

Tax Evasion and Productivity

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Abstract

Corporate tax evasion through cost overreporting spreads internationally causing governments significant tax revenue losses. Detecting and measuring the magnitude of tax evasion remains a challenge, even for the few studies on overreporting where researchers can exploit administrative data. Moreover, if this evasion strategy accounts for economic losses as large as reported, then cost overreporting might bias estimates of production functions, especially productivity. This paper addresses both issues. I first provide a novel strategy to estimate cost overreporting using commonly available firm-level data. I then formally show that ignoring cost overreporting leads to downward biased productivity estimates. Finally, I demonstrate how to recover productivity in the presence of tax evasion.

Keywords: Tax Evasion, Cost Overreporting, Production Function Estimation, Productivity

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TODO

- Correct tests:
 - Use corps to get beta, test on non-corps
 - Use corps to get beta, test on all
 - Bootstrap all (Use bayesian bootstrap)
- Why are there true negatives?
 - Only two industries: 323 Leather and Leather Products and 356 Plastic Products
- Other Data: Ecuador, (Spain?)
- Summarize main points of Lit in Slides

Updates

- Tax Evasion and Productivity: Using PF to identify tax evasion through input overreporting
 - CD
 - * Preliminary results of Tax Evasion: Moments, MLE (Truncated Normal and LogNormal)
 - * Identification of PF parameters when h AR(1)
 - * **New** Discussion: Variance of the error term and Choosing intermediates in GNR-CD.
 - * **New** Preliminary results of PF parameters
 - * **New** Productivity results
 - TransLog
 - * Identification strategy for tax evasion

- Leveraging Tax Policy change to identify changes in tax evasion
 - Why? Relax common technology assumption
 - Triple difference identification strategy (with CD)

Next Steps

- Tax Evasion and Productivity
 - Back of Envelope estimates for Gov Losses in Tax Revenue
 - App of Productivity: Aggregate Productivity and Resource Misallocations
 - Translog Tax Evasion results
 - Translog PF identification strategy
 - Translog PF results
 - Translog Productivity results
 - Using ELVIS to allow for more flexible functional forms of h the markov process of productivity
 - Non-parametric Deconvolution
 - Non-parametric PF
- DDD
 - Relaxing parallel trends: Linear trends that in the absence of policy change would have continued

1 A parsimonious model of tax evasion through input overreporting

Price-taking firms maximize expected after-tax profits. Firms choose the flexible input M_{it} to produce output Y_{it} given output and input prices $\{P_t, \rho_t\}$, a common technology, the production function (Equation ??), and their productivity ω_{it} .

$$Y_{it} = G(M_{it}) \exp(\omega_{it} + \varepsilon_{it}) \quad (1)$$

As standard in the literature, productivity ω_{it} is known to firms when they make input decisions. This is the well-known endogeneity problem of simultaneity. On the other hand, firms face output shocks. The output shock ε_{it} is not part of the firms' information set.

The model departs from the literature by allowing firms to overreport their inputs e_{it} to reduce their tax burden and optimize after-tax profits. Firms, then, consider in their optimization problem the profit tax τ , the evasion penalty/cost $\kappa(e)$, and the probability of detection $q(e_{it}|\theta_{it})$.

Firms solve Equation ??

$$\begin{aligned} \max_{M_{it}, e_{it} \in [0, \infty)} & [1 - q(e_{it}|\theta_{it})] [(P_t \mathbb{E}[Y_{it}] - \rho_t M_{it}) - \tau (P_t \mathbb{E}[Y_{it}] - \rho_t (M_{it} + e_{it}))] \\ & + q(e_{it}|\theta_{it}) [(1 - \tau)(P_t \mathbb{E}[Y_{it}] - \rho_t M_{it}) - \kappa(e)] \\ \text{s.t. } & Y_{it} = G(M_{it}) \exp(\omega_{it} + \varepsilon_{it}) \end{aligned} \quad (2)$$

The probability of detection $q(e_{it}|\theta_{it})$ is monotonically increasing in the amount evaded e_{it} , conditional on the type of the firm θ_{it} . Intuitively, for a given type, firms that evade more are more likely to get caught.

The type of the firm θ_{it} might be discrete, like the type of juridical organization, or continuous, like the level of revenue¹. Some types might be more likely to be detected if the firm engages in tax evasion. For example, in contrast to other types of juridical organizations in Colombia, corporations are closely supervised and are required to have an auditor. That is, for a given level of tax evasion e_0 and two different types $\theta' \neq \theta \in \Theta$, then $q(e_0|\theta') \geq q(e_0|\theta)$.

If the type θ is continuous, it might be a function of inputs; for example, level of revenue. Firms will then affect their probability of detection $q(e|\theta)$ in two ways: directly, by choosing how much they evade e ; and indirectly, when choosing inputs M .

The optimal decision of the firm will depend on the fiscal environment $\Gamma = \{\tau, \kappa, q\}$, namely the tax rates, the penalty/cost of detection, and the probability of detection.

The firms' problem (Equation ??) can be rewritten as follows,

$$\begin{aligned} \max_{M_{it}, e_{it}} \mathbb{E}[\pi_{it}|\Gamma] = & (1 - \tau) \left(\mathbb{E}[Y_{it}] - \frac{\rho_t}{P_t} M_{it} \right) + [1 - q(e_{it}|\theta_{it})] \left(\frac{\rho_t}{P_t} e_{it} \tau \right) - q(e_{it}|\theta_{it}) \kappa(e_{it}) \\ \text{s.t. } & Y_{it} = G(M_{it}) \exp(\omega_{it} + \varepsilon_{it}) \end{aligned} \quad (3)$$

Intuitively, if the firm overreports her inputs' cost, she will get the share of the value she overreported with probability $(1 - q)$ and she will be penalized with probability q .

Assuming well-behaved functions and no corner solutions, the first-order conditions lead to the following system of differential equations,

$$G_M(M_{it}) \exp(\omega_{it}) \mathcal{E} - \frac{\rho_t}{P_t} = \frac{1}{(1 - \tau)} \frac{\partial q(e_{it}|\theta_{it})}{\partial \theta_{it}} \frac{\partial \theta_{it}}{\partial M} \left[\frac{\rho_t}{P_t} e_{it} \tau + \kappa(e_{it}) \right] \quad (4)$$

¹Level of revenue is a common measure for fiscal authorities to determine a firm's taxes and/or level of scrutiny, e.g., Mexico, Spain, Colombia, and Ecuador.

$$[1 - q(e_{it}|\theta_{it})]\frac{\rho_t}{P_t}\tau - q(e_{it}|\theta_{it})\kappa'(e_{it}) = q'(e_{it}|\theta_{it})\left[\frac{\rho_t}{P_t}\tau e_{it} + \kappa(e_{it})\right] \quad (5)$$

where $E = \mathbb{E}[\exp(\varepsilon_{it})]$. The type of firms is continuous and increasing on the input. The probability of detection is increasing in the type continuum. In particular, $\frac{\partial q(e_{it}|\theta_{it})}{\partial \theta_{it}} \frac{\partial \theta_{it}}{\partial M} \geq 0$.

The left-hand side of Equation ?? is the familiar marginal output of inputs and the price ratio. In the absence of incentives' distortions induced by the fiscal environment, they are equal. But now, the equality holds no more. There's a wedge arising from the fiscal environment. The right-hand side of the equation is positive by the assumptions of the model.

Equation ?? solves the optimal evasion decision. The left-hand side is the marginal benefit net of the marginal cost of evasion. The right-hand side is the rate of change of the probability of detection due to a change in evasion weighted by the benefit and cost of evading.

1.1 Case 1 (Independence): $q(e|\theta) = q(e)$ and $\kappa(e) = \kappa_0$

Consider the case when the probability of detection is independent of type, $q(e|\theta) = q(e)$. This could be the case if the type is the juridical organization of the firm. Hence, the type of the firm, and thus the probability of detection, does not change with the firm's input decisions, $\frac{\partial q(e_{it}|\theta_{it})}{\partial \theta_{it}} \frac{\partial \theta_{it}}{\partial M} = 0$. In addition, assume the evasion cost is constant, $\kappa(e) = \kappa_0$, for simplicity.

In this case, the first-order conditions of Equation ?? with respect to the input M_{it} and the tax evasion e_{it} yield the following

$$G_M(M_{it}) \exp(\omega_{it}) \mathcal{E} = \frac{\rho_t}{P_t} \quad (6)$$

$$e_{it} = \frac{1 - q(e_{it})}{q'(e_{it})} - \frac{\kappa_0}{\frac{\rho_t}{P_t} \tau} \quad (7)$$

Equation ??, the well-known optimality condition, says that the price ratio is equal to the marginal product of the inputs.

Likewise, Equation ?? reveals the firms' optimal tax evasion decision decreases if the probability of detection $q(e_{it})$ or the penalty of evading κ increases. Tax evasion also depends on how sensitive the probability of detection is to the level of evasion $q'(e)$. In particular, greater sensibility will result in lower levels of evasion.

Note that the net change of tax evasion due to an increase in the relative prices $\frac{\rho_t}{P_t}$ or the tax rate τ is not evident at first sight. The net effect will also depend on the change in the detection probability induced by the changes in the relative prices or the tax rate. In particular, an increase in relative prices $\frac{\rho_t}{P_t}$ or the tax rate τ will incentivize a higher tax evasion level, however, a higher tax evasion level will increase the probability of detection—depending on the shape of the probability as a function of e —, so it will deter higher levels of evasion. An increase in the tax rate, for instance, will only increase tax evasion if the change in the tax rates increases the incentives to evade more than the decrease in the incentives due to the changes in the detection probability.

Formally, suppose a firm increases its tax evasion, $e_1 - e_0 > 0$ because of an increase in taxes $\tau_1 > \tau_0$. Then, it follows that

$$\left(\frac{\tau_1 - \tau_0}{\tau_1 \tau_0} \right) \frac{P\kappa}{\rho} > \left(\frac{1 - q(e_1)}{q'(e_1)} - \frac{1 - q(e_0)}{q'(e_0)} \right)$$

The change in the probability of detection weighted by the slope of the probability function should be less than the change in the tax rate weighted by the penalty of evading and the relative prices².

1.2 Case 2 (Spain): Discrete increase in the probability of detection after a certain threshold of revenue

In Spain, the Large Taxpayers Unit (LTU) of the tax authority focuses exclusively on firms with total operating revenue above 6 million euros. The LTU has more auditors per taxpayer than the rest of the tax authority, and these auditors are on average more experienced and better trained to deal with the most complex taxpayers. This LTU creates a discontinuity in the monitoring effort of the tax authority. Consequently, at this arbitrary revenue level, the probability of detection increases discretely (?).

In this scenario, depending on the productivity shock, the firm might be better off choosing not to produce past the revenue threshold. Indeed, for a relevant range of productivity draws $\Omega^B = [\omega^L, \omega^H]$, the firms will not choose to grow past the revenue threshold if the expected after-tax profits of staying small are greater than the expected after-tax profits of growing.

In the model, there is now a threshold of revenue θ^L after which the probability of detection increases discretely. To make things simpler, assume that before the threshold, the prob-

²An analogous condition for an increase in relative prices leading to higher levels of tax evasion exists. Under this condition, the model is consistent with the literature that macroeconomic downturns lead to higher evasion.

ability changes as a function of evasion but does not vary conditional on size. After the threshold, the probability increases for every level of evasion but does not vary conditional on size.

Formally, let $\Theta_L = \{\theta_i : \theta_i < \theta^L\}$ and $\Theta_H = \{\theta_i : \theta_i \geq \theta^L\}$, then for all e_0 and $\theta'_i \neq \theta_i$, $q(e_0|\theta_i \in \Theta_k) = q(e_0|\theta'_i \in \Theta_k)$ with $k = \{L, H\}$, but $q(e_0|\theta'_i \in \Theta_H) \geq q(e_0|\theta_i \in \Theta_L)$.

Firms' revenue with productivity draw ω^L corresponds exactly to the enforcement threshold θ^L . Production and reporting decisions of firms with productivity draws below ω^L are not affected by the change in the probability of detection. Firms choose their inputs according to Equation ?? and their evasion decision according to Equation ?. Firms with productivity draws above ω^U

Firms with productivity $\omega_i \in \Omega^B$ will choose the input level \tilde{M}_i resulting in an expected revenue below the threshold $\theta_i < \theta^L$, if the expected after-tax profit of staying small are greater than growing, $\mathbb{E}[\pi_i|\Theta_L, \Omega^B] - \mathbb{E}[\pi_i|\Theta_H, \Omega^B] \geq 0$.

The optimal input choice M_i^* for firms with productivity $\omega_i \in \Omega^B$ implies an expected revenue greater than or equal to the threshold $\theta_i^* \geq \theta^L$. Let the expected profits given M_i^* and the optimal tax evasion in the range of size θ_l , e_{it}^* , is $\pi_l \equiv \mathbb{E}[\pi(M_{it}^*, e_{it}^*)|\theta_l]$. Let \tilde{M}_{it} be the input level such that the expected revenue is below the threshold $\tilde{s}_{it} < \theta^L$ and \tilde{e}_{it} be the optimal tax evasion in the range of size θ_s . Let also the expected profits of staying small are $\pi_s \equiv \mathbb{E}[\pi(\tilde{M}_{it}, \tilde{e}_{it})|\theta_s]$.

In this second case, therefore, firms might optimally choose to remain small if, for a low productivity shock, the expected profits of not growing are greater than the expected profits of growing $\pi_l < \pi_s$. Firms choosing to remain small will lead to a bunching below the threshold in the size distribution of firms.

Besides the higher levels of evasion before the threshold —simply because of the higher probability of detection—, we can also expect bunching firms to evade more than their similar-sized peers. At \tilde{M}_{it} , the optimization condition of Equation ?? no longer holds, hence, the marginal product of the input is now greater than the relative prices. Therefore, according to Equation ??, bunching firms would compensate for their *higher* costs by increasing overreporting.

1.2.1 Discussion

What is new in this paper relative to the literature is that it focuses on the production function framework using public data whereas ? and other papers use a bunching estimator with government administrative data which is difficult to access. Second, the paper focuses on input overreporting rather than on revenue underreporting, which is the relevant margin of evasion for manufacturing firms. More on this point in the revenue underreporting section. Finally, in contrast to ? where the authors conclude that misreporting does not imply real losses in production but only fictitious reduction of the real sales, firms might optimally forgo higher revenue levels if the expected profits of staying small and evade taxes by misreporting are greater than the expected profits of growing and avoid misreporting.

1.3 Case 3 (Colombia & Mexico): Discrete increase in the tax rate after a revenue threshold

1.3.1 Colombia, Individual Proprietorships

In Colombia between 1981 and 1991, individual firm proprietors were subject to the individual income tax schedule. Individuals had incentives to not form juridical organizations to avoid double taxation. The tax authority suffered from severe limitations and inefficiencies at the

time.

In this case, after the revenue threshold, the tax rate increases discretely but the probability of detection does not. The jump in the tax rate generates the incentive to increase evasion. However, a higher level of evasion increases the cost of evading by increasing the probability of detection. If the cost of an increased evasion outweighs the benefits of growing past the revenue threshold, the firms would bunch below the cutoff.

1.3.2 Mexico, Irreversible Change in Tax Regime after a Revenue Threshold

In Mexico, firms with annual revenues below 2 million pesos are taxed under the REPECO (*Regime de Pequeños Contribuyentes*) regime of small contributors at 2 percent of annual revenues, while firms above that threshold are taxed under the general regime at 30 percent. Firms must transition to the general regime if revenues increase beyond the threshold. Once in the general regime, firms cannot revert to the REPECO regime.

Firms' decision is now dynamic. Firms will maximize the sum of current and future after-tax profits. The discrete jump in the tax rate will lead to a bunching below the threshold. Moreover, the bunching will be exacerbated because firms will choose to grow past the cutoff only if the future productivity shocks allow the firm to continue to be profitable.

1.4 Case 4 (Colombia): Firms first choose type, input decisions do not affect the probability of detection

In Colombia between 1981 and 1991, Corporations were closely supervised by the Superintendent of Corporations and were required to have an auditor. All other firms were subject to the regular monitoring efforts of the tax authority, which suffered from severe limitations and inefficiencies at the time.

In the model, firms first choose their type. Input decisions do not affect the probability of detection. However, if the type is *Corporation* the probability of detection is higher than *Partnership*. Firms maximize the sum of their expected profits. In their optimization problem, firms will consider the sum of expected productivity shocks and their corresponding probability of detection. High-productivity firms will self-select into *Corporations*.

2 Lit Review Notes

For personal reference, to be removed from the final version.

2.1 ?

Productivity improvements by firms can causally lead to lower tax evasion, i.e., underreporting sales for tax purposes. In other words, an increase in firm productivity leads to a higher share of sales reported for tax purposes. The authors use the self-reported share of declared income as a proxy for tax evasion.

Limitations: Using the World Bank Enterprise (47 emerging and developing countries) survey question to get the percentage of sales that are reported for tax purposes, which is used to measure tax evasion. In addition, productivity is measured as the log of sales per worker. No access to quantities data, they use LP but with sales and workers.

Undisputed (?) facts: 1) Tax evasion is higher in poor countries; 2) Tax evading firms tend to be less productive (?).

Productivity gaps between firms that comply with taxes and regulations and those which do not are significant, (between 25 and 50 %) (Amin et al. 2019; Fajnzylber, 2011; Busso et al., 2012). These large productivity gaps can translate into low-economy wide productivity and

growth: 1) if, by not complying with existing taxes and regulations, firms enjoy a potentially large implicit subsidy that allows them to stay in business despite low productivity, or 2) to expand their market share at the expense of more productive firms (Farrell, 2004; Bobbio, 2016).

Almeida and Carneiro (2009): Tax evasion may also increase firms' efficiency (productivity?). Avoiding onerous taxes might confer firms more flexibility in their employment and production decisions.

Kleven et. al (2016): Larger firms are less likely to avoid taxes. The firm's use of business records increases as it hires more employees. They show that the government may be able to enforce higher tax compliance even in the presence of a low threat of audit due to the increased ease of whistleblowing as the firm grows.

Tax evasion is hidden: it is difficult to obtain precise measures (Kundt et al., 2017; Slemrod and Weber, 2012).

Loayza and Rigolini (2006) and La Porta and Shleifer (2014): informality is larger in lower GDPpC countries

Tax evasion and informality are associated with lower income levels and productivity (Dabla-Norris and Feltenstein, 2003; Loayza, 1996; Sarte, 2000)

2.2 ?

The author models discrepancies between tax records and income survey data as individual tax evasion and measurement error jointly. Using data for Estonia, the author finds that people in the bottom and the top part of earnings distribution evade much more and about 12% of wages and salaries in total are underreported. The identifying assumption, i.e.,

location condition, is that people working in the public sector cannot evade taxes. In other words, measurement error is unrelated to the sector where the individual works while the same does not hold for tax compliance.

The main constraint for empirical research on tax evasion is the lack of suitable data. This is worse for developing countries. A measure of undeclared income for individuals is needed. This kind of data is usually unreliable and very difficult/or expensive to obtain.

Baldini et al. (2009) use linked income data with tax records at the individual level to estimate tax evasion. However, the author assumes people report their true income in the survey to obtain a measure of non-reporting. Furthermore, in the measurement error literature, several studies assume the opposite, e.g. Krueger (1991) and Bollinger(1998); survey of lit Chen et al. (2011) and Bound et al. (2001). Linked data is referred to as Validation data in the measurement error literature. Later studies have started to consider matching errors (Kapteyn and Ypma, 2007; Meijer et al., 2012) and errors in register data (Abowd and Stinson, 2013).

Pissarides and Weber (1989) pioneered a methodology to study underreported income. They assume that the underreported income for one population group is inferred from a comparison with a reference group assumed to have negligible non-compliance but to be similar in other respects. These types of studies assume implicitly that the underreporting of income in a survey corresponds to the underreporting of income to the tax authority.

2.3 ?

Tax evasion policies have been a greater priority in developed countries since the 2008 Great Recession (IMF, 2015). For example, the UK announced efforts to combat tax avoidance, tax evasion, and ‘imbalances’ in the tax system that would bring in £5 billion in additional

revenue each year.

Tax evasion matters because it affects both the resource cost of raising taxes and the distribution of the tax burden, i.e. core interests of public economics. Policy question? Does curbing evasion improve the equity and efficiency of the public finances, given the cost of doing so? If so, what's the best way to go about it?

Tax evasion creates horizontal inequity because people evade taxes by different proportionate amounts because of differences in personal characteristics —risk aversion, tax system, and honesty— and because of different opportunities and potential rewards for evasion. Likewise, efforts to combat tax evasion might create vertical inequity. For example, going after lower-income households such as the IRS fighting fraud related to the earned income tax credit (EITC) than going after unreported foreign accounts that are more likely to be held by high-income households.

Tax evasion creates efficiency costs, e.g., the individual's cost of implementing and hiding non-compliance, the tax authority's cost of combating non-compliance, etc. Tax evasion also creates socially inefficient incentives to engage in activities that facilitate tax evasion. For example, tax-noncompliance opportunities attract people to self-employment who would otherwise be employees.

Tax evasion creates misallocations. For example, it can cause companies that otherwise would not find it attractive to do so to set up operations in a tax haven to facilitate or camouflage evasion.

Curtailing tax evasion cost shapes optimal policy —it is not optimal to completely eradicate tax evasion because of the cost of doing so. The presence of evasion also alters the optimal setting of tax rates —because it affects the marginal efficiency cost of so doing— and the

choice of the tax base —because different tax bases are more or less susceptible to evasion.

Slemrod and Weber (2012) review macroeconomic measures of the informal economy.

The standard tax evasion framework is the model of tax deterrence by Allingham and Sandmo (1972), who adapted Becker's (1968) model of criminal behavior to tax evasion. A relatively recent simplified framework is where the maximand is $y(1 - t) + te - c(e, \alpha)$, y is income, e is the understated tax liability, t is a linear tax rate, and c is the private cost of evasion which includes the utility cost of bearing risk and the expected value of punishment. The private cost may depend on the amount of attempted evasion and α , the vector of enforcement instruments, such as the extent of auditing.

The model predicts that a risk-averse agent will evade taxes if the expected return is positive. Additionally, an increase in either the probability of detection or the penalty if detected will reduce evasion. Importantly, the effect of the tax rate is less clear, for even its sign depends on the penalty function (Yitzhaki, 1974).

What matters about p is the perception of taxpayers. Much work is focused on what alters the taxpayer's perception of the enforcement environment.

This framework naturally applies to individuals, and small, single-owned firms. Large firms might be risk-neutral rather than risk-averse.

The challenge of measuring tax evasion is the incentive of evaders to conceal their behavior. Many taxpayers are unwilling to respond accurately maybe because of the threat of punishment or social shame. For this reason, even the most credible empirical studies of evasion don't actually have a reliable measure of evasion.

Tax gap studies by auditing (real differences between countries' tax gaps or differences in methodologies?):

-U.S. Overall gross tax-gap average over 2008-10 was \$458 billion USD, 18.3% of estimated tax liability (IRS, 2016). IRS National Research Program (NRP).

-UK: overall tax gap from 2013-2014 6.4% of true liability: 6.4% for the corporation income tax 5.0% individual income tax, and 11.1% value-added tax (VAT). Small- and medium-sized enterprises account for over half of the overall tax gap (HMRC, 2015). Her Majesty's Revenue and Customs (HMRC, 2015) Toro et al. (2013).

-Denmark's overall tax gap is only 2.2 percent of net income (Kleven et al., 2011).

-OECD(2015) other countries.

Sharp variations in the compliance rate by the extent of information reporting to the tax authority provide compelling evidence that the probability of successful evasion play a key role in the deterrence of tax evasion. For example, in the US, the estimated non-compliance rate when there is little to no third-party-reported information (e.g., self-employment income) is 63%, compared to 19% and 7% for 'some' and 'substantial' information reporting, respectively. Small businesses represent a large portion of the tax gap in individual income with approximately 47% of underreporting. In contrast, only 1 % of non-compliance is estimated for employee wages and salaries —substantial information reporting. Kleven et al. (2011) find a similar disparity, but lower, non-compliance rates in Denmark, where 14.9 % for self-employment income but less than 1% for third-party reported items. In other words, when the information reporting renders the likelihood of evading taxes successfully near zero, evasion is eradicated; when the likelihood is higher, evasion proliferates.

Administrative data: Canada, UK, Brazil, Chile, China, Costa Rica, Ecuador, India, Pakistan, Rwanda, Tunisia, and Uganda. [US access](#)

RCT's review: Hallsworth (2014)

Bérgolo et al. (2018) RCT with firms using letters. Value-added-tax compliance, 20,000 Uruguayan small and medium-sized firms. The authors find that adding to the baseline letter a paragraph with statistics about the probability of being audited and the penalty rates increased tax compliance by about 6.3 percent (about one-fourth of its current level), and adding a paragraph that informs firms that evading taxes increased the probability of being audited increased tax compliance by about 7.4 percent.

Ortega and Scartascini (2018) Colombia, individuals. Method of communication. The authors find sizable differences across delivery methods. Personal visits by a tax inspector are more effective than the impersonal methods; they are, alas, also much more expensive. In a follow-up study, Ortega and Scartascini (2015) find that the effect of phone calls falls between those of the impersonal methods and the personal visits.

Almunia and Lopez-Rodriguez (2018) study the behavior of Spanish firms in response to a notch in enforcement intensity that arises because the Spanish Large Taxpayer's Unit (LTU) monitors firms with revenues above €6 million for all major taxes. Sure enough, there is significant bunching of firms just below the threshold. The bunching is more pronounced for firms that produce intermediate goods, which makes sense because their transactions create more of a paper trail than firms that sell to final consumers, and thus the discontinuous increase in enforcement intensity affects these firms more than retailers.

Examining data from Ecuador, Carrillo, Emran, and Rivadeneira (2012) find evidence of bunching in reported tax liability just above the withholding threshold, suggesting that some firms manipulate their self-reported tax liability and possibly real economic choices to minimize expected tax payments subject to the discontinuity in the audit probability. Third-party data on sales and intermediate input costs reported by large firms who act as withholding agents indicate that bunching is indeed associated with tax evasion: self-reported

sales are smaller than third-party reports for more than 10 percent of firms. “withholding”, large firms withhold and remit to the tax authority a fixed share of their purchases from small firms (and individuals), who can then apply the withheld amount as a credit against their self-reported tax liability. While withholding does not change the firms’ true tax liability, there is typically a discontinuity in the audit probability at the withholding rate; firms seeking tax refunds (because self-reported tax liability is lower than the withheld amount) are audited at a higher rate than firms making additional tax remittances.

In Chile, firms can only claim tax credits for inputs bought from tax-compliant suppliers, the invoice–credit VAT system has a built-in (albeit imperfect) self-enforcement mechanism for firm-to-firm transactions. Pomeranz (2015) tests this hypothesis by mailing increased-audit-threat letters to over 100,000 randomly selected Chilean firms, using a sample of over 300,000 firms receiving no letter as the control group. Consistent with theoretical predictions on the self-enforcement mechanism, the increase in VAT receipts (and therefore the inferred level of evasion) induced by the letters is concentrated at the level of sales from firms to final consumers, for which there is no paper trail.

In Ecuador, Carrillo, Pomeranz, and Singhal (2017) examine the effect of a change in the tax authority’s use of third-party information on reported corporate tax revenues. In the authors’ study, the Ecuadorian tax authority informed some firms of the discrepancy between the two reports and offered them the opportunity to file an amended return. The authors compare the reporting behavior of firms before and after the notification and find that 24 percent of firms underreported revenue in the years when the government did not use the third-party-verified information. In the three rounds of the experiment, 11–19 percent of notified firms filed an amended return.

In amended returns, firms correctly reported their revenues but also increased their reported

costs almost one-for-one with the increase in reported revenues (96 cents for each dollar!), nearly eliminating any impact on apparent tax liability. This study also reveals that reported costs were lower than third-party information on costs, which at first blush seems to be at odds with a model of firms that seek to maximize after-tax profits. The authors propose that this behavior is consistent with firms who may believe that the probability of an audit is a function of firm size and profits. To appear small, and lower the odds of being audited, some firms may underreport both revenues and costs.

