Four types of life tables:

- Cohort general life tables
    - Uses longitudinal observations on a cohort
    - Continuous time duration data
    - Typically for a sample
    - Accommodates right-censored data
- Cohort demography life tables
    - Uses longitudinal observations on a cohort
    - Continuous time duration data
    - Typically for an entire population
    - Doesn’t (typically) accommodates right-censored data
- Period life tables
    - Uses period data
    - Continuous time duration data
    - Typically for an entire population
    - Doesn’t accommodate right-censored data
- Cohort discrete-event life tables
    - Uses longitudinal observations on a cohort
    - Discrete time duration data
    - Typically for a sample
    - Accommodates right-censored data
    - (We won’t look at these in this course)

Questions we can answer from a life table:

Life expectancy at some age *x*:

A person surviving to x would have e_x^o years to live, on average.

Probability of surviving to age some age *x*:

$$\Pr(X \geq x) = \frac{l_x}{l_0}$$

The probability of dying between ages [*x*, *y*):

E.g. from [15 to 45), $x = 15$, $y = 40$:

$$\Pr(y > X \geq x) = \frac{l_x}{l_0} - \frac{l_y}{l_0} = \frac{l_x - l_y}{l_0}$$

The probability of surviving to age *y* given that one has survived to age *x*:

$$\Pr(X \geq y | X \geq x) = \frac{l_y}{l_0} \Big/ \frac{l_x}{l_0} = \frac{l_y}{l_x}$$

(This is sometimes called the survival ratio)

The probability of dying in [*x*, *y*) given one has made it to age *z*:

E.g. Probability of dying from age 50 to 70 given one has survived to age 20

$$\Pr(y > X \geq x | X > z) = \frac{l_x - l_y}{l_z}$$

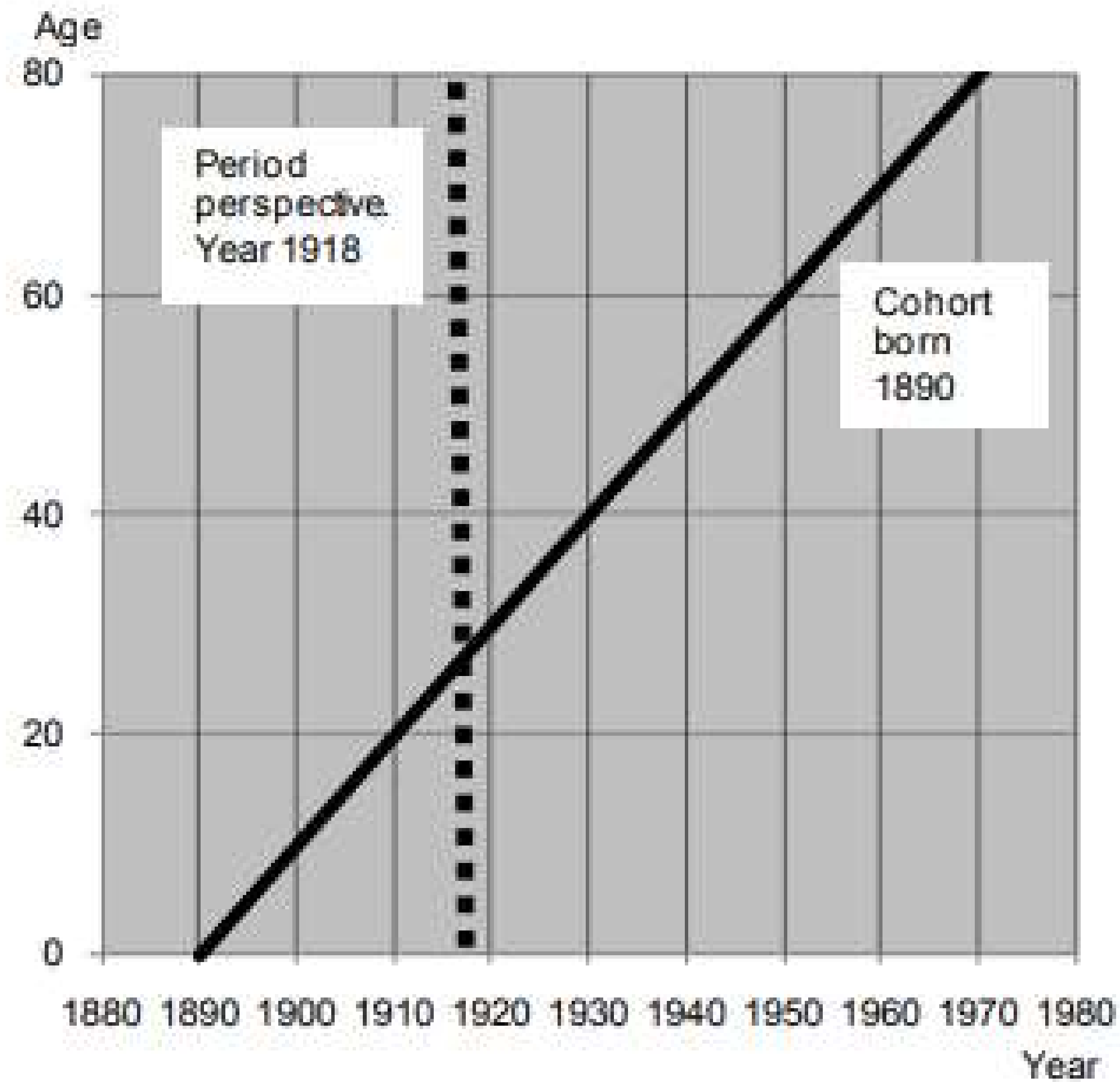
Period (or current) life tables

- These life tables make assumptions in order to present the same type of information as found in a cohort life table.
- The period life table shows **what would happen to a cohort if it were subjected to the mortality conditions of the period**.
- The imagined cohort is called a *synthetic* or *hypothetical* cohort
- Most life tables in demography are period life tables, in part because the data required to construct a cohort life table is rarely available from vital statistics

