Tax Evasion and Productivity

Hans Martinez*
Department of Economics, University of Western Ontario

September 8, 2025

Abstract

I propose a novel strategy that uses production functions to estimate corporate tax evasion through cost overreporting. Employing a structural production function approach using validation data with correctly reported costs, I investigate potential tax evasion by testing the presence of cost overreporting on a testing data, where the statistically significant differences on the output elasticities of the overreported input in the correct direction are interpreted as evidence of tax evasion. I apply this method to a well-known dataset where I find evidence suggesting that cost overreporting is widespread and quantitatively large. My results indicate that ignoring cost overreporting leads to consistently larger elasticities of intermediate inputs. The bias on the intermediate inputs spreads to the elasticities of labor and capital, whose bias direction varies by industry. Finally, I find significant differences in the productivity distributions. How much cheating there could be (back-of-envelope calculation, be careful

Keywords: Tax Evasion, Cost Overreporting, Production Function Estimation, Productivity

*email: hmarti33@uwo.ca.

Table of contents

Introduction

HM Note: I propose a novel method to estimate corporate tax evasion through cost overreporting using production functions. I first show how that cost overreporting lead to biased estimates of productivity and production function parameters. Then, I show how to recover unbiased estimates of the production function and productivity in the presence of tax evasion. With the production function parameters and productivity on hand, the production function can be inverted to recover the true inputs. Then, I estimate cost overreporting as the difference between observed and true inputs. I apply this method to a well-known dataset where I find evidence suggesting that cost overreporting is widespread and quantitatively large. I also find that ignoring cost overreporting leads to consistently larger elasticities of intermediate inputs. The bias on the intermediate inputs spreads to the elasticities of labor and capital, whose bias direction varies by industry. Finally, I find significant differences in the productivity distributions.

HM Note: Title:What is cost overreporting? Cost overreporting arises when firms acquire false invoices to claim additional tax deductions on value-added (VAT) and corporate income taxes (CIT). According to the OECD's document **?**, cost overreporting — also known as "fake invoicing", "ghost firms", "invoice mills", or "missing traders"— permeates internationally.

HM Note: Title:Why do we care? Revenue losses for govt's Reports from Latin America, Eastern Europe, Asia, and Africa claim cost overreporting led to annual tax revenue losses as large as 5.6% of the GDP, for example, in Poland, 2016 (Poland's Minister of Finance, 2018). Other reports show that cost overreporting led to revenue losses of 0.2% of Chile's GDP in 2004 (Gonzalez and Velasquez, 2013; Jorrat, 2001; CIAT, 2008); 0.2% of Colombia's GDP (Portafolio, 2019); and 0.03% of Mexico's GDP in 2018 (Senado de la Republica, 2019). HM Note: Replace percentages of GDP with USD. I care about Tax Revenue more than GDP. How much am I losing in tax revenue? Throw Poland. Don't say "up to". Give people a range. Compared to total tax revenue and the whole size of the economy GDP. 20-30% Tax Revenue relative to GDP.

HM Note: Title:Why do we care? Productivity biases Furthermore, ignoring tax evasion leads to biased estimates of productivity. In the proxy variable literature, productivity is measured as the residual of the production function, where the output of a firm is a function of the inputs, capital, labor, and intermediates. A key assumption is that intermediate input demand is strictly monotonic on the productivity of the firm (**???**). In other words, we expect highly productive firms will use fewer inputs to produce a given level of output. Intermediate inputs, however, are also the most likely input to be misreported. When firms overreport their inputs increasing their costs to claim additional tax deductions, their reported inputs are higher than their actual utilization, resulting in lower productivity estimates.

HM Note: Title:Overlooked by corp tax ev lit? Why? Despite its relevance, the literature on corporate tax evasion has mostly overlooked cost overreporting. The few studies focusing on this tax evasion strategy rely on exploiting detailed administrative data (**??**). Due to firms' confidentiality concerns, government tax authorities restrict access to kind of data. My approach complements these methods. It is not restricted to confidential tax records and can be applied using more commonly available data such as firm-level surveys.