Kumler, Verhoogen, and Frías (2013) study the effects on evasion of a 1997 pension reform in Mexico that tied younger workers' retirement benefits more closely to the wage reported by employers. This reform provided a new incentive for this group of workers to ensure that their employers accurately reported their wages, which in turn might lower payroll tax evasion by firms. To examine the impact of this initiative, the authors combine two sources of data on wages: administrative data from the Mexican Social Security Institute (IMSS) and household survey data from the Encuesta Nacional de Empleo Urbano (ENEU). The authors find that, consistent with a decline in evasion, the gap in median or mean wage within a cell between the ENEU data and the IMSS data fell for younger age groups after the pension reform but, as predicted, for older workers not affected by this reform there was no decrease in the gap between the two income reports.

In Pakistan, corporations either pay a tax on profits or turnover, depending on which liability is greater. This effectively implies that at a profit rate lower than the ratio of the turnover tax rate to the profit tax rate, firms cannot deduct costs. Because in Pakistan a large portion of evasion is through misreporting of costs, this tax regime trades off loss in production efficiency (as firms move from a neutral profit tax to a distorting output tax regime) for a gain in revenue collection. Best et al. (2015) use administrative data on the

universe of corporations in Pakistan to estimate the elasticity of taxable income using the bunching of firms below the threshold profit rate. They find clear evidence of such bunching, whose location shifts along with changes in tax rates that move the threshold. Using the analysis-of-bunching methodology, they estimate that turnover taxes reduce evasion by between 60–70 percent.

Relevance of tax evasion Widespreadness of tax evasion Seminal model of tax evasion Real effects of tax evasion firm-to-firm sales less likely to evade taxes small firms more likely to evade taxes evidence of misreporting of costs (Pakistan) Chile and Colombia tax evasion evidence

2.4 ?

The authors study tax evasion through the use of ghost firms — fake firms that issue fraudulent receipts so that their clients can claim false deductions. Using transaction-level data from Ecuador, the research documents that ghost transactions are widespread; prevalent among large firms —(!) measured as revenue level [related to size]— and firms with high-income owners —owners with experience in tax evasion? Sophisticated or less risk-averse owners?—; and exhibit suspicious patterns: bunching at the end of the fiscal year [tax evasion decision is realized post realization of productivity and true input optimization], at round numbers, and just below financial system thresholds [no real economic activity behind].

Globally widespread but extremely limited researched problem. “Ghost firms” —also known as “invoice mills”, or “missing traders”— issue fraudulent receipts that allow the receivers to claim additional tax deductions on value-added (VAT) and corporate income taxes (OECD, 2017; Keen and Smith, 2006) Appendix B.

Data: firm-to-firm transaction-level data with 800 ‘ghost firms’ and firm owners of Ecuadorian firms data.

Widespread and quantitatively important, not limited to small, semiformal firms, benefits those at the top of the income distribution. In 2015, 4.7% of potential clients took deductions from a ghost firm. Annual ghost deductions were 14.1% of the value of their purchase deductions [huge number!]. Larger firms are more likely to engage in ghost transactions and feature larger shares of ghost transactions in total input costs [a function of size]. Ghost deductions are disproportionately used by firms owned by high-income individuals.

Process: Given a list of identified ghost firms (2016) by the Ecuadorian Internal Revenue Service (SRI), the authors identified ghosts’ clients —firms with at least one purchase from ghosts firms— with administrative data -which includes annual corporate and individual income tax filings in Ecuador (2010-2017) and monthly purchase annexes (2010-2015). The sample is restricted to firms that are required to file purchase annexes: incorporated firms (88%) and large sole proprietorships (12%) (i.e., annual sales > \$100,000, annual costs > \$80,000, or capital above \$60,000)

1. Tax deductions based on fake receipts from ghost firms are widespread and large. 10.4% of unique firms filed deductions based on receipts from at least one identified ghost firm. On average, 3.6% of all purchases of ghost clients are from ghosts, amounting to 10.4% of the value of their purchase deductions. These shares are the highest, 4.6% and 14.1%, for 2015, the year before the ghost list was collected. Earlier ghosts might have disappeared by the time the list was established. Total tax evasion was \$2.1 billion in value, 1.7% of tax evaded for corporations and 11.5% for sole proprietorships.
2. Evasion through ghost firms is more prevalent among relatively large firms. Ghost clients have higher revenues, costs, and tax liabilities. The probability of engaging in

cost overreporting through ghost firms increases monotonically in firm revenue —maybe only because of more transactions. Share of ghost deductions also increases throughout much of the size distribution, except at the very top —very large corporations might have stronger incentives to avoid illegal behavior or they can use more sophisticated avenues of tax avoidance that do not require evasion using fake receipts (Bustos et al., 2022)

3. Ghost deductions are most prevalent in firms owned by high-income individuals. The probability of having ownership of a ghost client and the amount of ghost purchases attributed to individuals increases with individuals' income. The amount of additional tax as a share of owners' income is over 56 times higher in the top 5% than in the bottom 80% of the income distribution, and almost 170 times higher in the top 1%. Ghost-firm-enabled tax evasion tends to reduce disproportionately the effective taxation of firms owned by rich individuals.
4. The number and value of ghost transactions are clustered towards the end of the tax year —the same as the calendar year in Ecuador. In December, there are twice as many ghost transactions as during the six previous months of the year. Non-ghost transactions are only 6% higher in December. This is consistent with firms assessing their annual revenues at the end of the year and then utilizing ghost transactions to achieve a target reported profit level or rate for tax purposes. Similar to US firms spending more on capital investment towards the end of the fiscal year to reduce tax obligations (Xu and Zwick, 2022).
5. Ghost transactions bunch at round numbers more prevalently than non-ghost transactions. This is consistent with ghost transactions representing false activity (Klimek et al, 2018; Nigrim, 2018).

6. Ghost transactions exhibit bunching below the financial system payment threshold.

In Ecuador, transactions above \$5,000 (gross-of-VAT) are required by law to be made through the formal financial sector (e.g. electronic transfers, checks, or credit cards).

Ghost transactions bunch just below \$5,000. In contrast the distribution of non-ghost transactions is smooth through the \$5,000 threshold.

Lit on tax evasion via ghost firms is extremely sparse. Waseem (2020) Pakistan. Policy: VAT rates reduced to zero. Ghost firms decreased. Most ghost deductions are claimed by exporters. Ghost deductions are a large share of over-claimed refunds for exporters. Mittal et al. (2018) develop a Machine Learning algorithm to detect ghost firms.

Most recent work on tax evasion focuses on firms' revenue misreporting (Slemrod, 2019; Pomeranz and Vila-Belda, 2019), but few have studied firms' cost misreporting. Recent firms' revenue misreporting studies have found that enforcement policies targeting revenue underreporting led to increasing reported costs—to offset tax liabilities—, but there is no offsetting behavior—reduction in reported revenue—in the study by Carrillo et al. (2022). This is consistent with third-party information reporting—firm-to-firm transaction level data is used in this study— decreasing the incentive to evade by increasing the probability of getting caught.

Other studies finding bunching at the thresholds are: - VAT reporting thresholds: Onji, 2009; Asatryan and Peichl, 2017; Luksic and Mittal, 2019; Liu et al., 2021. - Different tax regimes and regulations: Hasegawa et al., 2013; Best et al., 2015; Almunia and Lopez-Rodriguez, 2018; Lopez-Luzuriaga, 2019; Chen et al., 2021; Clifford and Mavrokonstantis, 2021; Overview: Kleven, 2016.

Intervention: Tax authority challenged deductions from firms with ghost transactions (ghost

clients) during 2010-2015 by notifying them via email.

[What goes into the cost overreporting function? Tax authority chooses effort (b(type)-c(type))e (?)] Tax authorities target firms with higher potential tax recovery. In this study, notified firms were larger than typical incorporated ghost clients —authors only had access to the data on incorporated firms that were notified. Notified firms had 2.4 times higher median tax liability, the median amount of ghost deductions of these firms was \$181,000 (mean \$338,000) and the median share of ghost deductions out of their total purchases was 26% (mean 38%). [conditional on being large, the overreporting level does affect your probability of being audited.]

[Evidence of overreporting in intermediates] Adjusting firms reduced most strongly non-labor costs: domestic purchases, other production costs, and imports. Some offsetting behavior was observed; Firms increased labor, inventory, and financial costs. However, overall total costs still decreased on average by 72 cents for every dollar contested in the notification.

Many firms did not respond to the notifications. A portion might be explained by failure in communication (e.g., letters not reaching the firm or the right person within the firm). Firms might choose not to amend knowing the tax authority has limited capacity for follow-up enforcement. Adjusting firms tend to be somewhat smaller than all the notified firms.

The intervention resulted in large reductions in firms' reported costs. The reduction in costs increased tax liabilities by \$20.6 million, 81% increase on the \$25.4 million originally filed by adjusting firms. On average, reported costs were reduced by \$229,000

Firm-to-firm transactions —third-party reporting: firm's sales to other firms can be cross-checked— are less like to underreport revenue. Firms did not adjust revenues down to offset tax liability resulting from the decrease in costs —as in other studies where the enforcement

strategies pushing firms to truthfully report their revenues led firms to make large offsetting adjustments by increasing reported costs (Asatryan and Peichl, 2017; Carrillo et al., 2017; Slemrod et al., 2017; Almunia and Lopez-Rodriguez, 2018; Mascagni et al., 2018; Naritomi, 2019; Li and Wang, 2020)

No evidence that the intervention was followed by firms going out of business or de-formalizing; consistent with ghost clients being large and established.

International prevalence of false invoicing (ghost firms) - Mexico: between 2014 and 2018, the estimated annual tax revenue lost due to ghost firms was 5 billion MXN (SAT), 0.03% of Mexico's GDP; 8,500 identified ghost firms in 2018 (Senado de la Republica, 2019). - Chile: historically, false invoices represent between 15% and 25% of total VAT evasion. In 2004, fake-invoice tax evasion was 114 billion CLP, 0.2% of Chile's GDP (Gonzalez and Velasquez, 2013; Jorrat, 2001; CIAT, 2008). - Colombia: fake-invoice tax evasion was around 2,300 billion COP, 0.2% of Colombia's GDP (Portafolio, 2019). - Poland: In 2016, fake-invoice tax evasion amounted to 103,850 million PLN, 5.6% of Poland's GDP (Poland's Minister of Finance, 2018). - Asia: South Korea (Krever, 2014) and China (Hashimzada et al., 2010; Barboza, 2013). - Africa: Kenya (Kenya Revenue Authority; Mak'Osewe, 2019); Rwanda (Mascagni et al., 2019; 2021); African Tax Administration Forum (Brain and Nwankwo, 2018).

2.5 ?

2.5.1 Colombian Tax System

The tax system of Colombia has been under constant study and revision since the 1960s.

During the early 1960s, a Fiscal Survey of Colombia (1965) led by Professor Milton Taylor

was prepared as part of the ongoing program of the Organization of American States and the Inter-American Development Bank. This study led to important administrative innovations, but few concrete substantive reforms.

In 1970, Professor Richard Bird prepared a report, influential in terms of policy, based on his experience as a resident advisor to the government of Colombia on tax and fiscal policy during the period of 1964-1966.

In 1968, Professor Richard Musgrave was commissioned by President Carlos Lleras Restrepo to appraise the tax system and make recommendations for reform with the help of Colombian and foreign experts. The resulting document, Fiscal Reform for Colombia: Final Report and Staff Papers of the Colombian Commission on Tax Reform, significantly shaped the 1974 and subsequent tax reforms. This report has also become a classic in the field of tax reform in developing countries.

In late 1974 Colombia undertook a major fiscal reform drawing heavily on the recommendations of the Musgrave Commission.

Other influential documents during this period were La Reforma Tributaria 1974 (Gillis and McLure, 1974) and a World Bank report (McLure, 1982). Finally, there exists a report on the taxation of income from business and capital (McLure, Mutti, Thuronyi, and Zodrow, 1989?)

2.5.2 Income tax

At the time of the Taylor Mission, Colombia's system of *income and complementary taxes* consisted of a basic income tax, a net wealth tax, and an excess profit tax.

In the early 1960s, the base of the Colombian income tax was quite broad. It included all

“enrichment”; it included corporate dividends and shares in the earnings of limited liability companies and partnerships.

During this period, high-income individuals could evade the individual income tax on dividends and interest on bearer securities, since a withholding tax of only 12 percent was levied on such income.

The income of individuals was taxed under a graduated schedule consisting of 56 rates, ranging from 0.50 to 51 percent. There was a complex system of personal deductions for expenditures on medical, educational, and other professional services; these deductions depended on the income and number of children of the taxpayer.

Tax incentives were provided for, among others, industrial enterprises. In addition, income tax exemptions, limitations, or reductions were granted to oil companies, certain mining companies, large-scale agricultural improvements, cattle-raisers, Colombian airlines, certain basic industries, manufacturers using products from *Paz del Rio* steel plant, and exporters other than coffee, petroleum, bananas, hides and precious metals.

Juridical persons were subject to graduated rates. Corporate rates (on *Sociedades anonimas*), which applied to all foreign entities, were 12, 24, and 36 percent. The rates applied to limited liability companies (*Sociedades de Responsabilidad Limitada*), the business forms used for many of the most important Colombian business ventures, were only 4, 8, and 12 percent, and the income of partnerships was taxed at rates of 3 and 6 percent. Whereas the income of proprietorships was taxed only in the hands of the owner under the individual tax, the income of juridical persons was taxed directly to the firm. The entire net income of limited liability companies (like that of partnerships) was taxed to the partners, whether or not distributed; by comparison, only dividends were taxed to owners of corporate shares.

The tax system of Colombia suffered from many administrative problems in 1960. A serious problem involved the interplay of an overly short statute of limitations (two years), a system of penalties and interest that encouraged false and delinquent returns, and an overburdened tax administration. In extreme cases, taxpayers could avoid taxation completely by deliberately reporting false information, and waiting for the statute of limitations to preclude correction by the fiscal authorities.

2.5.3 Net wealth tax

Since 1953, Colombia has imposed a tax on net wealth (*impuesto complementario de patrimonio*), apparently to increase the tax burden related to income from capital and to induce a more productive use of land. The net wealth tax applies to individuals but not to corporations, partnerships, and limited liability companies. Taxable wealth was that held in Colombia, net of indebtedness. There were numerous exceptions, which paralleled, for the most part, those under the income tax. For example, there were exemptions for livestock kept for breeding purposes, certain agricultural investments, assets of Colombian airlines, investments in motor vehicle assembly or manufacturing plants, investments in securities of the *Paz del Rio* steel plant, and investments in certain basic industries and certain iron and steel fabricators.

2.5.4 Excess-Profits Tax

Colombia imposed an excess-profits tax beginning in 1935; this tax was repealed in 1974.

2.5.5 The Taylor Mission Proposals

The Taylor Mission produced an impressive analysis of the distortions and inequities that flow from the disparate treatment of individuals, corporations, limited liability companies, and

partnerships. It noted particularly the case for integration of the corporate and individual income taxes. However, it concluded that such a policy, integrating corporate and individual income taxes, was not warranted at the time. McLure rationalizes this contradiction with political realism because the double taxation had been recently introduced in 1953. The Mission's recommendation focused on the interplay between the corporate income tax, two of the supplementary taxes on income, and the excess-profits tax. The Mission favored a tax increase in limited liability companies, which it acknowledges were used as "tax avoidance device".

The Mission devoted particular attention to the agricultural sector because of the potential revenue being lost and the potential role of tax policy to stimulate efficient use of rural resources. The Mission proposed a presumptive calculation of income from agriculture equal to 10 percent of the assessed value of agricultural property.

The Taylor Mission also proposed administrative changes such as withholding for wages, salaries, interests, and dividends, the requirement of advance tax payments based on estimated liabilities, self-assessment, extension of the statute of limitations, and rigorous application of penalties.

2.5.6 Musgrave Commission's Proposals

For the most part, the Taylor and Musgrave recommendations for income tax reform were broadly consistent.

The Musgrave Commission repeated the call for heavier penalties for tax evasion. The Commission also suggested strategies to improve taxation of the *hard-to-tax* group such as small traders, independent professionals, and agriculture.

In contrast to the Taylor Mission, the integration of the taxes on business and on individuals

was rejected by the Musgrave Commission. Instead, the Commission proposed that the income of corporations and limited-liability companies be subject to a unified system of taxation. The proposed system would consist of taxation at the entity level and at the individual shareholder level only on income that is distributed. Partnerships would be taxed at the entity level at a substantial lower rate, and subject to certain conditions limited-liability companies would be given the option to being taxed like partnerships; the partners' share in partnership profits would also be subject to individual taxation, whether distributed or not.

2.5.7 Sales tax and VAT

Colombia introduced a national sales tax in 1965. This was a single-stage manufacturers' tax levied at rates of 3 to 10 percent on finished domestic goods and imports. Buyers' certification that goods would undergo further processing prevented the transaction from being taxed to avoid double taxation. This system led inevitably to double taxation and evasion. In 1966, this sales tax was converted into a crude credit-type value added tax (VAT), but one that extended only through the manufacturing level and allowed no credit for tax paid on capital goods.

2.5.8 1974 reforms

The 1974 reforms were greatly influenced by the Musgrave recommendations, which were considered and refined by local experts during the intervening half-decade. These reforms rationalized many aspects of the income and net wealth taxes, and eliminated the tax on excess profits, added a calculation of presumptive income based on net wealth, and further improved the system of internal indirect taxes.

Income. The 1974 reforms eliminated many exemptions and other forms of preferential treatment for non-labor income, including the exclusion of interest on public debt, exemptions for automobile producers and private electrical companies, and deductions for reserves for investment. Exemptions remained intact for the airlines, publishing, and reforestation sectors, and for various activities in selected regions (primarily “frontier” and other less developed ones).

The 1974 reforms replaced graduated rates with flat-rate taxes for business income. But the taxation of corporations and limited liability companies was not unified. Instead, the income of corporations was taxed at 40 percent and that of limited liability companies and partnerships at a rate of 20 percent. As before, shareholders of corporations paid tax only on dividends, whereas owners of limited liability companies and partnerships were required to pay tax currently on the share of the profits of the company imputed to them.

The 1974 reforms extended the concept of presumptive income from only to the agricultural sector (1973) to the economy as a whole. Income was presumed to be no less than 8 percent of net wealth, defined generally in the same way as for the net wealth tax.

Internal indirect tax. The 1974 reforms eliminated the possibility of buying otherwise taxed items on an exempt basis simply by certifying to the seller that they were to be processed further; under the new law, the tax credit (invoice) system was extended to all such transactions. This created administrative headaches because of the many claims for refunds that had to be processed.

The law expanded the base of the VAT to reduce the regressivity of the tax at the upper end of the income scale. The VAT was applied to many services, including parking lots, insurance, international air fares, photographic developing and photocopying, telegraphic

and telephone services, and fees of social clubs. Likewise, the degree of progressivity of the rate schedule applied to various goods, ranging from “wage” goods (6%) to luxuries (35%) was increased. The 1974 reforms also provided more widespread exemptions, for example, for almost all food and for selected agricultural machinery.

As under previous law, no credit was allowed on capital goods, in part to compensate for underpricing of capital resulting from various non-tax policies, including an undervalued exchange rate. On the other hand, the law contained anomalous provision that exempted imports of capital goods destined for “basic” industries. The combination of these policies placed domestic producers of capital goods at a competitive disadvantage relative to foreign producers of imported capital goods.

2.5.9 1974-1986 Period

During this period of continued tinkering with the tax system in Colombia, some changes represented improvements, but the more important ones represented retrogression. These *counter-reforms* were in large a response to the pleadings of politically powerful economic groups.

For example, law 20 of 1979 effectively exempted cattle raising from income tax and arbitrarily reduced the value of breeding stock and dairy cattle for net wealth and presumptive income tax purposes by 50%.

Law 9 of 1983 introduced a tax credit of 10% of dividends received was provided as a means of reducing double taxation of dividends.

In addition, Law 9 of 1983 instituted a measure of presumptive income equal to 2 percent of gross receipts to supplement the measure based on net wealth. This reform was aimed specifically at the commerce and service sectors; the former were thought to evade the

wealth-based presumptive tax by systematically understating inventories. In addition, it extended the presumptive income tax to limited liability companies.

Tax returns were not required if 80 percent of income was from labor subject to withholding, if the taxpayer was not an owner of shares in a limited liability company (ownership of corporate shares was allowed), and if the taxpayer was not liable for sales tax, among other requirements.

In 1983, the value-added tax (VAT) was extended to the retail level, with a *simplified system* being made available to small retailers in the effort to ease compliance costs and administrative burden. Services like hotels, computing services, maintenance, and rental of goods and fixtures were brought into the scope of the tax. Moreover, there was some unification of rates to alleviate the difficulties of dealing with differential rates at the retail level which led to an increase in the rate of (non-creditable) tax applied to domestically produced capital goods. Imported capital goods continued to be exempt from tax. In 1984, exemptions for agricultural machinery, transportation equipment, and certain other goods were eliminated.

2.5.10 The 1986 reforms

The 1986 reform reduced tax rates dramatically. The top tax rate applied to individual income was reduced from 49 to 30%; the same rate applied to the income of corporations and limited liability companies.

Expense deductions of independent professionals were limited to 50% of income in an effort to curtail a major source of abuse.

The 1986 reforms unified the taxation of corporations and limited liability companies by taxing both at a rate of 30%. It integrated the taxation of companies and individuals by

exempting corporate dividends and participation in profits of limited liability companies from tax at the individual shareholder/owner level. Consistent with this treatment, losses, exempt income, and tax credits of companies cannot be used to offset income of their owners.

3 Fiscal Survey of Colombia

Juridical persons in Colombia (p. 27-30).

- Corporation (*sociedad anónima*), the typical association of capital and is the counterpart of the United States corporation. The Colombian corporation is a separate juridical entity, having its own name, domicile, and property. Corporations are subject to the Superintendent of Corporations and are closely supervised, being required, for example, to have an auditor and to maintain a legal reserve. Foreign corporations, partnerships, and other companies or entities are taxed as corporations. Also taxable as corporations are limited stock partnerships (*sociedades en comandita por acciones*), which are limited partnerships with the partnership interest represented by shares of stock.
- General partnerships (*sociedades colectivas*), which include mining partnerships (*sociedades ordinarias de minas*), *de facto* or constructive partnerships (*sociedades de hecho*), organized associations, associations, and foundations.
- The third type of entity includes the ordinary limited partnership (*sociedad en comandita simple*), i.e., one other than a limited stock partnership; and the limited liability company (*sociedad de responsabilidad limitada*). The limited liability company is quite important (1960s) in Colombia. It combines certain features of a corporation

and a partnership. It is organized in the same manner as a general partnership but with the additional requirement that the articles of association must indicate that the personal liability of the partners is limited to the capital contributed. The limited liability company offers some advantages as compared to a corporation, chiefly, the lower cost of organization and taxation (until the 1980s reforms) and comparative freedom of operation. Under Colombian law, the limited liability company is an association of persons, not of capital, and it is not subject to the superintendent of Corporations unless more than one-third is owned by a corporation.

Profits from individual proprietorships [were] taxed only once under the income tax system, but the dividends of corporations [were] taxed in full, resulting in a double income tax burden on distributed corporate profits. On the other hand, for both partnerships and limited liability companies, all income after payment of the income tax at the business level is construed to be distributed and is taxed again at the individual level.

4 ?

Development of a Value Added Tax in Colombia

5 ?

5.1 Reforma 1983

El objetivo de la reforma de 1983 fue reducir el déficit fiscal y evitar financiación con recursos de emisión. Asimismo, la reforma de 1983 buscaba reactivar la economía aumentando la inversión mediante incentivos tributarios. Con el objetivo de la reactivación económica, se amplió y reestructuro la base tributaria y se simplificó el cobro de impuestos. Se redujo el

número de personas obligadas a declarar renta y se extendió la renta presuntiva al comercio y los intermediarios financieros. Se eliminó la doble tributación para las sociedades anónimas (Corporations) y se redujeron las tasas para las sociedades limitadas y las personas naturales.

El IVA se extendió al comercio y detal. Se unificaron y simplificaron las tarifas.

5.2 Reforma 1986

Objetivo: devolver la equidad y la neutralidad del impuesto de renta, fortalecer la administración tributaria y adecuar la estructura tributaria a la movilidad internacional de capitales.

Con este objetivo, se igualó las alternativas de inversión de las empresas (impuestos sobre rendimientos de capital), se unificaron las tarifas tributarias para todo tipo de sociedades tanto nacionales como extranjeras; se eximió del pago de la retención en la fuente al 90% de los asalariados del país.

En el campo administrativo, la contribución más importante fue la delegación en la red bancaria de la recepción y recaudo del impuesto de renta (income tax).

5.3 Reforma 1990

Objetivo: Incrementar el ahorro de la economía, desarrollar el mercado de capitales y fomentar la repatriación de los capitales fugados. El objetivo también incluía disminuir la dependencia fiscal de la actividad económica externa y adecuar la estructura tributaria a la apertura comercial.

Se desgravó la utilidad obtenida en la enajenación de acciones a través de bolsa, se exoneró del impuesto de renta a los fondos de inversión, fondos comunes y de valores y se redujeron

las tarifas de remesas y renta para la inversión extranjera. Se dieron amnistías fiscales y se redujo el impuesto de saneamiento para fomentar la repatriación de capitales.

Se amplió la base del IVA y se elevó la tarifa general de 10% al 12%.

5.4 Impacto en recaudos

Grafico 1. Se observa una caída en los ingresos tributarios como porcentaje del PIB desde 1980 hasta 1983, el punto más bajo. A partir de 1984, los ingresos incrementan hasta 1987, donde se observa un declive hasta 1990. Por último, se observa un incremento para 1991.

5.5 Evasión

Grafico 3. Impuesto a la renta (Income tax). Se muestra una reducción sostenida a lo largo del tiempo, de 1980 a 1991. Se muestra también un decremento de 1980 a 1983, luego un incremento desde 1983 hasta 1988, luego decremento del 88 al 90 y finalmente un gran incremento del 90 al 91.

Grafico 4. IVA, evasión y recaudo. Se aprecia un decremento del 42% de la tasa recaudo efectivo/recaudo potencial de 1984 hasta 1990 y luego un incremento en 1991.

6 ?

Effects of size-dependent tax enforcement on firm's tax compliance. The authors exploit quasi-experimental variation generated by a Large Taxpayers Unit (LTU) in Spain, which monitors firms with more than 6 million euros in reported revenue. Firms strategically bunch below the eligibility threshold in order to avoid stricter tax enforcement. The response is stronger in sectors where transactions leave more paper trail, suggesting that the monitoring

effort and the traceability of information reported by firms are complements.

Following the advice of multilateral organizations such as the IMF and the OECD, many tax administrations have established special units to monitor and improve the tax compliance of the largest taxpayers. The benefits of establishing Large Taxpayers Units (LTU) include higher tax revenues and efficiency.

In Spain, the LTU focuses exclusively on firms with total operating revenue above 6 million euros, a threshold established in 1995 and it has remained constant in nominal terms ever since. The LTU has more auditors per taxpayer than the rest of the tax authority, and these auditors are on average more experienced and better trained to deal with the most complex taxpayers. Thus, while firms above and below the eligibility threshold face the same tax schedule, monitoring effort changes discretely at this arbitrary cutoff.

In the model, firms make production and tax-reporting decisions given a tax rate on reported profits. Underreporting revenue lowers tax liabilities, but it also implies some resource costs (e.g., keeping two sets of accounting books or foregoing business opportunities). The detection probability increases endogenously with the amount evaded by firms, depending on the monitoring effort of the tax authority (resources devoted to analyzing tax returns and other paper trails generated by business activities) and the enforcement technology (technology to process the information generated by business transactions). The model assumes monitoring effort jumps up discretely at a certain threshold of reported revenue, while enforcement technology remains constant. This creates an incentive for firms to bunch below the LTU cutoff to avoid stricter tax enforcement. Firms are heterogeneous in their cost of evasion and in the effectiveness of monitoring efforts, which depend on the traceability of firms' transactions. Intermediate input sales (firm-to-firm) leave more paper trail and thus are easier to trace than firm-to-consumer sales. In line with this, the

model predicts a stronger bunching response by upstream (firm-to-firm) firms compared to downstream (firm-to-consumer) firms.

The authors use the Commercial Registry (Registro Mercantil) from the Banco de España. The dataset contains firm-level information on annual net operating revenue, input expenditures, number of employees, payroll taxes, total value added, and the tax base and liability in the corporate income tax, covering more than 80 percent of registered businesses in Spain with operating revenue between 3 and 9 million euros between 1995 and 2007. The LTU threshold remained constant at 6 million euros.

