Problem Set 1

Induction

1.)

Find the closed form of : $\sum_{i=1}^{n} 4^i$

Prove your closed-form formula via induction.

2.) Review 1

```
i <-- n;
while(i > 1) {
  j = i;      //%% CAUTION: this DOES NOT START AT 0
  while (j < n) {
    k <-- 0;
    while (k < n) {
      k = k + 2;
    }
    j <-- j * 2;
  }
  i <-- i / 2;
}
```

What is the asymptotic upper bound of the code above?

3.)Review 2

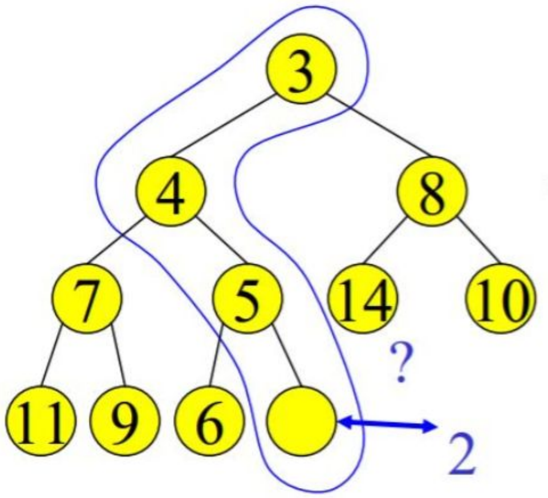
```
float useless(A){
  n = A.length;
  if (n==1){
    return A[0];
  }
  let A1,A2 be arrays of size n/2
  for (i=0; i <= (n/2)-1; i++){
    A1[i] = A[i];
    A2[i] = A[n/2 + i];
  }
  for (i=0; i<=(n/2)-1; i++){
     for (j=i+1; j<=(n/2)-1; j++){
        if (A1[i] == A2[j])
           A2[j] = 0;
     }
  }

  b1 = useless(A1);
  b2 = useless(A2);
  return max(b1,b2);
}
```

What recurrence equation describes the code above?

4.) MinHeap Review

a.) See the **min**Heap below. Please note that in this structure the parents are equal or smaller than their children. Show the resulting tree after you push(2), push(31), pop(), pop(), and update the key of 7 to -2.



b.) Given the following functions *push(a[1]), push(a[2]), push(a[3]) … push (a[n])* What is the asymptotic upper bound on the functions listed assuming the heap is initially empty.

5.)Analysis review

Given an analysis of the running time in Big-O for problems a-f

(a)
```
sum = 0;
  for( i = 0; i < n; ++i )
      ++sum;
```

(b)
```
sum = 0;
for( i = 0; i < n; ++i )
    for( j = 0; j < n; ++j )
        ++sum;
```

(c)
```
sum = 0;
for( i = 0; i < n; ++i )
 for( j = 0; j < n * n; ++j )
     ++sum;
```

(d)
```
sum = 0;
for( i = 0; i < n; ++i )
   for( j = 0; j < i; ++j )
       ++sum;
```

(e)
```
sum = 0;
for( i = 0; i < n; ++i )
        for( j = 0; j < i * i; ++j )
          for( k = 0; k < j; ++k )
              ++sum;
```

(f)
```
sum = 0;
    for( i = 1; i < n; ++i )
         for( j = 1; j < i * i; ++j )
               if( j \% i == 0 )
                    for( k = 0; k < j; ++k )
                         ++sum;
```

6.)

Is $\log_4 n = O(\log_{16} n)$? What about $\log_{16} n = O(\log_4 n)$? Why or why not?

7.)

Rank the following time bounds. That is write them as $f_1, f_2, ..., f_6$ and show that $f_i = O(f_{i+1})$ for all $1 \leq i \leq 5$ (You may use limit lemma theorem)

- $3n^4 + 6n$
- $n \log(n^{1000})$
- $7n^3 \log(n) + 1000$
- 3^n
- 6^n
- $1024n^2 + 4n + 460$