HM Note: Title: Why is it hard? To the best of my knowledge, no other study has attempted to structurally identify cost overreporting. A fundamental problem is that when it comes to corporate tax evasion, researchers have to account for an additional source of unobserved heterogeneity, productivity. Why? Because cost overreporting might be naively quantified as low productivity. Intuitively, for a given output level, high input utilization by a firm could be explained by either the amount of input the firm overreports to evade taxes or by a negative productivity shock.

HM Note: Title: How do I do it? To address this gap in the literature, first I formally show that ignoring tax evasion leads to downward biased productivity estimates. I then provide a new estimation strategy using production functions to jointly recover the densities of tax evasion and productivity. The intuition works as follows. In the absence of tax evasion, the first-order conditions of the firms' cost-minimization problem let us recover the common technology, the production function. Consequently, in the presence of cost overreporting, deviations from this common technology identify tax evasion up to the measurement error. Then, from a subset of non-overreporting firms, the strategy identifies the production function parameters and the density of the output shock. Finally, using non-parametric deconvolution techniques, I jointly recover the distributions of tax evasion and productivity.

HM Note: Title: What do I do? What do I find? Applying the method using firm-level data from Colombia between 1981 and 1991 —a commonly used dataset in the production function literature—, I find evidence suggesting that firms in four of the top five industries (8 of the top 20) engage in cost overreporting. These firms overreport up to 25% of their costs. My estimates suggest that the tax evading firms in the top 20 revenue industries caused the government of Colombia approximately XXXX in tax revenue losses. [exporters/importers/proprietorships/limited liability companies] are [less/more] likely to engage in cost overreporting. I also find that ignoring cost overreporting leads to consistently larger elasticities of intermediate inputs by [what factor]. The bias on the intermediate inputs spreads to the elasticities of labor and capital, whose bias direction varies by industry [what range?]. Lastly, I find significant differences in the productivity distributions. In particular, true productivity distributions are [how]. The differences between [exporters/corporations] and [importers/limited liability companies] are [what?].

HM Note: Next paragraph why do we care? - Two reasons: How much revenue governments are losing - Second: Productivity measurement bias -> potentially systematic wrong -> potential biases in productivity estimates might be behind the differences between different groups of firms. Do they differ due to inherent productivity differences or due to different incentives to evade taxes? Fav reasons for prd fns

Third, overlooked by the lit on corporate tax evasion why? Underreporting revenue. What lit? Big picture wise: Difficult to get data. People have done things like: experiments and have focus on revenue underreporting. Part of the reason, Access to tax data is hard. The few studies as a result. My method is complementary to these methods and can be applied to more commonly available data. STOP.

Why this is hard? A fundamental problem One partial reason of doing this. After making the point, explain how I do it.

Table: order testing and estimation data. order by magnitude one tail. Switch between quarters. Randomly rotate. Estimate with three quarters and test with the others. With replacement. Test approach

First productivity, Kernel density. How they differ. Eyeball test. Use pdf not cdfs. Compare prod densities. All vs. validation vs. overreporting.

What I really want is the joint distribution of productivity and tax evasion. How to get it???

Back of envelope calculation: tax revenue losses, how much is that.

Option 1: Focus on tax evasion, leave identification in the Appendix

1 A parsimonious model of tax evasion through input overreporting

Price-taking firms maximize expected after-tax profits. Firms choose the flexible input M_{it} to produce output Y_{it} given output and input prices $\{P_t, \rho_t\}$, a common technology, the production function (Equation **??**), and their productivity ω_{it}.

$$Y_{it} = G(M_{it}) \exp(\omega_{it} + \varepsilon_{it}) \tag{1}$$

As standard in the literature, productivity ω_{it} is known to firms when they make input decisions. This is the well-known endogeneity problem of simultaneity. On the other hand, firms face output shocks. The output shock ε_{it} is not part of the firms' information set.

The model departs from the literature by allowing firms to overreport their inputs e_{it} to reduce their tax burden and optimize after-tax profits. Firms, then, consider in their optimization problem the profit tax τ, the evasion penalty/cost $\kappa(e)$, and the probability of detection $q(e_{it}|\theta_{it})$.

