Lecture 7

5 Giant Planet Formation

For a very long time, there have been two compeeting theories for giant planet formation. In recent years, it seems more and more likely that gas giants, at least in Solar System like situations do not form via gravitational instability. Thus the emphasis has been on Core accretion. We will only go quickly over these theories in this course. There is much more to it and it quickly leads to a regime closely related to star formation where the internal processes of a proto-planet can be model with stellar evolution codes.

5.1 Core accretion

There are four stages in the core accretion model.

1. **Core formation**. A core needs to firm first. We generally assume that this happens in the same fashion as for a terrestrial planet. The only difference is that we need to make sure this happens fast enough so that a lot of gas is still around.

2. **Hydrodynamic growth** In this phase, the envelope and the gas in the disk are in hydrostatic equilibrium. The energy from the envelope is transported via radiative diffusion and convection. Note that the energy can come from multiple source, the collapsing envelope itself, the core or heating due to impacting planetesimals. In this phase the accretion is limited by cooling, not by supply.

3. **Runaway growth** After a critical mass is reached, the growth is no longer limited by cooling by by supply. It thus becomes a dynamical problem. Most of the mass of gas giants is accreted in this phase. It takes roughly 10^5 years.

4. **Termination.** At some point the accretion stops, namely when the supply is exhausted. This typically happens when a gap opens in the disk. What follows is a phase of cooling and contraction. This is very similar to the cooling curves of stars.

Let's estimate the transition radii. For a planet to hold an atmosphere, the escape speed from the planet must be larger than the sounds speed in the surrounding disk. In equations, we have

$$R_s \left(\frac{3}{4\pi} \frac{M_p}{\rho_m} \right)^{1/3}$$

$$v_{esc} = \sqrt{\frac{2GM_p}{R_s}}$$

$$c_s = \left(\frac{h}{r} \right) v_k$$

giving us

$$M_p > \left(\frac{3}{32\pi} \right)^{1/2} \left(\frac{h}{r} \right)^3 \frac{M_*^{3/2}}{\rho_M^{1/2} a^{3/2}}$$

For an icy body at 5 AU with $h = 0.05$ we get

$$M_p > 5 \cdot 10^{-4} M_{earth}$$

This is a very small mass. What really matters though is if the atmosphere that is retained is a significant fraction, ϵ, of the body's mass. Assuming an atmosphere in hydrostatic equilibium we can estimate that mass, the result being

$$M_p > \left(\frac{3}{4\pi\rho_m}\right)^{1/2} \left(\frac{c_s^2}{G}\right)^{3/2} \left(\ln\left(\frac{\epsilon\rho_m}{\rho_0}\right)\right)^{3/2}$$

where ρ_0 is the density in the midplane of the disk. For typical parameters and $\epsilon = 0.1$, we have $M_p > 0.2 M_{earth}$.

Thus, protoplanets need to grow up to about an Earth mass before the disk disappears before they can even start accreting gas. This turns out to be very hard to do inside the snowline (≈ 2.5" AU). Beyond that, it is possible.

We will only estimate the critical mass for the onset of runaway accretion to an order of magnitude. Imagine the following scenario.

The core establishes a hydrostatic envelope. On long timescales the core continues to accrete and grows. A more massive core means the atmosphere can also be a little more massive. One can convince oneselves that the fractional mass of the atmosphere over the mass of the core is an increasing function of the core mass.

Thus, at some point the core and the atmosphere have equal mass. At that point, adding a little bit more mass to the atmosphere will allow the atmosphere to become even bigger. A runaway growth starts. A numerical value for standard disk parameters is

$$M_{crit} \sim 10 M_{earth}$$

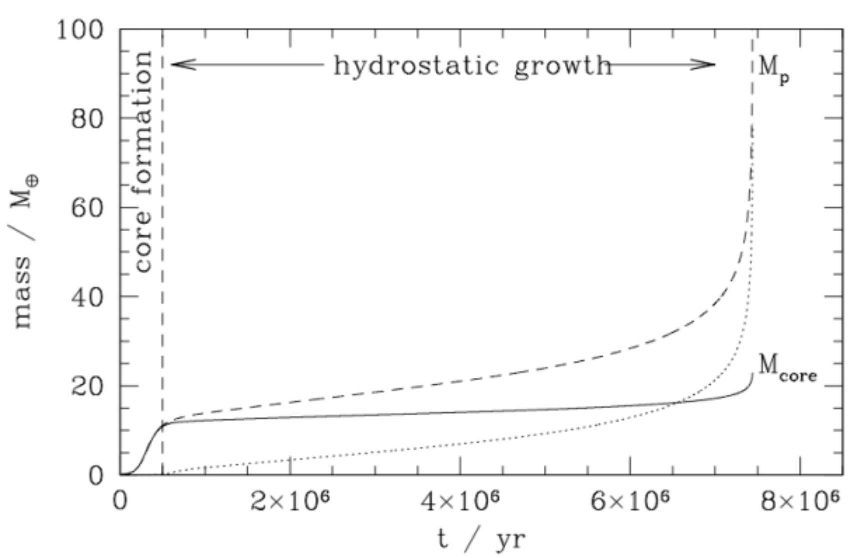

Figure 9: Summary of giant planet formation via core accretion. From Pollack et. al.

5.2 Gravitational instability

We already derived the Toomre Q parameter that determines if a disk is gravitationally unstable or not.

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma}$$

Gravitational instability sets in if $Q \sim 1 - 2$. Observationally this is only satisfied for the upper bound in the mass spectrum of disks. Naturally, the condition is mostly satisfied near the beginning of the planet formation phase. Note that it needs to be more than an order of magnitude higher than the Minumum Mass Solar Nebula.

The Q condition is necessary but not sufficient for collapse. The reasons are similar to those we discussed when we talked about the accretion disk formation and evolution around a star.

One reason is that gravitational instability will initially produce spiral waves in the disk. These non-axisymmetric perturbation can lead to angular momentum transports. Moving material in means tapping into an energy reservoir. The energy will eventually get dissipated into heat. However, this heat will cause Q to rise and thus eventually preventing gravitational instability. If the heating is large enough, nothing will ever collapse and the graviational instability will result in a quasi-steady-state turbulent disk.

We can quantify this by requiring that for fragmentation in protoplanetary disks, the cooling rate has to be smaller than the orbital timescale. Otherwise clumps will not be able to contract fast enough and will get sheared apart.