Advanced Metrics

Problem Set 2: Maximization

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MLE (Probit)

1. For the first question, I programmed the routine to estimate a probit model by MLE. I used the `CDF_Normal`, `PDF_Normal` and `BFGS` routines of the `Probability` and `Minimization` modules. Whenever the routine evaluated $\Phi(x'\beta)$, I used $\mu = 0$ and $\sigma = 1$, because $U_I \sim N(0,1)$. The coefficients are displayed in row 1 of the results table. To compare, the results of estimating the probit in R are the following:

```
# probit <- glm(V4~V2+V3, family = binomial(link=probit),data=data)
summary(probit)
```

```
##
## Call:
## glm(formula = V4 ~ V2 + V3, family = binomial(link = probit),
##     data = data)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.5734  -0.6464  -0.2295   0.6487   2.9027
##
## Coefficients:
##              Estimate Std. Error  z value Pr(>|z|)
## (Intercept)  6.38161    0.05034  126.764  < 2e-16 ***
## V2           0.12014    0.02376    5.055  4.3e-07 ***
## V3          -3.90106    0.02919 -133.665  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 68828  on 49999  degrees of freedom
## Residual deviance: 43351  on 49997  degrees of freedom
## AIC: 43357
##
## Number of Fisher Scoring iterations: 5
```

2. For the second question, according to Nail's notes (take it with caution), $n^{1/2}(\hat{\theta}_{MLE} - \theta_0) \sim N(0, B^{-1})$, where in the case of probit

$$B = E\left[xx' \frac{\phi(-x'\beta)^2}{\Phi(-x'\beta)\Phi(x'\beta)}\right]$$

To invert the variance-covariance matrix B, I used the Cholesky factorization using intel-LAPACK routine `POTRF` and `POTRI`. I tried using the `Matrix_Inverse` and `Matrix_Inverse_symmetric` provided in `Matrix` module but I was getting negative variances for some estimates. Using the Cholesky factorization, I could replicate the exact estimates as the `R` intrinsic program. The standard errors are shown in the table row 2.

3. Lastly, I bootstrapped the estimates 100 times. I used sampling from the uniform distribution and Halton sequences, by using`Sample_Uniform` and `Halton` from the `random` module, respectively. I report the unbiased bootstrapped estimators in the the table. Namely, $\tilde{\theta} = 2\hat{\theta} - \bar{\hat{\theta}}^{\star}$. Rows 3 to 6 display the results. Results are very close. Considering the standard errors, they are not significantly different from one another.

Table 1: MLE Probit

	Alpha	Lambda	Gamma
Coef	6.3816177	0.1201244	-3.9010558
S.E	0.05034238	0.02376460	0.02918537
Coef (BS)	6.322229	0.125762	-3.866202
S.E. (BS)	0.05003862	0.02369987	0.02897557
Coef (BSH)	6.3998747	0.1215389	-3.9130211
S.E. (BSH)	0.05052107	0.02376524	0.02927741