Advanced Macro

Assignment 1

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Competitive equilibrium for this economy

Definition: A sequential markets equilibrium is a a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, allocations for the representative agent $\{c_t, k_{t+1}^s\}_{t=0}^{\infty}$ and allocations for the representative firm $\{l_t^d, k_t^d\}_{t=0}^{\infty}$ such that:

1. Given k_0 and $\{w_t, r_t\}_{t=0}^{\infty}$, allocations for the representative agent $\{c_t, k_{t+1}^s\}_{t=0}^{\infty}$ solves the agent's maximization problem:

$$\begin{aligned} \max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t U(c_t) \\ s.t.\ & c_t + k_{t+1}^s = r_t k_t^s + w_t \\ & c_t, k_{t+1}^s \geq 0 \ \forall\, t = 0, 1, \ldots \\ & k_0 \text{ given} \end{aligned}$$

2. For each $t \geq 0$, given (w_t, r_t) the firm allocation (l_t^d, k_t^d) solves the firms' maximization problem:

$$\max_{k_t, l_t \geq 0} F(k_t^d, l_t^d) - r_t k_t^d - w_t l_t^d$$

3. Markets clear, for all $t \geq 0$

$$\begin{aligned} l_t^d &= 1 \\ k_t^s &= k_t^d \\ c_t + k_{t+1}^s &= F(k_t^d, l_t^d) \end{aligned}$$

Social Planner's Problem

$$\begin{aligned} w^T(\bar{k_0}) = \max_{\{k_{t+1}\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t U(f(k_t) - k_{t+1}) \\ s.a.\ & \\ & 0 \leq k_{t+1} \leq f(k_t) \\ & k_0 = \bar{k_0} > 0 \text{ given} \end{aligned}$$

Where $f(k_t) = F(k_t, 1)$.

Show that the equilibrium allocation of consumption, capital, and labor coincides with those of the planner's.

- CE FOC

$$\begin{aligned} -u'(c_t) + \beta u'(c_{t+1}) r_t &= 0 \\ -u'(c_t) + \beta u'(c_{t+1}) f'(k_{t+1}) &= 0 \quad \text{from firm's static optimization problem} \\ -u'(f(k_t) - k_{t+1}) + \beta u'(f(k_{t+1}) - k_{t+2}) f'(k_{t+1}) &= 0 \quad \text{from market clearing} \end{aligned}$$

- SPP FOC

$$-u'(f(k_t) - k_{t+1}) + \beta u'(f(k_{t+1}) - k_{t+2}) f'(k_{t+1}) = 0$$

Therefore, as long as the transversality condition holds, both solutions are equivalent.

$$\lim_{t \to \infty} \beta^t u'(f(k_t) - k_{t+1}) f'(k_t) k_t = 0$$

Planner's dynamic problem and the Bellman equation

$$v(k') = \max_{0 \geq k' \geq f(k)} \{U(f(k) - k') + \beta v(k')\}$$

Solve planner's problem.

Assume $u(c) = log(c),\ f(k, l) = zk^{\alpha} l^{1-\alpha}$.

Using the guess and verify method, let $v(k) = A + Bln(k)$.

FOC: $k' = \frac{B\beta z k^{\alpha}}{1 + B\beta}$

Plugging into orginial problem and solving for coefficients, we get:

$$\begin{aligned} A &= \frac{1}{1-\beta} \left[\frac{\alpha\beta}{1-\alpha\beta} ln(\alpha\beta) + ln(1-\alpha\beta) \right] \\ B &= \frac{\alpha}{1-\alpha\beta} \end{aligned}$$

Then, our value function and our policy function would be:

$$\begin{aligned} v^* &= A + Bln(k) \text{ with A and B as defined above} \\ g^*(k) &= \alpha\beta z k^{\alpha} \end{aligned}$$

Obtain {c,k,r,w,y} at the steady state

$$\begin{aligned} k^* &= (\alpha\beta z)^{\frac{1}{1-\alpha}} \\ r^* &= \beta^{-1} \\ w^* &= z(1-\alpha)(\alpha\beta z)^{\frac{\alpha}{1-\alpha}} \\ y^* &= z(\alpha\beta z)^{\frac{\alpha}{1-\alpha}} \\ c^* &= z(\alpha\beta z)^{\frac{\alpha}{1-\alpha}} - (\alpha\beta z)^{\frac{1}{1-\alpha}} \end{aligned}$$

Obtain the path of {c,k,r,w,y} in the following cases

Let $\alpha = \frac{1}{3}, z = 1$. Use planner's solution, starting from steady state:

A. Capital decreases to 80% of its steady state.

```
#Defining vars
a = 1/3;
b = 0.6;
k_star = (a*b)^(1/(1-a));
r = 1/b;

# Initializing array for loop
m, n = 100,2;
K = fill(0.0, (m, n));

#defining policy fn
g = x -> a*b*(x^a);

# loop for capital
k = 0.8*k_star;
for i in 1:m
    K[i, 1] = k
    K[i, 2] = g(K[i,1])
    global k = K[i, 2]
end

# defining fn's for the rest of the variables
w = x -> (1-a)x^a;
y = x -> x^a;
c = (p,q) -> y(p)-q;
r = x -> a*(x^(a-1));
int = r.(K[1:100,2]);
wage = w.(K[1:100,1]);
output = y.(K[1:100,1]);
consumption = c.(K[1:100,1],K[1:100,2]);

# importing packages for plotting
import Pkg; Pkg.add("Plots")
using Plots
gr();

# plotting capital
pcap = plot(1:10,fill(k_star,10), label="capital SS");
plot!(1:10,K[1:10,1], label="capital path", linestyle = :dot, legend = :bottomright);
xlabel!("t");
title!("Capital");
pcap
```

```
## Plot{Plots.GRBackend() n=2}
```