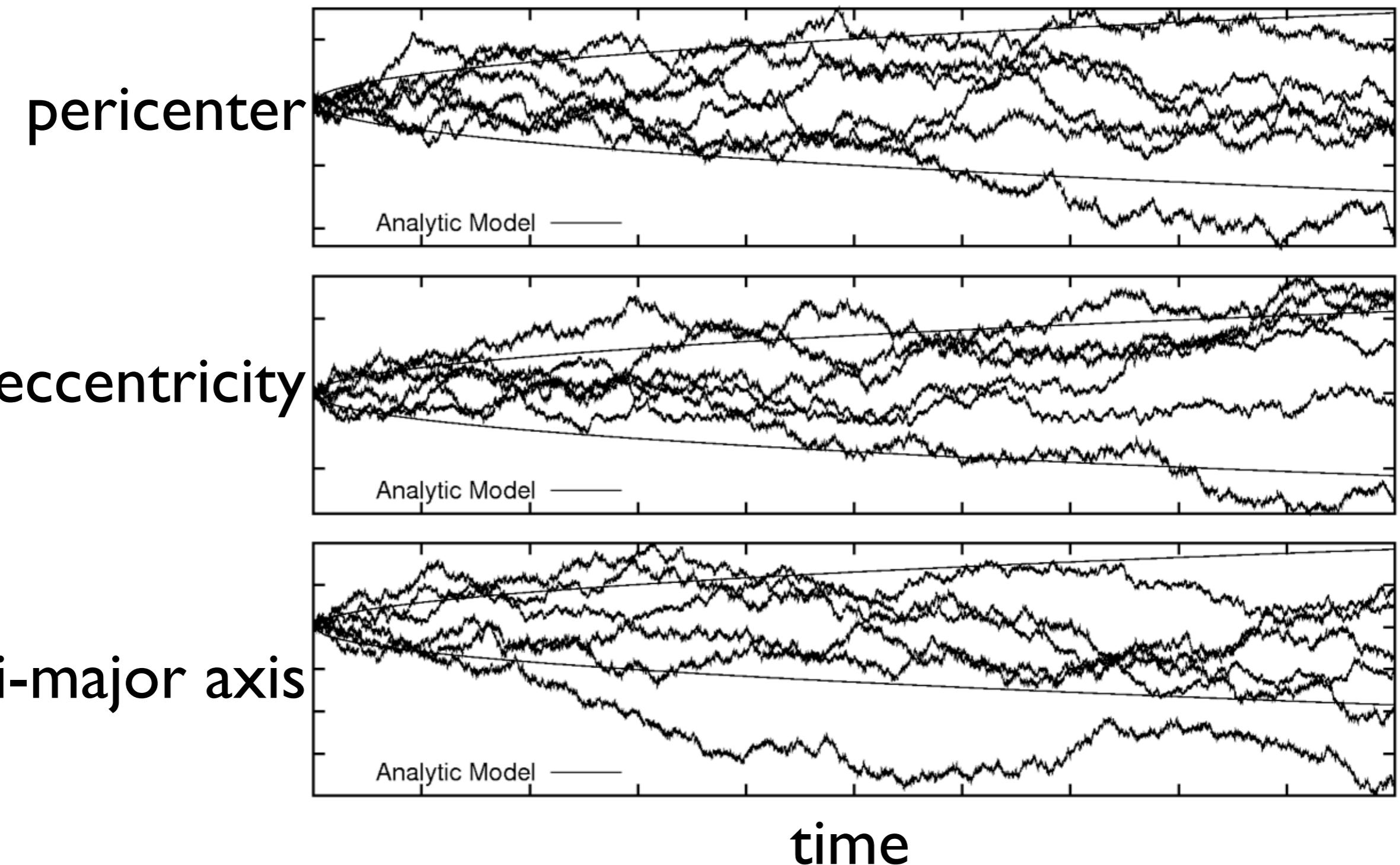
# Turbulent disc

- Angular momentum transport
- Magnetorotational instability (MRI)
- Density perturbations interact gravitationally with planets
- Stochastic forces lead to random walk
- Large uncertainties in strength of forces



Animation from Nelson & Papaloizou 2004  
Random forces measured by Laughlin et al. 2004, Nelson 2005, Oischi et al. 2007

