Quantum Mechanics I
Problem Set 3

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Due: Monday, January 27th 2014

There are four problems. Answer any three.

Problem 3.1

The momentum space wave function $\Phi(p)$ is given by

$$\Phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{ipx/\hbar} \Psi(x) dx.$$

Show that

$$\langle \Psi | x | \Psi \rangle = \int_{-\infty}^{\infty} \Phi^* \left(\frac{-\hbar}{i} \frac{\partial}{\partial p} \right) \Phi \; dp.$$

Problem 3.2

(a) Proof:

$$[AB, C] = A[B, C] + [A, C]B$$

(b) Proof:

$$[x^n, p] = i\hbar n x^{n-1}$$

(c) Proof:

$$[f(x), p] = i\hbar \frac{df}{dx}$$

Problem 3.3

Two operators A and B do not commute, $[A, B] \neq 0$. Proof that A and B can not have a complete set of common eigenfunctions. Hint: Assume the opposite.

Problem 3.4

Let A be an arbitrary operator. The exponential of an operators, e^A, is defined by the power series expansion:

$$e^A = \sum_{i=0}^{\infty} \frac{1}{n!} A^n = \mathbb{1} + A + \frac{1}{2}A^2 + \frac{1}{6}A^3 + \dots$$

Show that

$$f(x + x_0) = e^{ipx_0/\hbar} f(x)$$

where p is the momentum operator.