

# Quantum Mechanics I

## Problem Set 3

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Deadline: Monday, January 25th 2015, noon

### Problem 3.1

(a) Proof:

$$[AB, C] = A[B, C] + [A, C]B$$

(b) Proof:

$$[x^n, p] = i\hbar n x^{n-1}$$

(c) Proof:

$$[f(x), p] = i\hbar \frac{df}{dx}$$

### Problem 3.2

Two operators  $A$  and  $B$  do not commute,  $[A, B] \neq 0$ . Proof that  $A$  and  $B$  can not have a complete set of common eigenfunctions. Hint: Assume the opposite.

### Problem 3.3

Let  $A$  be an arbitrary operator. The exponential of an operators,  $e^A$ , is defined by the power series expansion:

$$e^A = \sum_{i=0}^{\infty} \frac{1}{i!} A^i = \mathbb{1} + A + \frac{1}{2}A^2 + \frac{1}{6}A^3 + \dots$$

Show that

$$f(x + x_0) = e^{ipx_0/\hbar} f(x)$$

where  $p$  is the momentum operator.