# Random walk

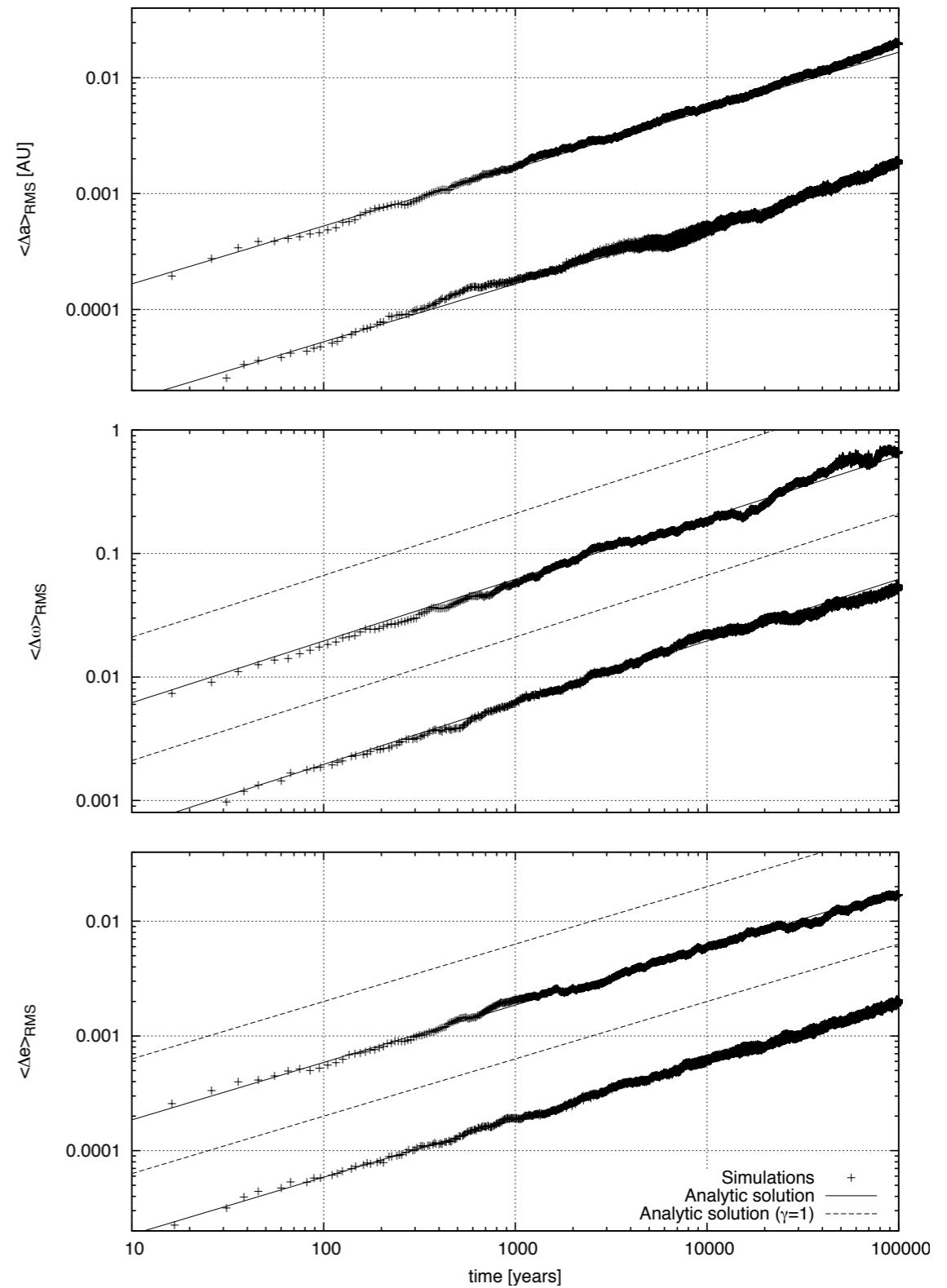


# Analytic growth rates for 1 planet

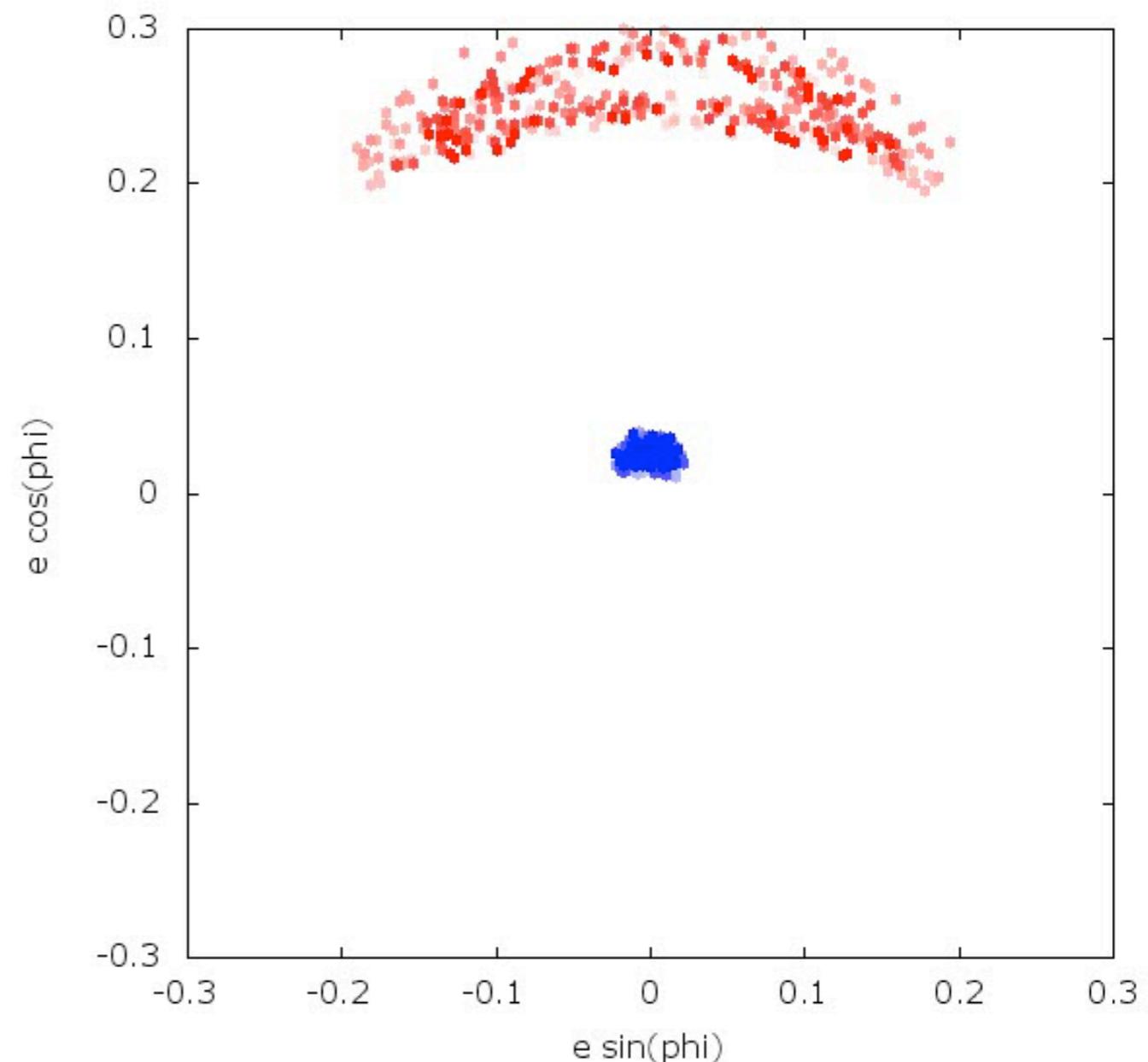
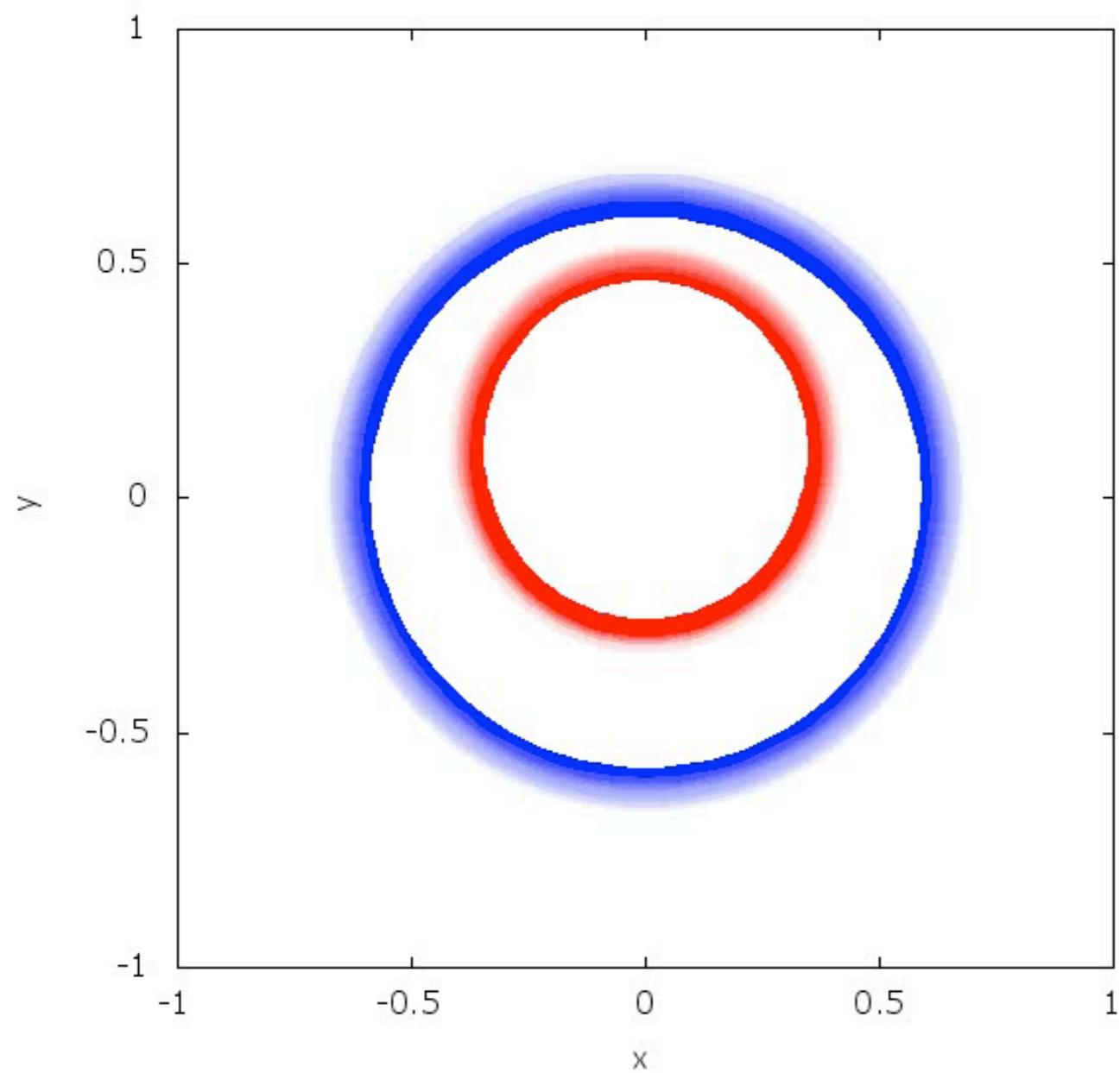
$$(\Delta a)^2 = 4 \frac{Dt}{n^2}$$

