Advanced Metrics

Problem Set 1

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Question 1

Comparing different quadrature rules to calculate the integral:

$$ \Phi_{\mu,\sigma} = \int_{-\infty}^{a} \phi(x)dx $$

1. Gauss-Legendre between $(\mu - 4\sigma, a)$.
2. Gauss-Chebyshev with change of variables (COV).
3. Gauss-Hermite with COV.

For the Gauss-Chevyshev COV, I used $\rho(y)=ln(y+1)-ln(2)+a$, which maps $[-1,1] \to [-\infty,a]$. For Gauss-Hermite, $\rho(y)=-e^{-y}+a$ which maps $[- \infty , \infty] \to [-\infty,a]$.

The next table summarizes the results.

μ	σ	a	Gauss-Legendre	Gauss-Chebyshev	Gauss-Hermite
-1	0.5	-2.5	0.01460856	0.01703302	0.01628041
-1	0.5	-1.75	0.14208332	0.14488280	0.14076644
-1	0.5	-1	0.49766113	0.50080450	0.49416151
-1	0.5	-0.25	0.85323895	0.85606443	0.84864305
-1	0.5	0.5	0.98071371	0.98223965	0.97563008
-1	5	-16	0.00000000	0.00000000	0.00000000
-1	5	-8.5	0.00039812	0.00039904	0.00039085
-1	5	-1	0.49999976	0.50394920	0.49813474
-1	5	6.5	1.00001041	0.52275421	1.06847568
-1	5	14	1.01309803	0.00007890	0.21901183
1	0.5	-0.5	0.01460856	0.01703302	0.01628041
1	0.5	0.25	0.14208332	0.14488280	0.14076644
1	0.5	1	0.49766113	0.50080450	0.49416151
1	0.5	1.75	0.85323895	0.85606443	0.84864305

μ	σ	a	Gauss-Legendre	Gauss-Chebyshev	Gauss-Hermite
1	0.5	2.5	0.98071371	0.98223965	0.97563008
1	5	-14	0.00000000	0.00000000	0.00000000
1	5	-6.5	0.00039812	0.00039904	0.00039085
1	5	1	0.49999976	0.50394920	0.49813474
1	5	8.5	1.00001041	0.52275421	1.06847568
1	5	16	1.01309803	0.00007890	0.21901183

We can observe that integral results from the three quadrature rules are very close across the whole distribution when σ is low. However, when the σ is high, we start getting odd results in the upper tail of the distribution, like probabilities above 1 for example.

Question 2