ECS 132 Fall 2020: Midterm Exam 2

December 4, 2020

Instructions

1. You may in no circumstances upload this midterm to private tutoring websites such as Course-Hero or Chegg. Remember all material related to this course is a property of the University of California and posting them is a violation of the copyright laws.
2. You may not upload the questions to any website and ask for help with the questions, nor may you send the exam to anyone else to solicit help.
3. The exam IS open-book, which means you may look up equations in the textbook, homework, discussions, course jupyter notebooks, or course notes
4. You may NOT help your peers or classmates under any circumstances. All questions should be directed to your TAs and professor
5. If you refer to a source (either a book or course notes), you must cite it.
6. You are MUST work these problems on your own - YOU MAY NOT WORK WITH PEERS.
7. Write your answers in GradeScope using markdown.
8. If you have questions, email your TAs and professor: sgburman@ucdavis.edu, dghosal@ucdavis.edu, davhaddad@ucdavis.edu, aacharyya@ucdavis.edu – You must email ALL of us, as only some of us will be responding at any given time. The subject of your email must begin with "ECS 132 Midterm Q: "

1 Problem

Unitrans' newly introduced spacecraft service involves spaceships that arrive arrive at the MU terminal at 15 minutes intervals starting at 8am (i.e., at 8am, 8:15am, 8:30am, 8:45am, and so forth). You arrive at the spaceship port at a time that is Uniformly distributed between 8 and 8:30am. Assuming that all waiting passengers immediately board the spaceship when it arrives, find the probability that you wait for...

1. less than 5 mins for a spaceship - 3pts
2. more than 10 mins for a spaceship - 3pts

Answer

Let X denote the number of minutes past 8 that you arrive at the spaceport.

1. Since $X \sim \text{Unif}(0, 30)$, you will have to wait less than 5 mins if you arrive between 8:10 and 8:15 or between 8:25 and 8:30. Hence the desired probability is given by

$$\begin{aligned}
P(10 < X \leq 15 \cup 25 < X \leq 30) &= P(10 < X \leq 15) + P(25 < X \leq 30) \\
&= F_X(15) - F_X(10) + F_X(30) - F_X(25) \\
&= \frac{15-0}{30-0} - \frac{10-0}{30-0} + \frac{30-0}{30-0} - \frac{25-0}{30-0} \\
&= \frac{1}{2} - \frac{1}{3} + 1 - \frac{5}{6} \\
&= \frac{2}{6} \\
&= \frac{1}{3}
\end{aligned}$$

2. In this case, you arrive between between 8:00 and 8:05 or between 8:15 and 8:20

2 Problem

In a bipolar-encoded transmission system, message bits are communicated using different voltages: 0s are encoded with -3 volts, and 1s are encoded with +3 volts. These voltages are modified by channel noise, which is added to the transmitted signal. This noise (N) is random and modeled by a Normal distribution with mean $\mu = 2$ and $\sigma = 2$. The receiver of the messages decodes the data using the following detection algorithm: if the received signal is greater than 0 volts, a 1 is recorded, and if the received signal is less than 0 volts, a 0 is recorded.

Two errors are possible: a 0 can be recorded as a 1, and a 1 can be recorded as a 0.

1. Given that a 0 was transmitted, what is the probability that a 1 was recorded? - 3pts
2. Given that a 1 was transmitted, what is the probability that a 0 was recorded? - 3pts

You may use R functions or Z table to compute the answer. Note that this is not a simulation question.

Answer

1. In this case, the transmitted signal was -3 volts but the received signal is greater than 0 volts, i.e. $-3 + N > 0 \iff N > 3$. So, the probability is $P(N > 3) = 1 - pnorm(3, 2, 2) = 0.3085$.
2. In this case, the transmitted signal was 3 volts but the received signal is less than 0 volts, i.e. $3 + N < 0 \iff N < -3$. So, the probability is $P(N < -3) = pnorm(-3, 2, 2) = 0.00620966532577613$

3 Problem

Requests arrive at a webserver following a Poisson process with rate parameter λ requests/sec. Hence, the time between requests (inter-request time) is Exponentially distributed with rate λ requests/sec. We are given that inter-request time is less than 0.75 seconds for 30% of the requests. What percentage of the inter-request times are greater than 1.5 seconds? - 4pts

Answer

We don't know λ but we are given that $P(X < 0.75) = 0.3$ this implies that

$$P(X <= 0.75) = F_X(x) = 1 - e^{-\lambda \times 0.75} = 0.3 \tag{1}$$

Solve for λ and then find $P(X > 1.5)$

$-e^{-\lambda \times 0.75} = -0.7$
$e^{-\lambda \times 0.75} = 0.7$
$\lambda = \frac{\ln(1/0.7)}{0.75}$
$\lambda = 0.476$
$P(X > 1.5) = e^{-\lambda \times 1.5}$
$P(X > 1.5) = 0.49$

4 Problem

Pedestrians approach a crossing from the left and right sides following independent Poisson processes with average arrival rates of $\lambda_L = 5$ and $\lambda_R = 1$ arrivals per minute. Each pedestrian then waits until a light is flashed, at which time all waiting pedestrians must cross to the opposite side (either from left to right or from right to left). Assume that the left and right arrival processes are independent, that the light flashes every T = 2 minutes, and that crossing takes zero time – it is instantaneous.

1. What is the probability that in a particular crossing, there are total 10 pedestrian and they are all crossing from left to right? - 4pts

Answer

1. This implies that 0 pedestrians are crossing from right to left, and 10 pedestrians crossing from left to right.

$$P = e^{-\lambda_R T} \times \frac{e^{-\lambda_L T}(\lambda_L T)^{10}}{10!} = dpois(0, 2) \times dpois(10, 10) = 0.0169318 \tag{2}$$