$$(\Delta\varpi)^2 = \frac{2.5}{e^2} \frac{\gamma Dt}{n^2 a^2}$$

$$(\Delta e)^2 = 2.5 \frac{\gamma Dt}{n^2 a^2}$$



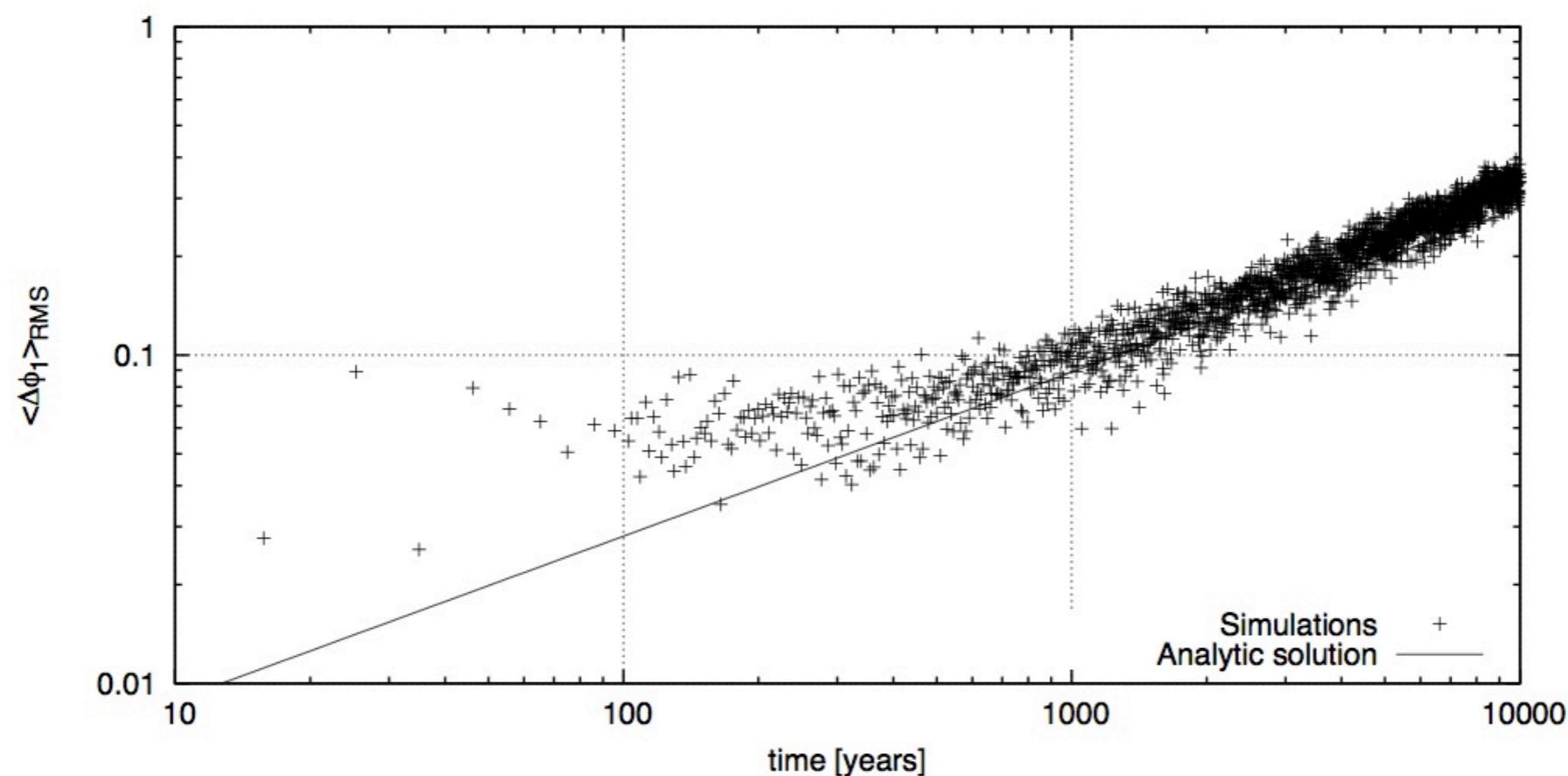
# Two planets: turbulent resonance capture



