ECS 122B: Conceptual Homework #4

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Summer Session #2 2020

1 Changelog

You should always refer to the latest version of this document.

- v.1: Initial version.

2 Grading

- **Due date**: the night of Saturday, 09/05. (Due to the "grace period", Gradescope will say 12:30 AM on Sunday, 09/06.)
- A subset of the problems will be graded for correctness. The rest will be graded on completion.

3 Submission Requirements

- Your submission must be created with LaTeX. **Handwritten/scanned solutions, or solutions not typed with LaTeX, will earn no credit.**
  - You should avoid doing this assignment at the last minute, because it will take a bit of time to get used to LaTeX and to type your answers up into a LaTeX document.
- Your submission must consist of one file, the PDF generated by your LaTeX file (`hw4_answers.pdf`). Since Gradescope does not let you submit files that are neither PDFs nor images, I will not have you submit the `.tex` file. However, if it is clear that your submission was not made with LaTeX, we may email you to ask that you send us your `.tex` file, and if you cannot (i.e. if it turns out that your PDF was in fact not created with LaTeX), you will get a zero on this assignment.
- You may be penalized if, when submitting on Gradescope, you mark the wrong page for a given homework problem, because dealing with this slows down grading. At the beginning of the 08/06 lecture, I talk about how to mark the pages of your submission on Gradescope, starting at around 3:20 in the video.
- When using LaTeX, you should make use of the math notation/mode where sensible. (The math mode refers to when you put mathematical equations, etc. between dollar signs so that LaTex makes them look nice.) Repeated failures to do this, to the point that the purpose of using LaTeX is defeated or the readability of your answers is impeded, may result in a penalty on this assignment.

4 Regarding Collaboration

- You may not copy answers from any sources, including online sources such as Chegg, StackOverflow, or any solutions manual of any textbook.
- You may partner with ***at most one*** other student on this assignment. In other words, you can work in *pairs*. You do not have to partner with anyone. (In fact, I think it is better that you do not.) If you partner with someone, it must be a committed partnership; that is, you two will have the same submission, and you must mark on Gradescope that you have partnered for this assignment by following the directions here.
- If students that were not in the same pair seem to have excessively similar answers, they will be reported to the OSSJA for suspicion of academic misconduct. Do not copy answers from (or share answers with) any student who you are not partnered with.

5 Identification

Enter the members of your pair. (You can partner with at most one other student.) If you are not partnered with anyone, then leave the second box empty. You can remove the use of `\vspace` in the `.tex` file.

Pair member #1:

Julio Beas

Pair member #2:

Han Nguyen

6 Problems

Place your answer into the answer boxes; you can remove the use of `\vspace` in the answer boxes in the `.tex` file. If you are using your own LaTeX template, you may find the `setcounter` directive useful; it lets you set the number of the next section, and you can read more here.

6.1 `PermuteWithoutIdentity()`

Consider the below procedure.

```
PermuteWithoutSorting(A):
    n := |A|
    For i = 1 .. (n-1):
        Swap A[i] and A[Random(i + 1, n)]
```

The goal of the above procedure is to randomly produce any permutation besides the identity permutation. Does the procedure achieve this goal?

No. For example, suppose that $A = [7, 8, 9]$. We expect that by using this algorithm, the permutation $[9, 8, 7]$ will be produced. Since $A[i] = 7$ at $i = 1$ is swapped in the first iteration, we want to swap with $A[3] = 9$ to retrieve the desired permutation. Now the permutation is $[9, 8, 7]$; however, we have to do the swap with $A[2] = 8$ and $A[3] = 7$ in the second iteration. That leaves us a permutation $[9, 7, 8]$. Hence, we conclude that the procedure does not achieve the goal.

6.2 `PermuteByCycle()`

Consider the below procedure for generating a uniform random permutation.

```
PermuteByCycle(A):
    n := |A|
    Let B[1 .. n] be a new array
    offset = Random(1,n)  # n is included in the range
    For i = 1 to n:
        dest = i + offset
        If dest > n:
            dest = dest - n
        B[dest] = A[i]
    Return B
```

Explain why each element $A[i]$ has a $\frac{1}{n}$ probability of winding up in any particular position in B. Explain why the above procedure generates a permutation that is not uniformly random. (That is, explain why the above procedure does not have an equal chance of generating each permutation.)

1. Suppose the random particular position in B is k. We observe that the probability such that $A[i] = B[k]$ is the

probably when $k = dest$. Since $dest = i + offset$ (if $dest \leq n$) or $dest = i + offset - n$ (if $dest > n$), and $offset$ has the probability of $1/n$, we say that the probability of $A[i] = B[k]$ is also $1/n$.
2. The above procedure generates a permutation that is not uniformly random because, with the example from 6.1, let $A = [7, 8, 9]$ and $B = [1, 2, 3]$. At $offset = 1$, we have $B = [9, 7, 8]$. At $offset = 2$, we have $B = [8, 9, 7]$. At $offset = 3$, we have $B = [7, 8, 9]$. We observe that $B = [7, 9, 8]$ is not generated after three cases of offset, hence the procedure does not have an equal chance of generating each permutation from A.

6.3 Markov Chain: Transition Matrix

Refer to the slide deck on probability and algorithms. On slide 21, an example set of observations $\mathbf{X} = (1, 2, 2, 0, 2)$ is shown. Could a Markov chain whose transition probabilities are given by the transition matrix on slide 23 have produced these observations? If so, what is the probability that the chain would produce those observations? If not, explain.

No, because there is no transition between 0 and 2, hence the probability for the observations will be 0.