Firms solve Equation ??

$$\max_{M_{it}, e_{it} \in [0,\infty)} [1 - q(e_{it}|\theta_{it})] \left[(P_t \mathbb{E}[Y_{it}] - \rho_t M_{it}) - \tau \left(P_t \mathbb{E}[Y_{it}] - \rho_t (M_{it} + e_{it}) \right) \right]$$
$$+ q(e_{it}|\theta_{it}) \left[(1-\tau)(P_t \mathbb{E}[Y_{it}] - \rho_t M_{it}) - \kappa(e) \right] \tag{2}$$
$$\text{s.t.} \quad Y_{it} = G(M_{it}) \exp(\omega_{it} + \varepsilon_{it})$$

The probability of detection $q(e_{it}|\theta_{it})$ is monotonically increasing in the amount evaded e_{it}, conditional on the type of the firm θ_{it}. Intuitively, for a given type, firms that evade more are more likely to get caught.

The type of the firm θ_{it} might be discrete, like the type of juridical organization, or continuous, like the level of revenue[1]. Some types might be more likely to be detected if the firm engages in tax evasion. For example, in contrast to other types of juridical organizations in Colombia, corporations are closely supervised and are required to have an auditor. That is, for a given level of tax evasion e_0 and two different types $\theta' \neq \theta \in \Theta$, then $q(e_0|\theta') \geq q(e_0|\theta)$.

If the type θ is continuous, it might be a function of inputs; for example, level of revenue. Firms will then affect their probability of detection $q(e|\theta)$ in two ways: directly, by choosing how much they evade e; and indirectly, when choosing inputs M.

The optimal decision of the firm will depend on the fiscal environment $\Gamma = \{\tau, \kappa, q\}$, namely the tax rates, the penalty/cost of detection, and the probability of detection.

[1]Level of revenue is a common measure for fiscal authorities to determine a firm's taxes and/or level of scrutiny, e.g., Mexico, Spain, Colombia, and Ecuador. HM Note: Chile (?)

The firms' problem (Equation **??**) can be rewritten as follows,

$$\max_{M_{it},e_{it}} \mathbb{E}[\pi_{it}|\Gamma] = (1-\tau)\left(\mathbb{E}[Y_{it}] - \frac{\rho_t}{P_t}M_{it}\right) + [1-q(e_{it}|\theta_{it})]\left(\frac{\rho_t}{P_t}e_{it}\tau\right) - q(e_{it}|\theta_{it})\kappa(e_{it})$$
$$\text{s.t.} \quad Y_{it} = G(M_{it})\exp(\omega_{it}+\varepsilon_{it}) \tag{3}$$

Intuitively, if the firm overreports her inputs' cost, she will get the share of the value she overreported with probability $(1-q)$ and she will be penalized with probability q.

Assuming well-behaved functions and no corner solutions, the first-order conditions lead to the following system of differential equations,

$$G_M(M_{it})\exp(\omega_{it})\mathcal{E} - \frac{\rho_t}{P_t} = \frac{1}{(1-\tau)}\frac{\partial q(e_{it}|\theta_{it})}{\partial \theta_{it}}\frac{\partial \theta_{it}}{\partial M}\left[\frac{\rho_t}{P_t}e_{it}\tau + \kappa(e_{it})\right] \tag{4}$$

$$[1-q(e_{it}|\theta_{it})]\frac{\rho_t}{P_t}\tau - q(e_{it}|\theta_{it})\kappa'(e_{it}) = q'(e_{it}|\theta_{it})\left[\frac{\rho_t}{P_t}\tau e_{it} + \kappa(e_{it})\right] \tag{5}$$

where $\boldsymbol{\mathcal{E}} = \mathbb{E}[\exp(\varepsilon_{it})]$. The type of firms is continuous and increasing on the input. The probability of detection is increasing in the type continuum. In particular, $\frac{\partial q(e_{it}|\theta_{it})}{\partial \theta_{it}}\frac{\partial \theta_{it}}{\partial M} \geq 0$.

The left-hand side of Equation **??** is the familiar marginal output of inputs and the price ratio. In the absence of incentives' distortions induced by the fiscal environment, they are equal. But now, the equality holds no more. There's a wedge arising from the fiscal environment. The right-hand side of the equation is positive by the assumptions of the model.

Equation **??** solves the optimal evasion decision. The left-hand side is the marginal benefit net of the marginal cost of evasion. The right-hand side is the rate of change of the probability of detection due to a change in evasion weighted by the benefit and cost of evading.

