

DP的**核心思想**类似于我们高中学习的数学归纳法：

1. 把一个大问题(size == n)的解决方案用比他小的问题（问题们）来解决，也就是思考从问题size = $n-1$ 增加到 size = n 的时候, 如何用小问题的solution构建大问题的solution。
2. 与recursion的关系：
 - 2.1. Recursion 从大到小来解决问题，不记录任何sub-solution只要考虑
 - 2.1.1. base case
 - 2.1.2. recursive rule
 - 2.2. DP 从小到大来解决问题，记录sub-solution
 - 2.2.1. 由size ($< n$) 的 subsolution(**s**) \rightarrow size (n) 的solution
 - 2.2.2. base case
 - 2.2.3. Induction rule

DP 的解题常用方法:

1. 一维的original data (such as a rope, a word, a piece of wood), 求MAX or MIN (cut, merge, etc..)
 - 1.1. if the **weight** of each smallest element in the original data is identical/similar
 - 1.1.1. e.g. **identical**: 1 meter of rope
 - 1.1.2. e.g. **similar**: a letter, a number

Then this kind of problem is usually simple:

Linear scan and look back to the previous element(s)

For example:

Longest Ascending Subarray (when at i, look back at i-1)

Longest Ascending Subsequence (when at i, look back at 1....i-1)

Cut rope

Cut palindrome

- 1.2. If the **weight** are not the same:
 - 1.2.1. e.g. DP1 课后题: 沙子归并
 - 1.2.2. e.g. 强化练习题: 切木头

从中心开花, [index = 0.1.2.3. N-1], for each $M[i, j]$, we usually need to try out all possible k that ($i < k < j$), $M[i, j] = \max (M[i, k] +/- * M[k, j])$ (for all possible k)

....Method1: Recursion

```
00 public int getMaxProduct(int n) {
01     if (n <= 1) {
02         return 0;    // base case;
03     }
04     int maxProduct = 0;
05     for (int i = 1; i < n; i++) { // i = meters of rope to cut off
06         int best = Math.max(n - i, getMaxProduct(n - i));
07         // (no cut) rope remains,    cut== subproblem
08         maxProduct = Math.max(maxProduct, i * best);
09     }
10     return maxProduct;
11 }
```

1 meter long rope

___ **M[1] = invalid**

2 meter long rope

___ ___ **M[2] = ?**

___ ___ There is only 1 possible way to cut two meter long rope, that is

Case 1: ___ | ___

max (1, M[1]) x	max (1,M[1])	= 1 x 1	= 1
左大段	右大段		

所谓的大段，是指我们可以靠读取M[i] 序列，而得到的值 (it is actually a sub-problem that we have solved before.)

3 meter long rope

____ M[3] = 2

|

____ There are only 2 possible ways to cut two meter long rope, that is

Case 1: ____ | ____

$$\max(1, M[1]) \times \max(2, M[2]) = 1 \times 2 = 2$$

Case 2: ____ | ____

$$\max(2, M[2]) \times \max(1, M[1]) = 2 \times 1 = 2$$

$$M[3] = \max(\text{Case1}, \text{Case2}) = \max(2, 2) = 2$$

Determine if you are able to reach the last index.

For example:

index 0 1 2 3 4

A = [2,3,1,1,4], return true.

B = [3,2,1,0,4], return false.

input= 0 1 2 3 4

I A = [2,3,1,1, 4], return true.

M = t t t t t

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Solution:

1. base case $M[n-1] = \text{true}$;
2. induction rule
 - a. $M[i]$ represents whether we could reach the target from i-th index
 - b. $M[i] = \begin{matrix} \text{true} & \text{iff there exists a } j, \text{ where } M[j] = \text{true, AND } j - i \leq \text{input}[i] \\ \text{false} & \text{otherwise} \end{matrix}$

Time = $O(n^2)$ or $O(k * n)$ where k is the largest value in input, AND $k < n$