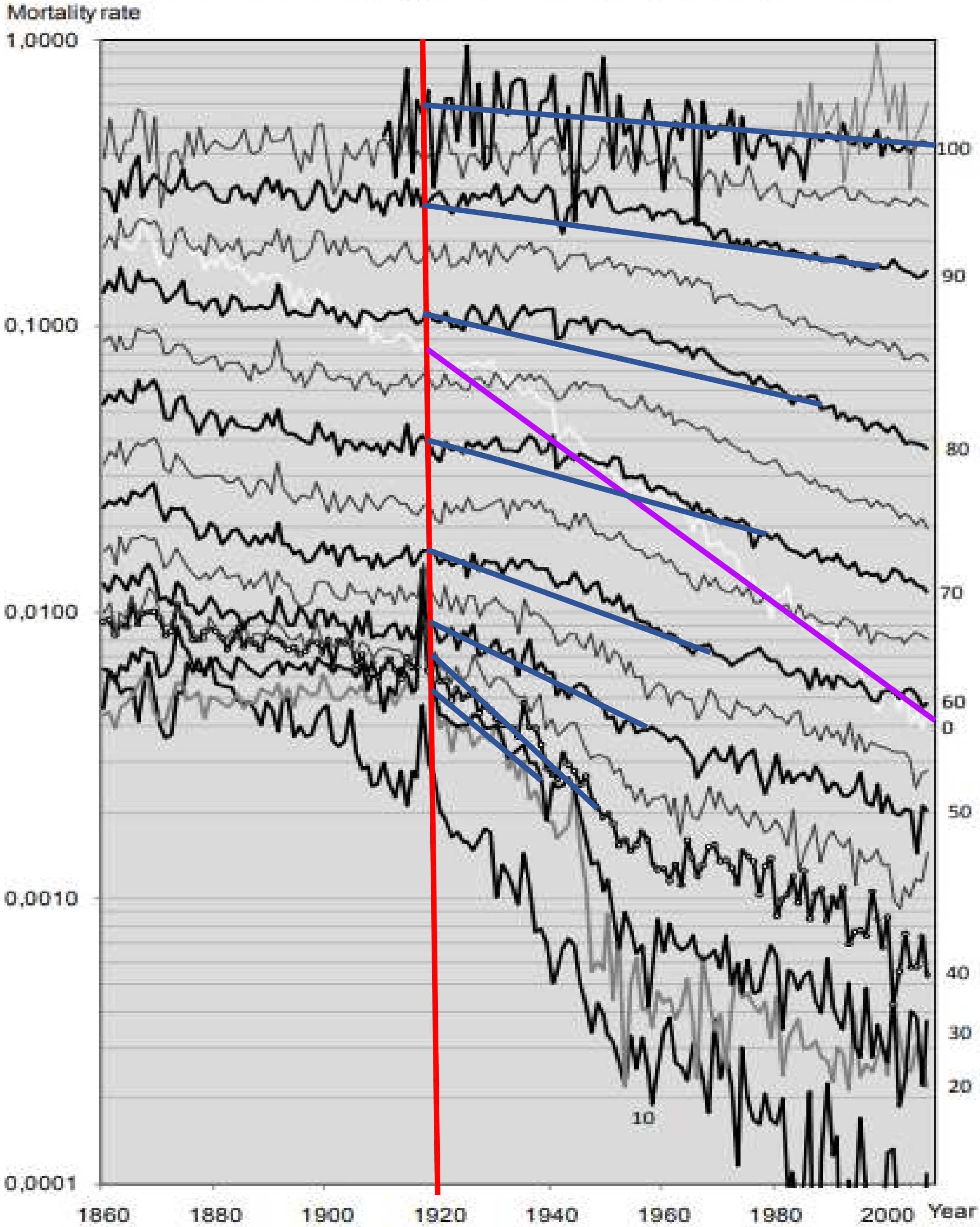
The raw data for a period life table:

- Period deaths and person-years to find ${}_nM_x = \frac{\textit{period deaths in the age interval } [x, x+n)}{\textit{person-yrs in the age interval } [x, x+n)}$ and later estimate ${}_nq_x$
  - Note that this is quite different from a cohort life table that starts with
    - Demographer's: ${}_nq_x = \frac{{}_nd_x}{l_x}$ and later estimates ${}_nm_x = \frac{{}_nd_x}{{}_nL_x}$
    - General: $q_i = d_i/n_i$ and later estimates $\hat{h}(x_{i_{mid}}) = \frac{d_i}{w_i(n_i - \frac{1}{2}d_i)}$
  - n_i and l_x refers to the number of cohort members at the *start* of the age interval

Figure 1.1
Period and cohort perspective. Calendar year 1918 and cohort born in 1890



Mortality rate for women by age for calendar years 1861–2008



A period life tables begins with a period measure, ${}_nM_x$, the set of age-specific death rates and converts them into ${}_nq_x$, age-specific probabilities of dying. How?

Recall from the cohort life table that

$$ {}_nq_x = \frac{{}_nd_x}{l_x} = \frac{\text{cohort deaths in the interval } [x, x+n)}{\text{cohort survivors to age } x} $$

and

$$ {}_nm_x = \frac{{}_nd_x}{{}_nL_x} = \frac{\text{cohort deaths in the interval } [x, x+n)}{\text{person−yrs lived in the cohort } [x, x+n)} $$

Our objective is to find an expression for ${}_nq_x$ that, ideally, only involves ${}_nm_x$.

