Quantum Mechanics I
Problem Set 7

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Due: Monday, March 7th 2016

Problem 7.1

From the last problem set, we know that

$$Y_2^1(\theta,\phi) = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\phi}.$$

(a) Explicitly calculate Y_2^2 by applying L_+ to Y_2^1. Don't worry about the normalization factor.

(b) Apply L_+ to Y_2^2.

Problem 7.2

Prove the rotational analog to Ehrenfest's theorem:

$$\frac{d}{dt}\langle\vec{L}\rangle = \langle\vec{N}\rangle$$

where

$$\vec{N} \equiv -\vec{r}\,\times\,\nabla V.$$

What happens for a spherically symmetric potential, $V(\vec{r}) = V(r)$.

Problem 7.3

The time-independent Schroedinger Equation for two particles is

$$-\frac{\hbar}{2m_1}\nabla_1^2\psi - \frac{\hbar}{2m_2}\nabla_2^2\psi + V\psi = E\psi.$$

Assume that the potential only depends on the distance between the two particles $r \equiv r_1 - r_2$. The reduced mass of the system is defined as $\mu \equiv m_1 m_2/(m_1+m_2)$.

(a) Rewrite the operators ∇_1, ∇_2 and then Schroedinger equation in the coordinates r and $R \equiv (m_1 r_1 + m_2 r_2)/(m_1+m_2)$ instead of r_1 and r_2.

(b) Use the following Ansatz to separate variables in the Schroedinger Equation:

$$\psi(R,r) = \psi_R(R)\psi_r(r)$$

(c) What does the equation for $\psi_R(R)$ describe?