Empirical finding 1: The authors find that a significant number of firms report revenues just below the LTU threshold. Their estimates indicate that firms reduce their reported revenue by 2 percent of their total revenue on average. The marginal buncher's response is bounded between 6.4 and 8.7 percent of the total reported revenue.

EF 2: The average bunching response is significantly higher in upstream sectors than in downstream sectors. The authors argue that the effectiveness of an additional monitoring effort is higher the easier it is to trace firms' transactions, which suggests that information trails and monitoring efforts are complements.

EF 3: The authors suggest that they identify patterns that are not compatible with firms lowering real output (real response) suggesting firms are merely underreporting revenue (evasion response). They find that at the LTU threshold, the average ratio of material expenditures over total revenue shifts down sharply, while the average ratio of the wage bill (net of payroll taxes) as a share of total revenue shifts upward. The argument is that if bunchers reduced their true output, both ratios would jump up at the threshold because they are more productive than other firms below the threshold (?). Firms might also overreport

material inputs and underreport labor costs to evade VAT and payroll tax (PRT). This doesn't make sense. This is precisely what would be expected if firms were reporting true output but overreporting inputs and underreporting labor.

The authors also document a positive persistent gap in the average reported taxable profit and significant jumps in the VAT and PRT between the two enforcement regimes.

? conclude that Spanish firms misreport both revenue and inputs. In their argument, the authors do not discard the case in which firms report their true output and only misreport inputs. However, upstream firms are likely to fall in this case, only input but not revenue misreporting. While buying firms have incentives to overreport input expenses, selling firms have incentives to underreport sales. Hence, due to opposite incentives, upstream firms might find it hard to underreport their revenue. If the expected benefits of staying below the LTU threshold and evading taxes outweigh the costs of forgone production and evasion, bunching upstream firms are likely to reduce their true output and misreport their inputs.

This is not so clear in the introduction, where the authors just state that the cross-sectional patterns they observe in the data are not compatible with a real response (firms lowering their true output to avoid being monitored by the LTU). Maybe the reason for not clarifying they find evidence of both revenue and input misreporting in the introduction is because they focus on revenue misreporting and have nothing to say about input misreporting.

The authors do not consider the case where firms only misreport inputs and reduce their true output to stay below the LTU threshold. To underreport revenue, firms need to coordinate with customers. Customers must agree to not require an invoice, to avoid leaving a paper trail. This is likely to happen with downstream firms, firms that sell to final customers. However, with upstream firms, firms selling to other firms, this is unlikely the case. The

reason is that, unlike final customers, other firms have incentives to overreport their input consumption to reduce their VAT and CIT.

It might be likely bunching upstream firms reduce their true output if the expected benefits of staying below the LTU threshold outweigh the cost of forgone production and evasion costs. The authors indeed find lower bunching for upstream firms than for downstream firms. Moreover, in the online appendix, the authors show that the ratios of intermediate expenses over revenue and labor expenses over revenue of wholesale (upstream) and retail (downstream) firms follow the general evasion patterns. Intermediates upward sloping then dropping at the cutoff; labor downward sloping then jumping at the cutoff.

I argue that they cannot distinguish between the scenario where firms only misreport inputs and the one where firms can misreport both revenue and inputs. Another point they missed is that the input expense over revenue ratio is equal to the output elasticity of inputs. In turn, this elasticity depends on the shape of the production function and is productivity-free if the productivity is Hicks-neutral. This is key to their argument, because, if it might be true that bunchers are more productive than firms below the LTU threshold, their input shares are not necessarily lower. If the production function is Cobb-Douglas, for example, the shares are constant below and above the threshold.

Under multiple taxes, firms have incentives to underreport revenue and overreport intermediate expenses to evade Value-Added Tax (VAT) and Corporate Income Tax (CIT), and to underreport labor expenditure to evade the Pay-Roll Tax (PRT). To study the mechanism of the bunching response under multiple taxes, the authors compare with the data their model's 'predictions' on the input expense share of revenue under three different scenarios. The idea is that in the absence of the LTU, firms around the threshold should have similar average ratios of input expenditure as a fraction of revenue. Since the probability of detection is

higher under the LTU, firms above the threshold have a smaller incentive to evade taxes. Once the LTU is introduced, some firms, the bunchers, above the threshold have incentives to bunch below to avoid stricter monitoring by the LTU and have larger expected profits. The heterogeneity arises because firms differ in monitoring effectivity by the authority and evasion costs.

Under the first scenario, there is no misreporting. The bunchers reduce their input consumption to lower their output and stay below the LTU threshold. The authors argue that because the bunchers are more productive than the firms below the threshold, their input expenses share of revenue ratio is lower. Therefore the average ratio of input expenses over revenue should be lower the closer to the cutoff. Then, at the threshold, there would be a discrete upward jump for both ratios. In brief, the predictions of this scenario are (i) a downward trend of both ratios as we get closer to the revenue threshold along the revenue axis, and (ii) a discrete upward jump at the cutoff. Gap: share equals output elasticity of inputs, do not depend on productivity, but on PF shape.

In the second scenario, the authors allow for revenue misreporting but no input misreporting. According to the authors, the bunchers' input expenses share of revenue ratios are mechanically greater than the firms below the cutoff. As a consequence, both ratios go up and then drop at the cutoff. Gap: share should be lower for bunchers than firms below the cutoff. Implicitly, authors are assuming that bunchers underreport revenue enough to make their input shares larger than firms below the cutoff, but is this guaranteed?

Finally, the authors allow for misreporting revenues and inputs. In this scenario, One goes up, the other goes down. Gap:

They conclude that it is likely that firms underreport revenue and misreport inputs (over-

reporting inputs, underreporting labor). This is not clear in the introduction as they have nothing to say about input misreporting. Moreover, they don't claim causality or significance. They just say that given their model's testable implications and what they observe in the data simply suggests the third scenario is the more likely to be happening.

They take the input over revenues and labor over revenue ratios and look at what would happen just before the LTU threshold and after the threshold. In their argument, higher productivity firms would use a lower amount of inputs, hence, in the absence of misreporting, these shares should be smaller for firms with higher revenues (higher productivity). However, they miss the point that actually these ratios are productivity free and depend on the output elasticity of inputs.

However, this ratio is equal to the output elasticity of inputs and it is free of productivity. Indeed, using the authors' notation, we have that for the intermediates,

$$\begin{aligned}\psi f_x(x, z) &\geq w \\ \frac{f_x(x, z)x}{f(x, z)} &\geq \frac{wx}{y}\end{aligned}$$

In this equation, equality holds only when firms produce the optimum quantity. When they don't, then the intermediate's input expense over the revenue ratio is lower than the elasticity. For the sake of clarity, suppose the production function is Cobb-Douglas, $f(x, z)\psi = x^\alpha z^{1-\alpha}\psi$. In this case, we have

$$\alpha \geq \frac{wx}{y}$$

Because they are producing less output than the optimal, bunchers' input cost over revenue is lower than the input elasticity of output.

7 ?

In the early 1990s, Colombia's publicly-owned power sector was financially distressed and poorly managed, becoming a risk for the country's macroeconomic stability. The sector reached a tipping point during extensive demand rationing episodes in 1992. Amidst the paradigm transformation promoted by the 1991 Constitution, Colombia launched an ambitious power sector reform package in 1994, introducing private sector participation, independent regulation of distribution and transmission, and competition in power generation and retailing.

Colombia's power sector was heavily distressed in the early 1990's. Financially unsustainable utilities placed an enormous burden on the country's fiscal gap. Efficiency was very low overall, particularly in areas with high electricity theft and fraud. Furthermore, two major blackout periods, first in 1983 and then in 1992-1993, pushed power sector reform to the top of the country's agenda in 1993 (Larsen, Dyner, Bedoya, & Franco, 2004).

To tackle these challenges, a comprehensive reform package was launched with enactment of Laws 142 and 143 of 1994. These laws established the framework for vertical unbundling and private sector participation, a new regulatory framework and rate-system governed by principles such as efficiency and financial sufficiency, and introduction of wholesale competition, among other measures.

8 Colombia 1981-1991

8.1 Colombian Corporate Tax System

The relevant corporate taxes for input overreporting in Colombia during this period are the Corporate Income Tax (CIT) and the Sales Tax. The Sales Tax gradually transformed into a kind of Value-Added Tax (VAT). Also relevant for the CIT are the different juridical organizations that exist in Colombia.

This period was characterized by high levels of overall tax evasion (?). The fiscal rules had a system of penalties and interest that encouraged false and delinquent returns (?). The fiscal authority was characterized by having an inefficient auditing system, being overburdened, and legal loopholes (?).

8.1.1 Juridical Organizations

In Colombia, there are five types of juridical organizations: Corporations, Partnerships, Limited Liability Companies, and Individual Proprietorships.

Corporations (*sociedad anónima*) are the typical associations of capital. They are the counterpart of the US corporation. The capital of a corporation is provided by the shareholders (no less than 5) and is divided into tradable shares of equal value. Shareholders' liability is limited to the capital contributed. Corporations are subject to the Superintendent of Corporations and are closely supervised, being required to have an auditor.

Joint Stock Companies (*sociedad en comandita por acciones*) comprises two or more managing partners who are jointly and severally liable, and five or more limited partners whose liability is limited to their respective contributions. Joint Stock Companies are taxed as Corporations. Its shares are tradable, like the shares of Corporations.

Partnerships are associations of two or more persons. Partners are jointly and severally liable for the partnership's operations. Partnerships include general partnerships (*sociedad colectiva*), de Facto partnerships (*sociedades de hecho*), and ordinary limited partnerships (*sociedad en comandita simple*).

A limited liability company (*sociedad de responsabilidad limitada*) is an association of two or more persons —not exceeding 20 (Fiscal Survey) or 25 (1992 *EAM* survey documents)—, whose shares cannot be traded. The personal liability of the partners is limited to the capital contributed. The Limited Liability Company is quite important in Colombia (Fiscal Survey).

Finally, proprietorships are individuals (natural persons) who allocate part of their assets to conduct commercial activities.

There are other juridical organizations in the data that will be excluded from the final analysis. These organizations are non-profit, like cooperatives and community enterprises, or state industrial enterprises, the proceeds of which come from taxes, fees, or special contributions.

8.1.2 Corporate Income tax

The juridical organizations were subject to different Corporate Income Tax rates. Corporations were taxed at a fixed rate of 40%, while Partnerships and Limited Liability companies at 20%. Individual proprietors were subject to the graduated Individual Tax Schedule consisting of 56 rates, ranging from 0.50 to 51 percent.

Corporations were taxed on their distributed dividends, while partnerships and limited liability companies were taxed on their profits, whether or not distributed. Owners of juridical organizations were double taxed, at the firm and the individual level, whereas the

income of proprietorships was taxed only once, at the individual level.

Since 1974, individuals and juridical organizations, except for limited liability companies, were subject to the minimum presumptive income. Rent (income and profits) was presumed to be no less than 8 percent of net wealth (assets less depreciation, real estate, livestock, securities).

Certain industries like airlines, publishing, and reforestation sectors, and various activities in selected regions (primarily “frontier” and other less developed ones) had their income tax exempted, limited, or reduced.

Table 1: Juridical Organizations in Colombia (1980s), A Summary

Organization	Corporate			
	In-			
	come			
	Tax	Liability	Capital	Owners
Corporation	40%	Limited to	Tradable	$N \geq 5$
	(on dis-	capital	capital	
	tributed	participation	shares	
	divi-			
	dends)			
Limited Co.	20%	Limited to	Non-	$2 \leq N \leq 20(25)$
	(on	capital	tradable	
	prof-	participation	capital	
	its)		shares	

	Corporate			
	In-			
	come			
Organization	Tax	Liability	Capital	Owners
Partnership	20%	Full	Not a	$N \geq 2$
	(on		capital	
	prof-		association	
	its)			
Proprietorship	Individual	Full	Owner	$N = 1$
	In-			
	come			
	Tax			

8.1.3 Sales taxes

Sales taxes were originally targeted at the manufacturing sector on finished goods and imports. Since 1974, manufacturers were allowed to credit taxes paid on any purchase made by the firm, except the acquisition of capital goods (?). The credits worked through a system of refunds. Consequently, the tax became a kind of value-added tax (VAT).

The basic rate was 15 percent. There was also a preferential rate of 6 percent for certain industries like clothing, footwear, and major inputs used for building popular housing, and a rate of 35 percent for luxury goods. Exports, foodstuff, drugs, and textbooks were excluded from the beginning. Also excluded were inputs, transportation equipment, agricultural machinery, and equipment.

8.1.4 Discussion

From Colombia's tax system, we can conclude that corporations are the least likely to evade taxes by misreporting because of at least three reasons. One, the Superintendent of Corporations closely monitored corporations by requiring them to have an auditor. In the model, this implies that the probability of detection is higher for them. Second, free tradable shares impose an incentive to be profitable because the market value of the shares might be negatively affected otherwise. In other words, if a corporation fakely reports lower profits, the value of its shares would likely decrease scaring away shareholders and potential investors. Joint stock companies have freely tradable shares too. Three, corporations pay CIT on distributed dividends, not on profits as Partnerships and LLCs did. Corporations have an additional margin regarding the income tax they pay because they decide when to pay dividends. This might generate other types of incentives that might be influenced by the corporation's policy regarding their dividends and the demands of their shareholders. On the other hand, Proprietorships and LLCs are subject to the incentive to evade CIT by artificially reducing their profits.

Moreover, Proprietorships, Partnerships, and Limited Companies had incentives to overreport inputs to evade VAT and CIT. The incentives to evade varied across industries because the sales tax rate differed between industries. The incentives to evade also varied within industry sectors because juridical organizations within an industry were subject to different CIT rates. There were additional sources of variation depending on the firm's location due to local exemptions and sales composition (inputs to other firms, to consumers, to the foreign market).

Lastly, Individual proprietorships were likely to bunch at the individual income thresholds because they were subject to individual income tax which was increasing by brackets.

8.2 Fiscal Reforms

During this period, Colombia went through three major fiscal reforms (1983, 1986, 1990).

8.2.1 1983

The 1983 reform tried to alleviate double taxation by introducing a tax credit of 10% of dividends received by shareholders of corporations.

In addition, Law 9 of 1983 instituted a new measure of presumptive income equal to 2 percent of gross receipts to supplement the measure based on net wealth. This reform was aimed specifically at the commerce and service sectors; the former was thought to evade the wealth-based presumptive tax by systematically understating inventories. Under the original presumptive income, a juridical entity cannot declare income less than 8% of its capital (wealth).

In addition, the 1983 reform extended the presumptive income tax to limited liability companies. Before this reform, all juridical organizations were subject to the presumptive income tax except for limited liability companies.

In 1983, the value-added tax (VAT) was extended to the retail level, with a *simplified system* being made available to small retailers to ease compliance costs and the administrative burden.

The 1983 reform relatively unified the value-added tax (VAT) rates by combining previously taxed goods at 6% and 15 percent into 10%. The number of the products and services that were levied expanded.

In 1984, VAT exemptions for agricultural machinery, transportation equipment, and certain other goods were eliminated.

8.2.2 1986

The 1986 reforms unified the taxation of corporations and limited liability companies by taxing both at a rate of 30%. The top tax rate applied to individual income was reduced from 49 to 30%.

Double taxation was eliminated. The reform exempted corporate dividends and participation in profits of limited liability companies from tax at the individual shareholder/owner level.

Lastly, the 1986 reform relocated the tax collection and reception of tax reports to the banking system, among other things.

8.2.3 1990

The 1990 reform increased the VAT from 10% to 12%.

In addition, the individual income obtained from the sale of shares through the stock market was exempted from taxation. Income tax was waived for investment funds, mutual funds, and securities, and the rates for remittances and income for foreign investment were reduced. Tax amnesties were granted, and the sanitation tax was reduced to encourage the repatriation of capital.

8.2.4 Discussion

Increases in the VAT would increase the incentives to evade. Decreases in CIT would decrease them. Intuitively, we expect higher levels of tax evasion if tax rates increase.

The CIT homogenization between Corporations and LLCs in 1986 would have motivated LLCs to incorporate. Likewise, the elimination of double taxation also in 1986 would have motivated proprietorships to become LLCs, Corporations, or Partnerships.

Table 2: Income Tax Reforms in Colombia (1980-1986)

Reform Year	J.O. Affected	Income Tax Change
1983	Individuals	8% increase in most scales; Max tax rate was reduced from 56 to 49%
	Ltd. Co.	Reduction from 20 to 18%; Now subject to presumptive income
1986	Individuals	Max tax rate applied was reduced from 49 to 30%
	Ltd. Co.	Increased from 18 to 30%
	Corporations	Decreased from 40 to 30%

On the other hand, reporting more information to the tax authority — like the banking system being responsible for the collection and reception of tax reports in 1986— would decrease tax evasion.

9 Data

The Colombian data is a well-known firm-level panel data set that has been used in the estimation of production functions and productivity before. The Colombian dataset comes from the Annual Survey of Manufacturing (EAM) and contains information about manufacturing firms with more than 10 employees from 1981 to 1991.

Besides the information on output, intermediates, capital, and labor, the dataset includes the type of juridical organization and the sales taxes. Table ?? offers some summary statistics.

Table 3: Sales Tax Reforms in Colombia (1980-1986)

P&T (1990)		SIC	Industry
Industry Description	Sales Tax Change		
Beverages and Tobacco	- to 35;10	313	Beverage Industries
	- to 35;10	314	Tobacco Manufactures
Textiles	6 to 10	321	Textiles
Paper	15 to 10	341	Paper and Paper Products
Other Chemical Products	15 to 10	351	Industrial Chemicals
Soap	6;15 to 10	352	Other Chemical Products
Oil and Coal Derivatives	10 to 14	354	Miscellaneous Products of Petroleum and C
Plastics	15 to 10	356	Plastic Products Not Elsewhere Classified
Iron and Steel; Nickel Smelting	6;15 to 10	371	Iron and Steel Basic Industries
Equipment and Machinery	6 to 10	382	Machinery Except Electrical
	6 to 10	383	Electrical Machinery Apparatus, Appliances
Transportation	6 to 10	384	Transport Equipment

Table 4: Summary Statistics, Manufacturing Firms in Colombia (1981-1991)

	Missing (%)	Mean	SD	Q1	Median	Q3
Share of Revenues						
Sales Taxes	0	0.066	0.051	0.006	0.069	0.100
Skilled Labor (Wages)	0	0.043	1.620	0.009	0.021	0.046
Unskilled Labor (Wages)	0	0.086	0.106	0.019	0.051	0.119
Capital	0	0.495	7.555	0.126	0.261	0.499
Intermediates						
Materials (M)	0	0.457	0.191	0.333	0.457	0.582
Electricity (E)	0	0.018	0.028	0.005	0.010	0.020
Fuels (F)	0	0.011	0.025	0.001	0.003	0.010
Repair & Maintenance (R&M)	0	0.007	0.013	0.001	0.003	0.009
Services (S)	0	0.103	0.167	0.052	0.084	0.130
Deductible Inter. (M+E+F+R&M)	0	0.493	0.180	0.371	0.488	0.610
Non-Deductible Inter. (S)	0	0.103	0.167	0.052	0.084	0.130
J. Org.	N	%				
Proprietorship	5343	12.877				
Ltd. Co.	25457	61.353				
Corporation	8761	21.114				
Partnership	1425	3.434				

Table 5: Top 10 industries in Colombia by revenues

description	sic_3	n_sic	n_Corp	Market Share
Food Manufacturing	311	912	150	23.5
Beverage Industries	313	130	74	9.5
Other Chemical Products	352	301	101	8.5
Industrial Chemicals	351	117	69	7.5
Textiles	321	476	71	7.5
Transport Equipment	384	238	42	6.5
Food Manufacturing	312	193	41	5.5
Paper and Paper Products	341	153	41	5.5
Fabricated Metal Products, Except Machinery and Equipment	381	633	85	3.5
Electrical Machinery Apparatus, Appliances and Supplies	383	213	57	3.5

9.1 Top Industries by Revenue in Colombia

9.2 Corporations by Industry

Looking into industry sectors.

9.3 The Fiscal Reforms

A simple graphical analysis shows that the average (of the log) intermediates cost share of sales started growing after 1983 and that it stabilized in 1988 after the policy changes of the 1986 reform settled in (Figure ??). The dataset does not capture any changes after the 1990 reform, although there is only one more year of data.

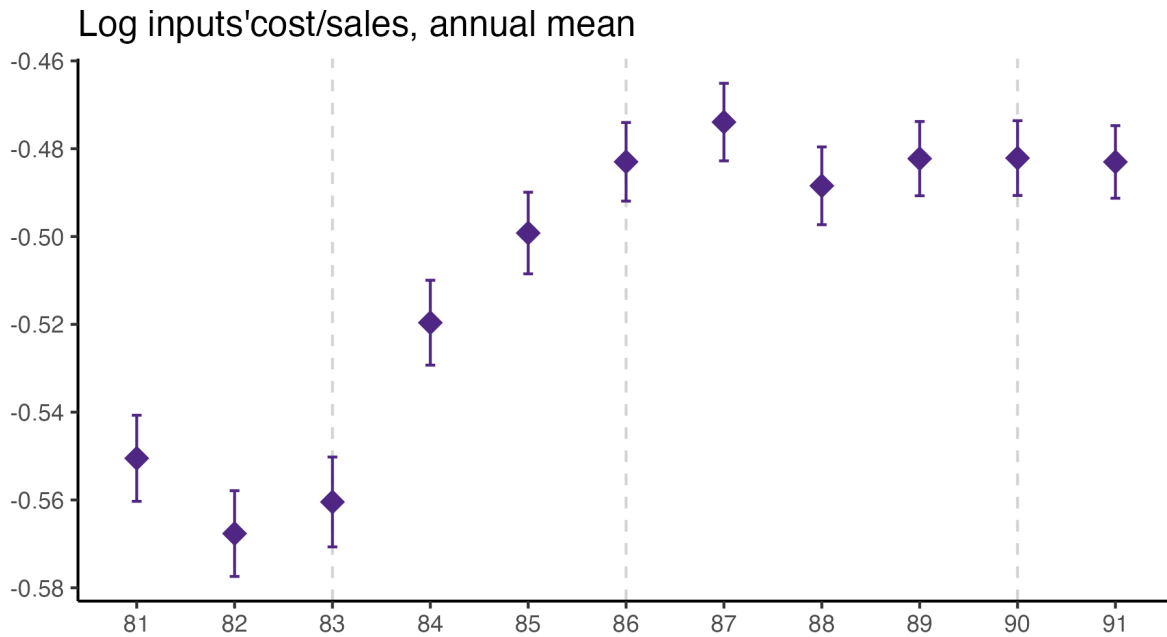


Figure 1: Input's cost share of sales, average by year of the logs.

As a validation exercise, we can see that the VAT changes induced by the three fiscal reforms are captured in the dataset. Figure ?? shows that the sales tax increased to 10% after the 1983 reform, and then around 12% after the 1990 reform.

Table 6: Corporations by Industry. Top 20 Industries in Colombia by number of firms.

Industry
Food Manufacturing
Wearing Apparel, Except Footwear
Fabricated Metal Products, Except Machinery and Equipment
Textiles
Machinery Except Electrical
Printing, Publishing and Allied Industries
Other Chemical Products
Other Non-Metallic Mineral Products
Plastic Products Not Elsewhere Classified
Transport Equipment
Electrical Machinery Apparatus, Appliances and Supplies
Food Manufacturing
Furniture and Fixtures, Except Primarily of Metal
Footwear, Except Vulcanized or Moulded Rubber or Plastic Footwear
Other Manufacturing Industries
Wood and Wood and Cork Products, Except Furniture
Paper and Paper Products
Beverage Industries
Industrial Chemicals
Leather and Products of Leather, Leather Substitutes and Fur, Except Footwear and Wearing Apparel

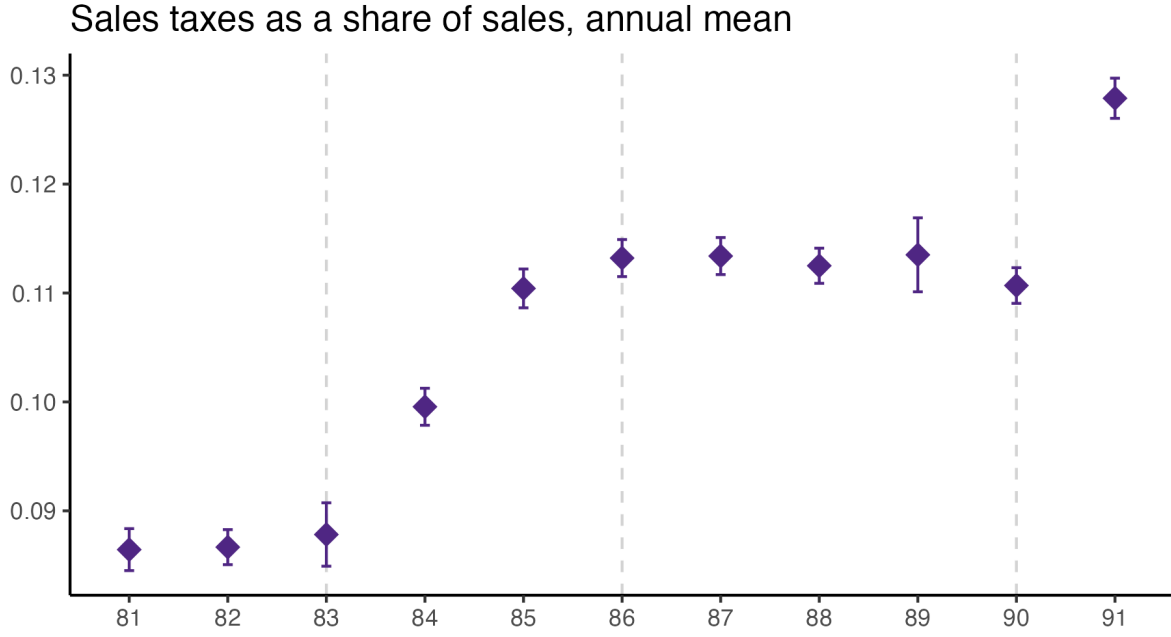


Figure 2: Sale taxes paid as share of sales, average by year.

Just as an exercise to see if other economic changes in this period were driving the apparent changes in overreporting, Figure ?? shows that sales, for instance, were not exactly following the changes in fiscal policy. Sales started to grow during 1983, the year of the reform, whereas the cost share of sales started to grow the year after. Likewise, sales fell in 1986, while the cost share seems to reduce its growth after 1986.

Finally, in a preliminary empirical assessment, I observe that the sales tax rate is a significant determinant of the log share of revenue and that non-corporations consistently use 13-17 percent more intermediates than Corporations for a rich set of controls Table ?. The results were estimated following Equation ??

$$\log(s_{it}) = \alpha_1 Tax_{it} + \beta'_1 JurOrg_i + \beta'_2 JurOrg_i \times \gamma_t + \gamma_t + \gamma_{ind} + \gamma_{metro} + \beta'_3 Z + \varepsilon_{it} \quad (8)$$

Although this is not deterministic evidence, it does not contradict the hypothesis that firms

Table 7: Effect of the Juridical Organization Type and Sales Tax on the Log Share of Intermediate Inputs.