After some algebra we come to the relationship:

$$ {}_nq_x = \frac{n \cdot {}_nm_x}{1 + (n - {}_na_x){}_nm_x} \qquad (1) $$

Recall ${}_na_x$ is the average years lived in the interval by those who die in $[x, x + a)$

Eq. (1) works if we have some way of determining ${}_na_x$.

With a schedule of period death rates and ${}_na_x$s, we can construct a period life table.

If we assume *uniform deaths* across the interval (${}_na_x$=n/2) then eq. (1) becomes

$$ {}_nq_x = \frac{2n \cdot {}_nm_x}{2 + n \cdot {}_nm_x} $$

Demographers have worked on what to do about ${}_na_x$ for a century years.

Here is a summary:

1. Direct observation. This requires knowing exact ages at death (64.71, 72.30, etc.).
   - Such information is rarely available
   - ${}_na_x$ will be affected by the underlying age structure in the interval.

2. Parametric assumptions (5 and greater)
   - Uniform distribution of death over interval gives: ${}_nq_x = \frac{2n \cdot {}_nm_x}{2 + n \cdot {}_nm_x}$
   - Constant risk of death gives

   $${}_na_x = n + \frac{1}{{}_nm_x} - \frac{n}{1 - e^{-n \cdot {}_nm_x}},$$

   so our conversion from ${}_nm_x$ to ${}_nq_x$ reduces to: ${}_nq_x = 1 - e^{-n \cdot {}_nm_x}$
   - Both of these assumptions work pretty well except for ages under 5 and at high mortality older ages.

3. Curve fitting across age intervals (age 5 and above):

- Greville (1943) fits a Gompertz distribution

$$ {}_nq_x = \frac{{}_nm_x}{\frac{1}{n} + \left[{}_nm_x\left(\frac{1}{2} + \frac{n}{12}\right)\right]\left({}_nm_x - \frac{19}{20}\right)} $$

- Keyfitz (1966) fits a 2nd degree polynomial to deaths:

$$ {}_na_x = \frac{-\frac{n}{24}\,{}_nd_{x-n} + \frac{n}{2}\,{}_nd_x + \frac{n}{24}\,{}_nd_{x+n}}{{}_nd_x} $$

  - Can't estimate ${}_na_x$ for first and last interval
  - Requires the same *n* for all groups

4) Borrow ${}_na_x$ from another population (Reed and Merrell 1939).

- Should be sex-specific
- Use a standard population similar to the target population

Table 3.2: *Average person-years lived between ages x and x + n for persons dying in the interval (${}_na_x$)*

e_0^o	Sweden, 1900 Males	Sweden, 1900 Females	Sweden, 1985 Males	Sweden, 1985 Females	United States, 1985 Males	United States, 1985 Females	Guatemala, 1985 Males	Guatemala, 1985 Females
	51.528	54.257	73.789	79.830	71.266	78.422	60.582	64.415
Age x	Average person-years lived for people dying in the interval x to $x + n$							
0	0.358	0.375	0.083	0.081	0.090	0.086	0.165	0.150
1	1.235	1.270	1.500	1.500	1.500	1.500	1.500	1.500
5	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500
10	2.456	2.469	3.006	2.773	3.014	2.757	2.469	2.390
15	2.639	2.565	2.749	2.617	2.734	2.644	2.711	2.665
20	2.549	2.536	2.569	2.578	2.564	2.552	2.628	2.601
25	2.481	2.514	2.561	2.665	2.527	2.588	2.573	2.563
30	2.505	2.509	2.600	2.649	2.571	2.632	2.593	2.627
35	2.544	2.521	2.638	2.625	2.622	2.678	2.545	2.566
40	2.563	2.522	2.695	2.662	2.666	2.706	2.541	2.543
45	2.572	2.561	2.705	2.722	2.688	2.702	2.604	2.592
50	2.574	2.578	2.706	2.694	2.684	2.683	2.596	2.627
55	2.602	2.609	2.687	2.670	2.657	2.671	2.623	2.661
60	2.602	2.633	2.673	2.689	2.626	2.650	2.635	2.623
65	2.591	2.628	2.643	2.697	2.608	2.642	2.616	2.676
70	2.561	2.585	2.607	2.706	2.571	2.631	2.557	2.607
75	2.500	2.517	2.547	2.650	2.519	2.614	2.486	2.532
80	2.415	2.465	2.471	2.607	2.460	2.596	2.409	2.447
85+	3.488	3.888	4.607	5.897	5.455	6.969	4.611	4.836

Source: Keyfitz and Flieger, 1968: 491; and 1990: 310, 348 and 528.

Comparing different assumptions about ${}_n a_x$

Table 7.3 Comparison of ${}_nq_x$ values from abridged life tables for Australia, 2005–2007 using three alternative methods of computation

	${}_nq_x$ values for males				${}_nq_x$ values for females		
x	Uniform	Constant	Greville	x	Uniform	Constant	Greville
5	0.00055	0.00044	0.00044	5	0.00045	0.00036	0.00036
10	0.00060	0.00060	0.00060	10	0.00045	0.00045	0.00045
20	0.00404	0.00404	0.00404	20	0.00150	0.00150	0.00150
30	0.00519	0.00519	0.00519	30	0.00215	0.00215	0.00215
40	0.00827	0.00827	0.00827	40	0.00479	0.00479	0.00479
50	0.01769	0.01769	0.01770	50	0.01099	0.01099	0.01099
60	0.04401	0.04400	0.04408	60	0.02684	0.02683	0.02686
70	0.11392	0.11379	0.11430	70	0.06922	0.06919	0.06938
80	0.31259	0.30961	0.31345	80	0.21814	0.21718	0.21906
90	0.69202	0.65290	0.66950	90	0.59841	0.57425	0.58731
e_0 (males)	78.9	78.9	79.0	e_0 (females)	83.6	83.6	83.6

Yusuf F, Martins JM, Swanson DA (2014) Methods of Demographic Analysis. New York: Springer.