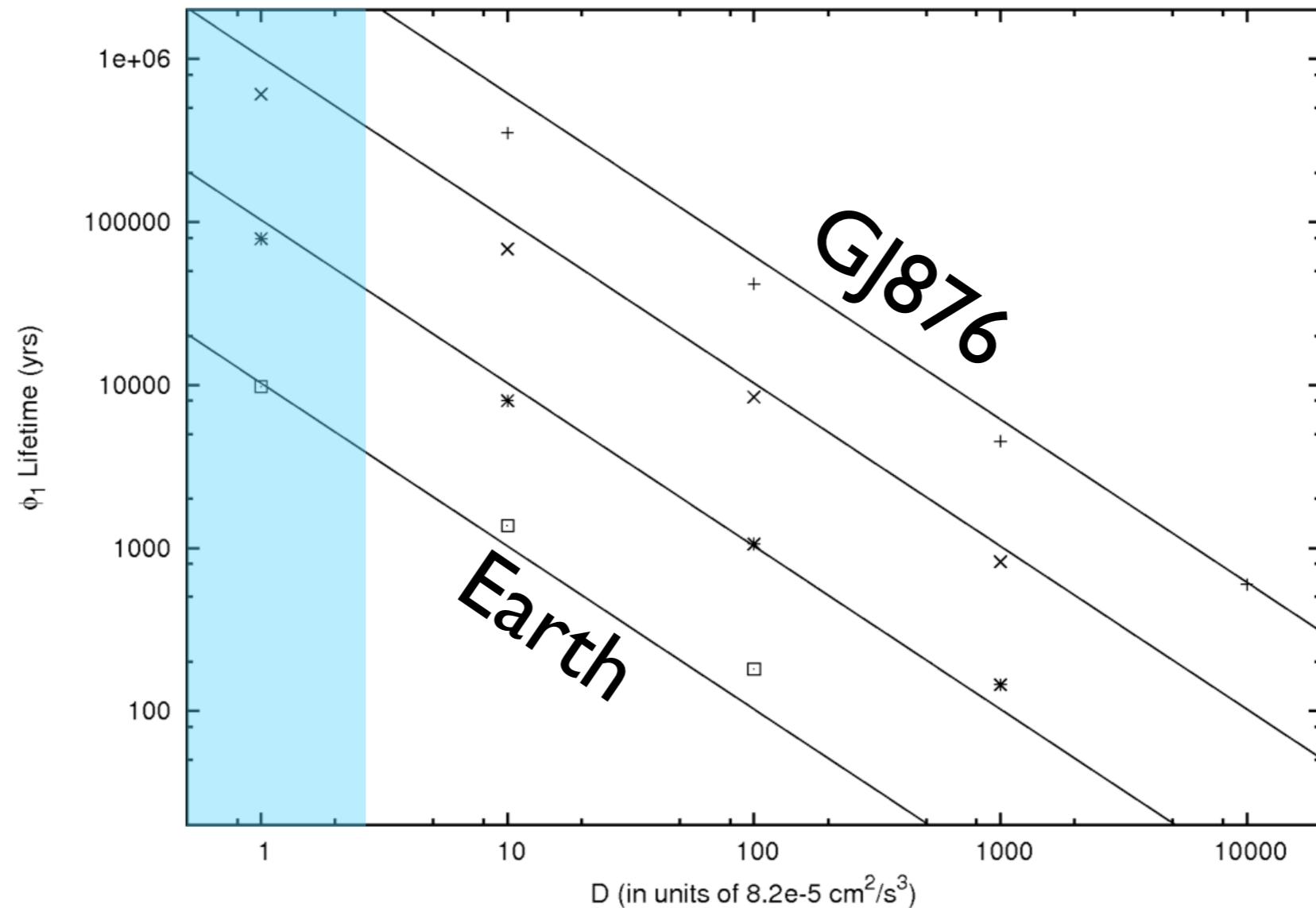
# Analytic growth rates for 2 planets

$$\frac{(\Delta\phi_1)^2}{(p+1)^2} = \frac{9\gamma_f}{a_1^2\omega_{lf}^2} D t$$

$$(\Delta(\Delta\varpi))^2 = \frac{5\gamma_s}{4a_1^2n_1^2e_1^2} D t$$



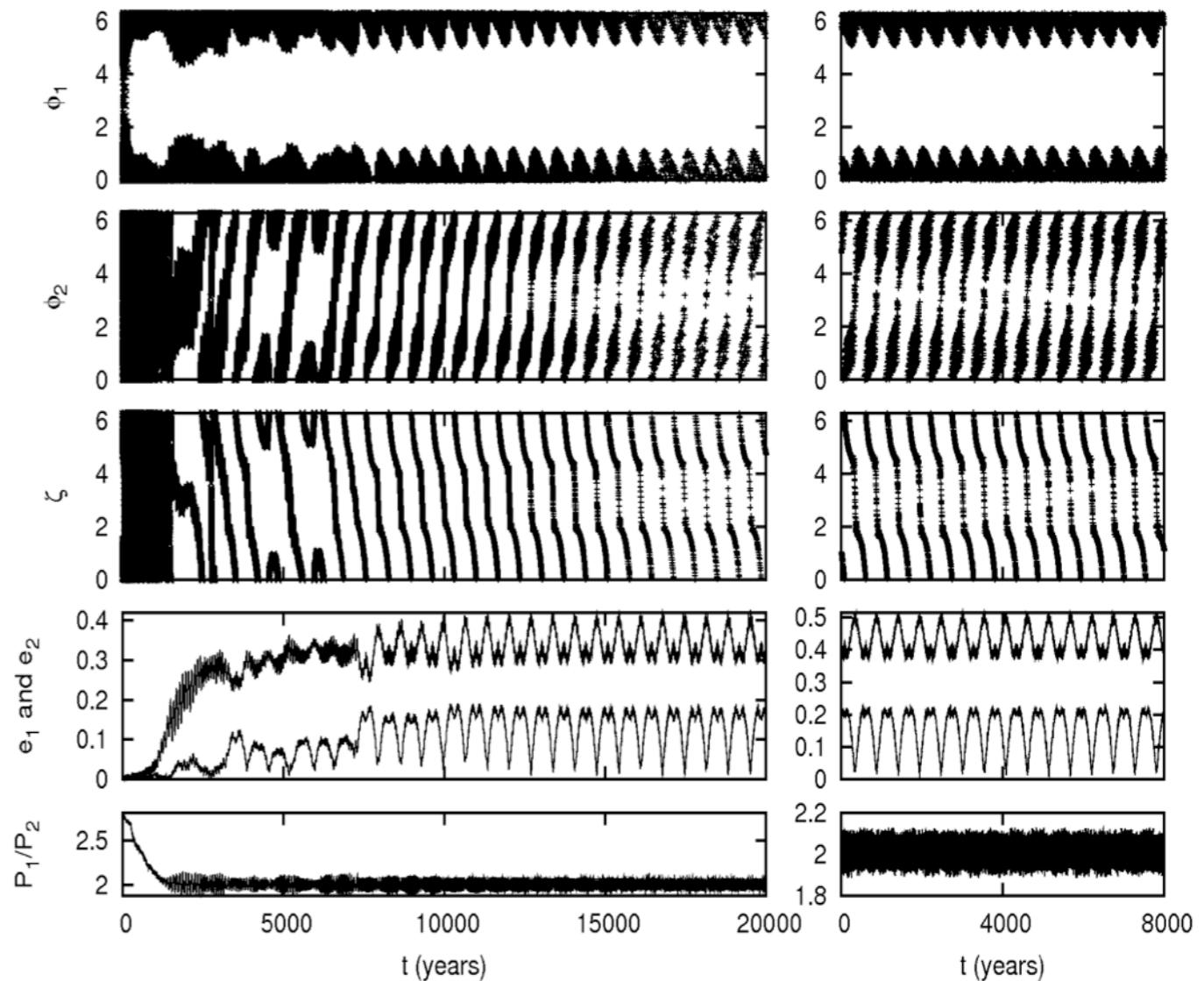
# Multi-planetary systems in mean motion resonance



- Stability of multi-planetary systems depends strongly on diffusion coefficient
- Most planetary systems are stable for entire disc lifetime

# Modification of libration patterns

- HD128311 has a very peculiar libration pattern
- Can not be reproduced by convergent migration alone
- Turbulence can explain it
- More multi-planetary systems needed for statistical argument



