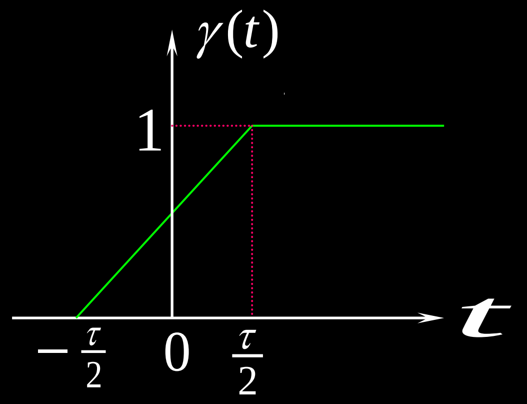
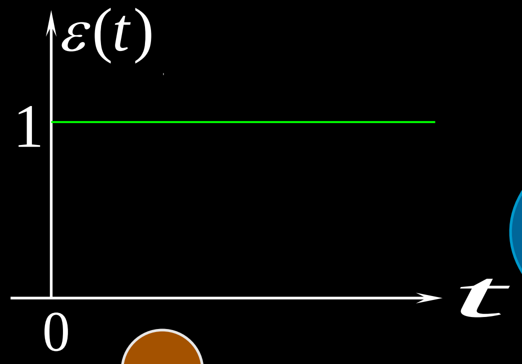
阶跃函数和冲激函数

- 单位阶跃函数$\varepsilon(t)$





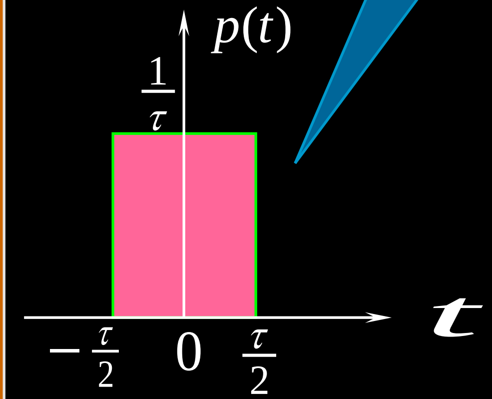
$$) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$