Uniform distribution of deaths: ${}_nq_x = \frac{2n \cdot {}_nm_x}{2 + n \cdot {}_nm_x}$

Constant risk of death: ${}_nq_x = 1 - e^{-n \cdot {}_nm_x}$

Greville (Gompertz fit): ${}_nq_x = \frac{{}_nm_x}{\frac{1}{n} + \left[{}_nm_x\left(\frac{1}{2} + \frac{n}{12}\right)\right]\left({}_nm_x - \frac{19}{20}\right)}$

Note that these methods are not applied below age 5 for mortality

${}_na_x$ at young ages

- Life expectancy is very sensitive to the procedures to estimate ${}_nq_x$ at the youngest, high mortality ages
- As infant mortality decreases, deaths become more concentrated at youngest side of the age interval
  - For [0, 1), about 90% of deaths occur at ages < 0.5. ${}_1a_0$ is typically about 0.125
  - For [1, 5), about 60% of deaths occur at ages < 3. ${}_4a_1$ is typically about 1.6.
- Ideally find ${}_na_x$ directly from infant/child ages-at-death.
- In absence of better information, use Coale and Demeny (1983) formulas:

Table 3.3: *Values of ${}_na_x$ for use below age 5*

	Males	*Females*
Value of ${}_1a_0$		
If ${}_1m_0 \geq .107$	.330	.350
If ${}_1m_0 < .107$	$.045 + 2.684 \cdot {}_1m_0$	$.053 + 2.800 \cdot {}_1m_0$
Value of ${}_4a_1$		
If ${}_1m_0 \geq .107$	1.352	1.361
If ${}_1m_0 < .107$	$1.651 - 2.816 \cdot {}_1m_0$	$1.522 - 1.518 \cdot {}_1m_0$

Source: Adapted from Coale and Demeny (1983) West model life tables.

“Closing” the life table

- With period life tables, the last interval, traditionally [80, ∞) or [85, ∞), must be handled differently.
- Note that n is, essentially, ∞.
- Simple procedure:
  1) All surviving people die in the interval, so set ${}_{\infty}q_{85} = 1$ and ${}_{\infty}p_{85} = 0$
  2) Compute $l_{85}=l_{80} \cdot {}_5p_{80}$
  3) Since all l_{85} individuals will die, $l_{85} = {}_{\infty}d_{85}$. This means we can compute ${}_{\infty}L_{85} = \frac{{}_{\infty}d_{85}}{{}_{\infty}m_{85}}$
  4) Other quantities can be computed as usual.

Austria, 1992 with l_x = 100,000

Example: Austria, males, 1992

Age x	${}_nN_x$	${}_nD_x$	${}_nm_x$	${}_na_x$	${}_nq_x$	${}_np_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	e^o_x
0	47,925	419	0.008743	0.068	0.008672	0.991328	100,000	867	99,192	7,288,901	72.889
1	189,127	70	0.000370	1.626	0.001479	0.998521	99,133	147	396,183	7,189,709	72.526
5	234,793	36	0.000153	2.500	0.000766	0.999234	98,986	76	494,741	6,793,526	68.631
10	238,790	46	0.000193	3.143	0.000963	0.999037	98,910	95	494,375	6,298,785	63.682
15	254,996	249	0.000976	2.724	0.004872	0.995128	98,815	481	492,980	5,804,410	58.740
20	326,831	420	0.001285	2.520	0.006405	0.993595	98,334	630	490,106	5,311,431	54.014
25	355,086	403	0.001135	2.481	0.005659	0.994341	97,704	553	487,127	4,821,324	49.346
30	324,222	441	0.001360	2.601	0.006779	0.993221	97,151	659	484,175	4,334,198	44.613
35	269,963	508	0.001882	2.701	0.009368	0.990632	96,492	904	480,384	3,850,023	39.900
40	261,971	769	0.002935	2.663	0.014577	0.985423	95,588	1,393	474,686	3,369,639	35.252
45	238,011	1,154	0.004849	2.698	0.023975	0.976025	94,195	2,258	465,777	2,894,953	30.734
50	261,612	1,866	0.007133	2.676	0.035082	0.964918	91,937	3,225	452,188	2,429,176	26.422
55	181,385	2,043	0.011263	2.645	0.054861	0.945139	88,711	4,867	432,096	1,976,988	22.286
60	187,962	3,496	0.018600	2.624	0.089062	0.910938	83,845	7,467	401,480	1,544,893	18.426
65	153,832	4,366	0.028382	2.619	0.132925	0.867075	76,377	10,152	357,713	1,143,412	14.971
70	105,169	4,337	0.041238	2.593	0.187573	0.812427	66,225	12,422	301,224	785,699	11.864
75	73,694	5,279	0.071634	2.518	0.304102	0.695898	53,803	16,362	228,404	484,475	9.005
80	57,512	6,460	0.112324	2.423	0.435548	0.564452	37,441	16,307	145,182	256,070	6.839
85	32,248	6,146	0.190585	5.247	1.000000	0.000000	21,134	21,134	110,889	110,889	5.247

Data source: United Nations, 1994.

