Quantum Mechanics I
Problem Set 5

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Due: Monday, March 3rd 2014

Problem 5.1

The momentum space wave function in three dimensions is the natural generalization of the one dimensional case:

$$\phi(\vec{p}) = \frac{1}{(2\pi\hbar)^{3/2}} \int e^{i\,\vec{p}\cdot\vec{r}/\hbar}\psi(\vec{r})d^3\vec{r}.$$

- Calculate the momentum space wave function for the ground state of the H-atom, ψ_{100}.
- Check that your result is normalized.
- Use the momentum space wave funtion to calculate $\langle p^2 \rangle$.
- What is the kinetic energy of the ground state, ψ_{100}?

Problem 5.2

You are given

$$Y_2^1(\theta,\phi) = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\phi}.$$

(a) Explicitly calculate Y_2^2 by applying L_+ to Y_2^1. Don't worry about the normalization factor.

(b) Apply L_+ to Y_2^2.

Problem 5.3

Prove the rotational analog to Ehrenfest's theorem:

$$\frac{d}{dt}\langle\vec{L}\rangle = \langle\vec{N}\rangle$$

where

$$\vec{N} \equiv -\vec{r}\,\times\,\nabla V.$$

What happens for a spherically symmetric potential, $V(\vec{r}) = V(r)$.