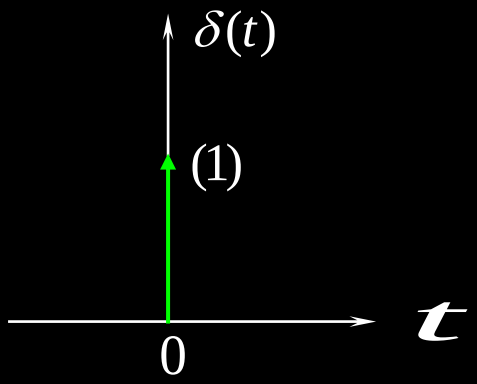
面积为1

δ(t)与ε(t)的关系：

$$\delta(t) = \frac{d\varepsilon(t)}{dt}$$

$$\varepsilon(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

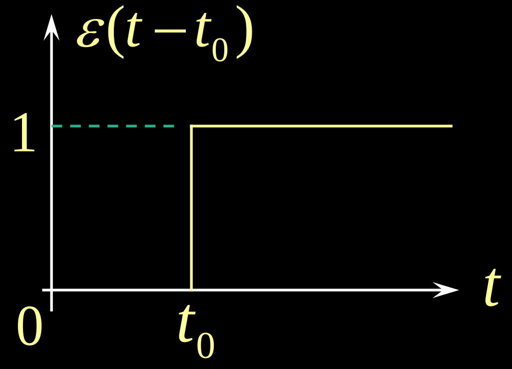




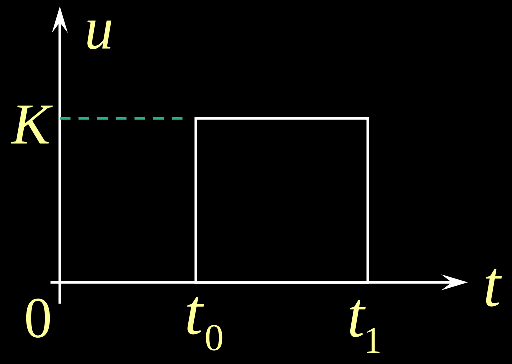
延迟的阶跃函数

延迟的阶跃函数定义为：

$$\varepsilon(t-t_0)=\begin{cases}0 & t<t_0\\ 1 & t>t_0\end{cases}$$



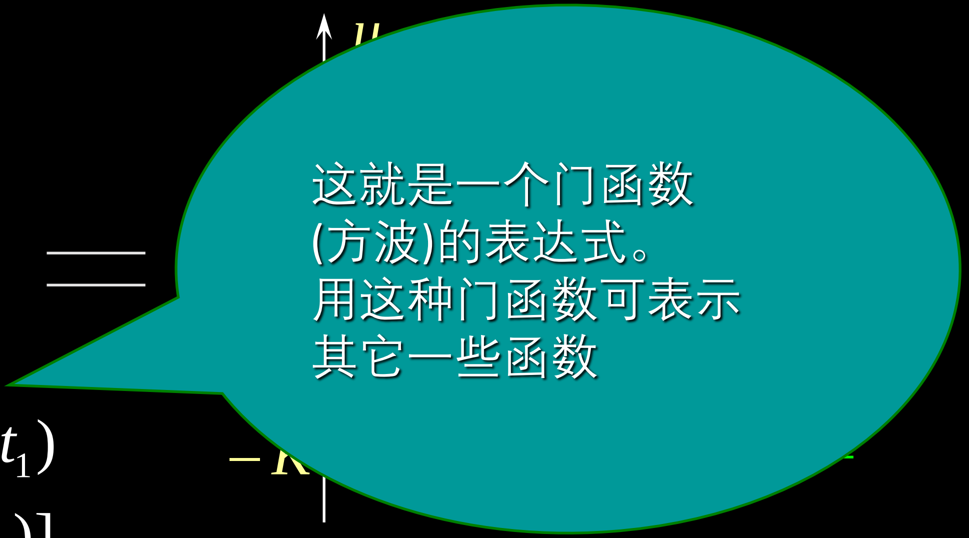
用阶跃函数可以表示方波或分段常量波形：



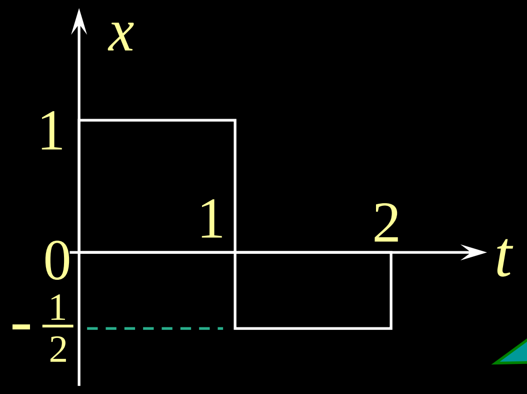
$$u=K\varepsilon(t-t_0)-K\varepsilon(t-t_1)$$

$$=K[\varepsilon(t-t_0)-\varepsilon(t-t_1)]$$

$=$



延迟的阶跃函数



$x=\varepsilon(t)-1.5\varepsilon(t-1)+0.5\varepsilon(t$

也可以用门函数的方法求:

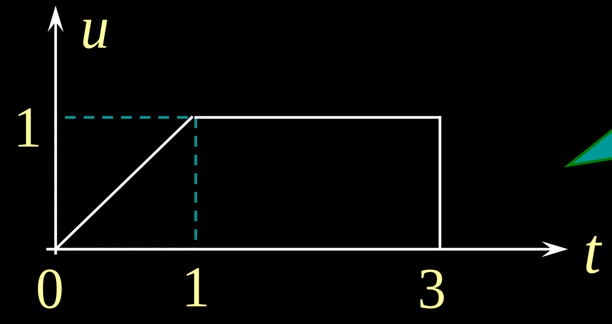
$x=[\varepsilon(t)-\varepsilon(t-1)]-0.5[\varepsilon(t-1)-\varepsilon(t-2)]$

$=\varepsilon($

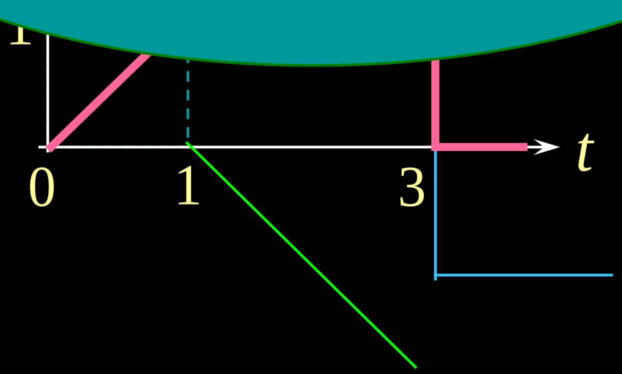
也可以用门函数的方法求:

$u=t[\varepsilon(t)-\varepsilon(t-1)]+[\varepsilon(t-1)-\varepsilon(t-3)]$

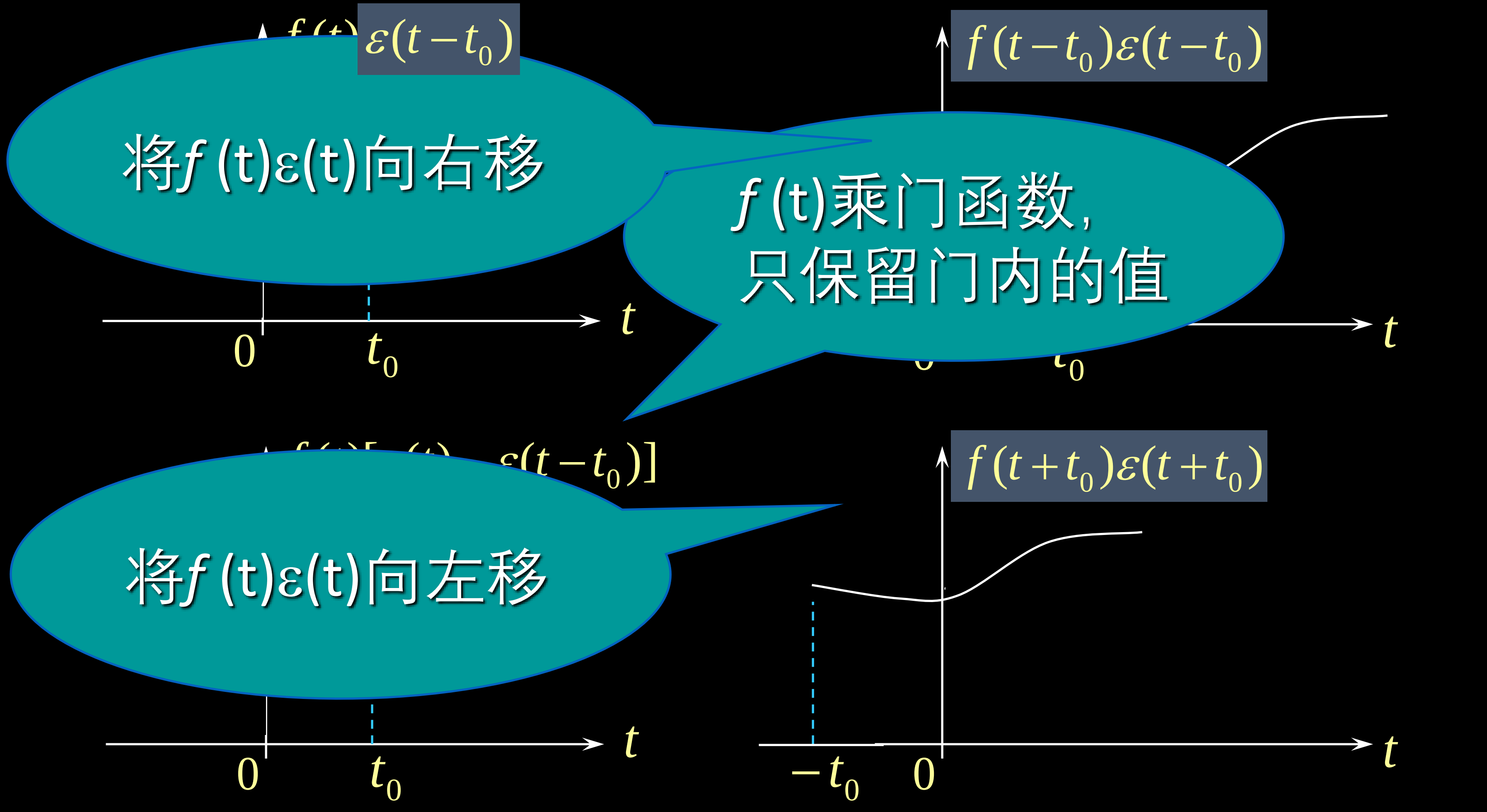
$=t\varepsilon(t)-(t-1)\varepsilon(t-1)-\varepsilon(t-3)$