Austria, 1992 with $l_x = 1.0$

Table 1. Life table for Austrian Males, 1992

x	n	${}_nm_x$	${}_na_x$	${}_nq_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	e_x
0	1	0.008743	0.068	0.008672	1.00000	0.00867	0.99192	72.8891	72.889
1	4	0.000370	1.626	0.001479	0.99133	0.00147	3.96183	71.8972	72.526
5	5	0.000153	2.500	0.000765	0.98986	0.00076	4.94742	67.9354	68.631
10	5	0.000193	3.143	0.000965	0.98910	0.00095	4.94375	62.9880	63.682
15	5	0.000976	2.724	0.004869	0.98815	0.00481	4.92980	58.0442	58.740
20	5	0.001285	2.520	0.006405	0.98334	0.00630	4.90108	53.1144	54.014
25	5	0.001135	2.481	0.005659	0.97704	0.00553	4.87128	48.2133	49.346
30	5	0.001360	2.601	0.006778	0.97151	0.00658	4.84176	43.3420	44.613
35	5	0.001882	2.701	0.009369	0.96493	0.00904	4.80385	38.5003	39.900
40	5	0.002935	2.663	0.014575	0.95589	0.01393	4.74687	33.6964	35.251
45	5	0.004849	2.698	0.023977	0.94195	0.02259	4.65778	28.9496	30.733
50	5	0.007133	2.676	0.035083	0.91937	0.03225	4.52189	24.2918	26.422
55	5	0.011263	2.645	0.054860	0.88711	0.04867	4.32096	19.7699	22.286
60	5	0.018600	2.624	0.089064	0.83845	0.07468	4.01481	15.4489	18.426
65	5	0.028382	2.619	0.132927	0.76377	0.10153	3.57713	11.4341	14.971
70	5	0.041238	2.593	0.187572	0.66225	0.12422	3.01224	7.8570	11.864
75	5	0.071634	2.518	0.304102	0.53803	0.16362	2.28404	4.8448	9.005
80	5	0.112324	2.423	0.435547	0.37441	0.16307	1.45182	2.5607	6.839
85	∞	0.190585	5.247	1.000000	0.21134	0.21134	1.10889	1.1089	5.247

Source Preston, Heuveline, and Guillot [3], page 49. The life table columns were calculated from the age-specific death rates and ${}_na_x$ values in the source using the formulas given in the source and the parameeters in Table 3.3. Minor discrepancies between the numbers shown here and the numbers in the source are due to rounding error.

Creating a period life table:

1. Pick interval widths (n) and create age intervals:

   Note that ω denotes the last age interval

	x	n
	0	1
	1	4
	5	5
	10	5
	15	5
	20	5
	25	5
	30	5
	35	5
	40	5
	45	5
	50	5
	55	5
	60	5
	65	5
	70	5
	75	5
	80	5
ω :	85	∞

2. If data come as ${}_nN_x$ (mid-year population) and ${}_nD_x$, deaths in ages $[x, x + n)$ during the year, compute

$${}_nM_x = \frac{{}_nD_x}{{}_nN_x}$$ as an estimate of ${}_nm_x$

Age x	${}_nN_x$	${}_nD_x$	${}_nm_x$
0	47,925	419	0.008743
1	189,127	70	0.000370
5	234,793	36	0.000153
10	238,790	46	0.000193
15	254,996	249	0.000976
20	326,831	420	0.001285
25	355,086	403	0.001135
30	324,222	441	0.001360
35	269,963	508	0.001882
40	261,971	769	0.002935
45	238,011	1,154	0.004849
50	261,612	1,866	0.007133
55	181,385	2,043	0.011263
60	187,962	3,496	0.018600
65	153,832	4,366	0.028382
70	105,169	4,337	0.041238
75	73,694	5,279	0.071634
80	57,512	6,460	0.112324
85	32,248	6,146	0.190585

Creating a period life table:

3. Compute ${}_nq_x$ values, which may require computing ${}_na_x$
   a) Do ages less than 5.

Table 3.3: *Values of ${}_na_x$ for use below age 5*

	Males	*Females*
Value of ${}_1a_0$		
If ${}_1m_0 \geq .107$	.330	.350
If ${}_1m_0 < .107$	$.045 + 2.684 \cdot {}_1m_0$	$.053 + 2.800 \cdot {}_1m_0$
Value of ${}_4a_1$		
If ${}_1m_0 \geq .107$	1.352	1.361
If ${}_1m_0 < .107$	$1.651 - 2.816 \cdot {}_1m_0$	$1.522 - 1.518 \cdot {}_1m_0$

Source: Adapted from Coale and Demeny (1983) West model life tables.

Then: $${}_nq_x = \frac{n \cdot {}_nm_x}{1+(n-{}_na_x){}_nm_x}$$

   b) Do age intervals 5 to ω - *n*
   - Use ${}_na_x$ from another population and ${}_nq_x = \frac{n \cdot {}_nm_x}{1+(n-{}_na_x){}_nm_x}$
   - OR assume ${}_na_x$:

     Uniform distribution of deaths: ${}_na_x = n/2$ and
$${}_nq_x = \frac{2n \cdot {}_nm_x}{2 + n \cdot {}_nm_x}$$

   c) Do age ω: ${}_\infty q_\omega = 1.0$

Age x	${}_nN_x$	${}_nD_x$	${}_nm_x$	${}_na_x$	${}_nq_x$
0	47,925	419	0.008743	0.068	0.008672
1	189,127	70	0.000370	1.626	0.001479
5	234,793	36	0.000153	2.500	0.000766
10	238,790	46	0.000193	3.143	0.000963
15	254,996	249	0.000976	2.724	0.004872
20	326,831	420	0.001285	2.520	0.006405
25	355,086	403	0.001135	2.481	0.005659
30	324,222	441	0.001360	2.601	0.006779
35	269,963	508	0.001882	2.701	0.009368
40	261,971	769	0.002935	2.663	0.014577
45	238,011	1,154	0.004849	2.698	0.023975
50	261,612	1,866	0.007133	2.676	0.035082
55	181,385	2,043	0.011263	2.645	0.054861
60	187,962	3,496	0.018600	2.624	0.089062
65	153,832	4,366	0.028382	2.619	0.132925
70	105,169	4,337	0.041238	2.593	0.187573
75	73,694	5,279	0.071634	2.518	0.304102
80	57,512	6,460	0.112324	2.423	0.435548
85	32,248	6,146	0.190585	5.247	1.000000

