Lecture 4

2.2 Sticking

This rarely happens in astronomy, but in this particular case, laboratory experiments are ahead of theoretical, numerical or observational constraints. The basic task is to study the sticking efficiency for particles as a function of size, velocity, impact parameter and composition.

Some number to keep in mind: 1) Micron sized dust will collide with a velocity of 1mm/s. 2) Meter sized rocks will collide with relative velocities of up to 100m/s.

Small particle stick due to surface forces. The larger particles get, the smaller the surface to volumne ratio gets. A micron meter sized particle can stick together in collisions if the impact speed is less than 1-2m/s. Assuming standard disk parameters, with moderate turbulence, we expect particle to form via coagulation up to a few milimeters in size. Beyond that size, particle-particle collisions become destructive.

2.3 Goldreich and Ward Mechanism

A currently less favoured model of planetesimal formation is the Goldreich and Ward Mechanism (Goldreich & Ward, 1973)

Assuming no (or almost no) turbulence, large particles settle to the mid plane within the life time of the disc. We can then treat the particle layer as a razor thing disk and model it with fluid equations. The velocity dispersion of the particles can be thought of as a sounds speed. Doing a linear stability analysis with the ansatz $\exp[i(kr - \omega t)]$ on those equations yields the following dispersion relation

$$\omega^2 = \Omega^2 + c_s^2 k^2 - 2\pi G \Sigma_0 |k|$$

The calculation is not complicated but long which is why we won't do it in class. For an instability to occur, we need an imaginary ω, thus the right hand side has to be negative. The first two terms, rotation and *pressure* are stabilizing. Only the third term, gravity, is destabilizing. The most unstable wavenumber (largest imaginary ω) is for

$$k_{\min} = \frac{\pi G \Sigma_0}{c_s^2}$$

No instability at all occurs if

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma_0} > 1$$

This is known as the Toomre Q criteria. We can estimate the size of planetesimals formed via this instability.

$$m_p \sim \pi \lambda^2 \Sigma_s \sim 4\pi^5 G^2 Q_{\text{crit}}^4 \frac{\Sigma_s^3}{\Omega^4}$$

Typical values at 1 AU in a standard protoplanetary disk give

$$m_p \sim 3 \cdot 10^{18}\ g$$

a body of 5-10km in size.

The thickness of the particle dist is roughly

$$h_d = \frac{c_s}{\Omega}$$

For an instability to occur, the soundspeed and thus the thickness need to be small. For $Q \sim 1$ and typical values of the protoplanetary disk, the particle layer thickness is 10^{-4} times the thickness of the gas disk:

$$\frac{h_d}{h} \sim 10^{-4}$$

This is razor thin. Saturn's rings are one example where we have such a thin disk ($h_d = 10m$).

Besides the Toomre Q criteria, a system also needs to satisfy that the Roche criteria:

$$\rho_d > \frac{M_*}{R^3}$$

It says that the tidal gravity

$$f_{tide} \sim \frac{GM_* l}{R^3}$$

is small compared to self gravity

$$f_{sg} \sim G\rho l$$

where l is some arbitrary lengthscale of an object.

Note that the Toomre Q criteria gives us a characteristic lengthscale, whereas the Roche criteria is scale free and only says something about the particle density. In Saturn's ring, the Toomre criteria is satisfied, but not the Roche limit (except at the outermost parts).

Intrinsic disk turbulence prevents the dust layer to form such an extremely thin disk for small particles, i.e. particles that are well coupled to the gas. However, for larger particles, radial drift becomes very fast.

One might argue that disk are not turbulent, at least in same parts. However, if they are not intrinsically turbulent, the thin dsut layer might make it turbulent. The reason for this is simple: if the particles dominate the mass in the midplane, then the gas will be dragged along by the particles on a Keplerian orbit. However, just above the midplane there is not dust and the gas is on a sub-Keplerian orbit. This vertical shear will be unstable to Kelvin-Helmholtz instabilities, thus inducing turbulent mixing and destroying the particle layer.

2.4 Radial pressure gradients

In the last lecture, we calculated the radial drift velocity of dust to be (in the limit of small particles, $\tau_{\text{fric}} \ll 1$):

$$v_r \approx v_{r,\text{gas}} - \eta \tau_{\text{fric}} v_k$$

where $\eta = n\, c_s^2/v_k^2$ and $P \propto r^{-n}$. Thus the velocity depends on the pressure gradient. Note that if we have a pressure profile that have a local maximum at some r_0, the radial velocity will be zero. Thus, particles will concentrate near pressure maxima.

2.5 Johansen Model of Planet formation

In recent years it became more and more obvious that turbulence might not be purely negative for planetesimal formation. Although turbulence has in general a diffusing effect on dust particles, it can also locally enhance the density and thus stimulating gravitational collapse.

It gets particularly interesting when the mass of the dust becomes locally comparable to the mass of the gas. In that case the dust can now drag the gas, not only the other way round. The interaction of gas and dust can be described with two coupled sets of fluid equations. The stopping time sets the strength of the interaction. Most interesting dynamics will occur when the interaction time is of order the orbital frequency. In standard disks, this corresponds to particles roughly 10cm-1m in size.

It turns out that this system is now unstable to what is referred to as streaming instability. A toy picture of the streaming instability is shown in Figure 6.

Note the zoo of instabilities we have collected:

- MRI, MHD turbulence
- Kelvin-Helmholtz instability
- Streaming instability

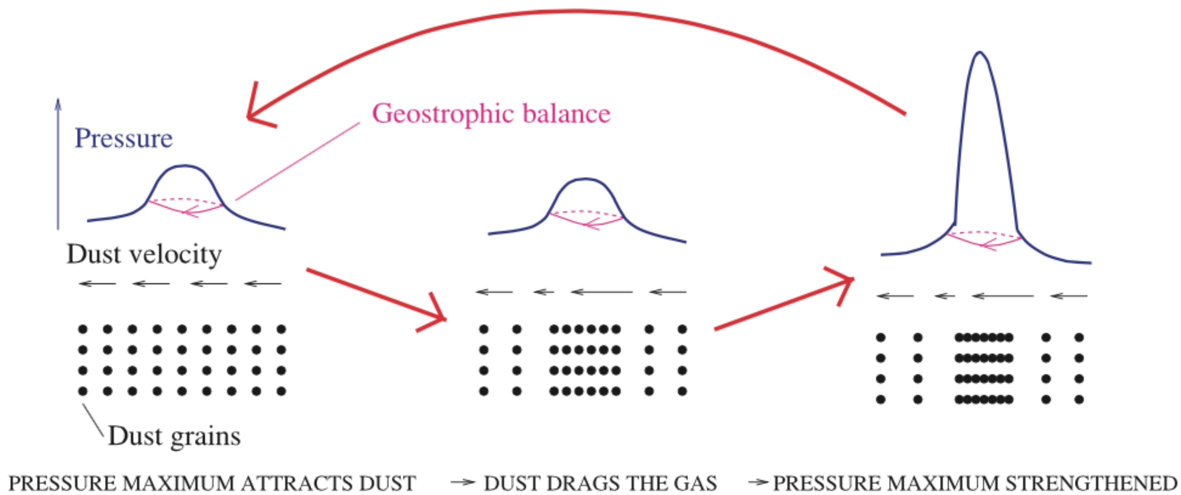

Figure 6: Toy model of the streaming instability (E. Jacquet, S. Balbus and H. Latter).