$u=t\varepsilon(t)-(t-1)\varepsilon(t-1)-\varepsilon(t-3)$

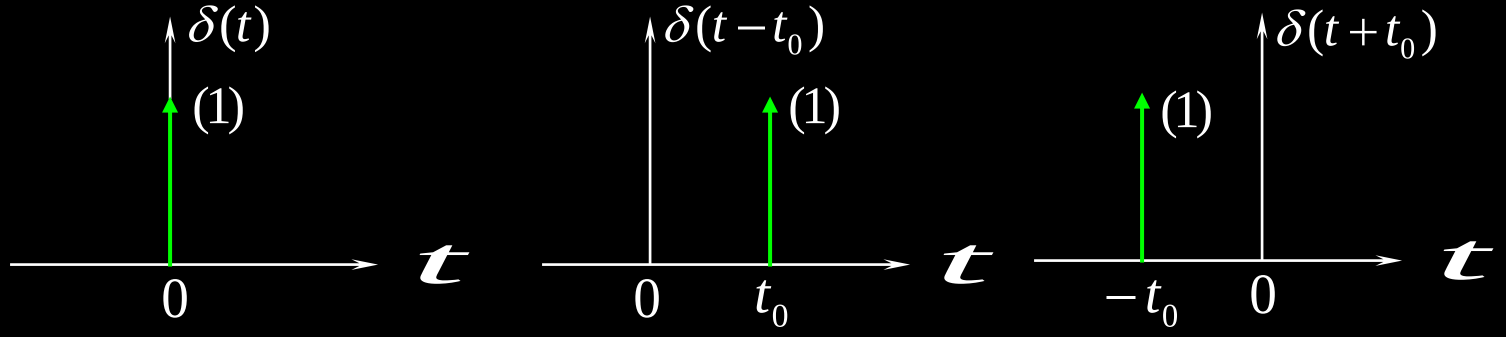


f(t)ε(t)的意义


$\varepsilon(t-t_0)$
将f(t)ε(t)向右移
t
0
t_0
$f(t-t_0)\varepsilon(t-t_0)$
f(t)乘门函数，
只保留门内的值
t
$\varepsilon(t-t_0)]$
将f(t)ε(t)向左移
t
0
t_0
$f(t+t_0)\varepsilon(t+t_0)$
t
$-t_0$
0

冲激函数的性质

- 延迟的冲激函数



- **加权特性**

$$f(t)\delta(t)=f(0)\delta(t);\quad f(t)\delta(t-t_0)=f(t_0)\delta(t-t_0)$$

- **抽样特性**

$$\int_{-\infty}^{\infty} f(t)\delta(t)\,dt=f(0)$$

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)\,dt=f(t_0)$$

是冲激函数的
严格的数学定义。

冲激函数的性质

- 单位冲激函数为偶函数$\delta(-t)=\delta(t)$

- **尺度变换**

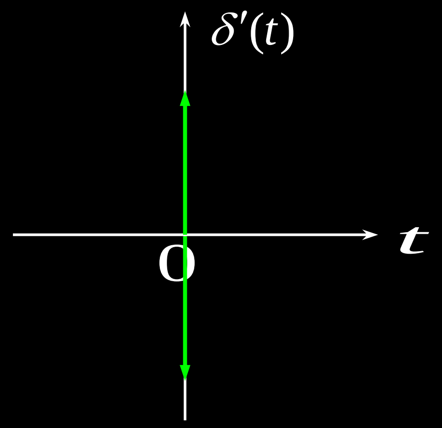
$$\delta(at)=\frac{1}{|a|}\delta(t) \qquad \delta(at-t_0)=\frac{1}{|a|}\delta(t-\frac{t_0}{a})$$

这里 a 和 t_0为常数，且$a\neq 0$。

$$\int_{-\infty}^{\infty} f(t)\delta(at)dt=\frac{1}{|a|}f(0) \qquad \int_{-\infty}^{\infty} f(t)\delta(at-t_0)dt=\frac{1}{|a|}f(\frac{t_0}{a})$$

- **δ(t)的导数及其性质**

定义：$\delta'(t)=\dfrac{d\delta(t)}{dt}$



称单位二次冲激函数或冲激偶。

冲激偶的性质

- 冲激偶的抽样特性

$$\int_{-\infty}^{\infty} f(t)\delta'(t)dt = -f'(0) \qquad \int_{-\infty}^{\infty} f(t)\delta'(t-t_0)dt = -f'(t_0)$$