1.1 Case 1 (Independence): $q(e|\theta) = q(e)$ and $\kappa(e) = \kappa_0$

Consider the case when the probability of detection is independent of type, $q(e|\theta) = q(e)$. This could be the case if the type is the juridical organization of the firm. Hence, the type of the firm, and thus the probability of detection, does not change with the firm's input decisions, $\frac{\partial q(e_{it}|\theta_{it})}{\partial \theta_{it}} \frac{\partial \theta_{it}}{\partial M} = 0$. In addition, assume the evasion cost is constant, $\kappa(e) = \kappa_0$, for simplicity.

In this case, the first-order conditions of Equation **??** with respect to the input M_{it} and the tax evasion e_{it} yield the following

$$G_M(M_{it}) \exp(\omega_{it}) \mathcal{E} = \frac{\rho_t}{P_t} \tag{6}$$

$$e_{it} = \frac{1 - q(e_{it})}{q'(e_{it})} - \frac{\kappa_0}{\frac{\rho_t}{P_t} \tau} \tag{7}$$

Equation **??**, the well-known optimality condition, says that the price ratio is equal to the marginal product of the inputs.

Likewise, Equation **??** reveals the firms' optimal tax evasion decision decreases if the probability of detection $q(e_{it})$ or the penalty of evading κ increases. Tax evasion also depends on how sensitive the probability of detection is to the level of evasion $q'(e)$. In particular, greater sensibility will result in lower levels of evasion.

Note that the net change of tax evasion due to an increase in the relative prices $\frac{\rho_t}{P_t}$ or the tax rate τ is not evident at first sight. The net effect will also depend on the change in the detection probability induced by the changes in the relative prices or the tax rate. In particular, an increase in relative prices $\frac{\rho_t}{P_t}$ or the tax rate τ will incentivize a higher tax evasion level, however, a higher tax evasion level will increase the probability of detection —depending on the shape of the probability as a function of e—, so it will deter higher levels of evasion. An increase in the tax rate, for instance, will only increase tax evasion if the change in the tax rates increases the incentives to evade more than the decrease in the incentives due to the changes in the detection probability.

Formally, suppose a firm increases its tax evasion, $e_1 - e_0 > 0$ because of an increase in taxes $\tau_1 > \tau_0$. Then, it follows that

$$\left(\frac{\tau_1 - \tau_0}{\tau_1 \tau_0}\right) \frac{P\kappa}{\rho} > \left(\frac{1 - q(e_1)}{q'(e_1)} - \frac{1 - q(e_0)}{q'(e_0)}\right)$$

The change in the probability of detection weighted by the slope of the probability function should be less than the change in the tax rate weighted by the penalty of evading and the relative prices[2].

1.2 Case 2 (Spain): Discrete increase in the probability of detection after a certain threshold of revenue

In Spain, the Large Taxpayers Unit (LTU) of the tax authority focuses exclusively on firms with total operating revenue above 6 million euros. The LTU has more auditors per taxpayer than the rest of the tax authority, and these auditors are on average more experienced and better trained to deal with the most complex taxpayers. This LTU creates

[2] An analogous condition for an increase in relative prices leading to higher levels of tax evasion exists. Under this condition, the model is consistent with the literature that macroeconomic downturns lead to higher evasion.

a discontinuity in the monitoring effort of the tax authority. Consequently, at this arbitrary revenue level, the probability of detection increases discretely (?).

In this scenario, depending on the productivity shock, the firm might be better off choosing not to produce past the revenue threshold. Indeed, for a relevant range of productivity draws $\Omega^B = [\omega^L, \omega^H]$, the firms will not choose to grow past the revenue threshold if the expected after-tax profits of staying small are greater than the expected after-tax profits of growing.

In the model, there is now a threshold of revenue θ^L after which the probability of detection increases discretely. To make things simpler, assume that before the threshold, the probability changes as a function of evasion but does not vary conditional on size. After the threshold, the probability increases for every level of evasion but does not vary conditional on size.

Formally, let $\Theta_L = \{\theta_i : \theta_i < \theta^L\}$ and $\Theta_H = \{\theta_i : \theta_i \geq \theta^L\}$, then for all e_0 and $\theta_i' \neq \theta_i$, $q(e_0|\theta_i \in \Theta_k) = q(e_0|\theta_i' \in \Theta_k)$ with $k = \{L, H\}$, but $q(e_0|\theta_i' \in \Theta_H) \geq q(e_0|\theta_i \in \Theta_L)$.