Creating a period life table:

4. Compute ${}_np_x$ values as ${}_np_x = 1 - {}_nq_x$
5. Compute l_x
   a) Select a suitable *radix* either 1.0, 100,000, etc. and put it in as l_0
   b) For all other age groups, $l_x = l_{x-n} \cdot {}_np_{x-n}$

Age x	${}_nm_x$	${}_na_x$	${}_nq_x$	${}_np_x$	l_x	l_x
0	0.008743	0.068	0.008672	0.991328	100,000	1.00000
1	0.000370	1.626	0.001479	0.998521	99,133	0.99133
5	0.000153	2.500	0.000766	0.999234	98,986	0.98986
10	0.000193	3.143	0.000963	0.999037	98,910	0.98910
15	0.000976	2.724	0.004872	0.995128	98,815	0.98815
20	0.001285	2.520	0.006405	0.993595	98,334	0.98334
25	0.001135	2.481	0.005659	0.994341	97,704	0.97704
30	0.001360	2.601	0.006779	0.993221	97,151	0.97151
35	0.001882	2.701	0.009368	0.990632	96,492	0.96493
40	0.002935	2.663	0.014577	0.985423	95,588	0.95589
45	0.004849	2.698	0.023975	0.976025	94,195	0.94195
50	0.007133	2.676	0.035082	0.964918	91,937	0.91937
55	0.011263	2.645	0.054861	0.945139	88,711	0.88711
60	0.018600	2.624	0.089062	0.910938	83,845	0.83845
65	0.028382	2.619	0.132925	0.867075	76,377	0.76377
70	0.041238	2.593	0.187573	0.812427	66,225	0.66225
75	0.071634	2.518	0.304102	0.695898	53,803	0.53803
80	0.112324	2.423	0.435548	0.564452	37,441	0.37441
85	0.190585	5.247	1.000000	0.000000	21,134	0.21134

Creating a period life table:

6. Compute ${}_nd_x$ as ${}_nd_x = l_x - l_{x+n}$

Note that the magnitude of ${}_nd_x$ depends on the radix.

Age x	${}_nq_x$	${}_np_x$	l_x	${}_nd_x$	l_x	${}_nd_x$
0	0.008672	0.991328	100,000	867	1.00000	0.00867
1	0.001479	0.998521	99,133	147	0.99133	0.00147
5	0.000766	0.999234	98,986	76	0.98986	0.00076
10	0.000963	0.999037	98,910	95	0.98910	0.00095
15	0.004872	0.995128	98,815	481	0.98815	0.00481
20	0.006405	0.993595	98,334	630	0.98334	0.00630
25	0.005659	0.994341	97,704	553	0.97704	0.00553
30	0.006779	0.993221	97,151	659	0.97151	0.00658
35	0.009368	0.990632	96,492	904	0.96493	0.00904
40	0.014577	0.985423	95,588	1,393	0.95589	0.01393
45	0.023975	0.976025	94,195	2,258	0.94195	0.02259
50	0.035082	0.964918	91,937	3,225	0.91937	0.03225
55	0.054861	0.945139	88,711	4,867	0.88711	0.04867
60	0.089062	0.910938	83,845	7,467	0.83845	0.07468
65	0.132925	0.867075	76,377	10,152	0.76377	0.10153
70	0.187573	0.812427	66,225	12,422	0.66225	0.12422
75	0.304102	0.695898	53,803	16,362	0.53803	0.16362
80	0.435548	0.564452	37,441	16,307	0.37441	0.16307
85	1.000000	0.000000	21,134	21,134	0.21134	0.21134

Note that ${}_nd_x$ is not the same as ${}_nD_x$, rather, ${}_nd_x$ is the number of deaths in interval [n, $n + x$) to our synthetic cohort of l_x individuals.

Creating a period life table:

7. Compute ${}_nL_x$ (person-years lived in interval) as:
   a) Intervals 0 through ω - n: $${}_nL_x = n \cdot l_{x+n} + {}_na_x \cdot {}_nd_x$$
   b) Interval ω: $${}_\infty L_\omega = \frac{l_x}{{}_\infty m_x}$$

Note that the magnitude of ${}_nL_x$ is dependent on the radix

x	n	${}_nm_x$	${}_na_x$	l_x	${}_nd_x$	${}_nL_x$	l_x	${}_nd_x$	${}_nL_x$
0	1	0.008743	0.068	100,000	867	99,192	1.00000	0.00867	0.99192
1	4	0.000370	1.626	99,133	147	396,183	0.99133	0.00147	3.96183
5	5	0.000153	2.500	98,986	76	494,741	0.98986	0.00076	4.94742
10	5	0.000193	3.143	98,910	95	494,375	0.98910	0.00095	4.94375
15	5	0.000976	2.724	98,815	481	492,980	0.98815	0.00481	4.92980
20	5	0.001285	2.520	98,334	630	490,106	0.98334	0.00630	4.90108
25	5	0.001135	2.481	97,704	553	487,127	0.97704	0.00553	4.87128
30	5	0.001360	2.601	97,151	659	484,175	0.97151	0.00658	4.84176
35	5	0.001882	2.701	96,492	904	480,384	0.96493	0.00904	4.80385
40	5	0.002935	2.663	95,588	1,393	474,686	0.95589	0.01393	4.74687
45	5	0.004849	2.698	94,195	2,258	465,777	0.94195	0.02259	4.65778
50	5	0.007133	2.676	91,937	3,225	452,188	0.91937	0.03225	4.52189
55	5	0.011263	2.645	88,711	4,867	432,096	0.88711	0.04867	4.32096
60	5	0.018600	2.624	83,845	7,467	401,480	0.83845	0.07468	4.01481
65	5	0.028382	2.619	76,377	10,152	357,713	0.76377	0.10153	3.57713
70	5	0.041238	2.593	66,225	12,422	301,224	0.66225	0.12422	3.01224
75	5	0.071634	2.518	53,803	16,362	228,404	0.53803	0.16362	2.28404
80	5	0.112324	2.423	37,441	16,307	145,182	0.37441	0.16307	1.45182
85	∞	0.190585	5.247	21,134	21,134	110,889	0.21134	0.21134	1.10889