- **冲激偶的加权特性**

$$f(t)\delta'(t) = f(0)\delta'(t) - f'(0)\delta(t)$$

$$f(t)\delta'(t-t_0) = f(t_0)\delta'(t-t_0) - f'(t_0)\delta(t-t_0)$$

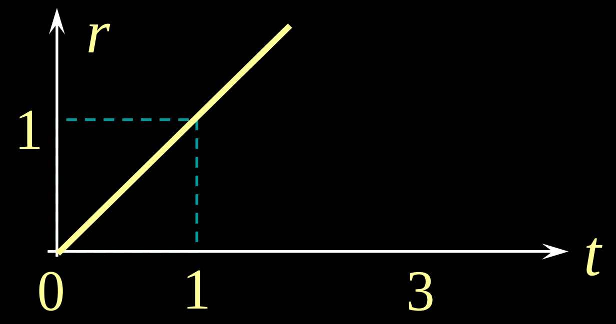
- **冲激偶δ'(t)是 t 的奇函数**

$$\delta'(t) = -\delta'(-t)$$

任何偶函数的导数为奇函数。

斜升函数

$$r(t)=\begin{cases} t & t\geq 0 \\ 0 & t<0 \end{cases}$$



或$r(t)=t\cdot\varepsilon(t)$

四种信号具有微积分关系

$$\delta'(t) = \frac{\mathrm{d}\delta(t)}{\mathrm{d}t}$$

$$\delta(t) = \int_{-\infty}^{t} \delta'(\tau)\mathrm{d}\tau$$

$$\delta(t) = \frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t}$$

$$\varepsilon(t) = \int_{-\infty}^{t} \delta(\tau)\mathrm{d}\tau$$

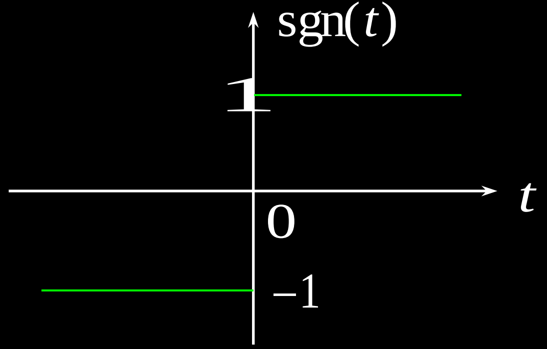
$$\varepsilon(t) = \frac{\mathrm{d}r(t)}{\mathrm{d}t}$$

$$r(t) = \int_{-\infty}^{t} \varepsilon(\tau)\mathrm{d}\tau$$

符号函数和抽样函数

- 符号函数

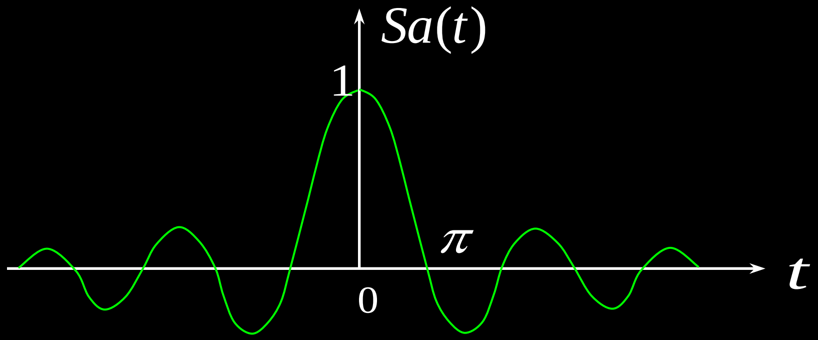
$$\operatorname{sgn}(t)=\begin{cases}1 & t>0\\ -1 & t<0\end{cases}$$



Sgn(t)是奇函数，可以表示成：sgn(t)= -1 +2ε(t)= ε(t)−ε(-t)

- **抽样函数**

$$Sa(t)=\frac{\sin t}{t}$$

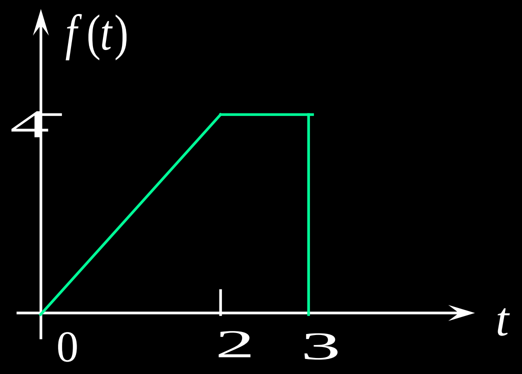


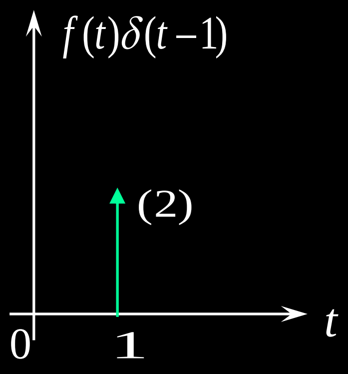
Sa(t)是偶函数，Sa(0)=1
t = nπ 时，Sa(t)=0,
t →∞ 时，Sa(t)→0

