ECS 122B: Conceptual Homework #2

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Summer Session #2 2020

1 Changelog

You should always refer to the latest version of this document.

- v.1: Initial version.

2 Grading

- **Due date**: the night of Sunday, 08/23. (Due to the "grace period", Gradescope will say 12:30 AM on Monday, 08/24.)
- A subset of the problems will be graded for correctness. The rest will be graded on completion.

3 Submission Requirements

- Your submission must be created with LaTeX. **Handwritten/scanned solutions, or solutions not typed with LaTeX, will earn no credit.**
  - You should avoid doing this assignment at the last minute, because it will take a bit of time to get used to LaTeX and to type your answers up into a LaTeX document.
- Your submission must consist of one file, the PDF generated by your LaTeX file (`hw2_answers.pdf`). Since Gradescope does not let you submit files that are neither PDFs nor images, I will not have you submit the `.tex` file. However, if it is clear that your submission was not made with LaTeX, we may email you to ask that you send us your `.tex` file, and if you cannot (i.e. if it turns out that your PDF was in fact not created with LaTeX), you will get a zero on this assignment.
- You may be penalized if, when submitting on Gradescope, you mark the wrong page for a given homework problem, because dealing with this slows down grading. At the beginning of the 08/06 lecture, I talk about how to mark the pages of your submission on Gradescope, starting at around 3:20 in the video.
- When using LaTeX, you should make use of the math notation/mode where sensible. (The math mode refers to when you put mathematical equations, etc. between dollar signs so that LaTex makes them look nice.) Repeated failures to do this, to the point that the purpose of using LaTeX is defeated or the readability of your answers is impeded, may result in a penalty on this assignment.

4 Regarding Collaboration

- You may not copy answers from any sources, including online sources such as Chegg, StackOverflow, or any solutions manual of any textbook.
- You may partner with ***at most one*** other student on this assignment. In other words, you can work in *pairs*. You do not have to partner with anyone. (In fact, I think it is better that you do not.) If you partner with someone, it must be a committed partnership; that is, you two will have the same submission, and you must mark on Gradescope that you have partnered for this assignment by following the directions here.
- If students that were not in the same pair seem to have excessively similar answers, they will be reported to the OSSJA for suspicion of academic misconduct. Do not copy answers from (or share answers with) any student who you are not partnered with.

5 Identification

Enter the members of your pair. (You can partner with at most one other student.) If you are not partnered with anyone, then leave the second box empty. You can remove the use of `\vspace` in the `.tex` file.

Pair member #1:

Julio Beas

Pair member #2:

Han Nguyen

6 Problems

Unless explicitly specified, you do not need to justify your answer. Place your answer into the answer boxes; you can remove the use of `\vspace` in the answer boxes in the `.tex` file.

6.1 Coin Changing Problem

6.1.1 OPT Array / DP Table / Memoization Array

Fill out the OPT array (shown below) for the coin changing problem, from $n = 0$ to $n = 15$. Assume the coin denominations (i.e. coin values) are ***instead*** 1, 4, and 5. (Regarding LaTeX: you will need to look closely at the `.tex` file to figure out how to put values in the cells. As with many things in LaTeX, you definitely don't need to know how it all works. I don't; I just know where to put the values that go in the cells.)

0	1	2	3	4	5	6	7
0	1	2	3	1	1	2	3

8	9	10	11	12	13	14	15
2	2	2	3	3	3	3	3

6.1.2 Solution Reconstruction #1

What is the optimal solution that corresponds to $OPT[9]$? In other words, when we are trying to find the minimum number of coins to make change for $n = 9$ cents, what coins are chosen in the optimal solution?

The coins chosen are 5, 4.

6.1.3 Solution Reconstruction #2

What is the optimal solution that corresponds to $OPT[15]$?

The coins chosen are 5, 5, 5.

6.2 Subset Sum Problem

Write pseudocode for a top-down implementation of the DP algorithm for solving the subset sum problem. You must use memoization.

```
function GetSubsetSum(array, sum):
    opt = {}
    solution = []

    for (int a = 0; a < array.length(); a++):
        for each b in array:
            if (GetSubsetSumAux(array, sum-b, opt, a+1) > 0):
                solution.append(b)
                sum -= b
    return solution

function GetSubsetSumAux(array, sum, opt, i):
    if (i >= array.length) and (sum == 0):
        return 1
    else if (i >= array.length) and (sum != 0):
        return 0

    if (i, sum) not in opt:
        min_adders = GetSubsetSumAux(array, sum, opt, i+1)
        min_adders = GetSubsetSumAux(array, sum-array[i], opt, i+1)

        opt[(i, sum)] = min_adders

    return opt[(i,sum)]
```

6.3 Set Cover Problem: Certificate

As stated on slide #11 of the computational complexity slide deck, a **certificate** is proof that the answer to a given decision problem with given inputs is "yes". For example, if I were trying to prove to you that a given graph $G = (V, E)$ had a hamiltonian path, then I could do so by telling you such a hamiltonian path P; this path P is the certificate.

Recall the Set Cover Problem (specifically the decision problem) that we talked about during the lectures on complexity classes. What would be an appropriate certificate for the Set Cover Problem? That is, given an instance of the Set Cover Problem (i.e. a set U of n elements, a collection $S_1, ..., S_m$ of subsets of U, and a number k), how could I prove to you that the answer is "yes" (if it is "yes" for a given instance)?

To prove that the answer is "yes" for the Set Cover Problem, we need to find that there exists an optimal solution that contains k subsets among $S_1, ..., S_m$ whose union could cover maximum elements in U.

6.4 Definition of $\mathcal{P}$

Consider the problem of finding the shortest path between two vertices in a graph. Is this problem in $\mathcal{P}$? Briefly justify your answer.

This problem is in $\mathcal{P}$. We would like to consider Dijkstra's Shortest Path algorithm to find the shortest path between two vertices in a graph, and its time complexity is polynomial $O(V^2)$, hence it is in $\mathcal{P}$.