Dependent Variable:	Interm.		
Model:	(1)	(2)	(3)
<i>Variables</i>			
Sales Tax Rate	-0.6387*** (0.1419)	-0.5235*** (0.1027)	-0.5521*** (0.1028)
Proprietorship	0.1654*** (0.0275)	0.1269*** (0.0180)	0.1237*** (0.0168)
LLC	0.1717*** (0.0286)	0.1333*** (0.0182)	0.1280*** (0.0180)
Partnership	0.1607*** (0.0295)	0.1248*** (0.0199)	0.1234*** (0.0191)
<i>Fixed-effects</i>			
Industry		Yes	Yes
Metro Area			Yes
<i>Fit statistics</i>			
Observations	41,467	41,467	41,467
R ²	0.60315	0.62848	0.63387
Within R ²		0.54989	0.55348

Clustered (Industry & Year) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

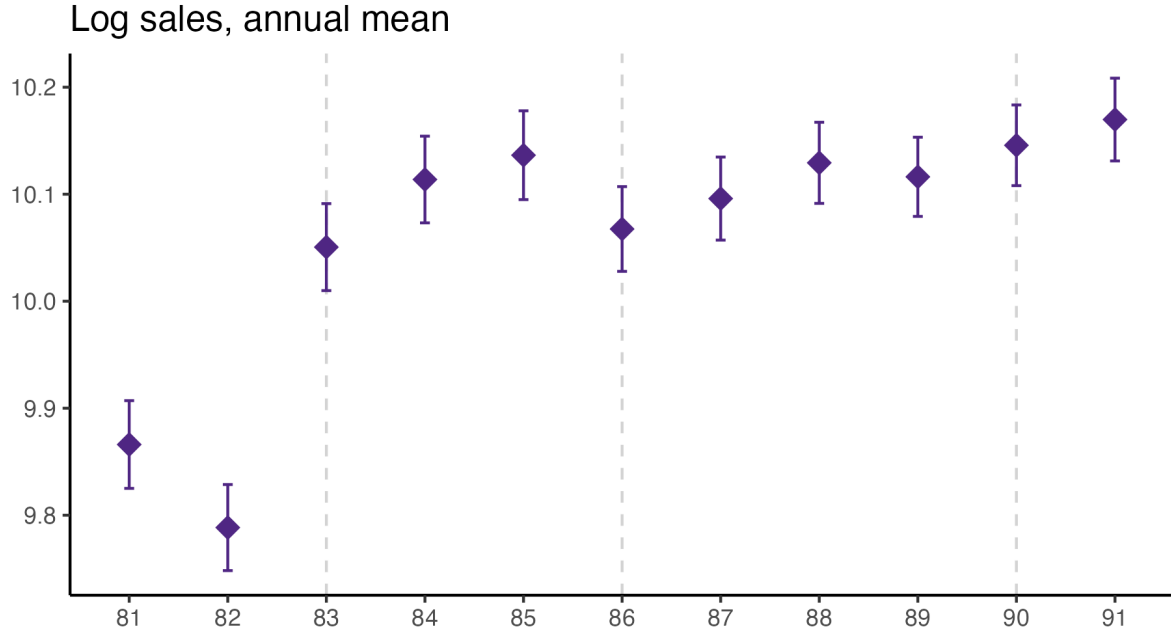


Figure 3: Sales in logs, annual mean.

other than corporations have incentives to overreport intermediates to evade taxes and that the higher the taxes the higher the incentives to evade by misreporting.

9.4 The Fiscal Reform of 1983 by Industry

10 Empirical Application

Given the fiscal reforms of 1983 and 1986 in Colombia, it is natural to think in a difference-in-difference empirical application to test if the fiscal reforms induced any change in the tax evasion behavior of the firms. We can expect that an increase in either the sales tax or corporate income tax rate would lead to higher levels of evasion.

Among other changes, the fiscal law of 1983 tried to homogenize the sales taxes of the manufacturing industry. The reform reduced from 15 to 10% for some industries; for others, it increased the sales tax from 6 to 10%. Still, some others, like the Food Products industry

Table 8: Fiscal reform of 1983. Sales tax share of revenue, mean by industry year. Top 20 industries with more firms.

SIC	Industry
311	Food Manufacturing
312	Food Manufacturing
322	Wearing Apparel, Except Footwear
381	Fabricated Metal Products, Except Machinery and Equipment
321	Textiles
382	Machinery Except Electrical
384	Transport Equipment
383	Electrical Machinery Apparatus, Appliances and Supplies
313	Beverage Industries
341	Paper and Paper Products
324	Footwear, Except Vulcanized or Moulded Rubber or Plastic Footwear
342	Printing, Publishing and Allied Industries
369	Other Non-Metallic Mineral Products
332	Furniture and Fixtures, Except Primarily of Metal
390	Other Manufacturing Industries
351	Industrial Chemicals
352	Other Chemical Products
356	Plastic Products Not Elsewhere Classified

were exempt and certain others were not affected. See Table ?? for a description of the changes documented in ?.

The 1983 reform also adjusted the income tax rates for limited liability companies and individuals. For individuals, the income tax rate increased by 8% in most scales, while the maximum was reduced from 56 to 49%. For limited liability companies, the CIT was reduced from 10 to 18%.

To evaluate the change in tax evasion by input cost overreporting due to the change in the sales tax, I apply a triple difference approach. I use corporations in the industries exempted from sales taxes the year before the policy change as the control group.

Formally, non-corporations in industry k , which might have received an increment or decrement in their sales tax rate,

$$s_{1,j,t}^k = \lambda_t^k + \mu_1^k + e_{j,t}^{VAT} + e_{j,t}^{CIT} + \varepsilon_{jt}$$

Corporations in industry k ,

$$s_{0,j,t}^k = \lambda_t^k + \mu_0^k + \varepsilon_{jt}$$

Likewise, Non-corporations and Corporations in an industry exempt from sales taxes

$$s_{1,j,t}^E = \lambda_t^E + \mu_1^E + e_{j,t}^{CIT} + \varepsilon_{jt}$$

$$s_{0,j,t}^E = \lambda_t^E + \mu_0^E + \varepsilon_{jt}$$

Taking the difference between time t' and t in industry k for both, corporations and non-corporations,

$$\mathbb{E}[s_{1,j,t'}^k] - \mathbb{E}[s_{1,j,t}^k] = \Delta_\lambda^k + \Delta_e^{VAT} + \Delta_e^{CIT}$$

$$\mathbb{E}[s_{0,j,t'}^k] - \mathbb{E}[s_{0,j,t}^k] = \Delta_\lambda^k$$

The diff-in-diff method will recover the joint effect of both policy changes,

$$\mathbb{E}[s_{1,j,t'}^k] - \mathbb{E}[s_{1,j,t}^k] - \left(\mathbb{E}[s_{0,j,t'}^k] - \mathbb{E}[s_{0,j,t}^k] \right) = \Delta_e^{VAT} + \Delta_e^{CIT}$$

The joint effect might be ambiguous because an increase in the sales tax rate will increase the incentive to overreport inputs cost but a decrease in the CIT might decrease the incentive.

To recover the effect of the change in the sales tax rate, we can use the firms of the industries that are exempted from the sales tax. Intuitively, exempted firms would not react to the change in the sales tax—which is industry-specific—, but only to the CIT—which affects all industries.

So we have,

$$\begin{aligned} & \mathbb{E}[s_{1,j,t'}^k] - \mathbb{E}[s_{1,j,t}^k] - \left(\mathbb{E}[s_{0,j,t'}^k] - \mathbb{E}[s_{0,j,t}^k] \right) \\ & - \left[\mathbb{E}[s_{1,j,t'}^E] - \mathbb{E}[s_{1,j,t}^E] - \left(\mathbb{E}[s_{0,j,t'}^E] - \mathbb{E}[s_{0,j,t}^E] \right) \right] = \Delta_e^{VAT} \end{aligned}$$

In regression form,

$$s_{jt} = \alpha [\mathbb{1}\{t = t'\} \times \mathbb{1}\{\text{treat} = \text{Non-Corp}\} \times \mathbb{1}\{k \neq E\}] + \beta_Z' Z_{jt} + \gamma_j + \gamma_t + \varepsilon_{jt}$$

10.1 Changes in Composition of Total Expenditure

Depending on the industry, firms might be adjusting different margins, raw materials or other expenses. For example, firms in the non-metallic mineral products industry might

Table 9: Log of Inputs Cost Share of Revenue by Industry. Triple diff-in-diff. The reference group is Corporations in industries exempted from the Tax Rate the year before the Reform of 1983 (1982).

Dependent Variables:	log_mats_share	log_energy_share	log_fuels_share	log_...
Industry				Wearing Apparel, Except Footwe...
Tax Change			Increased	
Model:	(1)	(2)	(3)	
<i>Variables</i>				
1984 \times Non-Corp. \times treat_3tax_treat	0.0842**	-0.0708	-0.0235	
	(0.0315)	(0.0496)	(0.0571)	
1985 \times Non-Corp. \times treat_3tax_treat	0.3449***	0.0248	0.2256**	
	(0.0628)	(0.0621)	(0.0871)	
1986 \times Non-Corp. \times treat_3tax_treat	0.1842***	0.0385	0.1604*	
	(0.0495)	(0.0733)	(0.0802)	
<i>Fixed-effects</i>				
plant	Yes	Yes	Yes	
Year	Yes	Yes	Yes	
<i>Fit statistics</i>				
Observations	10,399	10,410	9,320	
R ²	0.77685	0.85498	0.83131	
Within R ²	0.01625	0.01232	0.01154	

Clustered (Year) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1* 65

not fake raw materials but they can adjust deductible expenses. Another example might be the textile industry. Although the available records and data might not allow for separating expenditures precisely, the evidence shows that firms are significantly adjusting these margins.

? defines services as general expenditures minus machinery rental and interest payments. The Colombian survey uses industrial expenditure to calculate intermediate consumption. I identified potentially deductible expenses with information from ?.

10.2 Parallel Trends

11 Empirical Appendix

11.1 Changes in Sales Tax

12 Changes in Relative Prices

An alternative hypothesis is that changes in the sales taxes change the relative prices of inputs. It could be that the regressions are capturing not the changes in evasion by overreporting but the changes in relative prices.

Suppose we have two flexible inputs M and L . M is deductible but L is not. Equation ?? becomes then,

$$\begin{aligned} \max_{M_{it}, L_{it}, e_{it} \in [0, \infty)} & [1 - q(e_{it} | \theta_{it})] [P_t \mathbb{E}[Y_{it}] - \rho_t M_{it} - w_t L_{it} - \tau (P_t \mathbb{E}[Y_{it}] - \rho_t (M_{it} + e_{it}))] \\ & + q(e_{it} | \theta_{it}) [P_t \mathbb{E}[Y_{it}] - \rho_t M_{it} - w_t L_{it} - \tau (P_t \mathbb{E}[Y_{it}] - \rho_t M_{it}) - \kappa(e)] \quad (9) \\ \text{s.t. } & Y_{it} = G(M_{it}, L_{it}) \exp(\omega_{it} + \varepsilon_{it}) \end{aligned}$$

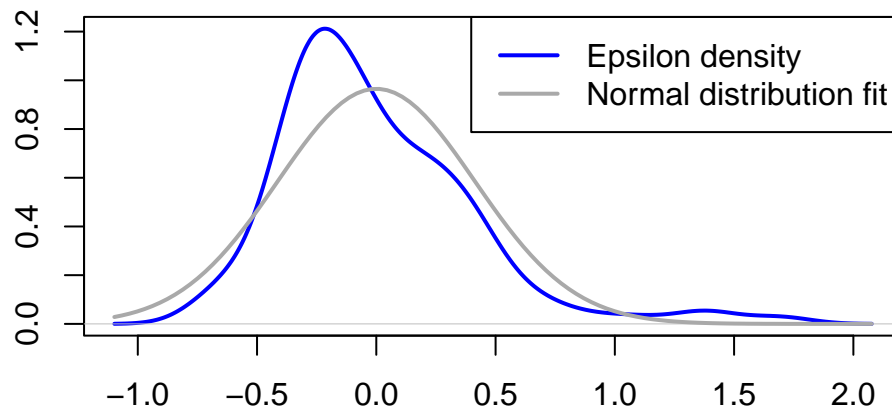
Table 10: Classifications of Expenditure.

Expenditure	Code	Services	Indust
Purchases of accessories and replacement parts of less than one year duration	c1		\$+
Purchases of fuels and lubricants consumed by the establishment	c2		\$+
Payments for industrial work by other establishments	c3		\$+
Payment of domestic workers	c4		\$+
Payments of third parties for repairs and maintenance	c5		\$+
Purchases of raw materials and goods sold without transformation	c6		\$+
TOTAL Industrial Expenditures (c1:c6)	c7		
Rent of fixed property	c8	\$+\$	
Payments for professional services	c9	\$+\$	
Machinery rental	c10		
Insurance, excl. employe benefits	c11	\$+\$	
Water, mail, telephone, etc.	c12	\$+\$	
Publicity and advertising	c13	\$+\$	
Interest payments	c14		
Royalty payments	c15	\$+\$	
Other expenditures	c16	\$+\$	
TOTAL General Expenditures (c8:c16)	c17		
TOTAL Expenditure (c7+c17)			

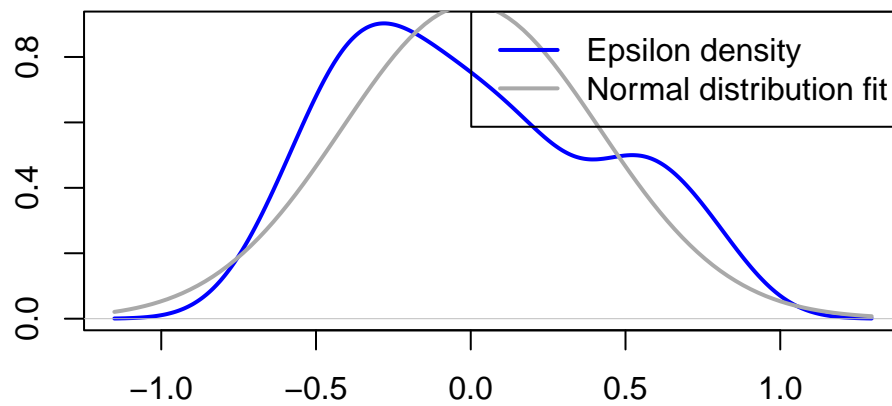
Table 11: Log of Types of Expenses Share of Total Expenses by Industry. Triple diff-in-diff.
The reference group is Corporations in industries exempted from the Tax Rate the year before the Reform of 1983 (1982).

Dependent Variables:	log_services_share	log_energy_share	log_fuels_share	log_machinery_share
Industry	322–Wearing Apparel, Except Footwear			
Tax Change	Increased			
Model:	(1)	(2)	(3)	(4)
<i>Variables</i>				
1984 × Non-Corp. × treat_3tax_treat	-0.1164**	-0.0641	-0.0235	
	(0.0469)	(0.0498)	(0.0571)	
1985 × Non-Corp. × treat_3tax_treat	0.0751	0.0530	0.2256**	
	(0.0560)	(0.0641)	(0.0871)	
1986 × Non-Corp. × treat_3tax_treat	0.0201	0.0737	0.1604*	
	(0.0503)	(0.0746)	(0.0802)	
<i>Fixed-effects</i>				
plant	Yes	Yes	Yes	
Year	Yes	Yes	Yes	
<i>Fit statistics</i>				
Observations	10,508	10,417	9,320	
R ²	0.72910	0.75102	0.83131	
Within R ²	0.01639	0.01122	0.01154	
<i>Clustered (Year) standard-errors in parentheses</i>				
<i>Signif. Codes: ***: 0.01, **: 0.05, *: 0.1</i>				

Industry 322



Industry 331



Industry 321

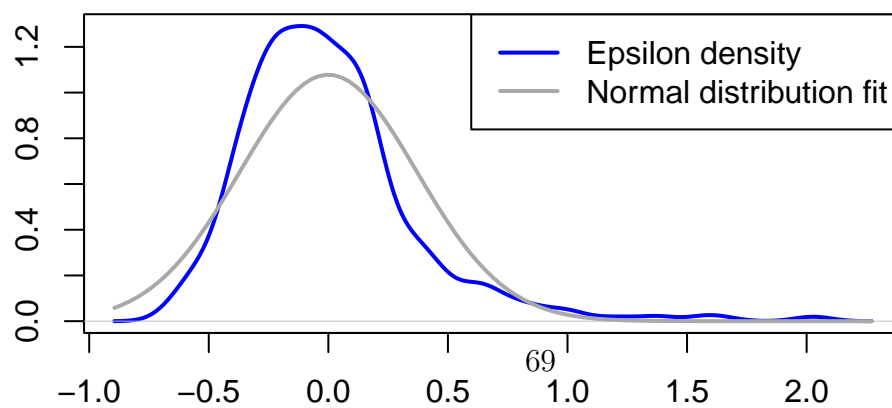


Table 12: DiD on Sales Taxes by Industry. The Control Group is Corporations in 1983 (1982).

Dependent Variable:		Sales Tax Rate			
Industry	311	312	322	381	
Tax Change	exempt		increased		
Model:	(1)	(2)	(3)	(4)	
<i>Variables</i>					
1984	-0.0001	-0.0007***	0.0351***	-0.0018**	
	(8.27×10^{-5})	(5.57×10^{-5})	(0.0012)	(0.0007)	
1985	-0.0005***	-0.0017***	0.0408***	-0.0014*	
	(0.0002)	(0.0002)	(0.0017)	(0.0007)	
1986	-0.0005**	-0.0026***	0.0382***	0.0017	
	(0.0002)	(0.0002)	(0.0020)	(0.0011)	
Non-Corp.	0.0015	-0.0020***	0.0009	0.0076	
	(0.0008)	(0.0006)	(0.0026)	(0.0056)	
1984 \times Non-Corp.	-0.0004**	-0.0001	-0.0038***	0.0020**	
	(0.0001)	(0.0002)	(0.0012)	(0.0007)	
1985 \times Non-Corp.	-0.0003	0.0018***	-0.0021	0.0101***	
	(0.0003)	(0.0003)	(0.0018)	(0.0006)	
1986 \times Non-Corp.	8.2×10^{-5}	0.0036***	0.0013	0.0083***	
	(0.0002)	(0.0001)	(0.0021)	(0.0011)	
<i>Fixed-effects</i>					
plant	Yes	Yes	Yes	Yes	
<i>Fit statistics</i>					
Observations	6.074	1.207	4.445	2.678	

In the case of Colombia, θ is their juridical organization. Assume firms choose their θ before the start of the period. I'll come to this decision later, but for now, it follows that $q(e|\theta) = q(e)$. In other words, firms cannot affect their detection probability when choosing inputs.

The first-order conditions now yield the following. For deductible flexible inputs, the optimality condition remains the same as Equation ??,

$$G_M(M_{it}, L_{it}) \exp(\omega_{it}) \mathcal{E} = \frac{\rho_t}{P_t} \quad (10)$$

However, for non-deductible flexible inputs, the sales tax induces a distortion in the optimality condition.

$$G_L(M_{it}, L_{it}) \exp(\omega_{it}) \mathcal{E} = \frac{w_t}{(1 - \tau)P_t} \quad (11)$$

Finally, the optimality condition for tax evasion becomes,

$$[1 - q(e_{it})] \frac{\rho_t}{P_t} \tau - q(e_{it}) \kappa'(e_{it}) = q'(e_{it}) \left[\frac{\rho_t}{P_t} \tau e_{it} + \kappa(e_{it}) \right] \quad (12)$$

What can we conclude? First, if the production function is a Cobb-Douglas, then we can independently solve for each input. In this case, changes in VAT or CIT would not affect the optimality condition of deductible flexible inputs because it would not affect the relative prices. Therefore, changes in the consumption of deductible flexible inputs due to changes in VAT or CIT can only be explained by an increase in the incentives to evade taxes by overreporting.

On the other hand, an increase in the VAT or CIT would increase the relative prices of non-deductible flexible inputs, leading to a decrease in their consumption.

Alternatively, if non-deductible flexible inputs become deductible the distortion in the optimality condition induced by the VAT and CIT would be eliminated. Eliminating the distortion would lead to a reduction of the relative prices inducing a higher consumption of these inputs.

Second, if the production function is not Cobb-Douglas, then the firm has to solve the system of equations. Theoretically, the changes in the relative prices of the non-deductible flexible inputs might affect the consumption of the deductible flexible inputs depending on whether these inputs are complements or substitutes.

Empirically, however, if I can observe deductible inputs separately from non-deductible inputs, I can still be able to run the share regression to recover the input elasticity of output. The observed consumed non-deductible inputs will capture the changes in relative prices.

If I want to run the share regression using the non-deductible flexible inputs, I have to be careful to account for the distortion induced by the changes in taxes. If I observe the sales taxes, I can still be able to control for changes in sales taxes and run this regression.

Theoretically, we would have,

$$\begin{aligned}\ln\left(\frac{\rho_t M_{it}}{P_t Y_{it}}\right) &= \ln D_M^\varepsilon(M_{it}, L_{it}) - \varepsilon_{it} \\ \ln\left(\frac{w_t L_{it}}{(1-\tau)P_t Y_{it}}\right) &= \ln D_L^\varepsilon(M_{it}, L_{it}) - \varepsilon_{it}\end{aligned}$$

Note that while we can use the gross revenue of sales for deductible flexible inputs, we would have to use the net of tax sales revenues for non-deductible flexible inputs.

More importantly, note that the practitioner observes only $\ln M^* = \ln M + e$ but not M . In section Section ??, I show how to recover these functions.

13 Do Corporations in Colombia have different technologies than other juridical organizations?

An implicit assumption in the identification strategy is that the subset of non-evaders has the same common technology as evaders. In the case of Colombia, this implies that Corporations have the same common technology as Proprietorships, Limited Companies, and Partnerships. However, we can think that firms with better technology are self-selecting into corporations, and thus, firms with low-performance technology will be mislabeled as evaders.

The key question is how firms are choosing their juridical organization, and in particular, how firms are choosing to become corporations. I argue that choosing the juridical organization is a result of the capital needs of the firm and it is affected by the preferences of the owners over their corporate liability and their social connections.

From the definitions of the juridical organizations, we can expect corporations to be the largests in terms of capital, followed by LLCs and Partnerships, and Proprietorships to be smallest. The reason is simply that more people can participate by contributing their capital to the firm. This ordering in turn would suggest a growth path for the firms.

Turning to the data, I find some firms change their juridical organization type. I build a juridical organization transition matrix from the previous year to the next one at the firm level. Table ?? shows the transition matrix. Although the transition matrix suggests that firms mostly stay in their juridical organization, a growth trend emerges. It looks like

Table 13: Transition Matrix

	Proprietorship	Partnership	Ltd. Co.	Corporation	Total
Proprietorship	94.8	0.7	4.5	0.1	4404
Partnership	0.5	92.7	5.6	1.3	1241
Ltd. Co.	0.2	0.1	98.6	1.0	22154
Corporation	0.0	0.1	0.5	99.4	7805

proprietorships turn into LLCs, and LLCs into Corporations.

Looking into the capital distributions of proprietorships, LLCs, and corporations we find that corporations are the largest, followed by LLCs, and proprietorships are the smallest Figure ?? . However, there are considerable overlaps. The same is true for revenues (in logs) Figure ?? . This is not surprising, as the need for larger capital increases, the more people would need to participate.

However, when we look at the frequency distributions of the capital over revenue ratio in logs, the distributions perfectly overlap Figure ?? . If corporations had different technologies we would expect to see two different distributions. In particular, the distribution of corporations would have been farther to the right than the distributions of proprietorships and LLCs.

These patterns remain even after controlling for industry, metropolitan area, and year Table ?? .

13.0.1 Model of firm (capital) growth (sketch)

A limited liability company looking to acquire more capital had three options.

First, partners can increase their capital participation, using personal wealth or through a

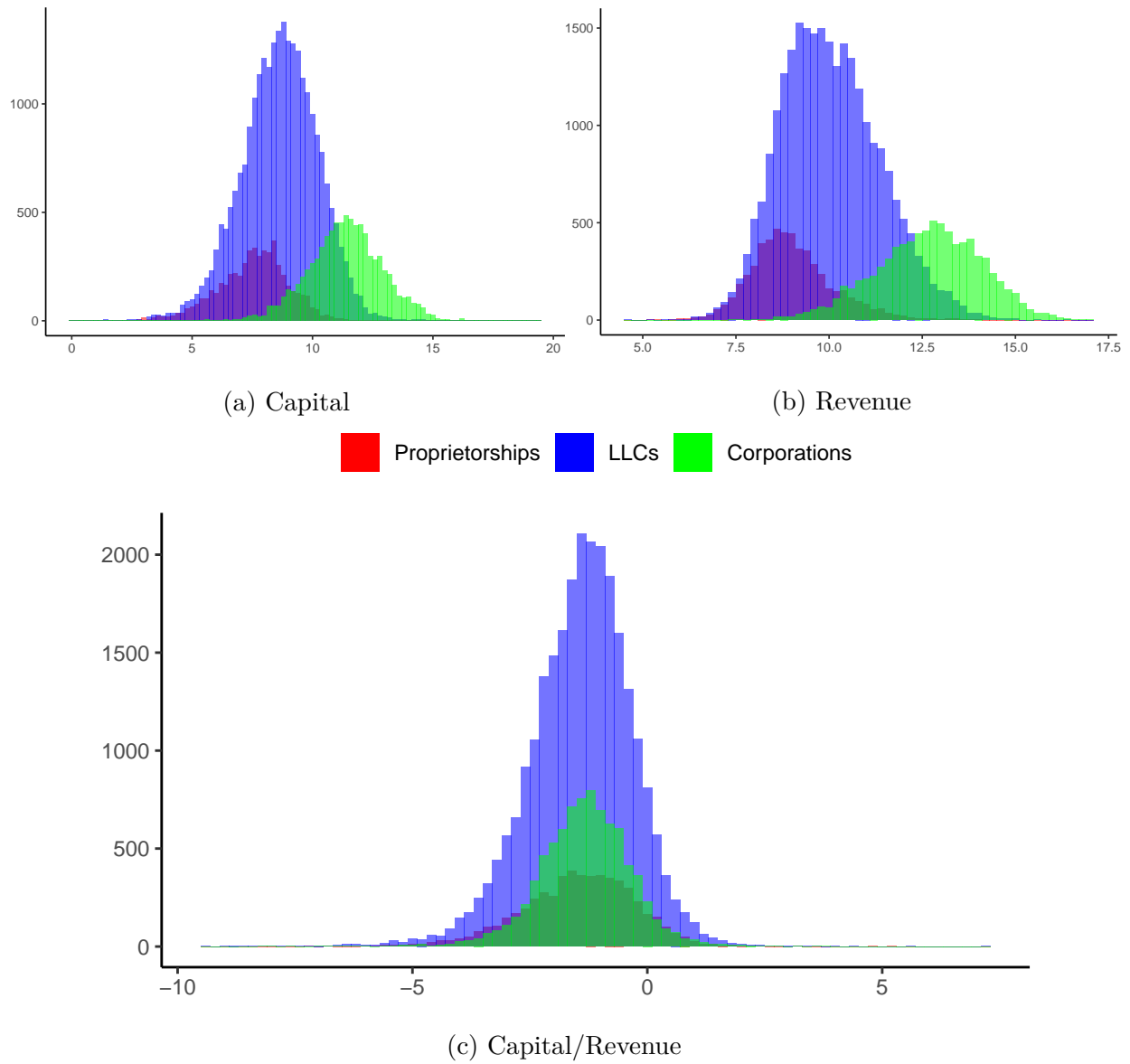


Figure 4: Frequency histograms of capital, revenue and capital/revenue (in logs) of Corporations, LLCs, and Proprietorships.

Table 14: Testing Differences in the log of Capital, Revenue, and Capital/Revenue for Corporations and Other Juridical Organizations

Dependent Variables:	log(capital)	log(sales)	log(capital/sales)
Model:	(1)	(2)	(3)
<i>Variables</i>			
LLC	-2.446*** (0.0743)	-2.424*** (0.0639)	-0.0212 (0.0601)
Proprietorship	-3.346*** (0.1672)	-3.372*** (0.1220)	0.0265 (0.0897)
<i>Fixed-effects</i>			
Industry	Yes	Yes	Yes
Metro Area	Yes	Yes	Yes
Year	Yes	Yes	Yes
<i>Fit statistics</i>			
Observations	39,628	39,628	39,628
R ²	0.44574	0.51147	0.12003
Within R ²	0.31430	0.39972	0.00025

Clustered (Industry & Year) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

bank credit. This option however will increase the partners' liability. The more capital I bring the greater my liability is.

Second, the LLC might increase its capital by inviting additional partners. As the firm increases its capital, inviting more partners will become more difficult. The funding partners risk losing control of their firm, as more people participate in the firm's management and decision-making.

Third, the firm can incorporate. This will bring capital to the firm. Anyone in the market for the shares of the firm can participate. Shares can be traded. A CEO reporting to the shareholders might be appointed. Liability is limited to shares.

The decision to incorporate is a decision on how to acquire capital. Firms still need to consider the stream of future productivity shocks, but the decision also depends on other unobservables such as risk-aversion (increasing the partners' liability), the partners' ability to convince more partners to join the LLC, and preferences over the control of the firm's management.

Assume there's an unobserved scalar ζ_i that captures all of the owners' heterogeneity in preferences over liability risk, access to networks of potential investors, and preferences over the control of the firm's management. Let a higher value of ζ capture less risk aversion, a wider network of potential investors, or a higher preference for retaining control over the firm. Consider the case in which two LLCs firms are looking to optimally increase their capital by the same quantity, but the owners are different in ζ . In particular, let the owner of firm 1 have a higher $\zeta_1 > \zeta_2$ than the owner of firm 2. The firms also have the same technology and productivity shocks. The owner of firm 1 ζ_1 would be able to convince additional investors to join their LLC, but the owner 2 ζ_2 would not. ζ_2 owner would have

to choose to incorporate to get the capital for his firm or forgo growth.