Firms' revenue with productivity draw ω^L corresponds exactly to the enforcement threshold θ^L. Production and reporting decisions of firms with productivity draws below ω^L are not affected by the change in the probability of detection. Firms choose their inputs according to Equation **??** and their evasion decision according to Equation **??**. Firms with productivity draws above ω^U

Firms with productivity $\omega_i \in \Omega^B$ will choose the input level $\tilde{M}_i$ resulting in an expected revenue below the threshold $\theta_i < \theta^L$, if the expected after-tax profit of staying small are greater than growing, $\mathbb{E}[\pi_i|\Theta_L, \Omega^B] - \mathbb{E}[\pi_i|\Theta_H, \Omega^B] \geq 0$.

The optimal input choice M_i^* for firms with productivity $\omega_i \in \Omega^B$ implies an expected revenue greater than or equal to the threshold $\theta_i^* \geq \theta^L$. Let the expected profits given M_i^* and the optimal tax evasion in the range of size θ_l, e_{it}^*, is $\pi_l \equiv \mathbb{E}[\pi(M_{it}^*, e_{it}^*)|\theta_l]$. Let $\tilde{M}_{it}$ be the input level such that the expected revenue is below the threshold $\tilde{s}_{it} < \theta^L$ and $\tilde{e}_{it}$ be the optimal tax evasion in the range of size θ_s. Let also the expected profits of staying small are $\pi_s \equiv \mathbb{E}[\pi(\tilde{M}_{it}, \tilde{e}_{it})|\theta_s]$.

In this second case, therefore, firms might optimally choose to remain small if, for a low productivity shock, the expected profits of not growing are greater than the expected profits of growing $\pi_l < \pi_s$. Firms choosing to remain small will lead to a bunching below the threshold in the size distribution of firms.

Besides the higher levels of evasion before the threshold —simply because of the higher probability of detection—, we can also expect bunching firms to evade more than their similar-sized peers. At $\tilde{M}_{it}$, the optimization condition of Equation **??** no longer holds, hence, the marginal product of the input is now greater than the relative prices. Therefore, according to Equation **??**, bunching firms would compensate for their *higher* costs by increasing overreporting.

1.2.1 Discussion

What is new in this paper relative to the literature is that it focuses on the production function framework using public data whereas **?** and other papers use a bunching estimator with government administrative data which is difficult to access. Second, the paper focuses on input overreporting rather than on revenue underreporting, which is the relevant margin of evasion for manufacturing firms. More on this point in the revenue underreporting section. Finally, in contrast to **?** where the authors conclude that misreporting does not imply real losses in production but only fictitious reduction of the real sales, firms might optimally forgo higher revenue levels if the expected profits of staying small and evade taxes by misreporting are greater than the expected profits of growing and avoid misreporting.

1.3 Case 3 (Colombia & Mexico): Discrete increase in the tax rate after a revenue threshold

1.3.1 Colombia, Individual Proprietorships

In Colombia between 1981 and 1991, individual firm proprietors were subject to the individual income tax schedule. Individuals had incentives to not form juridical organizations to avoid double taxation. The tax authority suffered from severe limitations and inefficiencies at the time.

In this case, after the revenue threshold, the tax rate increases discretely but the probability of detection does not. The jump in the tax rate generates the incentive to increase evasion. However, a higher level of evasion increases the cost of evading by increasing the probability of detection. If the cost of an increased evasion outweighs the benefits of growing past the revenue threshold, the firms would bunch below the cutoff.

1.3.2 Mexico, Irreversible Change in Tax Regime after a Revenue Threshold

In Mexico, firms with annual revenues below 2 million pesos are taxed under the REPECO (*Regime de Pequeños Contribuyentes*) regime of small contributors at 2 percent of annual revenues, while firms above that threshold are taxed under the general regime at 30 percent. Firms must transition to the general regime if revenues increase beyond the threshold. Once in the general regime, firms cannot revert to the REPECO regime.

Firms' decision is now dynamic. Firms will maximize the sum of current and future after-