0.1 The extraction of gravitational waves

In `SACRA`, we extract the out going component of the complex Weyl scalar, Ψ_4 at finite radii $r_{\rm ex}$ and project it onto the spin–weighted spherical harmonic functions, ${}_{-2}Y_{lm}$, namely,

$$\Psi_4\left(r_{\rm ex},\theta,\varphi,t\right)=\sum_{l}\sum_{m=-l}^{m=l}\Psi_4^{lm}(r_{\rm ex},t)_{-2}Y_{lm}\left(\theta,\varphi\right). \tag{1}$$

We extrapolate Ψ_4 to null infinity following [Lusto et. al 2010]. Using the areal radius defined by

$$D=r_{\rm ex}\left(1+\frac{M_0}{2r_{\rm ex}}\right)^2, \tag{2}$$

, where M_0 is the ADM mass of the system, we compute

$$\begin{aligned}\hat{\Psi}_4^{lm}\left(t\right): &= D\Psi_4^{lm}\left(D,t\right)|_{D\to\infty}\\ &= \left(1-\frac{2M_0}{D}\right)\left[D\Psi_4^{lm}\left(D,t\right)-\frac{(l-1)(l+2)}{2}\int\Psi_4^{lm}\left(D,t'\right)dt'\right].\end{aligned} \tag{3}$$

To ease the notation, we suppress the 'hat' sign on $\hat{\Psi}_4^{lm}$ in the following.

The gravitational waveforms are obtained by integrating Ψ_4^{lm} twice in time, namely,

$$h^{lm}\left(t\right)=h_{+}^{lm}\left(t\right)-ih_{\times}^{lm}\left(t\right)=\int\left(\int\Psi_4^{lm}\left(t''\right)dt''\right)dt'. \tag{4}$$

Instead of integrating Ψ_4 directly in the time domain, the integrations are performed in the frequency domain. The time–domain data are transformed to frequency–domain data by,

$$\tilde{\Psi}_4^{lm}\left(f\right)=\int w\left(t\right)\Psi_4^{lm}\left(t\right)e^{-2\pi ift}dt, \tag{5}$$

where $w\left(t\right)$ is a tapered–cosine filter of the form

$$w\left(t\right)=\left\{\begin{array}{cc}\left\{1-\cos\left[\pi\left(t-t_i\right)/\Delta t\right]\right\}/2 & \left(t_i\le t<t_i+\Delta t\right)\\ 1 & \left(t_i+\Delta t\le t<t_f-\Delta t\right)\\ \left\{1-\cos\left[\pi\left(t_f-t\right)/\Delta t\right]\right\}/2 & \left(t_f-\Delta t\le t<t_f\right)\end{array}\right. . \tag{6}$$

Here, t_i and t_f are the initial and the final times of the data, respectively, and $\Delta t'$ is the width of the tapering region. We use fixed–frequency integration [Reisswig and Pollney 2011] to compute the gravitational waves, namely,

$$h^{lm}\left(t\right)=-\int\frac{\tilde{\Psi}_4^{lm}\left(f\right)}{\left[2\pi\max\left(f,f_0\right)\right]^2}e^{2\pi ift}df. \tag{7}$$

A fixed frequency f_0 is introduced to suppress unphysical drift associated with spurious contributions from low–frequency components. We set $f_0=0.8\left(2\Omega_0/2\pi\right)$ to compute the precessing–frame waveforms, where Ω_0 is the initial orbital angular frequency of the simulation, because we are focusing on the $(l,m)=(2,\pm2)$ modes in the precessing frame.

0.2 Precessing–frame waveforms

As long as the angular velocity of the orbital precession is small enough compared with the orbital angular velocity, it is expected that the gravitational waves are emitted in the similar manner of the non–precessing cases instantaneously: The quadrupole emission with respective to the instantaneous orbital plane may dominate the gravitational–wave emission. Therefore, one could expect that the waveforms obtained by the observer who moves to face the instantaneous plane are similar to the waveforms obtained by non-precessing cases. Indeed, this idea is studied many groups in the context of precessing black hole–black hole binary mergers, and they found some approximately connections between those waveforms obtained by orbital–plane facing observer, which are so called the *precessing frame* waveforms, and non–precessing waveforms [e.g., Schmidt et. al 2015]. We expect that there also exist some approximately connection between the precessing–frame waveforms and non–precessing ones for black hole–neutron star mergers, for which case that the tidal effect is also present.

Through the frame rotation $\mathcal{R}\left(\alpha,\beta,\gamma\right)$ characterized by Euler angles, α, β, and γ , h^{lm} transforms as

$$\tilde{h}^{lm} = \sum_{m'} h^{lm'} D^{l}_{m'm}\left[\mathcal{R}^{-1}\left(\alpha,\beta,\gamma\right)\right], \tag{8}$$

where $D^{l}_{m'm}$ is the Wigner D–matrix defined by

$$\begin{aligned} D^{l}_{m'm}\left[\mathcal{R}^{-1}\left(\alpha,\beta,\gamma\right)\right] &= e^{im'\alpha} d^{j}_{m'm}\left(-\beta\right) e^{im\gamma}, &(9)\\ d^{j}_{m'm}\left(-\beta\right) &= \sum_{k=\max(0,m-m')}^{\max\left(l+m,l-m'\right)} (-1)^{k} \frac{\sqrt{(l+m)!\,(l-m)!\,(l+m')!\,(l-m')!}}{(l+m-m')!\,(l-m'-k)!k!\,(k+m'-m)!} \\ &\times \cos^{2l+m-m'-2k}\left(-\frac{\beta}{2}\right)\sin^{2k+m'-m}\left(-\frac{\beta}{2}\right). &(10) \end{aligned}$$

The precessing frame is obtained by finding the rotation which maximized the amplitude of $(l,m)=(2,\pm 2)$ modes:

$$\max_{\mathcal{R}}\left|\tilde{h}^{22}+\tilde{h}^{2-2*}\right|, \tag{11}$$

where $*$ denotes the complex conjugate.