The 20 pp difference in CIT rate between LLCs and Corporations might have deterred firm growth. The CIT was homogenized to 30% in 1986. What was the capital misallocation due to the tax rate differential? Are more firms incorporating after the reform?

13.0.2 Evasion vs. Production Technology

Production technology is industry-specific, but evasion technology is the same across industries.

Take the face value of the regressions and assume, without conceding, that corporations have different technologies. This will imply that corporations have technologies that are consistently 15% more productive than non-corporations regardless of the industry. In other words, it does not matter if the firms produce canned food, shoes or furniture, corporations have technologies consistently more productive. However, this is difficult to believe because there could be industries in which such an improvement in technology is difficult to imagine. For example, coffee grain mills (food products), steel foundries (steel basic products), plastic molding and extrusion (plastic products), or cement (non-metallic mineral products).

However, these cross-sectional differences across industries could be better explained by the evasion technology. We would expect the evasion technology to be the same across industries. It does not matter what I produce and what technology my firm employs, to overreport inputs all that is needed is a fake invoice.

13.0.3 Inputs Cost Share of Revenues

The evidence suggests that Corporations have similar technologies to the rest of the firms. I look at the share of revenues of different input costs. In doing so, I consider the incentives

generated by the fiscal environment in Colombia. Firms had the lowest incentives to evade taxes by misreporting capital, consumed energy, and skilled labor. The cost share of revenue of these inputs suggests that Corporations and the rest of the firms had the same technologies.

13.0.3.1 Capital

Firms could not deduct capital goods from sales taxes. However, firms of capital-intensive industries might have had incentives to underreport capital because capital was used to set the minimum income tax. Income tax could not be less than 8 percent of the capital.

13.0.3.2 Energy

Before the electrical energy reform in 1994, the power market was almost exclusively supplied by public companies with negligible participation from private firms. The public companies were inefficient and suffered financial distress due to poor management and low tariffs set by political actors. Two major blackouts in 1983 and 1992-1993 led to the 1994 reform that opened the sector to private participation.

It is unlikely that firms overreported energy, as they have to purchase from a Corporation or a public company. Both of those organizations had no incentives to cooperate and provide fake invoices.

In the data, energy sold by corporations accounted on average for 73% of the total energy sold, but only 1.7% of the total energy purchased. In addition, corporations sold energy at 12 times the market price on average Table ???. The high price might suggest corporations had market power in the electric energy market, however, this is unlikely to be relevant to affect any estimations as their overall market share is small.

Table 15: Electric energy market in Colombia (1981-1991).

	Sold/Purchased Price Ratio	Sold Energy (% of Total Sold Energy)	Sold Energy (%)
Corporation	11.9	70.3	
Ltd. Co.	7.4	2.1	
Other	2.5	27.6	
Partnership	1.0	0.0	
Proprietorship	1.0	0.0	

13.0.3.3 Labor

Firms might have incentives to underreport labor to evade Payroll Taxes (PRT) if the expected benefits of evading the PRT outweigh the opportunity costs of evading Corporate Income Tax (CIT) by overreporting labor costs and the expected cost of evading PRT (?).

Firms are more likely to underreport unskilled rather than skilled labor. Skilled employees are less likely to cooperate with firms in underreporting their wages. Firms might offer employees cash compensations in order for employees to accept lower reported wages or report their wages at all. The cost for employees is that these payroll taxes provide them with social benefits such as social security or public health access. These benefits are not obvious in the short run. It is more likely that unskilled labor to be short-sighted and accept to cooperate with firms to underreport their wages. Skilled workers, on the other hand, are less likely to accept these conditions and to have outside options and move if the conditions are not favorable to them.

13.0.3.4 Services and other expenditures

The Colombian dataset allows, to some extent, separating expenditures, including services,

into deductible and non-deductible expenses. In contrast to non-evading firms, evading firms should use a higher share of deductible expenses. The reason is that firms have incentives to overreport deductible expenditure to evade ICT and VAT.

The Colombian data separates firms' total expenditures into general and industrial expenditures. Industrial expenditure is defined as the indirect costs and expenditures incurred by the firm in order to perform its industrial activity. The data lists the purchase of accessories and replacement parts, fuels and lubricants consumed by the establishment, industrial work by other establishments, and third-party repairs and maintenance, among others, as industrial expenditures³. All other expenses, including insurance (excluding employee benefits) and machinery rentals, are considered general expenses.

Most services were excluded from sales taxes. Some non-excluded services in this period were insurance premiums (excluding life insurance), repair and maintenance, national and international telegrams, telex, and telephone, and rental of goods and chattels, including financial leasing.

I classify as deductible expenses the following industrial expenditures: purchase of fuels and lubricants, payments to third parties for repair and maintenance, purchase of raw materials and goods sold without transformation; and the following general expenditures: machinery rental, insurance excluding employee benefits, and water, mail and telephone expenses.

13.0.3.5 Technical note

According to the notes on the dataset, the quantity of energy consumed by firms is estimated as the difference between purchased plus generated energy and sold energy. In contrast, the

³After 1992, industrial expenditure included rent of machinery and property, payment for professional services, insurance (excluding employee benefits), water, mail, and telephone.

value of consumed energy is the difference between the value of purchased and sold energy. Consequently, using the calculated value over the calculated quantity of consumed energy ignores the quantity of generated energy and the increase in the price of sold energy by corporations.

In ?, services are defined as general expenditures minus machinery rental and interest payments. However, this approach does not include industrial expenditures which are closely linked to the production process. In the Colombian data, the intermediate consumption is defined as raw materials, purchased electric energy, and industrial expenditure. This definition is close to what is commonly defined as intermediate inputs.

13.0.3.6 Results

Table ?? shows that in terms of the capital and consumed energy share of revenue, corporations are not different from other types of juridical organizations. Capital and consumed energy are reliable indicators that Corporations have similar technologies to other firms because firms have no incentives to overreport capital or consumed energy. On the other hand, the raw materials share of revenues is 3 percent higher for non-corporations. Firms have incentives to overreport raw materials to evade sales taxes and CIT. Hence, materials are not a good reference to compare technology between Corporations and other types of firms.

Table ?? shows that the skilled labor share of revenue for corporations is not statistically different from other types of firms. On the other hand, unskilled labor is significantly lower for non-corporations. Unskilled workers are more likely to cooperate with non-corporations to underreport their employment/wages. An alternative explanation is that corporations employ more unskilled workers because they are bigger. Therefore, skilled labor is a more

reliable indicator of technology and shows that corporations have similar technology to other firms.

Table ?? also shows that the total expenditure share of revenue is 8 percent lower for non-corporations on average. On one hand, this might be due to size. Corporations tend to be larger than non-corporations. However, the composition of the expenditure matters.

?@tbl-reg-shares-3 shows that the deductible expenses share of total expenditure is 6 percent significantly higher for non-corporations. This is not the case if we classify expenditures by industrial and general expenses. Services as defined in GNR, that is general expenses minus machinery rental and interest payments, as a share of total expenditure is 5 percent higher for non-corporations. Services include deductible expenses such as insurance excluding employee benefits, and telegrams and telephone services.

In summary, looking at the inputs that firms are less likely to misreport due to tax evasion incentives, the evidence suggests that Corporations have similar technologies to the other types of firms. In addition, I find that the cost share of inputs that are likely to be misreported due to tax evasion incentives are significantly different from corporations and in the expected direction. In particular, it looks like firms overreport materials and deductible expenses, and underreport unskilled labor.

14 Intermediate Inputs

What are intermediate inputs? Intermediate inputs are flexible and in our case firms should have incentives to overreport them. Raw materials, electricity, fuels, and services are what researchers have in mind when they refer to intermediates. In the context of tax evasion in Colombia, materials are the input of interest for several reasons.

Table 16: Shares of Revenue for different inputs by Corporations and Non-Corporation.

Dependent Variables:	Capital	energy_share	Materials	fuels_share
Model:	(1)	(2)	(3)	(4)
<i>Variables</i>				
Non-Corporation	0.1524	0.0010	0.0173**	-5.62×10^{-5}
	(0.1063)	(0.0018)	(0.0072)	(0.0011)
<i>Fixed-effects</i>				
Industry	Yes	Yes	Yes	Yes
Metro Area	Yes	Yes	Yes	Yes
Year	Yes	Yes	Yes	Yes
<i>Fit statistics</i>				
Observations	42,145	42,145	42,145	42,145
R ²	0.00180	0.10438	0.23606	0.22140
Within R ²	5.74×10^{-5}	0.00022	0.00147	8.62×10^{-7}

Clustered (Industry & Year) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 17: Shares of Revenue for different inputs by Corporations and Non-Corporation.

Dependent Variables:	Skilled Labor	Unskilled Labor	services_share	repair_maint_share
Model:	(1)	(2)	(3)	(4)
<i>Variables</i>				
Non-Corporation	0.0134	0.0261***	-0.0324***	-0.0011
	(0.0201)	(0.0062)	(0.0048)	(0.0007)
<i>Fixed-effects</i>				
Industry	Yes	Yes	Yes	Yes
Metro Area	Yes	Yes	Yes	Yes
Year	Yes	Yes	Yes	Yes
<i>Fit statistics</i>				
Observations	42,145	42,145	42,145	42,145
R ²	0.00192	0.36301	0.02607	0.07357
Within R ²	9.65×10^{-6}	0.01266	0.00539	0.00095

Clustered (Industry & Year) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Study	Intermediates
Levinsohn and Petrin (2003)	Materials, Electricity, and Fuels
Gandhi, Navarro, & Rivers (2020)	Materials, Energy (Electricity and Fuels), and Services

First, firms have incentives to overreport production expenses to evade CIT, but only VAT deductible intermediates reduce their VAT burden. In fact, Colombia used system of refunds to companies with VAT tax balances in their favor. Raw materials are VAT deductible during the whole period of our data, 1981-1991. In addition, materials are the greater share of the firm's expenses among the other intermediate inputs.

Electricity was mainly provided by government companies during this period. Even though there were private participation in the market, the data shows that corporations sold most of the private electricity. In our paper, we argue that Corporations are the firms with the lowest incentives to misreport their commercial activity. Therefore, it could have been relatively riskier to fake an electric bill than other inputs. Why? Because any electricity bill not coming from a Corporation or a Government company might look suspicious. In addition, it is unclear that firms could deduct from their taxes their electric bill. Finally, electricity is a small share of the firm's expenses in the data.

Fuels consumed in production were deductible however these represent a non-significant share of the firms' expenses.

Finally, only repair and maintenance services were taxed from the beginning during this period. The authority gradually introduced services they could control easily like telephone and telecommunication, insurances, airline tickets, and parking, among others. The rest of

the services look more like fixed costs and not like intermediate inputs for production, or at least they do not necessarily increase with production. Take for instance, telephone bills. They might increase if an increase of output comes from handling more clients, but they might keep the same and production might increase if current clients increase their volume of demanded products.

15 Industries

Given that I'll focus on materials, the industries I pick would have to make sense. Meaning, that firms should have nontrivial incentives to overreport raw materials to evade taxes and raw materials should represent a nontrivial share of their total production expenses.

To start, unprocessed primary goods such as mining, agricultural, fishing, and forestry were exempt of sales taxes. If the main raw materials of an industry is a primary good, then firms in that industry would have no incentive to overreport their raw materials because they are not paying sales taxes on their raw material that could be credited towards the taxes they have to pay for their sales. Industries like concrete and clay used in construction (369-Non-Metallic Minerals) and leather products (323-Leather tanning and dyeing, Fur skins) might fall into this category.

For other industries, few specialized suppliers provide their main raw materials. These firms might have to import a non-trivial share of their materials. It would be highly suspicious to fake invoices for raw materials that come from few known local and international suppliers. For example, plastic pellets the main raw material in injection molding and plastic products (356) are supplied internationally by a handful of suppliers located in the US and Europe. Likewise, industries exempt of sales tax would have relative lower incentives to evade than

non-exempt. In Colombia, firms producing foodstuffs (311 and 312), drugs, textbooks, transportation equipment (384), agricultural machinery (382-Non-Electrical Machinery including Farm Machinery), and equipment (383-Electrical Equipment) were exempt of sales taxes since 1974. Exemption to Agricultural machinery and transportation equipment was eliminated in 1984. Almost half-way through the data.

(In addition, exporters were exempted from sales taxes.)

Lastly, it would be better to avoid industries selling a significant share of their output directly to consumers and retailers. Tax evasion through underreporting of sales severely affected Colombia during this time (?). It has been documented elsewhere that revenue underreporting is most significant for firms selling directly consumers. Why? Firms might offer consumers a discount if they do not request an invoice. Since the consumer is the final payer of the sales tax, they are more like to cooperate. On the other hand, firms selling to other firms face counterincentives. The firm buying the product is interested in having an invoice to report the sales tax pay to reduce the balance of the sales tax they collect when selling. In addition, during this period there was a tendency to create fictitious distributors to reduce the tax base.

A report based on a sample from that period in ? reveals that industries 322 (Wearing Apparel — 67.5%), 324 (Footwear — 49%), 342 (Publishing — 53.9%), 369 (Non-Metallic — 50%), and 390 (Other Manufacturing — 61.3%) sell half or more of their output to retailers and directly to consumers.

Table 19: Distribution of Sales from Manufacturing Firms, by type of Purchaser (in percentage) (?)

SIC	Total no. of firms	% to retailers	% to public	% retailers and public combined	% gov- ernment	% whole- salers
312	25	24.16	15.72	39.88	1.20	58.92
313	3	-	29.33	29.33	0	70.67
321	8	15.06	0.11	15.17	1.17	83.67
322	2	52.92	14.58	67.50	0	32.50
324	5	45.00	4.00	49.00	0	51.00
332	8	10.85	20.15	31.00	5.92	63.08
341	6	5.00	0	5.00	0	95.00
342	9	19.44	34.44	53.89	7.22	38.89
351	5	15.20	6.00	21.20	2.80	76.00
352	8	37.81	5.50	43.31	4.47	52.22
353	3	5.67	5.00	10.67	0	89.33
355	5	21.00	0	21.00	0	79.00
362	1	10.00	0	10.00	0	90.00
369	8	21.25	28.75	50.00	12.88	37.13
372	3	10.00	0	10.00	1.67	88.33
381	22	12.41	14.73	27.14	2.50	70.36
382	20	11.20	30.00	41.20	9.00	49.80
383	15	13.87	7.13	21.00	9.07	69.93
384	7	0	34.57	34.57	0	65.43
390	3	58.00	3.33	61.33	3.33	35.33

SIC	Total no. of firms	% to retailers	% to public	% retailers and public combined	% gov- ernment	% whole- salers
356	11	15.55	6.00	21.55	3.18	75.27

16 Identification Strategy

Because the firms' optimization decisions depend on the fiscal environment, the identification strategy should be motivated by the fiscal environment Γ . In particular, the identification strategy will be as good as how well we can tell apart a subset of firms that have the highest incentive to not evade. For example, in the case of Spain, the firms above the revenue LTU threshold. In the case of Colombia, the corporations.

Assumption 16.1: Non-Evaders

Based on the fiscal environment Γ , the researcher can identify a subset of firms $\theta_i \in \Theta^{NE} \subset \Theta$ that does not evade taxes by overreporting inputs.

For those firms, then $\mathbb{E}[e_{it} | \theta_i \in \Theta^{NE}] = 0$

In addition, I impose the following timing assumption.

Assumption 16.2: Independence

Firms choose overreporting e_{it} *before* the output shock ε_{it}

Assumption 10.2 implies that input overreporting is independent of the current period output shock, $e_{it} \perp \varepsilon_{it}$. In the literature is not rare to assume that the output shock is not part of the information set of the firms, $\varepsilon_{it} \notin \mathcal{I}_t$ (?). Timing and information set assumptions are not uncommon for identification strategies in production functions and demand estimation (??).

16.1 Identifying the production function parameters

Assume the production function is Cobb-Douglas, $G(M_{it}, K_{it}, L_{it}) \exp(\omega_{it} + \varepsilon_{it}) = M_{it}^\beta K_{it}^{\alpha_K} L_{it}^{\alpha_L} \exp(\omega_{it} + \varepsilon_{it})$.

The econometrician observes output, capital, labor, and prices of intermediate inputs and output $\{Y_{it}, K_{it}, L_{it}, \rho_t, P_t\}$, but she only observes overreported intermediate inputs, $M_{it}^* = M_{it} \exp(e_{it})$ ⁴.

Then, for the case of Colombia, equation ?? applies since the type of firms is the juridical organization and the non-evaders are Corporations. By multiplying both sides by the intermediate inputs and dividing by the output, we get

$$\begin{aligned} \ln \left(\frac{\rho_t M_{it}}{P_t Y_{it}} \right) &= \ln \beta + \ln \mathcal{E} - \varepsilon_{it} \\ &\equiv \ln D^\mathcal{E} - \varepsilon_{it} \end{aligned} \tag{13}$$

$$\ln(M_{it}^*) = \ln(M_{it}) + e_{it}$$

$$\ln \left(\frac{\rho_t M_{it}^*}{P_t Y_{it}} \right) = \ln \beta + \ln \mathcal{E} - \varepsilon_{it} + e_{it}$$

where, $\mathcal{E} \equiv \mathbb{E}[\exp(\varepsilon_{it}) | \mathcal{I}_{it}] = \mathbb{E}[\exp(\varepsilon_{it})]$.

We can use Equation ?? and assumption 10.1 to recover the production function parameter

β

$$\mathbb{E} \left[\ln \left(\frac{\rho_t M_{it}^*}{P_t Y_{it}} \right) \middle| \Theta^{NE} \right] = \ln D^\mathcal{E} \tag{14}$$

⁴Note we can always rewrite $M^* + e = M^* \exp\{a\}$, then $\exp\{a\} = \frac{e}{M^*} + 1$.

The constant \mathcal{E} is also identified (?),

$$\mathcal{E} = \mathbb{E} \left[\exp \left(\ln D^{\mathcal{E}} - \ln \left(\frac{\rho_t M_{it}^*}{P_t Y_{it}} \right) \right) | \theta^{NE} \right] = \mathbb{E} [\exp(\varepsilon_{it}) | \theta^{NE}] = \mathbb{E}[\exp(\varepsilon_{it})] \quad (15)$$

and, thus, the output elasticity of input, β , is also identified,

$$\beta = \exp \left(\ln D^{\mathcal{E}} - \ln \mathcal{E} \right). \quad (16)$$

16.2 Identifying Tax Evasion

Having recovered both the flexible input elasticity, β , and the constant \mathcal{E} , for all firms, I can form the following variable using observed data.

$$\begin{aligned} \mathcal{V}_{it} &\equiv \ln \left(\frac{\rho_t M_{it}^*}{P_t Y_{it}} \right) - \ln \beta - \ln \mathcal{E} \\ &= -\varepsilon_{it} + e_{it} \end{aligned}$$

Tax evasion therefore can be recovered up to an independent random variable. We can do better however. By assumption 10.2, the tax evasion, ε_{it} , is independent of e_{it} . Note that, from Equation ??, we also recovered $f_{\varepsilon}(\varepsilon)$ the distribution of ε . By using deconvolution methods, I can recover the tax evasion apart from the output shock.

In particular, from probability theory,

Definition 16.1.

Definition: Convolution

The density of the sum of two **independent** random variables is equal to the **convolution** of the densities of both addends; hence

$$h = f * m = \int f(\mathcal{Z} - \varepsilon)m(\varepsilon)d\varepsilon$$

where f is the density of \mathcal{Z} (Meister, 2009)

16.3 Identifying Productivity

Here, I show how to recover the rest of the parameters of the production function, including productivity and its Markov process. We can do it in several ways, depending on our object of interest.

In the literature, it is not uncommon to assume that productivity follows a Markov process. That is,

$$\omega_{it} = h(\omega_{it-1}) + \eta_{it} \tag{17}$$

Let $\ln X_{it} = x_{it}$. With Equation ??, we can form the following observed variable,

$$\begin{aligned} \mathcal{W}_{it} &\equiv y_{it} - \beta(m_{it}^* - \mathcal{V}_{it}) \\ &= y_{it} - \beta(m_{it} + e_{it} - e_{it} + \varepsilon_{it}) \\ &= y_{it} - \beta m_{it} - \beta \varepsilon_{it} \\ &= \alpha_K k_{it} + \alpha_L l_{it} + \omega_{it} + (1 - \beta)\varepsilon_{it} \end{aligned} \tag{18}$$

Here, intuitively, we are trading unobservables, the unobserved tax evasion for the output shock, just to make our lives easier. We could have use use directly m^* since we know the distribution of tax evasion e , which is equivalent to the distribution of $f_{m^*|m}(m^*|m)$. Also

note that is it not necessary to learn tax evasion to recover productivity. This is useful if a practitioner is only interested in productivity and not on tax evasion.

By replacing ω_{it} from Equation ?? in Equation ??, we get

$$\mathcal{W}_{it} = (1 - \beta)\varepsilon_{it} + \alpha_K k_{it} + \alpha_L l_{it} + h(\mathcal{W}_{it-1} - (1 - \beta)\varepsilon_{it-1} - \alpha_K k_{it-1} - \alpha_L l_{it-1}) + \eta_{it} \quad (19)$$

If $h(\xi) = \delta_0 + \delta_1 \xi$, to learn α and h , we need to find instruments Z such that,

$$\mathbb{E}[\eta_{it} + (1 - \beta)\varepsilon_{it} + \delta_1(1 - \beta)\varepsilon_{it-1} | Z] = 0. \quad (20)$$

Table ?? displays candidates for Z

Table 20: Orthogonality by Residuals

	Orthogonality
η_{it}	$k_{it}, l_{it}, \varepsilon_{it}, k_{it-1}, l_{it-1}, \varepsilon_{it-1}, \omega_{it-1}, \mathcal{W}_{it-1}, m_{it-1}, m_{it-1}^*$
ε_{it}	$k_{it}, l_{it}, \omega_{it}, m_{it}, m_{it}^*, \eta_{it}, k_{it-1}, l_{it-1}, \varepsilon_{it-1}, \omega_{it-1}, \mathcal{W}_{it-1}, m_{it-1}, m_{it-1}^*$
ε_{it-1}	$k_{it}, l_{it}, \omega_{it}, \varepsilon_{it}, m_{it}, m_{it}^*, \eta_{it}, k_{it-1}, l_{it-1}, \omega_{it-1}, \mathcal{W}_{it}, m_{it-1}, m_{it-1}^*$

The possible instruments are $Z_{it} = \{k_{it}, l_{it}, k_{it-1}, l_{it-1}, m_{it-1}^*, \mathcal{W}_{it-2}\}$.

During estimation, I use labor and capital as instruments for themselves $Z_{it} = \{k_{it}, l_{it}\}$, and the constant and the lag of observed overreported intermediates as instrument for the AR(1) productivity Markov process $h(\cdot)$.

Once the paramaters of the production function are identified, we can use the following observed variable to recover ω_{it}

$$\begin{aligned}
\widetilde{\mathcal{W}}_{it} &\equiv \mathcal{W}_{it} - \alpha_K k_{it} - \alpha_L l_{it} \\
&= \omega_{it} + (1 - \beta)\varepsilon_{it}
\end{aligned} \tag{21}$$

16.4 Identification with two Flexible Inputs

Suppose now that M_{it} and L_{it} are flexible inputs, but K_{it} is not. Furthermore, M_{it} is deductible and L_{it} is not. Under a Cobb-Douglas production function, Equation ?? and Equation ?? become,

$$\begin{aligned}
\ln \left(\frac{\rho_t M_{it}}{P_t Y_{it}} \right) &= \ln \beta + \ln \mathcal{E} - \varepsilon_{it} \\
&= \ln D_M^\mathcal{E} - \varepsilon_{it}
\end{aligned} \tag{22}$$

$$\begin{aligned}
\ln \left(\frac{w_t L_{it}}{(1 - \tau) P_t Y_{it}} \right) &= \ln \alpha_L + \ln \mathcal{E} - \varepsilon_{it} \\
&= \ln D_L^\mathcal{E} - \varepsilon_{it}
\end{aligned} \tag{23}$$

Note that firms do not have incentives to overreport non-deductible flexible inputs. So the observed non-deductible flexible inputs are the true inputs consumed in production. For the deductible flexible inputs, we still observe the overreported inputs, $M_{it}^* = M_{it} \exp(e_{it})$.

So, Equation ?? becomes,

$$\ln \left(\frac{\rho_t M_{it}^*}{P_t Y_{it}} \right) = \ln D_M^\mathcal{E} - \varepsilon_{it} + e_{it} \tag{24}$$

The constant $\ln D_L^\mathcal{E}$ is identified by a linear regression from Equation ??. Likewise, using assumption 10.1, we can identify the constant $\ln D_M^\mathcal{E}$ by running a linear regression of Equation ?? for the non-evaders.

To identify the elasticities from the constant \mathcal{E} , let $\hat{\varepsilon}_{it}^M$ and $\hat{\varepsilon}_{it}^L$ be the residuals of the regressions of Equation ?? and Equation ??, respectively. Then, we can form the following observed variable,

$$\varepsilon_{it} = \begin{cases} \lambda_{it}\hat{\varepsilon}_{it}^M + (1 - \lambda_{it})\hat{\varepsilon}_{it}^L & \text{if } i \text{ is a Non-evader} \\ \hat{\varepsilon}_{it}^L & \text{otherwise} \end{cases} \quad (25)$$

where λ_{it} is a weight that can be chosen by the practitioner. For example, it can be set to the share of the cost of both flexible inputs in the total cost of production, $\lambda_{it} = \frac{\rho_t M_{it}^*}{\rho_t M_{it}^* + w_t L_{it}}$. In this way, we would have a time-firm specific weight that captures the relative importance of each flexible input in the production process. alternatively, we can have time or firm fixed weights, $\lambda_t = N^{-1} \sum_i \lambda_{it}$ or $\lambda_i = T^{-1} \sum_t \lambda_{it}$, respectively.

Another option is to use the optimal weight to minimize the variance of ε_{it} ,

$$\lambda = \arg \min_{\lambda} \mathbb{E} [\varepsilon^2 | \Theta^{NE}]$$

$$\begin{aligned} \mathbb{E} [\varepsilon^2 | \Theta^{NE}] &= \lambda^2 \mathbb{E} [(\hat{\varepsilon}_{it}^M)^2 | \Theta^{NE}] + (1 - \lambda)^2 \mathbb{E} [(\hat{\varepsilon}_{it}^L)^2 | \Theta^{NE}] \\ &\quad + 2\lambda(1 - \lambda) \mathbb{E} [\hat{\varepsilon}_{it}^M \hat{\varepsilon}_{it}^L | \Theta^{NE}] \end{aligned}$$

Deriving the first order condition, we get

$$2\lambda \mathbb{E} [(\hat{\varepsilon}_{it}^M)^2 | \Theta^{NE}] + 2(1 - 2\lambda) \mathbb{E} [\hat{\varepsilon}_{it}^M \hat{\varepsilon}_{it}^L | \Theta^{NE}] - 2(1 - \lambda) \mathbb{E} [(\hat{\varepsilon}_{it}^L)^2 | \Theta^{NE}] = 0$$

Finally, solving for λ gives us the optimal weight,

$$\lambda^* = \frac{\mathbb{E} \left[\left(\hat{\varepsilon}_{it}^L \right)^2 | \Theta^{NE} \right] - \mathbb{E} \left[\hat{\varepsilon}_{it}^M \hat{\varepsilon}_{it}^L | \Theta^{NE} \right]}{\mathbb{E} \left[\left(\hat{\varepsilon}_{it}^M \right)^2 | \Theta^{NE} \right] + \mathbb{E} \left[\left(\hat{\varepsilon}_{it}^L \right)^2 | \Theta^{NE} \right] - 2 \mathbb{E} \left[\hat{\varepsilon}_{it}^M \hat{\varepsilon}_{it}^L | \Theta^{NE} \right]}. \quad (26)$$

This might work only if the covariance of the residuals is less than the variance of any residual. In theory, the covariance cannot be larger than the variance of either residual. In that case, it means the both residuals are good estimates of the output shock ε_{it} and we can use either of them to recover the output shock ε_{it} .