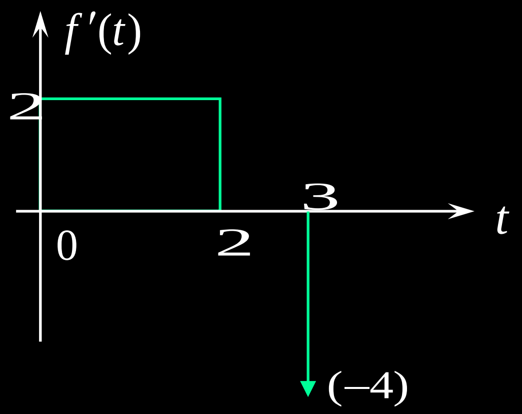
练习1

已知信号 $f(t)=2t\varepsilon(t)-2(t-2)\varepsilon(t-2)-4\varepsilon(t-3)$

求$f(t)$、$f(t)\delta(t-1)$、$f'(t)$的波形。







练习2

下列各表达式中错误的是______。　**C**

$$(A)\ \int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$$

$$(B)\ \int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$$

$$(C)\ \int_{-\infty}^{\infty} f(t-t_0)\delta(t)dt = f(t_0)$$

$$(D)\ \int_{-\infty}^{\infty} f(t-t_0)\delta(t-t_0)dt = f(0)$$

$$(C)\ \int_{-\infty}^{\infty} f(t-t_0)\delta(t)dt = \int_{-\infty}^{\infty} f(-t_0)\delta(t)dt = f(-t_0)$$

练习3

下列各表达式中错误的是______。　**B**

(A) $\delta'(t) = -\delta'(-t)$

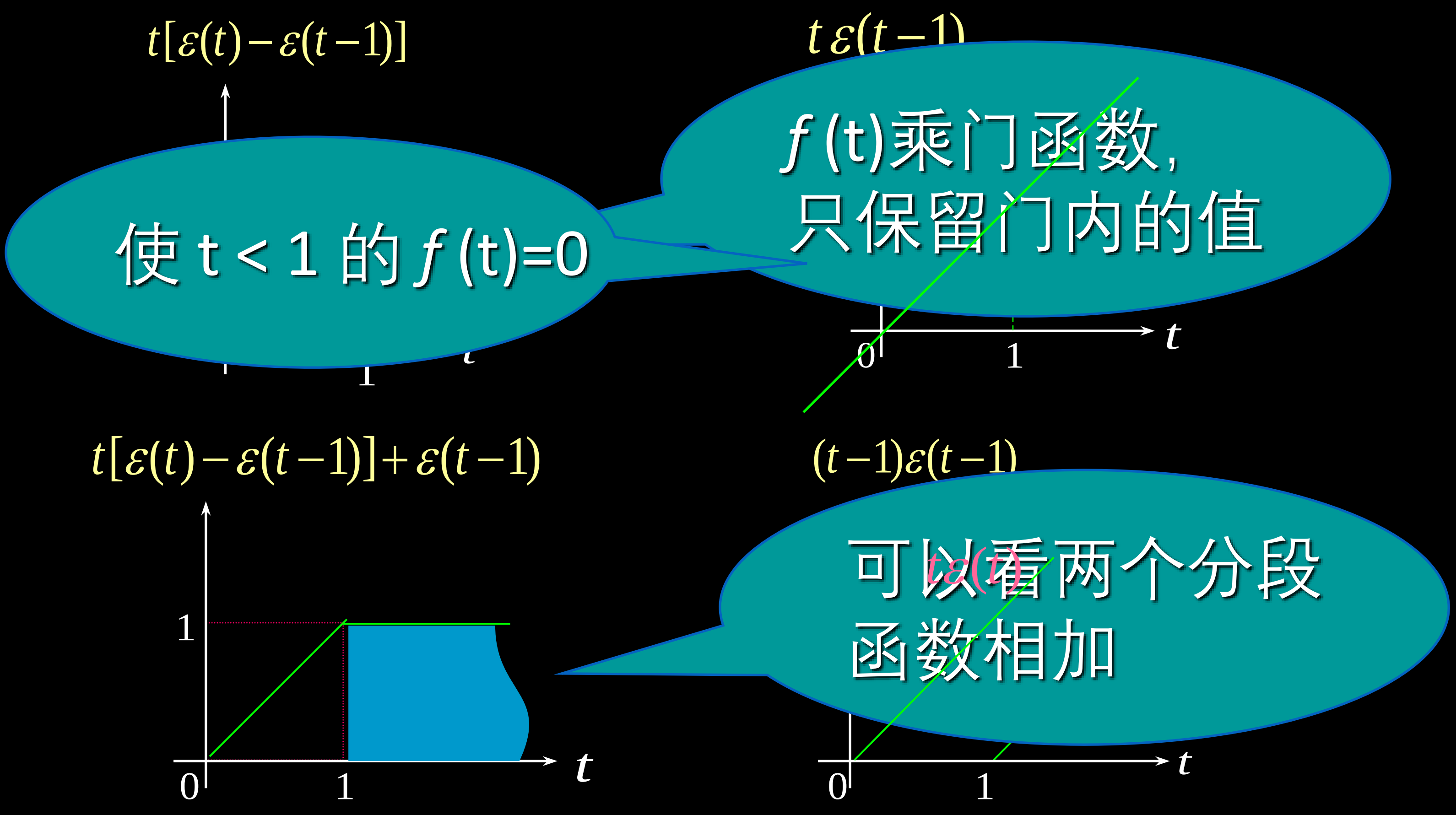
(B) $\delta'(t - t_0) = \delta'(t_0 - t)$　　(B) $\delta'(t - t_0) = -\delta'(t_0 - t)$