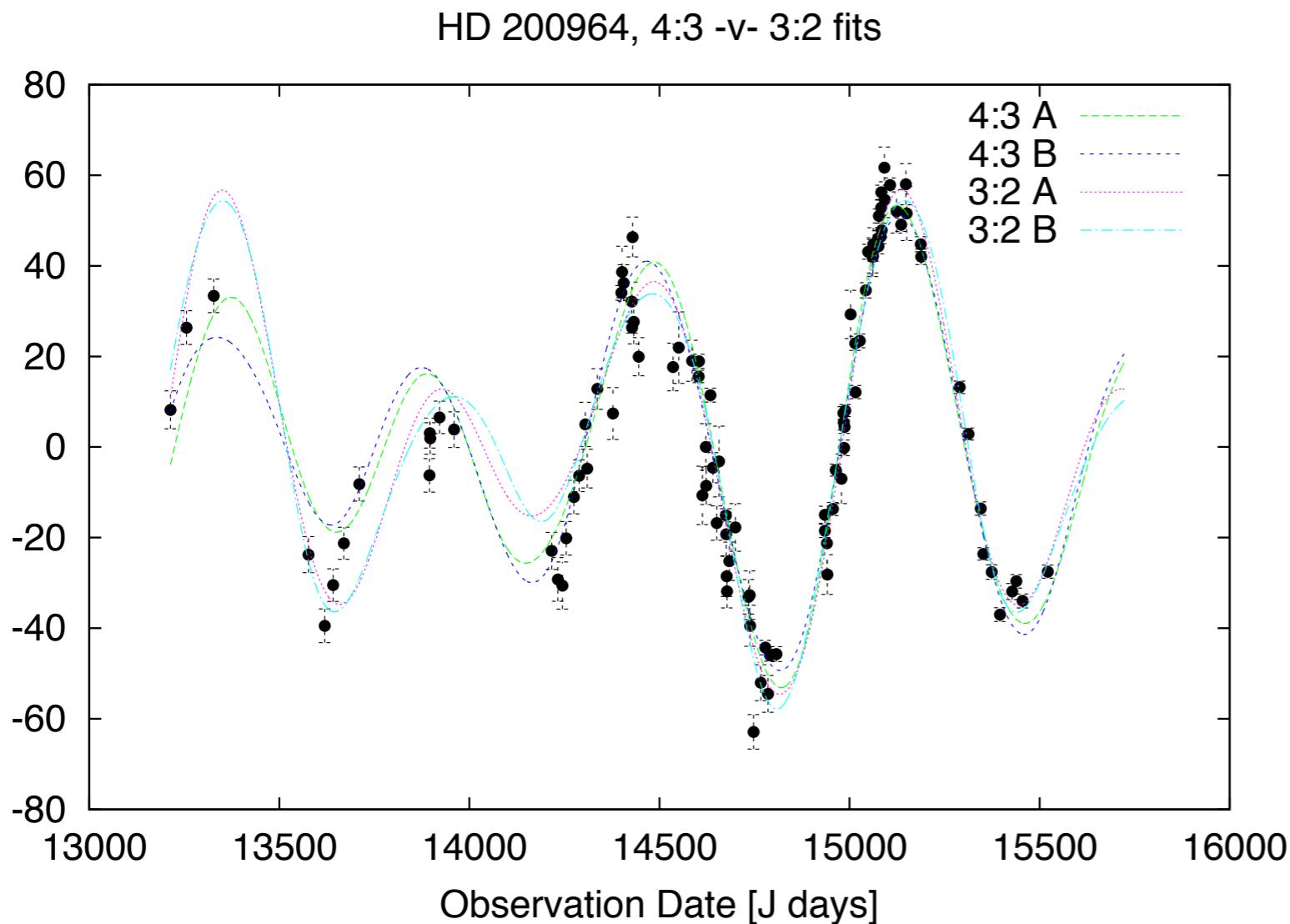
## Take home message II

Migration scenarios can explain  
the dynamical configuration of  
many systems in amazing detail

# HD200964

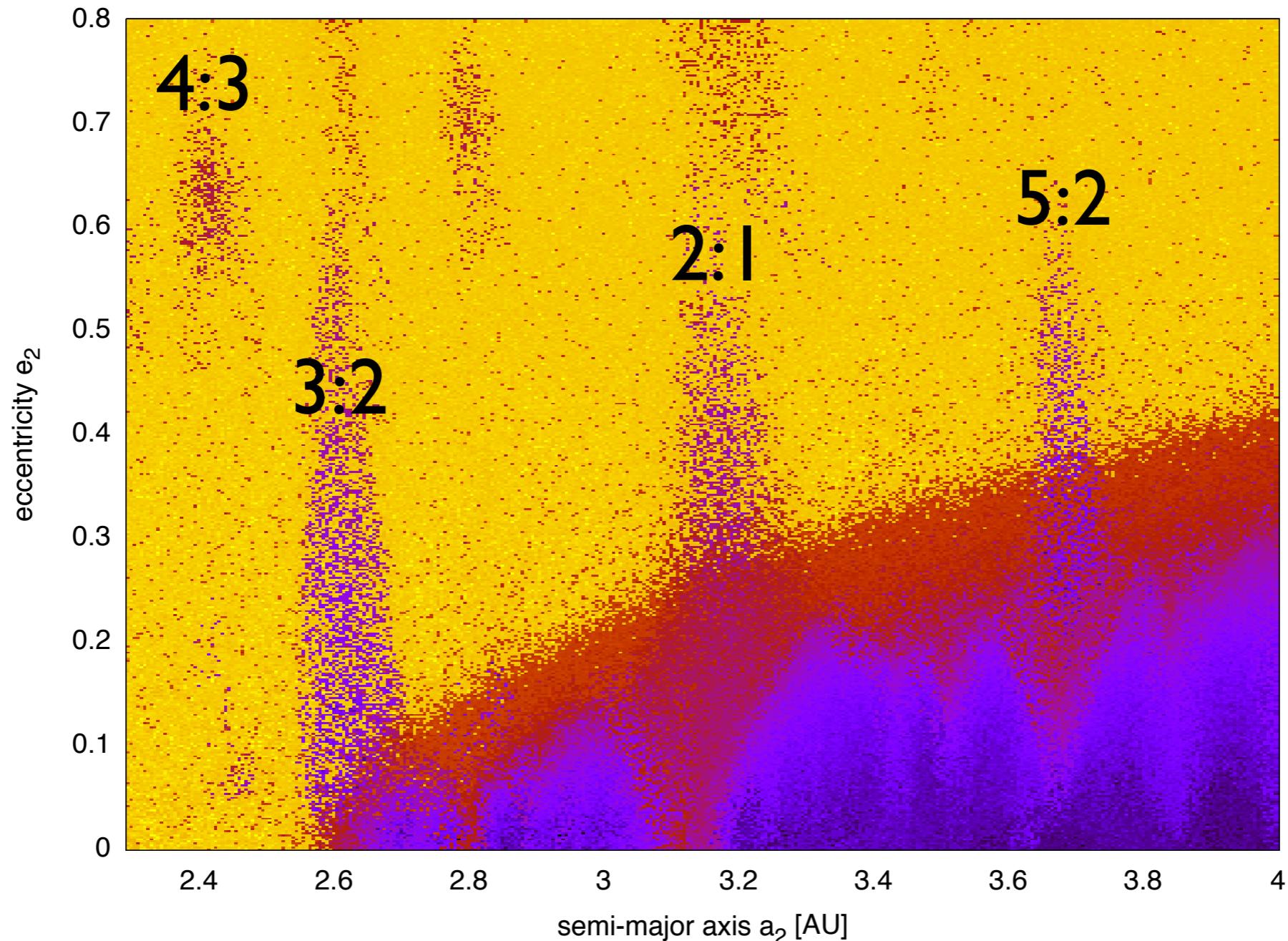
## The impossible system?

