Quantum Mechanics I
Problem Set 2

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Due: Monday, January 20th 2014

Problem 2.1

Find the eigenvalues and eigenvectors of the following matrix.

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

Is the matrix diagonalizable? If so, find the similarity matrix S to bring A into diagonal form: $D = SAS^{-1}$.

Problem 2.2

(a) Show that any linear combination of two square integrable functions is also square integrable. Hint: use the Schwarz inequality.

(b) To show that the set of all square-integrable functions is a vector space check all 8 axioms. Short answers are ok!

(c) Show that the following standard definition of the inner product satisfies the three axioms for an inner product. Short answers are ok!

$$\langle f|g\rangle = \int f(x)^* g(x) dx$$

Problem 2.3

Given an arbitrary operator E, proof or disproof that

(a) $E + E^\dagger$ is hermitian.

(b) $E - E^\dagger$ is hermitian.

(c) $i(E - E^\dagger)$ is hermitian.

(d) $EE^\dagger$ is hermitian.

(e) $iEE^\dagger$ is hermitian.

Problem 2.4

(a) Show that the sum of two hermitian operators is hermitian.

(b) Under what condition is the product of two hermitian operators hermitian?

(c) Show that the position operator x and the Hamiltonian operator $\frac{d^2}{dx^2} + V(x)$ are hermitian.