(C) $\int_{-\infty}^{\infty} \delta'(t)dt = 0$

(D) $\int_{-\infty}^{t} \delta'(\tau)d\tau = \delta(t)$

例

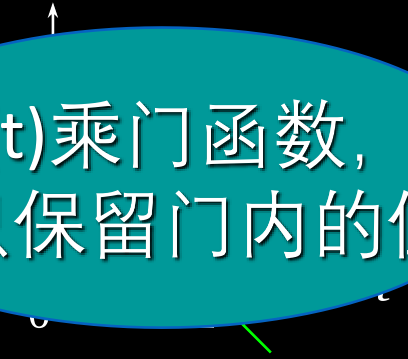
绘出下列各时间函数的波形，注意它们的区别：



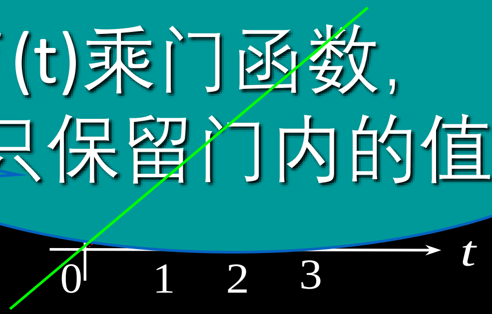
例 3

绘出下列各时间函数的波形，注意它们的区别：

$-(t-1)[\varepsilon(t)-\varepsilon(t-1)]$



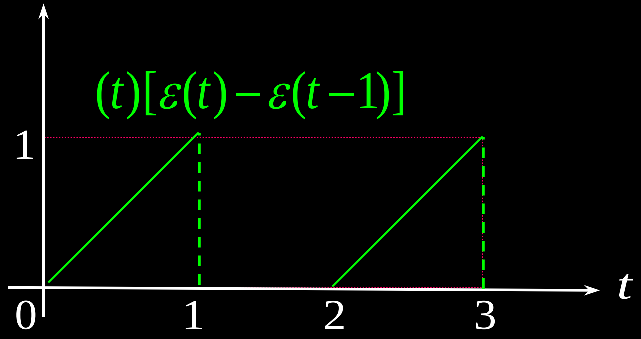
$t[\varepsilon(t-2)-\varepsilon(t-3)]$



f(t)乘门函数，只保留门内的值

f(t)乘门函数，只保留门内的值

$(t-2)[\varepsilon(t-2)-\varepsilon(t-3)]$

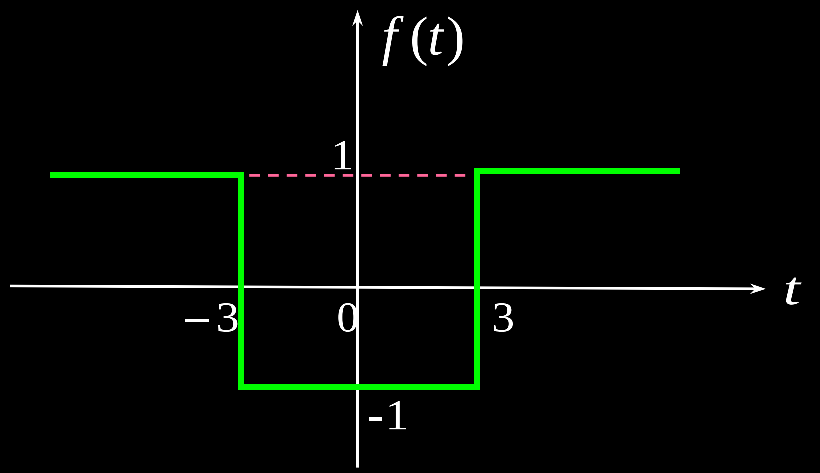


例　4

绘出下列函数的波形。　　(1) $f(t)=Sgn(t^2-9)$

$(t^2-9)=(t+3)(t-3)>0$时：有　$t>3$　和 $t<-3$时　$f(t)=1$

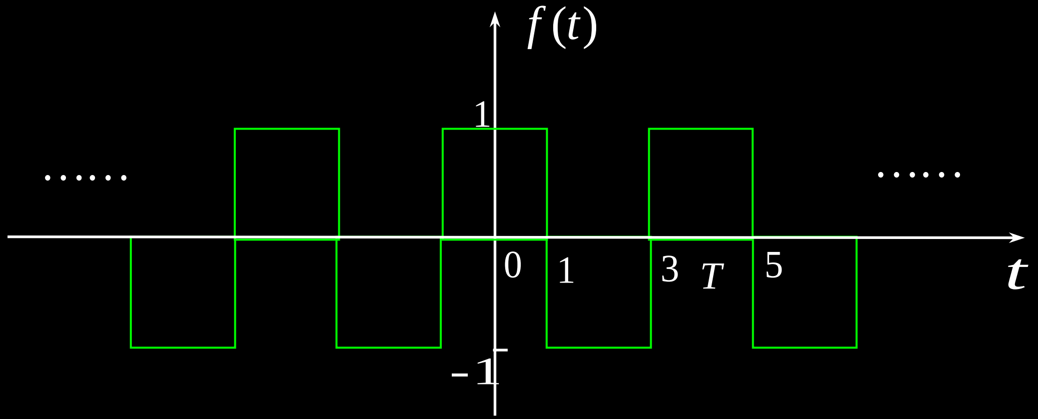
$(t^2-9)=(t+3)(t-3)<0$时：有　$t<3$　和 $t>-3$时　$f(t)=-1$