# Radial velocity curve of HD200964



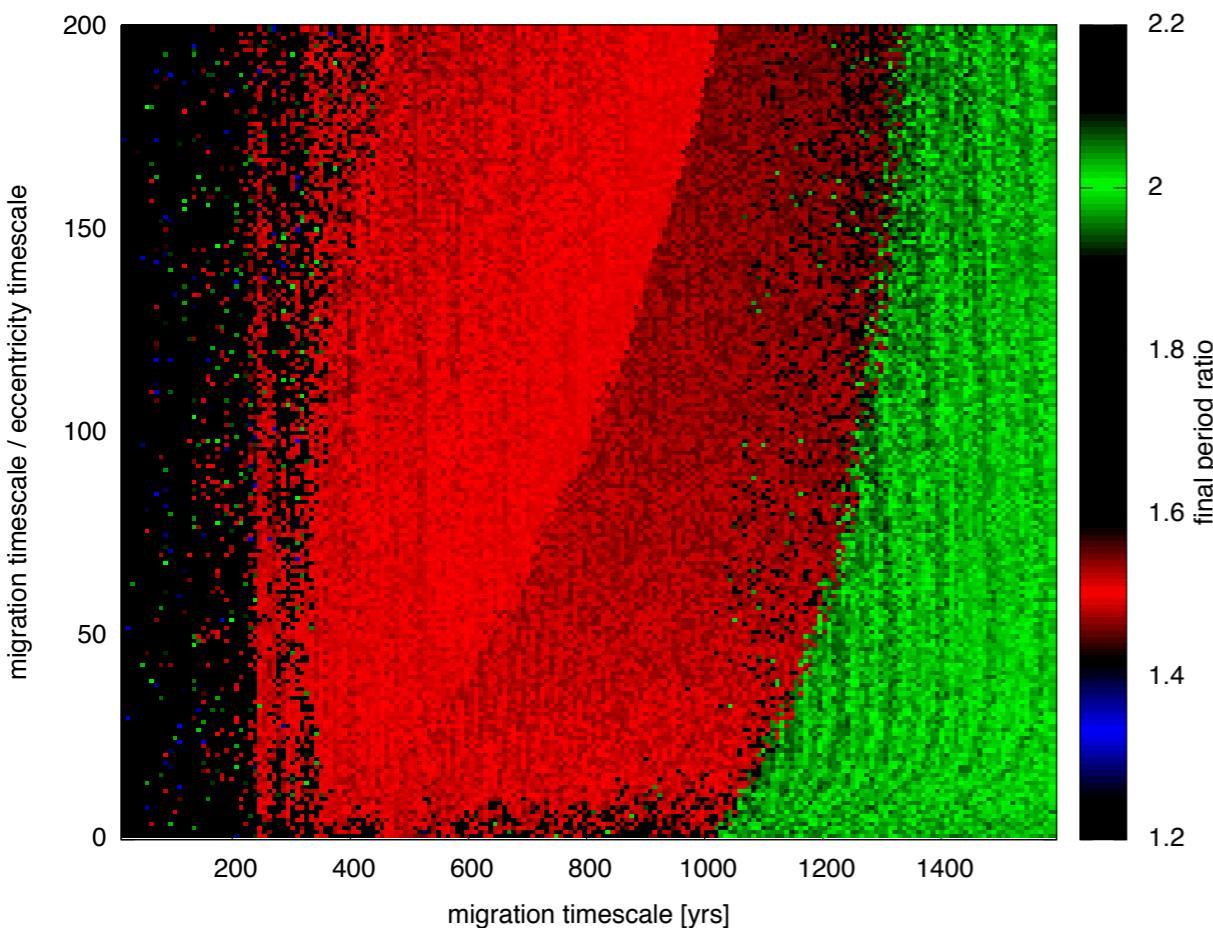
- Two massive planets  $1.8 M_{Jup}$  and  $0.9 M_{Jup}$
- Period ratio either 3:2 or 4:3
- Another similar system, to be announced soon
- How common is 4:3?
- Formation?