Then, we can recover $\mathcal{E} = \mathbb{E}[\exp(\varepsilon_{it})]$ and the elasticities $\beta = \exp(\ln D_M^\mathcal{E} - \ln \mathcal{E})$ and $\alpha_L = \exp(\ln D_L^\mathcal{E} - \ln \mathcal{E})$.

16.5 Identifying Tax Evasion with Two Flexible Inputs

$$\begin{aligned} \mathcal{V}_{it} &\equiv \ln \left(\frac{\rho_t M_{it}^*}{P_t Y_{it}} \right) - \ln D_M^\mathcal{E} + \varepsilon_{it} \\ &= \ln \left(\frac{\rho_t M_{it}}{P_t Y_{it}} \right) - \ln D_M^\mathcal{E} + \varepsilon_{it} + e_{it} \\ &= -\varepsilon_{it} + \varepsilon_{it} + e_{it} \\ &= e_{it} \end{aligned}$$

In the case of the two flexible inputs, we can form a firm-time specific variable that captures the tax evasion by overreporting the deductible flexible input, $\mathcal{V}_{it} = e_{it}$.

16.6 Identifying Productivity with Two Flexible Inputs

To recover α_K , we can now form the following observed variable,

$$\begin{aligned} \mathcal{Y}_{it} &\equiv y_{it} - \beta(m_{it}^* - e_{it}) - \alpha_L l_{it} - \varepsilon_{it} \\ &= y_{it} - \beta m_{it} - \alpha_L l_{it} - \varepsilon_{it} \\ &= \alpha_K k_{it} + \omega_{it} \end{aligned}$$

Then, using orthogonality of η_{it} ,

$$\mathbb{E}[\mathcal{Y}_{it}|Z_{it}] = \alpha_K k_{it} + \mathbb{E}[h(\mathcal{Y}_{it-1} - \alpha_K k_{it-1})|Z_{it}]$$

where $Z_{it} = \{k_{it}, k_{it-1}, \mathcal{Y}_{it-1}\}$.

16.7 Translog Production Function

To identify tax evasion and relaxing the assumption of a CD production function, we would need a flexible input for which firms have no incentives to overreport. Firms might face flexible inputs that they cannot deduct from their VAT or CIT, for example. If this is the case, then firms would have no incentives to overreport non-deductible flexible inputs.

Assume now that L_{it} is a non-deductible flexible input and, without loss of generality, there are only two inputs (L_{it}, M_{it}) . Let's assume the production function is now *translog*. We have,

$$\ln G(l, m) = \beta_0 m_{it} + \beta_1 m_{it} l_{it} + \beta_2 m_{it} m_{it} + \beta_3 l_{it} + \beta_4 l_{it} l_{it}$$

Then, equation Equation ?? becomes

$$\begin{aligned} s_{it}^L &= \ln(\beta_3 + 2\beta_4 l_{it} + \beta_1(m_{it}^* - e_{it})) + \ln \mathcal{E} - \varepsilon_{it} \\ &\equiv \ln D^{\mathcal{E}}(l_{it}, m_{it}^* - e_{it}) - \varepsilon_{it} \end{aligned} \tag{27}$$

where $s_{it}^L \equiv \ln\left(\frac{w_t L_{it}}{(1-\tau)P_t Y_{it}}\right)$.

Note that by assumption 10.1, $D^{\mathcal{E}}$ and the density of ε are still identified.

16.7.1 Testing for Tax Evasion

We can design a test for tax evasion by forming the analog of Equation ??,

$$\begin{aligned}
\mathcal{V}_{it}^L &= \ln D^\mathcal{E}(l_{it}, m_{it}^*) - s_{it}^L \\
&= \ln D^\mathcal{E}(l_{it}, m_{it} + e_{it}) - s_{it}^L \\
&= \ln D^\mathcal{E}(l_{it}, m_{it} + e_{it}) - \ln D^\mathcal{E}(l_{it}, m_{it}) + \varepsilon_{it} \\
&= \ln \left(\frac{D^\mathcal{E}(l_{it}, m_{it} + e_{it})}{D^\mathcal{E}(l_{it}, m_{it})} \right) + \varepsilon_{it}
\end{aligned}$$

Note that $\ln \left(\frac{D^\mathcal{E}(l, m+e)}{D^\mathcal{E}(l, m)} \right) \geq 0$ is always positive because $e_{it} \geq 0$ and $D^\mathcal{E}$ is strictly increasing in m . Thus, $\frac{D^\mathcal{E}(l, m+e)}{D^\mathcal{E}(l, m)} \geq 1$.

16.7.2 Identifying Tax Evasion

Once we recovered the parameters of Equation ??, we can invert it to solve for the true inputs m^* as a function of observed data and the output. Then, the difference between the observed and true inputs is by definition the tax evasion by overreporting.

$$\begin{aligned}
m_{it}^* &= m_{it} + e_{it} \\
&= D^{-1}(\exp(s_{it}^L - \ln \mathcal{E} + \varepsilon_{it}), l_{it}) + e_{it}
\end{aligned}$$

Again, having recovered the distribution of the output shock $f_\varepsilon(\varepsilon)$, we can use deconvolution techniques to learn the distribution of e . In particular, to learn the p.d.f of tax evasion $f_e(e)$, we can deconvolute it by

$$\int f_e(m_{it}^* - D^{-1}(\exp(s_{it}^L - \ln \mathcal{E} - \varepsilon_{it}), l_{it})) f_\varepsilon(\varepsilon) d\varepsilon$$

16.8 Non-Parametric Identification of Tax Evasion

The previous result also suggests a non-parametric identification strategy, as long as D^ε is monotonic in m . This identification strategy is analogous to ?, where the authors also require monotonicity and independence to recover a nonparametric function of m^* with nonclassical measurement error.

In our case, intuitively, if we know the density of ε and the function D^ε , the variation left is due to e , which can be recovered as long as we can vary D^ε by moving e .

To see why the non-deductible flexible input is needed to identify tax evasion consider the following. Suppose that only the input M is flexible and deductible.

$$\ln \left(\frac{\rho M}{PY} \right) = \ln D^\varepsilon(K, L, M) - \varepsilon$$

$D^\varepsilon(K, L, M)$ is still identified by assumption 10.1, however, when we form the analogous of Equation ??, we now have

$$\begin{aligned}
& \ln \left(\frac{\rho M^*}{PY} \right) - \ln D^\varepsilon(K, L, M^*) = \\
& \quad \ln \left(\frac{\rho(M+e)}{PY} \right) - \ln D^\varepsilon(K, L, M+e) \\
& = \ln \left(\frac{\rho(M+e)}{PY} \right) - \ln D^\varepsilon(K, L, M+e) \\
& \quad + \left[\ln \left(\frac{\rho M}{PY} \right) - \ln D^\varepsilon(K, L, M) \right] \\
& \quad - \left[\ln \left(\frac{\rho M}{PY} \right) - \ln D^\varepsilon(K, L, M) \right] \\
& = \ln \left(\frac{\rho M}{PY} \right) - \ln D^\varepsilon(K, L, M) \\
& \quad + \left[\ln \left(\frac{\rho(M+e)}{PY} \right) - \ln \left(\frac{\rho M}{PY} \right) \right] \\
& \quad - \left[\ln D^\varepsilon(K, L, M+e) - \ln D^\varepsilon(K, L, M) \right] \\
& = -\varepsilon \\
& \quad + \left[\ln D^\varepsilon(K, L, M+e) - \varepsilon - \ln D^\varepsilon(K, L, M) + \varepsilon \right] \\
& \quad - \left[\ln D^\varepsilon(K, L, M+e) - \ln D^\varepsilon(K, L, M) \right] \\
& = -\varepsilon(K, L, M)
\end{aligned}$$

Now, we are not able to separate the variation of ε from e .

17 Implementation

We are interested in the distribution of tax evasion e but it cannot be observed. What is observed is the contaminated version \mathcal{V} Equation ???. Evasion e and the output shock ε are independent [10.2] with probability density distributions f_e and f_ε . Then, from Definition ??

$$f_{\mathcal{V}}(\mathcal{V}) = \int f_e(\mathcal{V} + \varepsilon) f_\varepsilon(\varepsilon) d\varepsilon$$

where $f_{\mathcal{V}}$ denotes the density of \mathcal{V} .

17.1 Parametric Deconvolution

Assume a functional form for $f_{\varepsilon}(\cdot; \gamma)$ that depends on known parameters γ . Assume a known functional form for the density $f_e(\cdot; \lambda)$ that depends on unknown parameters λ . We can estimate parameters λ by

$$\hat{\lambda} = \arg \max_{\lambda} \sum_{i=1}^n \log \left(\int f_e(\mathcal{V}_i + \varepsilon; \lambda) f_{\varepsilon}(\varepsilon; \gamma) d\varepsilon \right) \quad (28)$$

Properties of MLE with unobserved scalar heterogeneity have been derived elsewhere before (??).

17.2 Non-Parametric Deconvolution by Splines

Following ?, consider the following logspline model:

$$f_{e|\Theta}(e) = \exp(s(e; \theta) - C(\theta)), \quad L < e < R$$

where, $[L, R]$ define the support of f_e ,

$$s(e; \theta) = \sum_{j=1}^{K-1} \theta_j B_j(e),$$

$\{B_j(E), j = 1, 2, \dots, K\}$ is a sequence of cubic B-spline basis functions, and

$$C(\theta) = \log \left(\int_L^R \exp(s(e; \theta)) de \right) < \infty.$$

Then $f_{e|\Theta}(e)$ is a positive density function on (L, R) . The logspline model assumes that f_e is a twice continuously differentiable function. The last basis function is not used for identifiability purposes, given that by the property of B-splines, $\sum_{j=1}^K B_j(e) = 1$ for all $e \in (L, R)$.

The log likelihood of the observed variable \mathcal{V} is

$$\begin{aligned} l_{\mathcal{V}}(\theta) &= \sum_{i=1}^n \log \left(\int f_{\varepsilon}(e - \mathcal{V}_i) f_{e|\theta}(e) de \right) \\ &= \sum_{i=1}^n \log \left(\int f_{\varepsilon}(e - \mathcal{V}_i) \exp(s(e; \theta) - C(\theta)) de \right) \\ &= \sum_{i=1}^n \log \left(\int f_{\varepsilon}(e - \mathcal{V}_i) \exp(s(e; \theta)) de \right) - nC(\theta) \end{aligned}$$

The usual maximum likelihood estimate $\hat{\theta}$ is the maximizer of $l_{\mathcal{V}}(\theta)$.

To avoid overfitting, I add a penalty term to the log-likelihood function. The penalty term is a linear combination of the squared second derivatives of the B-spline basis functions. The second derivative of the B-splines is approximated by finite differences (?).

The penalty term is given by

$$\text{Penalty}(\theta) = \lambda \cdot \|\mathbf{D}\theta\|^2$$

Intuitively, we want to penalize excessive curvature in the log-density $\log f_e(e) \approx s(e; \theta) = \sum_j \theta_j B_j(e)$, where B_j are B-spline basis functions.

The second derivative of a smooth function is a measure of curvature. Since we don't have an analytical form for $s''(e)$, we can approximate it using finite differences on the coefficients θ .

Given a vector of spline coefficients $\theta = (\theta_1, \theta_2, \dots, \theta_k)$, define the second-order differences as:

$$\Delta^2\theta_j = \theta_j - 2\theta_{j+1} + \theta_{j+2}$$

These values approximate the second derivative of the spline function at positions along the domain.

We can write all second differences as a matrix operation:

Let $D \in \mathbb{R}^{(k-2) \times k}$, then:

$$D\theta = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & -2 & 1 & \dots & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & \dots & 0 & 1 & -2 & 1 & \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_k \end{bmatrix}$$

So $D\theta$ is a vector of second differences:

$$[\theta_1 - 2\theta_2 + \theta_3, \theta_2 - 2\theta_3 + \theta_4, \dots]$$

The roughness penalty becomes:

$$\text{Penalty} = \lambda \cdot \|D\theta\|^2 = \lambda \cdot \sum_{j=1}^{k-2} (\Delta^2\theta_j)^2$$

where λ is a smoothing parameter that can be chosen based on BIC/AIC or cross-validation.

The final log-likelihood function to be maximized is:

$$l_{\mathcal{V}}(\theta) = \sum_{i=1}^n \log \left(\int f_{\varepsilon}(e - \mathcal{V}_i) \exp(s(e; \theta)) de \right) - nC(\theta) - \lambda \cdot \|D\theta\|^2$$

17.2.1 Simulations

Before deconvoluting with the actual data, I run some simulations to check the performance of my implmenetation in R.

I simulated a thousand observations of $\varepsilon \sim \mathcal{N}(0, 0.35)$ and $\log(e) \sim \mathcal{N}(-1.8, 0.95)$, and then I generated $\mathcal{V} = e - \varepsilon$. Figure ?? shows the histogram of the simulated data.

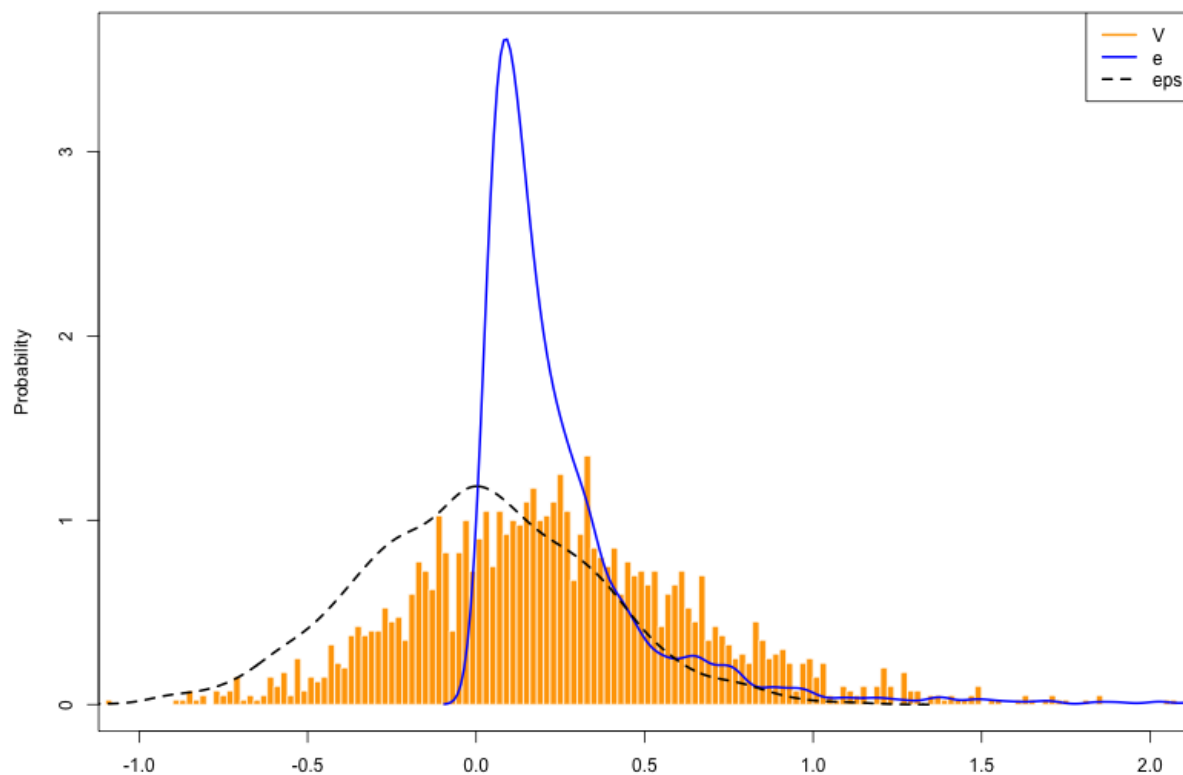


Figure 5: Simulated data

I then used the logspline model to estimate the parameters of the distribution of e . I used a B-spline basis with 10 knots. The knots were placed at equally spaced percentiles of the non-negative V observations. For the integral, I used Gauss Legendre quadrature. To ensure good coverage of the high probability region of the density, I used a recursive function to form a grid placing a higher number of nodes in the high probability region. I set the penalty parameter λ to 0.1. The estimated density is shown in Figure ??.

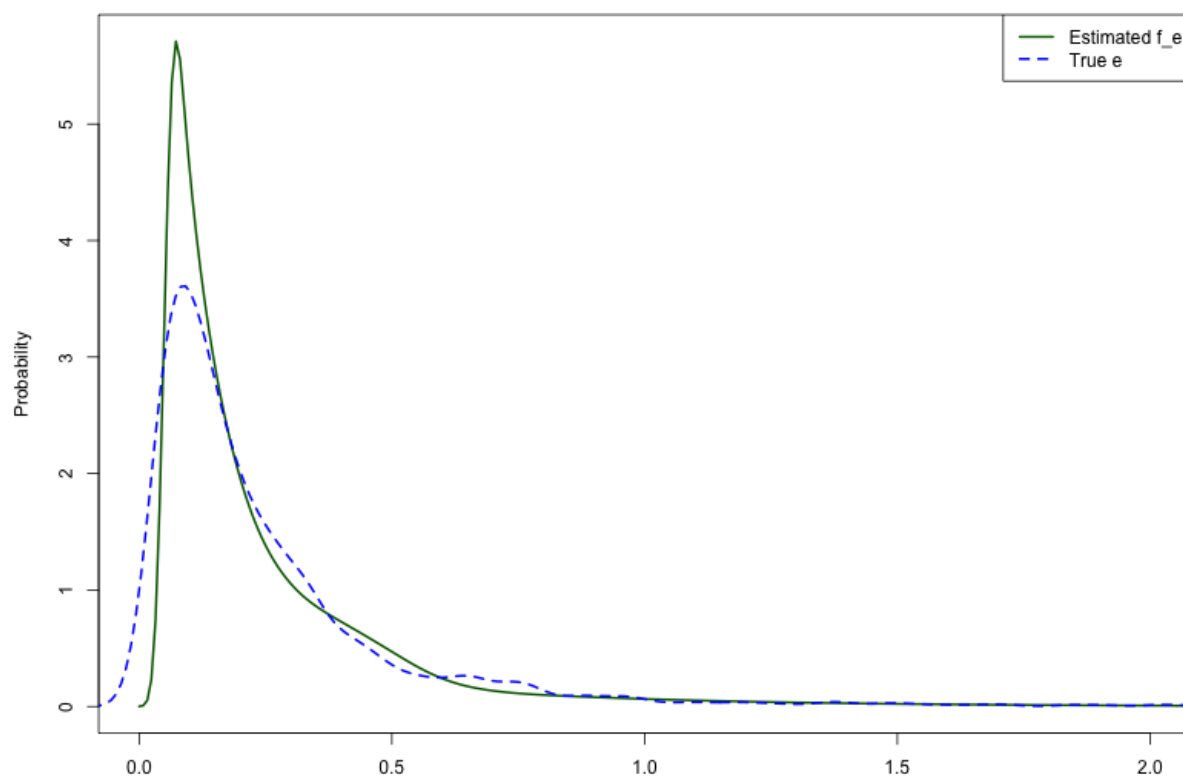


Figure 6: LogSpline Deconvolution

18 Results

19 Getting the Story Straight

Firms face incentives to evade sales taxes by overreporting their input costs. Firms overreport their cost by acquiring fake invoices to claim additional deductions of their sales taxes, maximizing after-tax profits. The higher the sales tax, the greater the incentive to evade. However, the probability of detection and the threat of penalties limit how much firms overreport.

The key incentive for firms is the **effective sales tax rate** — the amount firms ultimately pay or receive from the tax authority. The effective tax rate is calculated as the difference between the total sales tax charged by the firm on its sales and the total sales tax paid by the firm on its inputs, divided by total sales. A positive difference indicates what the firm should pay to the government. If the difference is negative, the firm receives a refund from the tax authority.

$$\begin{aligned}\tau &= \frac{\tau_0 P_t Y_{it} - \tau_1 \rho_t M_{it}}{P_t Y_{it}} \\ &= \tau_0 - \tau_1 \beta\end{aligned}$$

In Colombia, the total amount charged by the firm as sales tax depends on the statutory tax rate of each product sold domestically; **exports are exempt** from sales tax. Low statutory sales tax rates and exempt products result in lower sales tax collected by the firm and thus, a lower effective tax rate. Consequently, firms whose products are exempt from sales tax or taxed with **low statutory rates** have low incentives to evade. Likewise, as the share of their exports increases, exporter firms bear decreasing incentives to evade taxes because a larger portion of their sales is exempt from sales tax, reducing the effective sales tax rate.

On the other hand, the total amount paid by the firm as sales tax depends on the statutory tax rate of each of its inputs, including imports. **Imports**, in contrast to exports, are **not exempt**. Hence, the composition of the foreign and domestic inputs does not affect the effective sales tax rate, nor the incentives to evade. However, the tax system imposed a zero sales tax rate on **unprocessed primary inputs**, such as forestry, mining, fishing, and agriculture. Therefore, firms consuming a high share of primary inputs face a higher effective sales tax rate and thus, greater incentives to evade.

Note that the effective sales tax rate might naturally be **negative** for firms in certain industries. Before the 1983 reform in Colombia, there were four different statutory sales tax rates: 4, 10, 15 and 25 percent. Because the sales tax was applied at the product level, *the total amount paid as sales tax by the firms on its inputs could be larger than the total amount charged by the firm as sales tax on its products*. This situation would result in a negative effective sales tax rate. Firms in this situation would receive a refund from the tax authority.

Would firms with negative effective tax rates have incentives to overreport inputs to increase their refunds? Unlikely. Although the refunds had to be made within 30 days, *the procedure was complicated, and the tax authority delayed, on average, one year to process the payment*. Suspicion of fraudulent activities might further delay the process. Therefore, firms in this situation did not have incentives to overreport inputs to increase their refunds.

Whether firms in certain industries have incentives to overreport their inputs to evade sales taxes might **not** be **obvious** *ex-ante*. The tax code might be complex, with many exemptions and different statutory rates. Different taxes might generate overlapping or conflicting incentives. Available data might lack details on the products or inputs used by firms, making it difficult to identify the effective sales tax rate.

A first contribution of this paper is to provide practitioners and policymakers a way to identify industries where tax evasion through cost-overreporting might be non-trivial and can be identified from data. The simple model employed in this paper to investigate the mechanisms of tax evasion provides conditions that can be tested with data. As a result, I propose a statistical test where the null hypothesis states that firms in an industry of interest do not evade sales taxes by overreporting; the alternative states that firms do commit tax evasion Section ??.

Failing to reject the null hypothesis does not necessarily imply that firms in this industry are not overreporting. If a researcher suspects this practice on a particular industry, he would have to investigate if the assumptions of the model are not unreasonable for said industry. The main assumption comes from the production function method, even though they might differ in their productivity, evaders and non-evaders share a common technology. With this in mind, in any dataset using SIC codes, it should be considered that the SIC codes group firms with similar products, but some categories might group products that require different technologies. Furthermore, most firms are multiproducts, and might be classified under a certain SIC code because it produces more of a certain product marginally.

19.1 Showing the Evidence

31 Food & Beverages:

- 311 Food Products (Meat, Dairy, Sugar, Bakery, etc.)
- 312 Food Products (Animal feeds, Others)
- 313 Beverages (Soft drinks, Alcoholic beverages, etc.)

In the Food and Beverages industry classification, industry 313 beverages displays suspicious

Table 21: Intermediate elasticities for Corporations (Non-Evaders) and non-Corporations (Evaders) for different industries. Estimates were obtained using GNR (2020) method assuming a CD functional form. Intermediates are defined as raw materials. ε is defined as **measurement error**. The Tax Evasion Test displays the coefficient of the statistical test, where the null hypothesis states that there is no tax evasion through cost overreporting. Among the industry characteristics, Sales is the industry's market share of the total sales in Colombia between 1981 and 1991. Sales Tax Rate is the average sales tax reported rate. Exports is the average share of exports of the industry total sales, and Exporters is the share of firms exporting 10 percent or more of their total sales. Bias-corrected bootstrap confidence intervals at 95% significance level are displayed in brackets. *, **, *** denote significance at the 10%, 5%, and 1% levels (one-sided).

SIC	Elasticities		Test				Sales Tax Rate		
	Corps.	Others	Tax Ev.	N	N (Corps.)	Sales (Mkt %)	Effective	Sales	Pu
311	0.6	0.61	0.02*	888	147	23.6	-0.1	0.2	
	[0.57, 0.62]	[0.59, 0.62]	[-0.02, 0.08]						
313	0.29	0.35	0.18***	130	74	9.7	7.7	8.4	
	[0.26, 0.3]	[0.32, 0.39]	[0.07, 0.32]						
321	0.39	0.44	0.13***	470	71	7.4	6.2	8.0	
	[0.36, 0.42]	[0.42, 0.45]	[0.08, 0.24]						
351	0.34	0.37	0.09	116	68	7.5	5.9	7.0	
	[0.28, 0.37]	[0.29, 0.43]	[-0.14, 0.33]						
352	0.41	0.44	0.06**	300	100	8.5	5.2	6.8	
	[0.38, 0.43]	[0.42, 0.47]	[0, 0.16]						

Table 22: Intermediate elasticities for Corporations (Non-Evaders) and non-Corporations (Evaders) for different industries. Estimates were obtained using GNR (2020) method assuming a CD functional form. Intermediates are defined as raw materials. ε is defined as **measurement error**. The Tax Evasion Test displays the coefficient of the statistical test, where the null hypothesis states that there is no tax evasion through cost overreporting. Among the industry characteristics, Sales is the industry's market share of the total sales in Colombia between 1981 and 1991. Sales Tax Rate is the average sales tax reported rate. Exports is the average share of exports of the industry total sales, and Exporters is the share of firms exporting 10 percent or more of their total sales. Bias-corrected bootstrap confidence intervals at 95% significance level are displayed in brackets. *, **, *** denote significance at the 10%, 5%, and 1% levels (one-sided).

SIC	Elasticities		Test				Sales Tax R		
	Corps.	Others	Tax Ev.	N	N (Corps.)	Sales (Mkt %)	Effective	Sales	
31 Food & Beverages									
311	0.6	0.61	0.02*	888	147	23.6	-0.1	0.2	
	[0.57, 0.62]	[0.59, 0.62]	[-0.02, 0.08]						
312	0.58	0.57	-0.01	188	41	5.3	-0.3	0.2	
	[0.49, 0.64]	[0.51, 0.63]	[-0.11, 0.16]						
313	0.29	0.35	0.18***	130	74	9.7	7.7	8.4	
	[0.26, 0.3]	[0.32, 0.39]	[0.07, 0.32]						
32 Textile, Apparel & Leather									
321	0.39	0.44	0.13***	470	71	7.4	6.2	8.0	
	[0.36, 0.42]	[0.42, 0.45]	[0.08, 0.24]						
322	0.35	0.42	0.2***	737	22	2.5	6.5	8.1	
	[0.3, 0.4]	[0.4, 0.43]	[0.07, 0.35] 111						
323	0.56	0.47	-0.19	99	13	1.2	5.3	8.1	
	[0.5, 0.58]	[0.45, 0.5]	[0.08, 0.25]						

activity of overreporting. This finding is consistent with the model because, in contrast to 311 and 312, industry 313 is not exempt from the sales tax. In fact, the effective sales tax rate for industries 311 and 312 is negative, implying that these firms were receiving refunds from the government. In comparison, industries in 313 industry would have higher incentives to overreport because of their positive effective sales tax.

32 Textile, Apparel & Leather:

- 321 Textiles
- 322 Wearing apparel
- 323 Leather and Fur Products
- 324 Footwear

Table ?? also shows evidence for potential tax evasion by overreporting in industries 321, 322, and 324. Again, from the theory perspective, these firms face positive effective tax rates, tax rates generate incentives to evade. This is consistent with the model. Corporations in industries 321, 322, and 324 display intermediate elasticities statistically smaller than the rest of the firms and the tax evasion test display a significantly positive coefficient.

What about industry 323? The effective sales tax rate is positive but does not display evidence of evasion according to the test, moreover the coefficient of the test is negative and statistically significant as indicated by the confidence intervals.

Most likely, it is not a good idea to compare corporations with the rest of the firms in this industry. First, looking at the sample size, we note that the number of corporations in this industry is small. While this is also true for industry 324, looking more closely, we can also note that industry 323 groups classifications 3231 (Leather Tanning and Finishing), 3232 (Fur dressing and Dyeing), and 3233 (Leather products, except footwear

and wearing apparel). In contrast, industry 324, only includes 3240 (Footwear, except plastic). The two most important groups of 323 are 3231 (30% of all the firms) and 3233 (60%). However, the composition of these subindustries is different between corporations (60/40) and non-corporations (22/73).