例　4

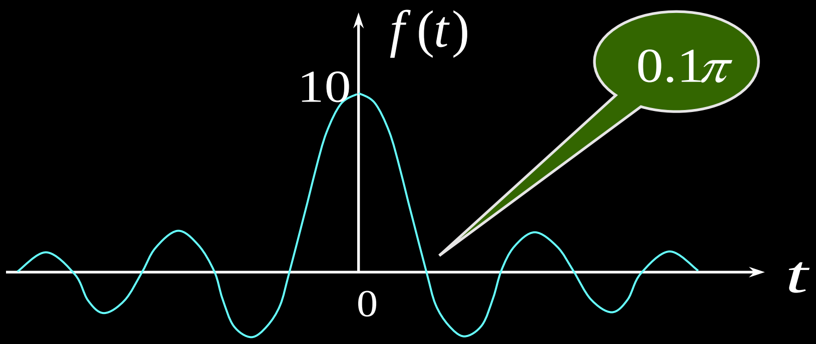
绘出下列函数的波形。

(2) $f(t)=Sgn(\cos\frac{\pi}{2}t)$



(3) $f(t)=\dfrac{\sin 10t}{t}$

$$f(t)=10\frac{\sin 10t}{10t}=10Sa(10t)$$



课堂练习题

计算下列各题。

(1) $4t^2\delta(2t-4)=(4t^2)(0.5)\delta(t-2)=4(2)^2(0.5)\delta(t-2)=8\delta(t-2)$

(2) $\int_0^{\infty}4t^2\delta(t+1)dt\ =0$ 因为 冲激位于积分范围之外。

(3) $\int_{-2}^{2}[(t-3)\delta(2t+2)+8\cos(\pi t)\delta'(t-0.5)]dt$

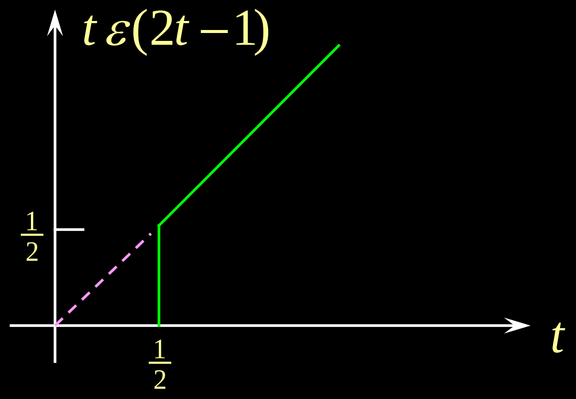
$$\delta(2t+2)=0.5\delta(t+1),\quad (t-3)\delta(2t+2)=-2\delta(t+1)$$

$$原式=-2-[8\cos(\pi t)]'_{t=0.5}=-2+8\pi\sin 0.5\pi=-2+8\pi$$

课堂练习题

画出下列信号的波形。

(1) $t\,\varepsilon(2t-1)$



(2) $\sin\pi(t-1)[\varepsilon(2-t)-\varepsilon(-t)]$