Creating a period life table:

8. Compute T_x (person-years lived [x,∞)) as:

$$T_x = \sum_{a=x}^{\infty} {}_nL_a$$

Note that the magnitude of T_x is dependent on the radix

Age x	${}_nL_x$	T_x	${}_nL_x$	T_x
0	99,192	7,288,901	0.99192	72.8891
1	396,183	7,189,709	3.96183	71.8972
5	494,741	6,793,526	4.94742	67.9354
10	494,375	6,298,785	4.94375	62.9880
15	492,980	5,804,410	4.92980	58.0442
20	490,106	5,311,431	4.90108	53.1144
25	487,127	4,821,324	4.87128	48.2133
30	484,175	4,334,198	4.84176	43.3420
35	480,384	3,850,023	4.80385	38.5003
40	474,686	3,369,639	4.74687	33.6964
45	465,777	2,894,953	4.65778	28.9496
50	452,188	2,429,176	4.52189	24.2918
55	432,096	1,976,988	4.32096	19.7699
60	401,480	1,544,893	4.01481	15.4489
65	357,713	1,143,412	3.57713	11.4341
70	301,224	785,699	3.01224	7.8570
75	228,404	484,475	2.28404	4.8448
80	145,182	256,070	1.45182	2.5607
85	110,889	110,889	1.10889	1.1089

Creating a period life table:

9. Compute e_x^o (expectation of life at age x) as:

$$e_x^o = \frac{T_x}{l_x}$$

The magnitude of e_x^o is *not* dependent on the radix

Age x	l_x	T_x	l_x	T_x	e_x^o
0	100,000	7,288,901	1.00000	72.8891	72.889
1	99,133	7,189,709	0.99133	71.8972	72.526
5	98,986	6,793,526	0.98986	67.9354	68.631
10	98,910	6,298,785	0.98910	62.9880	63.682
15	98,815	5,804,410	0.98815	58.0442	58.740
20	98,334	5,311,431	0.98334	53.1144	54.014
25	97,704	4,821,324	0.97704	48.2133	49.346
30	97,151	4,334,198	0.97151	43.3420	44.613
35	96,492	3,850,023	0.96493	38.5003	39.900
40	95,588	3,369,639	0.95589	33.6964	35.252
45	94,195	2,894,953	0.94195	28.9496	30.734
50	91,937	2,429,176	0.91937	24.2918	26.422
55	88,711	1,976,988	0.88711	19.7699	22.286
60	83,845	1,544,893	0.83845	15.4489	18.426
65	76,377	1,143,412	0.76377	11.4341	14.971
70	66,225	785,699	0.66225	7.8570	11.864
75	53,803	484,475	0.53803	4.8448	9.005
80	37,441	256,070	0.37441	2.5607	6.839
85	21,134	110,889	0.21134	1.1089	5.247