Although technologies between small and large firms were similar at the core, the processes were significantly different which raises the question on the validity of the comparison between corporations and the other firms. During this period, small firms were typically artisanal producers. These smaller establishments relied primarily on traditional local tanning methods based on vegetable extracts such as quebracho and chestnut. The process is long, typically requiring several weeks, however the products are highly valued because their durability and the distinctive patina it acquires as it ages. Their products were destined for the local market.

In contrast, large firms used a chrome-based mineral tanning, mechanized drums and automated finishing machinery. Chromium tanning is faster and allows for mass scale consistent quality. The product is softer and water resistant. These characteristics make chrome-based tanning leather suitable for exporting to the US and European markets. However, chromium disposal requires advanced waste water treatment necessary to remove heavy metals and prevent chromium discharge into the environment. Furthermore, these firms need to comply with strict environmental regulations including regular monitoring, hazard assessment, and reporting.

The data shows that corporations are more oriented to the foreign market than the rest of the firms, with highest average share of exports (28 vs. 13%), highest share of exporters (80 vs 35%).

33 Wood & Wood Products; 34 Paper & Printing:

- 331 Wood Products except furniture (Sawmills, Containers, etc.)
- 332 Wood Furniture (Wooden furniture and fixtures)
- 341 Paper and Paper Products
- 342 Printing, publishing and allied industries

In these categories, we find that industries 331 and 342 display suspicious activity of overreporting. However, results should be taken with caution in particular for industry 331 because the sample of corporations is relatively small.

Again, failing to reject the null hypothesis of no tax evasion could only mean that cost overreporting cannot be identified with the data available using corporations as reference group.

35 Chemicals, Petroleum, Rubber & Plastic; 36 Non-Metallic Mineral Products:

- 351 Industrial Chemicals (Basic chemicals, Fertilizers, Synthetic resins and fibers)
- 352 Other Chemicals (Paints and lacquers, Drugs, Soap and cosmetics, etc.)
- 356 Other Plastic Products (not classified elsewhere)
- 369 Non-Metallic Mineral Products (Structural clay products, Cement and plaster, Other non-metallic mineral products)

I find significant evidence for industry 369 Non-Metallic Mineral Products, and some evidence for industry 352. Results should be taken with caution as both industries include different types of products, processes might vary significantly from one another. On the other hand, the sample of corporations is not small and there are observations across all subindustries, suggesting that the comparison between corporations and the rest of the firms might not be unreasonable.

Regarding industry 356, the perspicaz reader might notice that the coefficient of the tax evasion test is negative and statistically significant. However, in this category is not clear the composition of the subcategories included, as it covers other plastic products not classified elsewhere. Unlike other categories, this one only includes one subcategory, 3560 (Plastic products, not elsewhere classified). As in other industries, corporations have higher average share of exports and their share of exporters is higher than the non-incorporated firms. Does this matter? Likely. How? It is not clear.

38 Fabricated Metal Products, Machinery & Equipment; 39 Other Manufacturing:

- 381 Fabricated Metal Products (Structural metal products, Cutlery, Hand tools, etc.)
- 382 Machinery and Equipment (Agricultural machinery, Metalworking machinery, etc.)
- 383 Electrical Machinery and Apparatus (Electric motors, Generators, etc.)
- 384 Transport Equipment (Motor vehicles, Aircraft, Ships, etc.)
- 390 Other Manufacturing (Jewelry, Musical instruments, etc.)

I cannot reject the null hypothesis of no tax evasion by overreporting for these industries. It is interesting to note that agricultural machinery, transportation equipment, and equipment were exempt from sales tax since 1974. In 1984, exemptions for agricultural machinery and transportation equipment were removed.

If these results are valid, this could imply that there is a dynamic component to the tax evasion decision. Meaning that the desicion of tax evasion in the current period depends critically on the past periodl. For example, if a firm did not evade in the previous year, it might be less likely to evade in the current year. The fact that there is no evidence of tax evasion might suggest that this effect is persistent over time.

How does this dynamic decision could enter the economic model? Firms in the agricultural machinery and transportation equipment sectors started facing incentives to evade after 1984. However, they have been providing truthful information to the tax authority for over a decade. They know that the probability of detection for them is high, given that the authority can easily compare their records with previous years and detect anomalies.

19.1.1 Results Summary

I find that four out of the top five (nine of the to top twenty) industries display activities consistent with tax evasion through cost overreporting in Colombia between 1981 and 1991. Firms might overreport up to 22 percent of their costs. Consisten with the model, firms in industries with exempt products and negative effective sales tax rates display minor or no evidence of tax evasion. Furthermore, the evidence could suggest that the tax evasion decision is dynamic, as there is a lack of evidence of tax evasion in industries that lost exemption after a decade.

Of course, the results should be taken with the usual precautions. The main assumption of the model is that evaders and non-evaders share a common technology. This assumption might be not unreasonable in most cases. However, commonly available data might group firms with similar products, but the technological processes might differ.

On the other hand, failing to reject the null hypothesis of no tax evasion does not necessarily mean that firms in this industry are not overreporting; it might mean that overreporting cannot be identified with the data available at this industry level using corporations as the reference group. A researcher suspecting tax evasion in a particular industry should exhaustively investigate if the assumptions of the model are not unreasonable for said industry. In particular, she should inform herself about the industry technological processeses

[illegible]

Figure 8

Table ?? displays the results of estimating the production function parameters following the identification strategy described above. I use an AR(1) model for the Markov process of productivity. I employed $Z = \{k_{it}, l_{it}\}$ as instruments for themselves. One additional instrument to control for the endogeneity between ε_{it-1} and \mathcal{W}_{it-1} is needed in the first stage of ‘TE+GNR’. The instrument used is specified in the table Table ??.

Table 23: Comparison of different methods to estimate the bias corrected confidence intervals and the p-values. The null hypothesis states that firms do not commit tax evasion through cost-overreporting. The alternative is that they do. The first column is a difference of the means test (includes error of the estimator, but assumes independence of the samples). The different methods shown in the rest of the columns are combinations of two different procedures when sampling the bootstrap—sampling from corporations and non-corporations, or sampling full data— and two different methods to estimate the coefficient of the test— using all the firms or only the non-incorporated firms. When not fixing the number of corporations, Bayesian sampling was used to ensure there would be at least one corporation needed to estimate the first-stage parameter. The numbers in parenthesis are the standard errors of the mean difference test. The numbers in brackets represent the bias-corrected bootstrap confidence intervals at 95% significance level. *, **, *** denote significance at the 10%, 5%, and 1% levels (one-sided).

SIC	Mean Diff.	Bootstrap			
		Two Group Sampling		Bayesian	
		All	Others	All	Others
311	0.02**	0.02*	0.02*	0.02	0.02
	(0.011)	[-0.01, 0.06]	[-0.01, 0.08]	[-0.03, 0.07]	[-0.04, 0.08]
312	-0.01	-0.01	-0.01	-0.01	-0.01
	(0.034)	[-0.09, 0.11]	[-0.15, 0.14]	[-0.16, 0.12]	[-0.2, 0.15]
313	0.18***	0.07***	0.18***	0.07**	0.18***
	(0.026)	[0.03, 0.12]	[0.05, 0.32]	[0.01, 0.12]	[0.03, 0.34]
321	0.13***	0.11***	0.13***	0.11***	0.13***
	(0.018)	[0.06, 0.19]	[0.09, 0.24]	[0.02, 0.18]	[0.02, 0.25]
322	0.2***	0.2***	0.2***	0.2**	0.2**
	(0.039)	[0.06, 0.32]	[0.07, 0.32]	[0.03, 0.35]	[-0.03, 0.36]
323	0.19	0.15	0.19	0.15	0.19

Table 24: Comparison of industry characteristics between corporations and non-corporations.

SIC	N		Avg. Age		Sales (Sales Tax Rate)		Purchases (Sales Tax Rate)		Mkt %
	Corp's	Others	Corp's	Others	Corp's	Others	Corp's	Others	Corp's
311	147	783	27	20	0.5	0.1	0.9	0.4	15.2
312	41	161	26	21	0.2	0.2	1.2	0.7	3.9
313	74	61	31	31	4.5	14.8	1.6	3.5	7.7
321	71	422	26	17	8.2	8.0	3.8	4.8	5.1
322	22	724	24	14	7.9	8.1	4.1	4.6	0.5
323	13	90	35	17	6.5	8.5	4.8	5.0	0.7
324	9	186	35	14	8.1	8.0	5.5	4.8	0.6
331	13	160	21	18	6.6	5.5	1.9	1.8	0.4
332	12	187	25	15	9.5	10.6	4.3	4.0	0.1
341	41	119	26	20	8.7	9.1	3.8	5.0	4.5
342	40	324	25	22	5.1	8.0	2.9	3.8	1.3
351	68	55	24	17	7.3	6.4	3.0	3.6	6.5
352	100	216	30	24	7.1	6.7	3.8	3.8	7.3
356	53	241	20	15	9.6	10.2	5.7	6.1	2.1
369	58	223	29	19	4.9	5.7	1.7	2.0	3.0
381	85	569	24	17	9.0	9.4	4.8	4.9	2.6
382	46	330	26	20	8.1	7.8	4.2	4.0	1.0
383	57	168	23	18	9.3	8.8	4.5	4.6	3.1
384	42	209	26	17	8.7	7.4	5.0	5.0	5.9
390	18	163	24	21	9.6	11.4	4.1	4.1	0.4

Table 25: Rejection frequencies of the \mathcal{V}_{it} for Corporations and Non-Corporations. The rejection frequencies of the one tail test are the average number of observations that fell above the 95% region of the $f_{\varepsilon}(\varepsilon)$ distribution recovered in the first stage. The rejection frequencies of the two tail test are the average number of observations that fell above the 97.5% or below the 2.5% region.

SIC	One Tail		Two Tail	
	Corp	Non-Corp	Corp	Non-Corp
311	5.1	9.3	5.1	8.7
312	5.1	3.2	5.1	6.8
322	4.4	26.3	3.5	8.9
381	4.9	4.6	4.9	6.3
321	4.9	21.4	5.1	12.7
382	5.0	5.2	5.0	5.4
384	5.0	0.6	4.7	4.3
383	4.9	2.6	5.1	2.9
313	5.0	17.9	5.0	12.9
341	5.0	4.4	5.0	8.1
324	5.1	10.4	5.1	11.2
342	5.1	6.3	5.1	5.9
369	5.1	12.5	4.8	12.8
332	5.3	4.3	5.3	8.5
390	4.4	1.1	4.4	1.0
351	4.8	11.2	4.8	10.5
352	4.9	12.1	5.0	9.9
356	5.2	3.9	5.2	8.6
331	5.8	26.1	4.8	22.7
323	4.7	1.5	3.8	12.6

To compare, I also include the uncorrected estimates using GNR and a simple OLS. In the case of GNR estimation method, the Markov process of productivity is a third degree polynomial, and capital and labor, the instruments.

I focus on the top 5 industries in terms of revenue: 1) 311 - Food Products, 2) 313 Beverages, 3) 352 Other Chemical Products. 4) 351 Industrial Chemicals, and 5) 321 Textiles.

From Table ??, we can conclude that m_{t-1}^* is a better than \mathcal{W}_{t-2} , k_{t-1} and l_{t-1} . The following discussion will be focusing on the results relating using m_{t-1}^* as the instrument.

First, because tax evasion artificially inflates the intermediates's cost share of revenues, lower output elasticities of intermediates than traditional methods are expected. In table Table ?? shows that correcting for tax evasion results in smaller intermediates' output elasticity with respect with OLS. With respect to GNR, all estimates are statistically lower except one. The output elasticity of intermediates of the Textile industry (321) is significantly greater than the GNR result.

Three of the five intermediate elasticities resulted as not statistically different from zero. These zeros are not exclusive to the method correcting by tax evasion, as we can see that GNR for Cobb-Douglas approach zero for some industries. These zeros could be the result of the functional form. A Cobb-Douglas specification leaves out the cross-elasticities which could lead to omitted variables bias. In other words, we effectively ignore complementarity or substitutability between the inputs. This CD omitted variable bias could be accentuated because we defined raw materials as intermediates leaving out others, such as energy, fuels, and services.

The elasticities of capital and labor vary across industries, with some being lower, some higher, and others quite similar. This is expected. The direction of the bias in the elasticities

Table 26: Cobb-Douglas production function parameters estimates, correcting intermediates —raw materials— for tax evasion vs. naive estimation and OLS. h is an AR(1) process for ‘Tax Evasion + GNR’ but a third degree polynomial for ‘GNR’. $Z = \{k_{it}, l_{it}\}$ for GNR. One additional instrument to GNR’s Z control for the endogeneity between ε_{it-1} and \mathcal{W}_{it-1} in the first stage of ‘TE+GNR’. The instrument used is specified in the table.

Inds.	Input	Tax Evasion + GNR				GNR	OLS
		k_{it-1}	l_{it-1}	m^*_{it-1}	\mathcal{W}_{it-2}		
322	m	0.3123	0.3123	0.3123	0.3123	0.3683	0.5669
	k	0.0911	0.0540	0.1698	0.1875	0.1921	0.0477
	l	0.3125	0.2986	0.4076	0.4106	0.3608	0.4119
331	m	0.2937	0.2937	0.2937	0.2937	0.3485	0.5593
	k	0.0388	0.0000	0.1156	0.0721	0.1043	0.0636
	l	0.5819	0.3383	0.6179	0.5809	0.5063	0.4311
321	m	0.3544	0.3544	0.3544	0.3544	0.3869	0.6616
	k	0.0697	0.0000	0.3389	0.3214	0.2604	0.0966
	l	0.1324	0.2780	0.2987	0.3383	0.3412	0.2796
342	m	0.3096	0.3096	0.3096	0.3096	0.3350	0.5789
	k	0.0414	0.0916	0.2836	0.3016	0.2426	0.0716
	l	0.2016	0.2105	0.3703	0.4034	0.4698	0.4147
313	m	0.2701	0.2701	0.2701	0.2701	0.2826	0.7497
	k	0.0160	1.0000	0.4610	0.0000	0.3395	0.1171
	l	0.0283	0.1592	0.1229	0.2157	0.2943	0.2702
311	m	0.5530	0.5530	0.5530	0.5530	0.5630	0.7907
	k	0.2196	0.2768	0.2661	0.2716	0.2443	0.0559
	l	0.2393	0.0990	0.1963	0.1688	0.2084	0.1750
381	m	0.3743	0.3743	0.3743	0.3743	0.3719	0.6449
	l	0.0641	0.0000	0.2450	0.2420	0.2890	0.0560

Table 27: Cobb-Douglas production function parameters estimates, correcting intermediates —raw materials— for tax evasion vs. naive estimation and OLS. h is an AR(1) process for ‘Tax Evasion + GNR’ but a third degree polynomial for ‘GNR’. $Z = \{k_{it}, l_{it}\}$ for GNR. One additional instrument to GNR’s Z control for the endogeneity between ε_{it-1} and \mathcal{W}_{it-1} in the first stage of ‘TE+GNR’. The instrument used is specified in the table. ε_{it-1} is defined as measurement error.

Inds.	Input	Tax Evasion + GNR				GNR	OLS
		k_{it-1}	l_{it-1}	m^*_{it-1}	\mathcal{W}_{it-2}		
322	m	0.3469	0.3469	0.3469	0.3469	0.4218	0.5669
	k	0.0865	0.0520	0.1612	0.1781	0.1759	0.0477
	l	0.2968	0.2837	0.3871	0.3900	0.3302	0.4119
331	m	0.3209	0.3209	0.3209	0.3209	0.3989	0.5593
	k	0.0373	0.0000	0.1112	0.0693	0.0962	0.0636
	l	0.5596	0.3253	0.5941	0.5587	0.4671	0.4311
321	m	0.3869	0.3869	0.3869	0.3869	0.4316	0.6616
	k	0.0662	0.0000	0.3218	0.3052	0.2414	0.0966
	l	0.1257	0.2640	0.2838	0.3213	0.3163	0.2796
342	m	0.3431	0.3431	0.3431	0.3431	0.3642	0.5789
	k	0.0394	0.0854	0.2699	0.2870	0.2319	0.0716
	l	0.1919	0.2007	0.3523	0.3839	0.4492	0.4147
313	m	0.2894	0.2894	0.2894	0.2894	0.3091	0.7497
	k	0.0155	1.0000	0.4489	0.0000	0.3270	0.1171
	l	0.0276	0.1531	0.1197	0.2100	0.2834	0.2702
311	m	0.5991	0.5991	0.5991	0.5991	0.6108	0.7907
	k	0.1971	0.2483	0.2386	0.2436	0.2176	0.0559
	l	0.2144	0.0888	0.1761	0.1514	0.1856	0.1750
331		0.4086	0.4086	0.4086	0.4086	0.4065	0.6449

Table 28: First stage ‘Weak Instruments’ F-test of the 2SLS-IV estimation. The null hypothesis is that the coefficients are equal to zero, in other words, uncorrelated with the endogenous variable \mathcal{W}_{it-1} .

Ins.	Inds.	df1	df2	statistic	p-value	stars
k_{t-1}	322	1	3581	1419.5262	0.0000	***
	331	1	785	121.4029	0.0000	***
	321	1	2339	3714.9184	0.0000	***
	342	1	1707	4800.6672	0.0000	***
	313	1	985	2758.2637	0.0000	***
	311	1	5102	5.4289	0.0198	**
	381	1	3060	3212.6117	0.0000	***
l_{t-1}	322	1	3581	3840.2193	0.0000	***
	331	1	785	778.9743	0.0000	***
	321	1	2339	4176.4138	0.0000	***
	342	1	1707	7325.5613	0.0000	***
	313	1	985	435.1505	0.0000	***
	311	1	5102	213.1251	0.0000	***
	381	1	3060	7087.4688	0.0000	***
\hat{m}_{t-1}	322	1	3581	4963.1390	0.0000	***
	331	1	785	416.9415	0.0000	***
	321	1	2339	420.5456	0.0000	***
	342	1	1707	844.1156	0.0000	***
	313	1	985	458.2628	0.0000	***
	311	1	5102	1344.3363	0.0000	***
	381	1	3060	1275.1243	0.0000	***
\mathcal{W}_{t-2}	322	1	2949	8673.2398	0.0000	***
	331	1	626	1061.0846	0.0000	***

Table 29: Estimates Omega Distribution by Industry after Deconvoluting from the Output Shock.

	μ	σ
322	5.43	0.58
342	4.25	0.47
313	3.29	0.45
351	3.47	0.65
331	4.04	0.38

of capital and labor depends on whether these inputs are complements or substitutes of intermediate inputs. This relationship will vary by industry. Put differently, for different industries, the cross-elasticity between intermediates and capital, and intermediates and labor will be different in direction and magnitude. Since these cross-elasticities are not included in the CD production function, the elasticities of capital and labor should reflect different degrees of these variations once intermediates are corrected for overreporting.

19.3 Productivity

Once, the production function parameters are known, we can recover ω by deconvolution using Equation ???. Here I assume ω follows a normal distribution.

?@tbl-boot-omega displays the estimates of the distribution parameters of ω corresponding to the instrument used in the previous section and the parametric specification of an AR(1).

To compare these productivity estimates to uncorrected ones, I simulate productivity using the parameters estimated in **?@tbl-boot-omega**. I use 1000 independent draws from ω and ε . Then, I computed productivity, $\varphi_{it}^S = \exp\{\omega_{it}^S + \varepsilon_{it}^S\}$.

Table 30: Summary statistics of the productivity estimates using GNR correcting and not correcting for tax evasion. Tax-evasion corrected estimates were simulated using the parameters of $\theta_{\text{tbl-boot-}\omega}$, with 1000 draws.

inds	Method	Mean	SD	Q1	Median	Q3
313	GNR	70.6	44.7	45.1	59.0	82.7
	TE: \hat{m}_{t-1}^*	55.8	39.9	29.3	47.0	70.2
321	GNR	23.6	11.0	17.4	21.5	27.3
	TE: \hat{m}_{t-1}^*	21.6	13.1	12.6	18.2	26.7
322	GNR	41.6	14.3	32.2	39.1	48.6
	TE: \hat{m}_{t-1}^*	83.1	54.5	47.2	68.1	106.0
331	GNR	61.1	23.5	46.3	56.5	70.3
	TE: \hat{m}_{t-1}^*	73.8	44.5	42.9	63.7	93.0
342	GNR	23.1	8.3	18.1	21.7	26.3
	TE: \hat{m}_{t-1}^*	32.1	18.4	18.5	27.7	40.7

Table ?? compares summary statistics the simulated productivity using the estimated parameters after correcting for tax evasion with the summary statistics of the productivity estimates of GNR.

Industries 321 Textiles, and 351 Industrial Chemicals, and 313 Beverages have means lower or equal to the uncorrected ones for both instruments considered. Industry 311 Food Products display a higher mean for both estimates, while industry 352 Other Chemicals displays a higher or lower mean depending on the instrument.

With respect to standard deviations, industries 321 Textiles, 351 Industrial Chemicals, and 352 Other Chemicals display lower magnitudes, while industries 311 Food Products display

significantly higher magnitudes. Lastly, industry 313 Beverages standard deviation is lower or higher depending on the instrument.

19.4 Testing for the Presence of Tax Evasion Through Overreporting

Equation ?? suggest a way to test the presence of tax evasion through cost overreporting. Let $\mathbb{E}[\mathcal{V}_{it}] \equiv \mu_{\mathcal{V}}$. Define the null hypothesis as the absence of cost overreporting, $H_0 : \mu_{\mathcal{V}} = 0$, and the alternative hypothesis as the presence of cost overreporting, $H_1 : \mu_{\mathcal{V}} > 0$. Consequently, we can use a one-sided t-test to verify for the presence of tax evasion by overreporting.

Under the null hypothesis, there is no tax evasion. Then,

$$\begin{aligned} z &= \frac{\bar{\mathcal{V}}}{\hat{\sigma}_{\mathcal{V}}/\sqrt{N}} = \frac{\mathbb{E}[\mathcal{V}]}{\sqrt{\mathbb{V}\mathcal{D} \setminus [\mathcal{V}]/N}} \\ &= \frac{-\mathbb{E}[\varepsilon] + \mathbb{E}[e]}{\sqrt{\mathbb{V}\mathcal{D} \setminus [\varepsilon] + \mathbb{V}\mathcal{D} \setminus [e]/N}} \\ &= \frac{-\mathbb{E}[\varepsilon]}{\sqrt{\mathbb{V}\mathcal{D} \setminus [\varepsilon]/N}} \sim N(0, 1) \end{aligned}$$

The previous test ignores the sample variance of the β estimator in the first stage.

Alternatively, we can do a difference in means test to account for this variation. This test is equivalent to a two sample GMM test, where in one moment the corporations are used to estimate $\hat{\beta}$ and the second is a test moment where we use $\hat{\beta}$ from the first one. Under the null, the two moment should be equal to zero.

Let $s_{it} = \log\left(\frac{\rho_t M_{it}^*}{P_t Y_{it}}\right)$ and $\bar{s}_G = \frac{1}{N_1 T_1} \sum_{i=1}^{N_1} \sum_{t=1}^{T_1} s_{it}^G$ and $\bar{s}_B = \frac{1}{N_1 T_1} \sum_{i=1}^{N_1} \sum_{t=1}^{T_1} s_{it}^B$ be the sample means for good guys (corporations) and the tax evasion guys, respectively.

Then, the test statistic is,

$$t = \frac{\bar{s}_B - \bar{s}_G}{\sqrt{\hat{\sigma}_{s_B}^2/N_2 + \hat{\sigma}_{s_G}^2/N_1}} \sim N(0, 1) \text{ under } H_0$$

where $\hat{\sigma}_{s_B}^2$ and $\hat{\sigma}_{s_G}^2$ are the sample variances of s_{it}^B and s_{it}^G , respectively.

Of course, the assumption of independent samples is likely incorrect. For example, firms in an industry might get contemporaneous shocks affecting everyone in one period, like a price shock in inputs. Productivity is usually assumed to be persistent over time, but the effects of productivity of one firm on the other firms is commonly ignored. In addition, if the data uses SIC codes, firms within sub-industries are more likely to be correlated than firms in different sub-industries.

I test for the presence of tax evasion for different classifications of intermediates. Intermediates include raw materials, energy and services. Deductibles include raw materials and deductible expenses. Materials include only raw materials. The items included as deductible expenses or services are detailed in Table ??

Table ?? shows that the null hypothesis of no tax evasion is rejected at the 1% significance level for twelve of the top twenty manufacturing industries.

In particular, there is no evidence of tax evasion for most industries in which products or raw materials are exempt of sales taxes, such as 312 (Other Food Products), 382 (Non-Electrical Machinery), 384 (Transport Equipment), 323 (Leather Products), and 369 (Non-Metallic Mineral Products); there is also no evidence of materials overreporting for 356 (Plastic Products) industry, whose main raw materials are likely to be specialized and supplied by few local and international suppliers.

Among the industries with exempted products, 311 (Food Products) and 383 (Electrical Machinery) there is evidence of tax evasion but as we'll see later the average overreporting

Table 31: Tax Evasion Through Cost-Overreporting One-Side t-Test by Industry in Colombia.

Under the null hypothesis, there is no tax evasion. Values of the statistic were computed from Equation ?? for different intermediate inputs. Standard errors shown in parenthesis. Stars indicate significance level at the 1% (***), 5% (**), and 10% (*).

SIC	Category	Deductibles	Materials	Electricity	Fuels
311	1 Exempt Product	0.0294 (0.005)***	0.1111 (0.0045)***	2.1331 (0.0201)***	2.7032 (0.0158)***
312	1 Exempt Product	0.0251 (0.0115)**	-0.0098 (0.0158)	3.2172 (0.0702)***	3.4251 (0.0251)***
383	1 Exempt Product	0.056 (0.0101)***	0.0471 (0.0108)***	2.3743 (0.1024)***	3.7437 (0.0312)***
322	4 Direct Customer Sales	0.6404 (0.0068)***	1.27 (0.0068)***	2.1725 (0.068)***	3.7666 (0.0187)***
342	4 Direct Customer Sales	0.0711 (0.0077)***	0.0936 (0.0082)***	2.2239 (0.0814)***	3.3124 (0.0251)***
390	4 Direct Customer Sales	0.0866 (0.0124)***	0.0761 (0.0131)***	2.2658 (0.0478)***	3.8163 (0.0187)***
321		0.0885 (0.007)***	0.1262 (0.0078)***	1.4567 (0.0255)***	2.77 (0.0158)***
313		0.0377 (0.0111)***	0.0659 (0.0123)***	1.7785 (0.0315)***	2.0103 (0.0158)***
351		0.0171 (0.0145)	0.1116 (0.0221)***	1.6454 (0.0406)***	2.7385 (0.0158)***
352		0.0462 (0.0079)***	0.0414 (0.0083)***	2.8723 (0.0918)***	3.8478 (0.0187)***

Table 32: Tax Evasion Through Cost-Overreporting One-Side t-Test by Industry in Colombia.

Under the null hypothesis, there is no tax evasion. Values of the statistic were computed from Equation ?? . Standard errors shown in parenthesis. Stars indicate significance level at the 1% (***), 5% (**), and 10% (*).

sic_3	n	n_Corp	share_sales_tax	gross_output	sales	deductible_intermediates	materials
311	888	147	0.0	0.2	0.2	0.3	0.3
312	188	41	0.0	0.0	0.1	0.1	0.1
313	130	74	0.1	0.1	0.1	0.1	0.1
321	470	71	0.1	0.1	0.1	0.1	0.1
322	737	22	0.1	0.0	0.0	0.0	0.0
323	99	13	0.1	0.0	0.0	0.0	0.0
324	192	9	0.1	0.0	0.0	0.0	0.0
331	172	13	0.1	0.0	0.0	0.0	0.0
332	193	12	0.1	0.0	0.0	0.0	0.0
341	153	41	0.1	0.1	0.1	0.1	0.1
342	347	40	0.1	0.0	0.0	0.0	0.0
351	116	68	0.1	0.1	0.1	0.1	0.1
352	300	100	0.1	0.1	0.1	0.1	0.1
356	272	53	0.1	0.0	0.0	0.0	0.0
369	274	58	0.1	0.0	0.0	0.0	0.0
381	633	85	0.1	0.0	0.0	0.0	0.0
382	364	46	0.1	0.0	0.0	0.0	0.0
383	213	57	0.1	0.0	0.0	0.0	0.0
384	238	42	0.1	0.1	0.1	0.1	0.1
390	179	18	0.1	0.0	0.0	0.0	0.0

is low compared to other industries.

