Quantum Mechanics I
Problem Set 4

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Deadline: Monday, February 1st 2015, noon

Problem 4.1

Assume that a particle is in the state

$$\Phi(x,t) = a\, e^{-b((mx^2/\hbar + it))}$$

where a, b and m are real positive numbers.

(a) What is the value of a and why?

(b) What is the expectation value of x, p, x^2 and p^2?

(c) Which Schroedinger Equation does this wavefunction satify? What is the potential?

Problem 4.2

The momentum space wave function $\Phi(p)$ is given by

$$\Phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-ipx/\hbar} \Psi(x) dx.$$

Show that

$$\langle \Psi | x | \Psi \rangle = \int_{-\infty}^{\infty} \Phi^* \left(\frac{-\hbar}{i} \frac{\partial}{\partial p} \right) \Phi \; dp.$$

Problem 4.3

We are working with the three dimensional Schroedinger Equation.

(a) Calculate

$$\frac{d}{dt} \langle \mathbf{r} \rangle$$

and express the results in terms of $\langle \mathbf{p} \rangle$. Note that because we are in three dimensions, both the $\mathbf{p}$ and $\mathbf{r}$ operators have three components.

(b) Calculate

$$\frac{d}{dt} \langle \mathbf{p} \rangle .$$

Problem 4.4

Let us work in cartesian coordinates and assume that the potential is given as

$$V(x,y,z) = \begin{cases} 0 & \text{if } |x| < \frac{a}{2} \text{ and } |y| < \frac{a}{2} \text{ and } |z| < \frac{a}{2} \\ \infty & \text{otherwise} \end{cases}$$

(a) Use an Ansatz in the following form to find all solutions to the three dimensional time independent Schroedinger equation with the above potential.

$$\psi(x,y,z) = \psi_x(x)\psi_y(y)\psi_z(z)$$

(b) What are the allowed energy levels of those states?

(c) What is the degeneracy of the first 6 energy levels (how many different states have the same energy)?

Problem 4.5

When solving the time-independent Schroedinger Equation in spherical coordinates, we separate variables:

$$\psi(r,\theta,\phi) = R(r)Y(\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi).$$

Show that

$$\Theta(\theta) = A\ln(\tan(\theta/2)), \qquad A \in \mathbb{R}$$

satisfies the equation for Θ (see lecture notes or equation 4.25 in Griffiths) for $l = m = 0$. Why is it not a physical solution?