Visualizing the life table

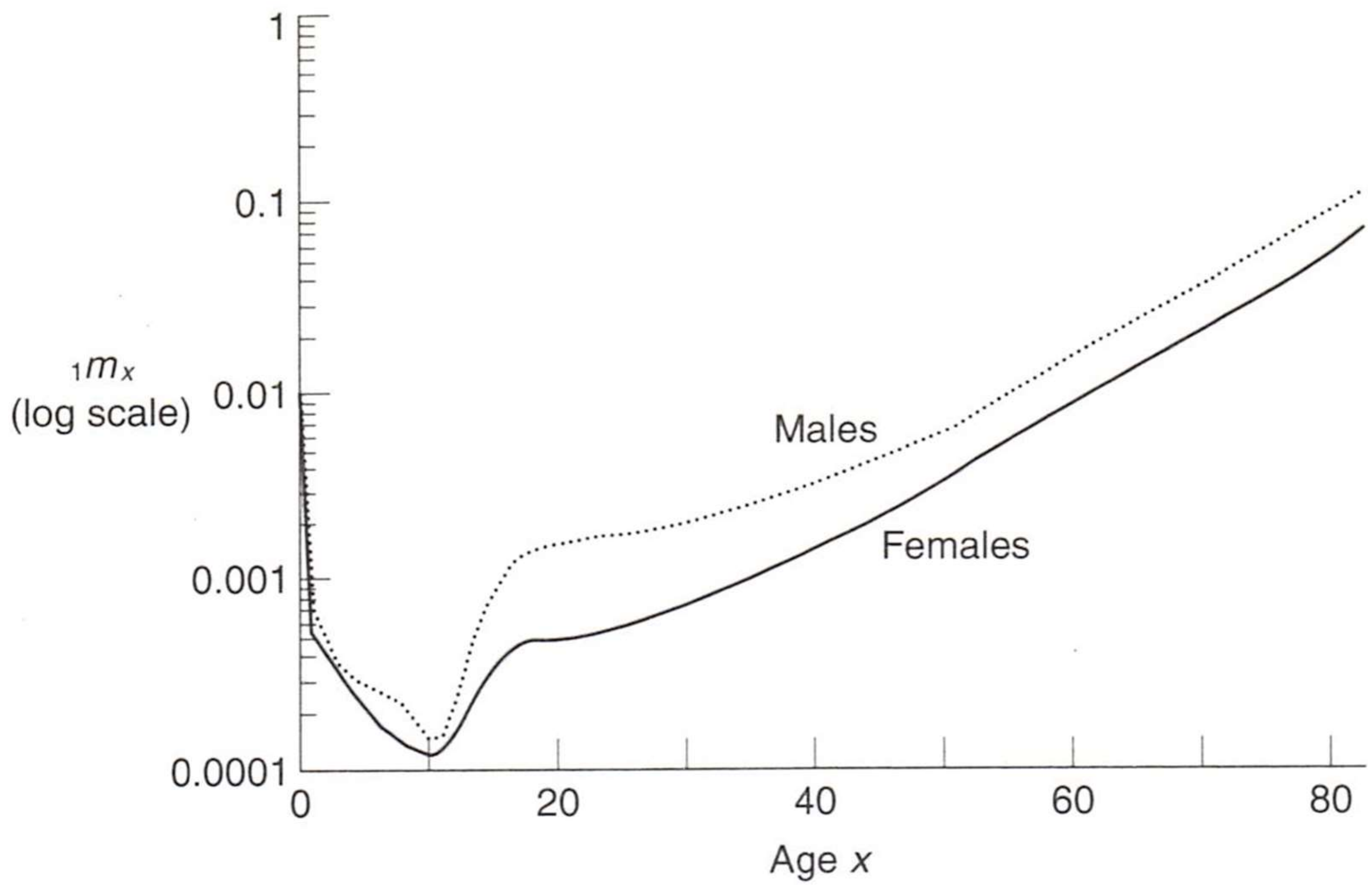


Figure 3.2 Age-specific death rates ($_1m_x$) by age, US, 1992, males and females
Data source: National Center for Health Statistics, 1996.

$_nm_x$ is analogous to the hazard function in the general cohort life table.

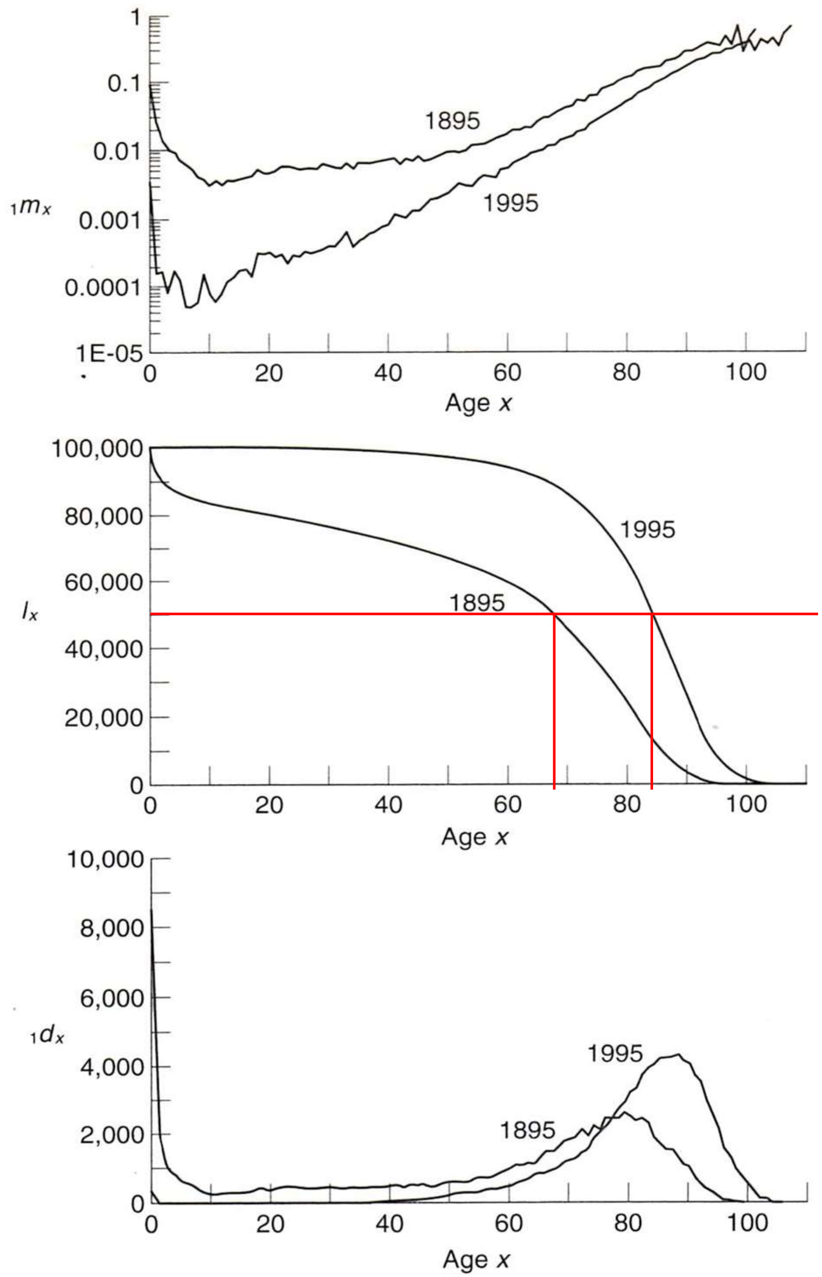


Figure 3.3 Age-specific death rates $({}_1m_x)$, survivors (l_x), and deaths $({}_1d_x)$ in Swedish female life tables, 1895 and 1995

Data source: Statistiska Centralbyrån and Berkeley Mortality Database.
http://demog.berkeley.edu/wilmoth/mortality