# Stability of HD200964

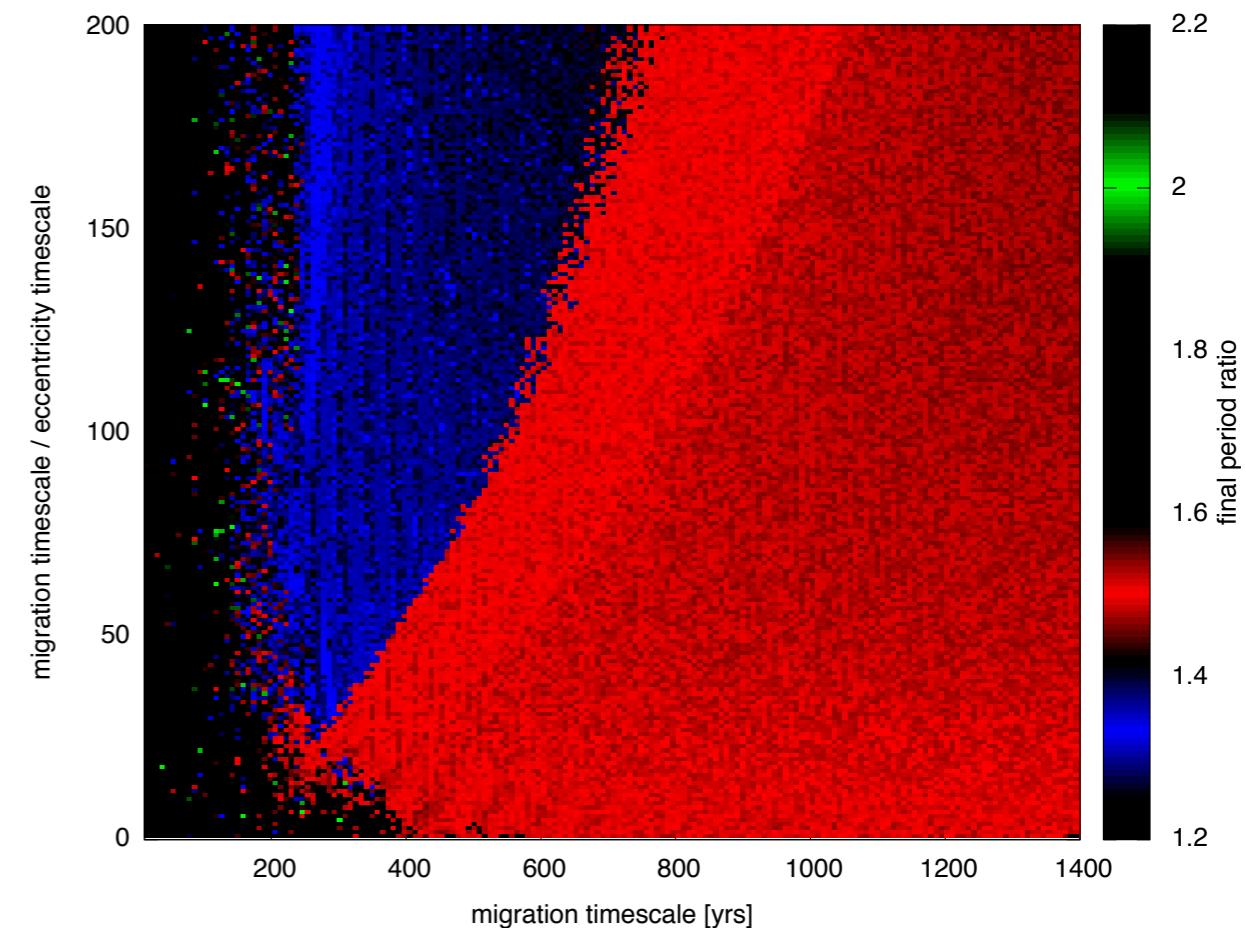


# Standard disc migration doesn't work

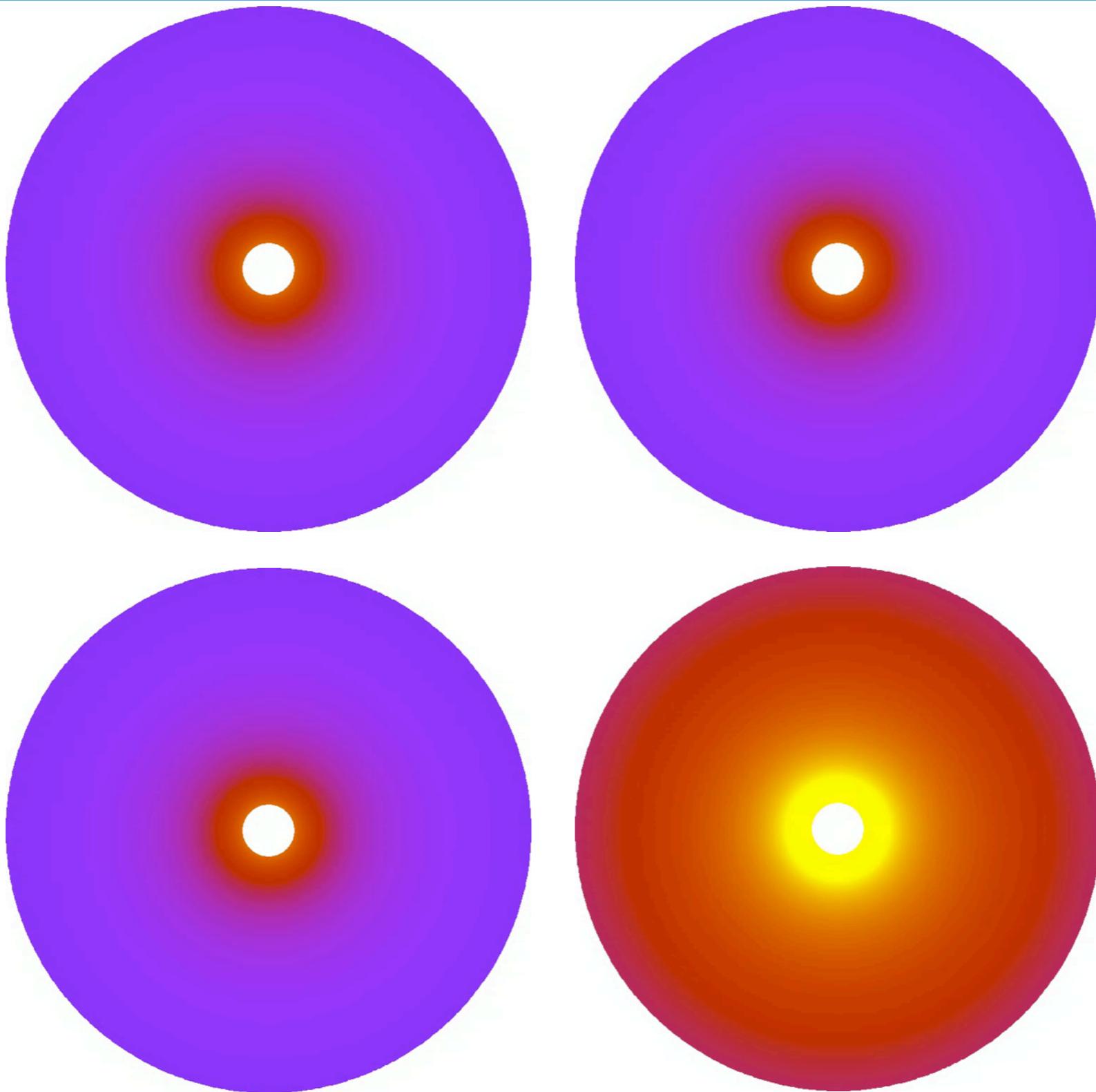
observed masses



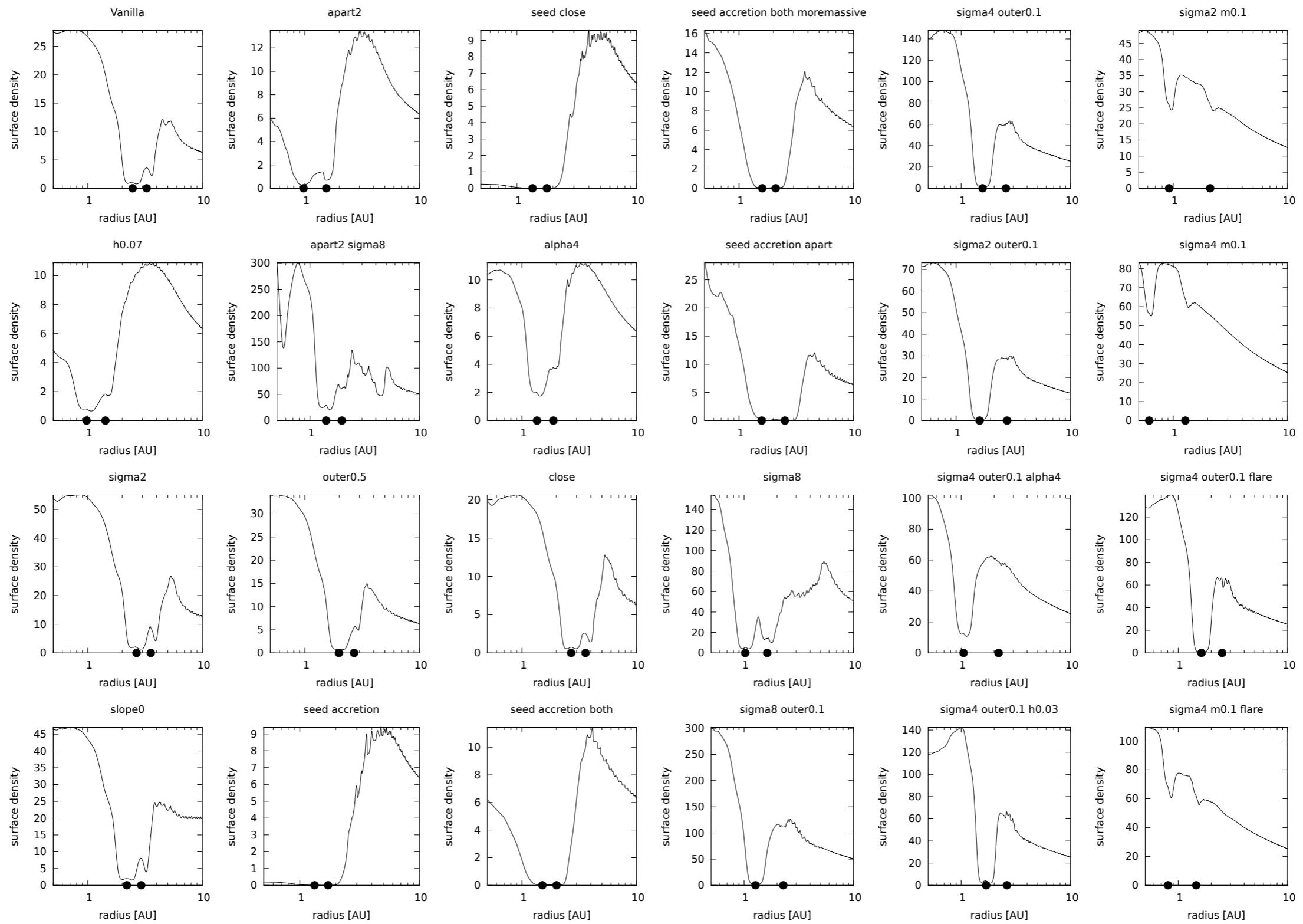
reduced masses



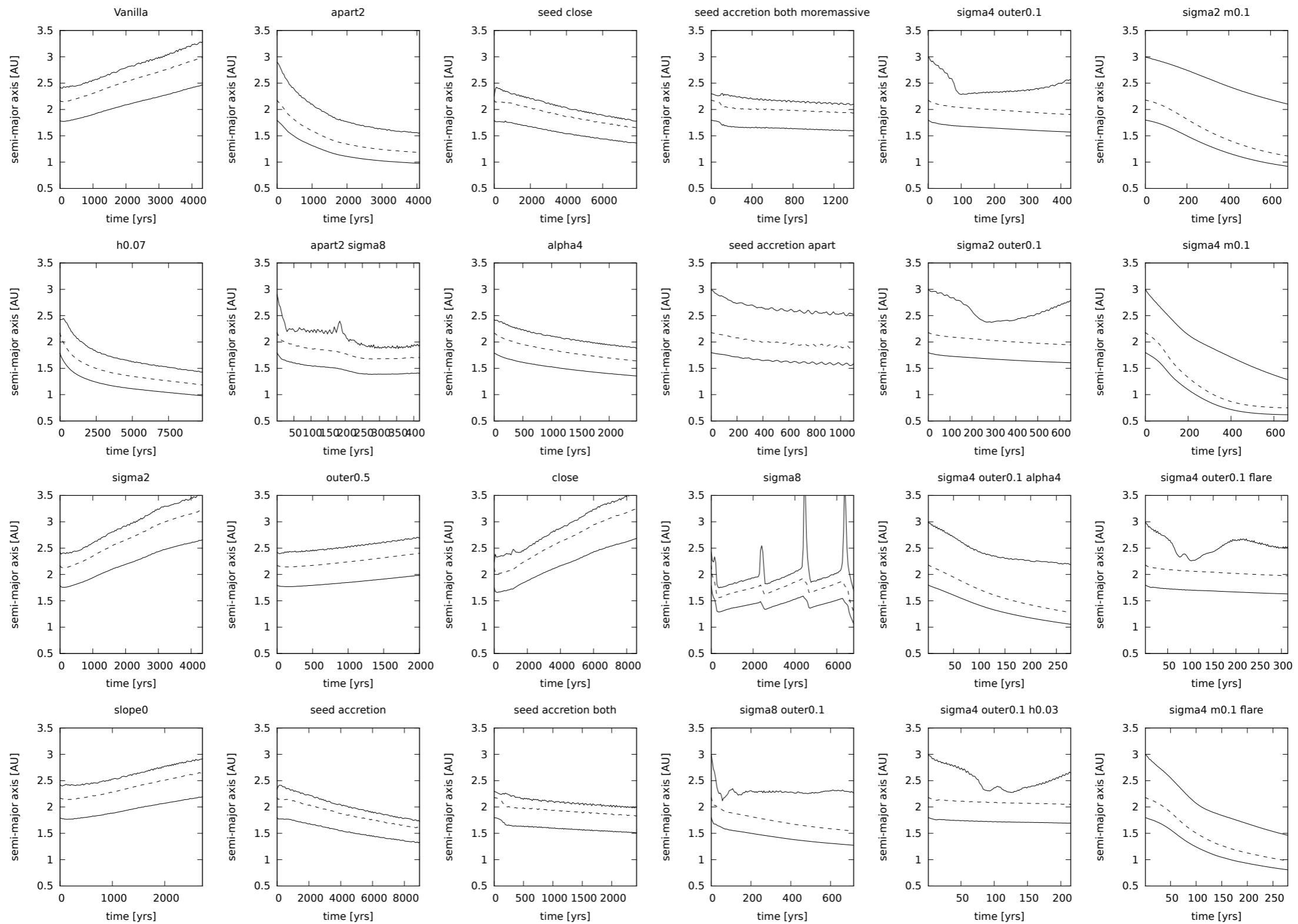
# Hydrodynamical simulations I



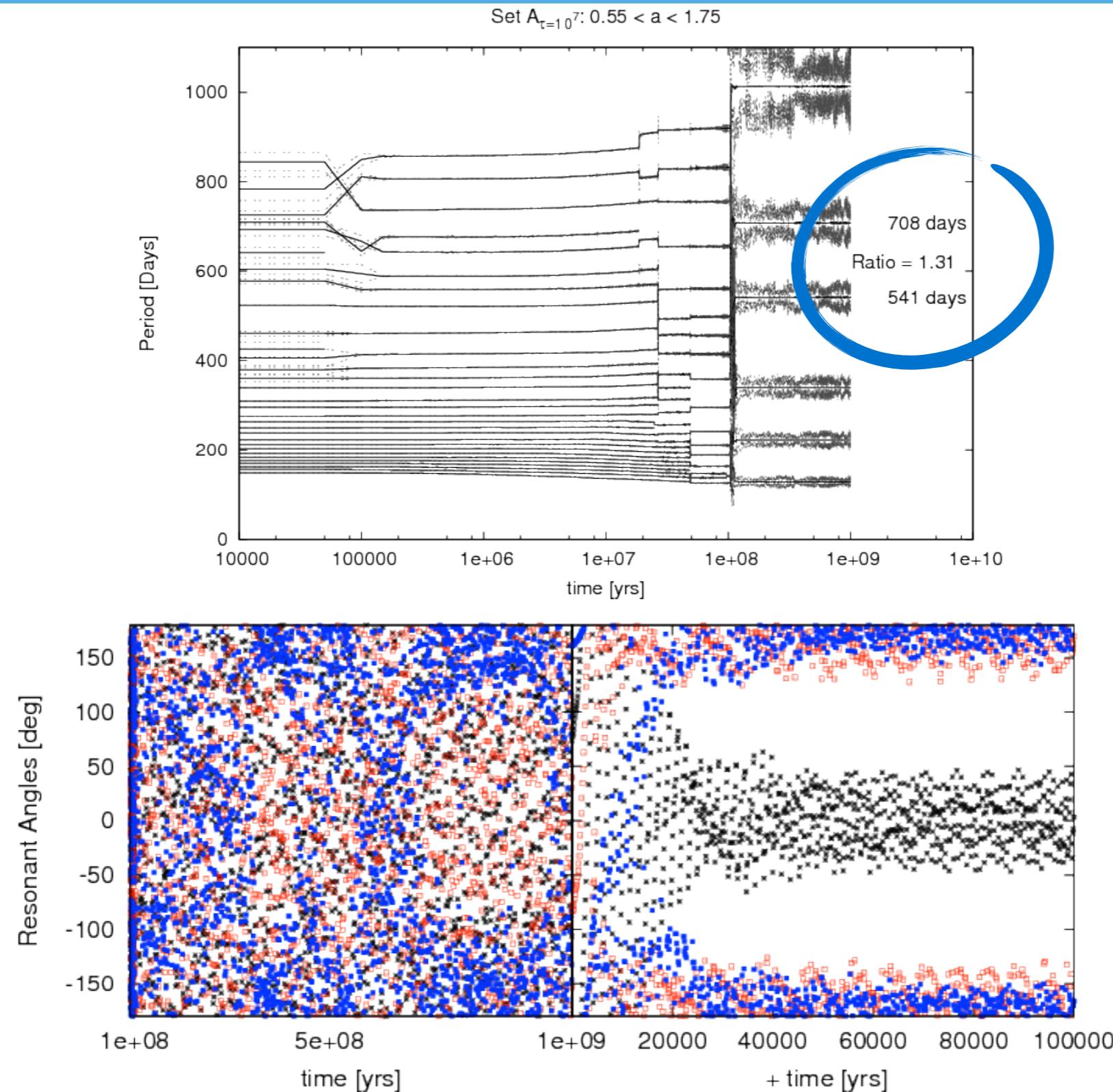
# Hydrodynamical simulations II



# Hydrodynamical simulations III



# Scattering of embryos



# HD200964

- In situ formation?
- Main accretion while in 4:3 resonance?
- Planet planet scattering?
- A third planet?
- Observers screwed up?



## Take home message III

We don't understand  
everything (yet).

# Conclusions

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## Formation of multi-planetary systems

Number of system increases almost every week.

Kepler has large number of planets, but not very suitable for detailed analysis

Multi-planetary system provide insight in otherwise unobservable formation phase

Dynamical configuration keeps a record of history

GJ876            formed in the presence of a disc and dissipative forces

HD128311        formed in a turbulent disc

HD45364        formed in a massive disc

HD200964        did not form at all