As expected, the evidence is stronger particularly for the other industries that do not fall in the previous categories. Namely, there is evidence of tax evasion for industries 313 (Beverages); for 321 (Textiles), 322 (Wearing Apparel), and 324 (Footwear); 331 (Wood Products except Furniture) and 332 (Wood Furniture); for industry 341 (Paper) and 342 (Publishing); 351 (Industrial Chemicals) and 352 (Other Chemicals); 381 (Metal Products Except Machinery), and 390 (Other Manufacturing Industries).

19.5 Deconvoluting Tax Evasion Using Moments

One simple way to start with the deconvolution is using moments. In particular, for every n -th moment $\mathbb{E}[\varepsilon_{it}^n | \Theta^{NE}] = \mathbb{E}[\varepsilon_{it}^n | t] = \mathbb{E}[\varepsilon_{it}^n]$

Therefore, any moment of the tax evasion e_{it} distribution $\forall t \in T$ can be estimated in theory.

Namely, from Equation ??, we can estimate the average tax evasion by

$$\begin{aligned}\mathbb{E}[e_{it}|t] &= \mathbb{E}[\mathcal{V}_{it}|t] + \mathbb{E}[\varepsilon_{it}] \\ &= \mathbb{E}[\mathcal{V}_{it}|t] + \mu_\varepsilon\end{aligned}$$

Note that we learned the distribution $f_\varepsilon(\varepsilon)$ of ε from the first stage, so μ_ε and σ_ε are known.

Table ?? displays the estimated average of the log fraction that firms increase their costs of raw materials to evade taxes by claiming a greater deductible amount of their owed sales taxes.

The top five tax evading industries, 322 (Wearing Apparel), 342 (Publishing), 313 (Beverages), 351 (Industrial Chemicals), and 331 (Wood Products), display an average tax evasion e greater than 12%, which is non-trivial.

Table 33: Average tax evasion by Industry. Estimates show the average tax evasion from the output shock in Equation ???. LCI and UCI are the bias-corrected bootstrap confidence intervals at the 10% significance level with 250 bootstrap replicates. Intermediates are defined as raw materials.

SIC	Inter	Mean	LCI	UCI
322	Deductibles	0.1589	0.0373	0.2199
	Materials	0.1955	0.0910	0.2895
331	Deductibles	0.1205	0.0140	0.2458
	Materials	0.2177	0.0475	0.3956
321	Deductibles	0.0885	0.0530	0.1465
	Materials	0.1094	0.0721	0.1782
342	Deductibles	0.0436	-0.0255	0.1162
	Materials	0.0597	0.0005	0.1572
313	Deductibles	0.0377	0.0058	0.0759
	Materials	0.0659	0.0320	0.1077
311	Deductibles	0.0071	-0.0360	0.0242
	Materials	0.0193	-0.0074	0.0634
381	Deductibles	-0.0035	-0.0460	0.0425
	Materials	-0.0053	-0.0494	0.0457

Although deconvolution by moments is the simplest method, it displays the undesirable characteristic that estimate of variances frequently result with a negative sign. In the next section, I address this problem by using parametric MLE.

19.6 Deconvoluting by Parametric MLE

I use parametric MLE to obtain better estimates of the variances using equation Equation ??.

I assume that the error term ε follows a normal distribution.

For tax-evasion, theory suggests that firms only have incentives to overreport costs, not to underreport them. Therefore, as explained elsewhere, it might be expected that overreporting $e \geq 0$ is non-negative. In addition, it might also be expected that most firms overreport a little and a few firms overreport greater amounts. Therefore, a lognormal or a truncated normal distribution might be more appropriate.

By definition, if a random variable U is log-normal distributed, then $\log(U) \sim N(\mu, \sigma)$. Thus, we cannot directly compare the parameters of the log-normal distribution to our previous estimates. We can however, use the parameters to compute any moment of the log-normally distributed variable U by

$$E[U^n] = e^{n\mu + \frac{1}{2}n^2\sigma^2}$$

In particular, the first moment and the variance are computed as follows,

$$E[U] = e^{\mu + \frac{1}{2}\sigma^2}$$

$$Var[U] = E[U^2] - E[U]^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1).$$

In addition, the mode and the mean by

$$\text{Mode}[U] = e^{\mu - \sigma^2}$$

$$\text{Med}[U] = e^{\mu}$$

Likewise, the probability density function of random variable U with a normal distribution with parameters $\tilde{\mu}$ and $\tilde{\sigma}$ and truncated from below at zero is

$$f(u; \tilde{\mu}, \tilde{\sigma}) = \frac{\varphi(\frac{u - \tilde{\mu}}{\tilde{\sigma}})}{\tilde{\sigma}(1 - \Phi(\alpha))}$$

where

$$\varphi(\xi) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}\xi^2)$$

is the probability density function of the standard normal distribution, $\Phi(\cdot)$ is its cumulative distribution function, and $\alpha = -\tilde{\mu}/\tilde{\sigma}$

Then, the mean becomes

$$E[U|U > 0] = \tilde{\mu} + \tilde{\sigma} \frac{\varphi(\alpha)}{1 - \Phi(\alpha)}$$

and the variance, median, and mode become

$$\text{Var}[U|U > 0] = \sigma^2[1 + \alpha\varphi(\alpha)/(1 - \Phi(\alpha)) - (\varphi(\alpha)/[1 - \Phi(\alpha)])^2]$$

$$\text{Median}[U|U > 0] = \tilde{\mu} + \Phi^{-1}\left(\frac{\Phi(\alpha) + 1}{2}\right) \tilde{\sigma}$$

$$\text{Mode}[U|U > 0] = \tilde{\mu}$$

Table ?? displays the estimates of the parameters for both the log-normal and truncated normal distributions using an MLE approach. Other moments of the densities are computed as explained previously and shown for reference.

Table 34: Estimates of Deconvoluting Tax Evasion from the Output Shock in Equation ?? by Parametric MLE Using A Log-Normal and a Truncated-Normal Distribution. μ and σ are the parameters of the densities. Moments displayed are estimated using the distribution paramters as explained above. Intermediates are defined as raw materials.

sic_3	dist	mu	sigma	mean	sd	mode	median
311	lognormal	-1.7460	0.9181	0.2659	0.3059	0.0751	0.1745
	truncated normal	0.2280	0.0894	0.2294	0.0876	0.2280	0.2286
313	lognormal	-1.4006	0.9855	0.4005	0.5131	0.0933	0.2465
	truncated normal	0.3285	0.1647	0.3376	0.1549	0.3285	0.3332
321	lognormal	-1.4011	0.8883	0.3655	0.4006	0.1119	0.2463
	truncated normal	0.3183	0.1302	0.3209	0.1269	0.3183	0.3195
352	lognormal	-1.5406	0.9517	0.3370	0.4091	0.0866	0.2143
	truncated normal	0.2847	0.1218	0.2879	0.1180	0.2847	0.2862
383	lognormal	-1.5748	0.9288	0.3187	0.3730	0.0874	0.2071
	truncated normal	0.2718	0.1230	0.2761	0.1180	0.2718	0.2739

Both the log-normal and the truncated normal distributions point to similar means. The differences between the other moments of the distributions can be explained by the differences in the shapes of their probability density functions. The standard deviation is larger in the

log-normal distribution than in the truncated normal distribution which can be explained by the asymmetry of the log normal distribution which is skewed to the right. With respect to the mode, it is the same to the mean in the case of the truncated normal distribution, while it is lower than the median for the log-normal. Finally, the median is lower than the mean in the case of the log-normal, but higher than the mean in the case of the truncated normal because of the truncation from below.

Both distributions show higher estimates of the average overreporting in logs than using only moments. Now it looks like firms evade taxes by inflating their cost of their materials by 40 percent or more.

Table ?? shows the bias corrected bootstrap confidence intervals for the mean of each distribution by industry using 200 replicates.

Table 35: Estimates of Deconvoluting Tax Evasion from the Output Shock in Equation ?? by Parametric MLE Using A Log-Normal and a Truncated-Normal Distribution. LCI and UCI are the bias corrected bootstrap confidence intervals at the 5 percent significance level using 250 replicates.

SIC	Density	Mean	SD
311	lognormal	0.2659	0.3059
		[0.2285, 0.2999]	[0.2069, 0.3536]
	truncated normal	0.2294	0.0876
		[0.2116, 0.2597]	[0.0791, 0.0991]
	lognormal	0.4005	0.5131
		[0.3407, 0.468]	[0.3279, 0.6767]

	truncated normal	0.3376	0.1549
		[0.2965, 0.3831]	[0.1343, 0.1791]
352	lognormal	0.337	0.4091
		[0.2718, 0.3804]	[0.2442, 0.4747]
	truncated normal	0.2879	0.118
		[0.2509, 0.3287]	[0.1046, 0.134]
321	lognormal	0.3655	0.4006
		[0.3176, 0.4179]	[0.2702, 0.4448]
	truncated normal	0.3209	0.1269
		[0.2909, 0.3751]	[0.1183, 0.1443]
383	lognormal	0.3187	0.373
		[0.2613, 0.3727]	[0.2391, 0.4416]
	truncated normal	0.2761	0.118
		[0.226, 0.3368]	[0.1082, 0.1351]

The difference between our previous estimates using moments can be explained by the fact that the log-normal and truncated distribution are restricted to positive values while we did not apply the restriction when using moments. Were we to take the average of only the positive values, the mean of the moments' method would be higher and closer to the log-normal and truncated normal distributions.

From the perspective of theory, firms would have incentives to overreport their materials.

Table 36: Semiparametric Deconvolution of Tax Evasion using Penalized B-Splines. The error density function is assumed to be normally distributed with mean and variance obtained from the residuals of the first stage conditional on firms beign Non-Evaders.

sic_3	mean	sd
322	0.199	0.074
331	0.226	0.110
321	0.113	0.053
342	0.058	0.013
313	0.072	0.037
311	0.021	0.004
381	0.015	0.002

Therefore, using moments to estimate overreporting without restricting to positive values like in the case of the MLE method using a log-normal or truncated distribution might underestimate tax evasion through cost overreporting.

19.7 Non-Parametric Deconvolution using Penalized B-Splines

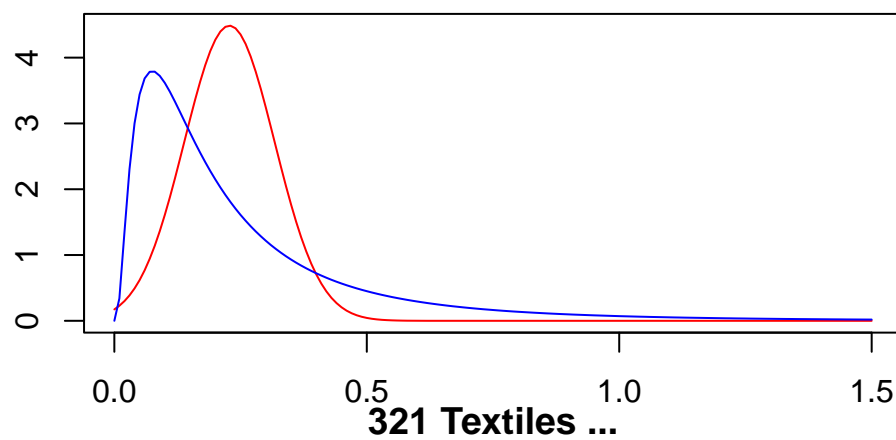
20 Tax Evasion and Resource Missallocation

A popular way to decompose total productivity introduced by ? is the following :

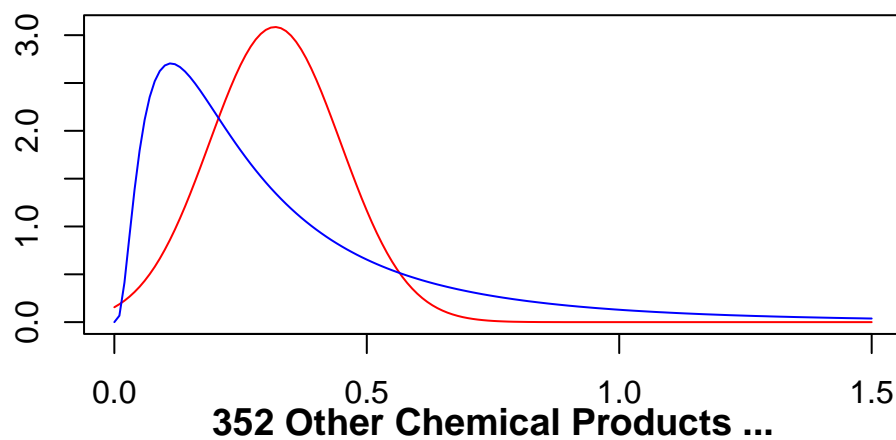
$$\varphi_t = \bar{\varphi}_t + \sum_{i=1}^I (s_{it} - \bar{s}_t) (\varphi_{it} - \bar{\varphi}_t)$$

where $\varphi_{it} = \exp\{\omega_{it} + \varepsilon_{it}\}$ is the firm-level productivity, s_{it} is firm' i share of output at time t . $\bar{\varphi}_t$ and \bar{s}_t represent the unweighted mean productivity and unweighted mean share.

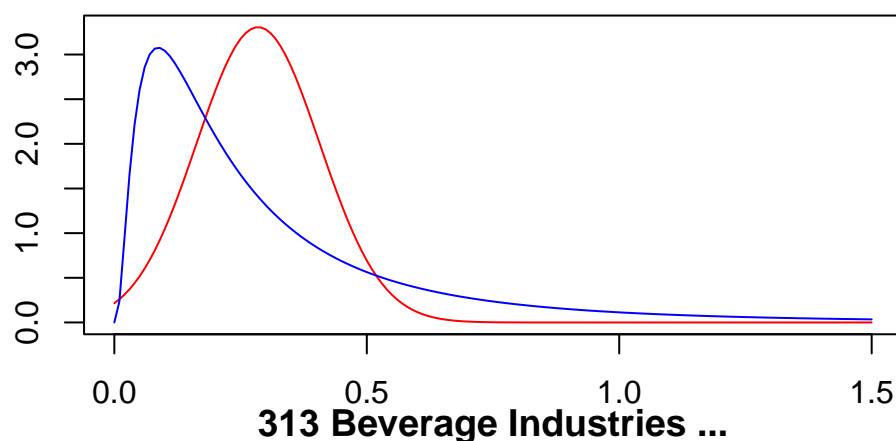
311 Food Manufacturing ...



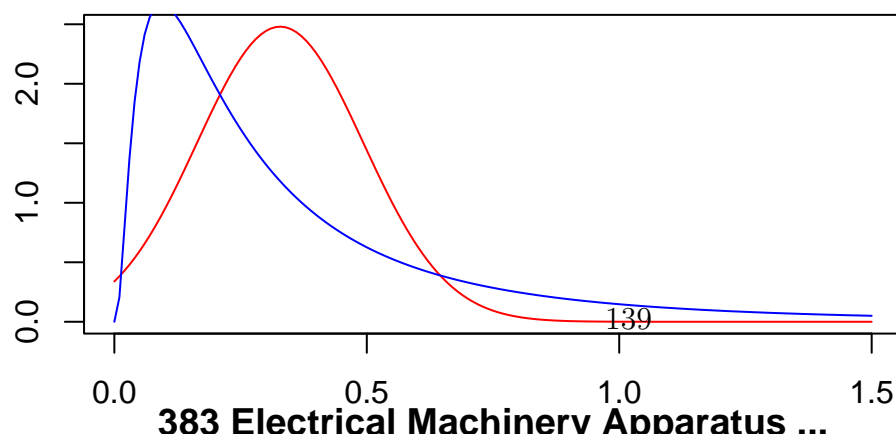
321 Textiles ...



352 Other Chemical Products ...



313 Beverage Industries ...



383 Electrical Machinery Apparatus ...

Intuitively, the more output is allocated to high productivity firms the higher the total productivity will be. Alternatively, the smaller the shares allocated to low productivity firms the greater the total productivity.

Ideally, we would like resources to be allocated to high productivity firms. Hence, the link between the decomposition of total aggregate productivity and missallocation.

In the tax evasion context, the productivity of misreporting firms would look lower than it really is. How would this affect our measures of total productivity? If misreporting firms have small shares, their corrected productivity measure might be closer to the mean reducing their contribution to total productivity, leading to lower measured levels of total productivity.

In other words, tax evasion through cost overreporting might lead to biased measures of total aggregate productivity.

However, if we think about the marginal firm that is struggling to stay in the market, the additional income gained through misreporting might help her survive longer. Therefore, tax evasion increases missallocation in the sense that a share of the output is allocated to firms that otherwise would not be in the market.

There are alternative ways to decompose total productivity that account for the entry and exit of firms (Foster et al. 2008). In addition, alternative measures to aggregate productivity have been used elsewhere (e.g. aggregate demand shocks in ?).

For example, we can decompose aggregate productivity like this

$$\Delta\varphi_t = \sum_{i \in C} \bar{s}_i \Delta\varphi_{it} + \sum_{i \in C} (\bar{\varphi}_i - \bar{\varphi}) \Delta s_{it} + \sum_{i \in N} s_{it} (\varphi_{it} - \bar{\varphi}) - \sum_{i \in X} s_{it-1} (\varphi_{it-1} - \bar{\varphi})$$

where Δ is the change from $t - 1$ to t , and the sets C, N , and X represent the continuing, entering, and exiting firms.

In the case of corporate tax evasion a relevant measure could be aggregate profitability. Low productivity firms are surviving not because they are technologically efficient but because their profitability is marginally higher when they evade taxes. Decomposing an aggregate profitability measure by market shares and firm dynamics would indicate how exiting firms are contributing to the aggregate measure. Intuitively, again higher levels of aggregate profitability correspond to larger shares of the market being allocated to high profitability firms.

In the absence of tax evasion, firms overreporting costs would have lower after-tax profits. The aggregate profitability level would be higher if these firms have small market shares and remain in the sample. On the other hand, the aggregate profitability level would decrease if the small-share and low-profitability firms were to exit the market.

Therefore, in the case of tax evasion, the key aspect of aggregate profitability is the counterfactual decision of low-profitability firms to exit the market in addition to the corrected profits if firms were not to overreport their costs.

Consequently, the economic model should account for firm entry and exit along with tax evasion.

Table 37: Fortran implementation of the Non-Parametric GNR(2020) method to estimate production functions. Comparison with Stata code and replication data from GNR(2020), and the working data of this paper and my own Fortran implementation code.

NP GNR									
	Replication Data			Working Data					
	Stata			Stata			Fortran		
	m	k	l	m	k	l	m	k	l
311	0.67	0.12	0.22	0.67	0.12	0.22	0.67	0.12	0.22
321	0.54	0.16	0.32	0.54	0.16	0.32	0.54	0.16	0.32
322	0.52	0.05	0.42	0.52	0.05	0.42	0.52	0.05	0.42
331	0.51	0.04	0.44	0.51	0.04	0.44	0.51	0.04	0.44
381	0.53	0.10	0.43	0.53	0.10	0.43	0.53	0.10	0.42

TABLE 2
AVERAGE INPUT ELASTICITIES OF OUTPUT
(Structural vs. Uncorrected OLS Estimates)

	Industry (ISIC Code)											
	Food Products (311)		Textiles (321)		Apparel (322)		Wood Products (331)		Fabricated Metals (381)		All	
	GNR	OLS	GNR	OLS	GNR	OLS	GNR	OLS	GNR	OLS	GNR	OLS
Colombia:												
Labor	.22 (.02)	.15 (.01)	.32 (.03)	.21 (.02)	.42 (.02)	.32 (.01)	.44 (.05)	.32 (.03)	.43 (.02)	.29 (.02)	.35 (.01)	.26 (.01)
Capital	.12 (.01)	.04 (.01)	.16 (.02)	.06 (.01)	.05 (.01)	.01 (.01)	.04 (.02)	.03 (.01)	.10 (.01)	.03 (.01)	.14 (.01)	.06 (.00)
Intermediates	.67 (.01)	.82 (.01)	.54 (.01)	.76 (.01)	.52 (.01)	.68 (.01)	.51 (.01)	.65 (.02)	.53 (.01)	.73 (.01)	.54 (.01)	.72 (.00)
Sum	1.01 (.01)	1.01 (.01)	1.01 (.02)	1.03 (.01)	.99 (.01)	1.01 (.01)	.99 (.04)	1.00 (.02)	1.06 (.01)	1.05 (.01)	1.04 (.00)	1.04 (.00)
Mean (capital)/mean (labor)	.55 (.08)	.27 (.07)	.49 (.09)	.27 (.06)	.12 (.04)	.04 (.02)	.08 (.05)	.08 (.05)	.23 (.04)	.11 (.04)	.40 (.03)	.23 (.01)
Chile:												
Labor	.28 (.01)	.17 (.01)	.45 (.03)	.26 (.02)	.45 (.02)	.29 (.02)	.40 (.02)	.20 (.01)	.52 (.03)	.32 (.02)	.38 (.01)	.20 (.01)
Capital	.11 (.01)	.05 (.00)	.11 (.01)	.06 (.01)	.06 (.01)	.03 (.01)	.07 (.01)	.02 (.01)	.13 (.01)	.07 (.01)	.16 (.00)	.09 (.00)
Intermediates	.67 (.00)	.83 (.01)	.54 (.01)	.75 (.01)	.56 (.01)	.74 (.01)	.59 (.01)	.81 (.01)	.50 (.01)	.71 (.01)	.55 (.00)	.77 (.00)
Sum	1.05 (.01)	1.05 (.00)	1.10 (.02)	1.06 (.01)	1.08 (.02)	1.06 (.01)	1.06 (.01)	1.04 (.01)	1.15 (.02)	1.10 (.01)	1.09 (.01)	1.06 (.00)
Mean (capital)/mean (labor)	.39 (.03)	.28 (.03)	.24 (.04)	.22 (.04)	.14 (.03)	.12 (.03)	.18 (.03)	.12 (.05)	.25 (.03)	.21 (.04)	.43 (.02)	.42 (.02)

NOTE.—Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates. For each industry, the numbers in the first column are based on a gross output specification using a complete polynomial series of degree two for each of the two nonparametric functions (D and C) of our approach (GNR). The numbers in the second column are also based on a gross output specification and are estimated using a complete polynomial series of degree two with OLS. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry. The “Sum” row reports the sum of the average labor, capital, and intermediate-input elasticities, and the “Mean (capital)/mean (labor)” row reports the ratio of the average capital elasticity to the average labor elasticity.

Figure 10: ? Table 2 (GNR2020)

Table 38: Fortran implementation of the Non-Parametric GNR(2020) method to estimate production functions. Comparison of the error standard deviation and \mathcal{E} between the Stata code and replication data from GNR(2020), and the working data of this paper and my own Fortran implementation code.

NP GNR						
	Replication Data		Working Data			
	Stata		Stata		Fortran	
	\mathcal{E}	err sd	\mathcal{E}	err sd	\mathcal{E}	err sd
311	1.05	0.24	1.05	0.24	1.05	0.24
321	1.03	0.23	1.03	0.23	1.03	0.23
322	1.02	0.22	1.02	0.22	1.02	0.22
331	1.04	0.25	1.04	0.25	1.04	0.25
381	1.02	0.20	1.02	0.20	1.02	0.20

Table 39: Fortran implementation of the Cobb-Douglas GNR(2020) method to estimate production functions. Comparison with Stata code and replication data from GNR(2020), and the working data of this paper and my own Fortran implementation code.

CD GNR									
	Replication Data			Working Data					
	Stata			Stata			Fortran		
	m	k	l	m	k	l	m	k	l
311	0.62	0.20	0.20	0.62	0.20	0.20	0.62	0.20	0.20
321	0.52	0.21	0.27	0.52	0.21	0.27	0.52	0.21	0.27
322	0.44	0.19	0.29	0.44	0.19	0.29	0.44	0.19	0.29
331	0.46	0.10	0.44	0.46	0.10	0.44	0.46	0.10	0.44
381	0.51	0.18	0.36	0.51	0.18	0.36	0.51	0.18	0.36

Table 40: Fortran implementation of the Cobb-Douglas GNR(2020) method to estimate production functions. Comparison of the error standard deviation and \mathcal{E} between the Stata code and replication data from GNR(2020), and the working data of this paper and my own Fortran implementation code.

CD GNR						
	Replication Data		Working Data			
	Stata		Stata		Fortran	
	\mathcal{E}	err sd	\mathcal{E}	err sd	\mathcal{E}	err sd
311	1.11	0.33	1.11	0.33	1.11	0.33
321	1.06	0.30	1.06	0.30	1.06	0.30
322	1.16	0.43	1.16	0.43	1.16	0.43
331	1.10	0.39	1.10	0.39	1.10	0.39
381	1.05	0.29	1.05	0.29	1.05	0.29

Table 41: First State of Cobb-Douglas production function intermediate elasticities estimates using GNR Stata estimator and R estimator with trimmed data at 0.05. **All firms.**

Industry	Inter.	Working Data					
		Stata			R		
		m	\mathcal{E}	err sd	m	\mathcal{E}	err sd
311	Intermediates	0.65	1.07	0.31	0.65	1.07	0.31
	Materials	0.56	1.08	0.35	0.56	1.08	0.35
	Deductibles	0.56	1.12	0.39	0.56	1.12	0.39
321	Intermediates	0.52	1.06	0.30	0.52	1.06	0.3
	Materials	0.39	1.12	0.41	0.39	1.12	0.41
	Deductibles	0.43	1.09	0.37	0.43	1.09	0.37
322	Intermediates	0.45	1.13	0.41	0.45	1.13	0.41
	Materials	0.37	1.15	0.45	0.37	1.15	0.45
	Deductibles	0.37	1.15	0.45	0.37	1.15	0.45
331	Intermediates	0.47	1.09	0.38	0.47	1.09	0.38
	Materials	0.35	1.14	0.48	0.35	1.14	0.48
	Deductibles	0.39	1.12	0.43	0.39	1.12	0.43
381	Intermediates	0.51	1.05	0.29	0.51	1.05	0.29
	Materials	0.37	1.09	0.39	0.37	1.09	0.39
	Deductibles	0.41	1.07	0.35	0.41	1.07	0.35

Table 42: First State of Cobb-Douglas production function intermediate elasticities estimates using GNR Stata estimator and R estimator with trimmed data at 0.05. **Only Corporations.**

Industry	Inter.	Working Data					
		Stata.			R		
		m	\mathcal{E}	err sd	m	\mathcal{E}	err sd
311	Intermediates	0.67	1.06	0.28	0.67	1.06	0.28
	Materials	0.55	1.08	0.35	0.55	1.08	0.35
	Deductibles	0.57	1.09	0.35	0.57	1.09	0.35
321	Intermediates	0.51	1.03	0.24	0.51	1.03	0.24
	Materials	0.35	1.09	0.37	0.35	1.09	0.37
	Deductibles	0.40	1.07	0.33	0.4	1.07	0.33
322	Intermediates	0.25	1.59	0.82	0.25	1.59	0.82
	Materials	0.31	1.11	0.41	0.31	1.11	0.41
	Deductibles	0.34	1.09	0.38	0.34	1.09	0.38
331	Intermediates	0.50	1.03	0.24	0.5	1.03	0.24
	Materials	0.29	1.09	0.42	0.29	1.09	0.42
	Deductibles	0.37	1.05	0.30	0.37	1.05	0.3
381	Intermediates	0.54	1.04	0.27	0.54	1.04	0.27
	Materials	0.37	1.09	0.40	0.37	1.09	0.4
	Deductibles	0.41	1.08	0.37	0.41	1.08	0.37

Table 43: First State of Cobb-Douglas production function intermediate elasticities estimates using GNR Stata estimator and R estimator with trimmed data at 0.05. **Corporations vs. All.** ε is the measurement error.

Industry	Inter.	Working Data (R)					
		Corps.			All		
		m	\mathcal{E}	err sd	m	\mathcal{E}	err sd
311	Intermediates	0.71	1	0.28	0.69	1	0.31
	Materials	0.6	1	0.35	0.61	1	0.35
	Deductibles	0.62	1	0.35	0.63	1	0.39
321	Intermediates	0.53	1	0.24	0.55	1	0.3
	Materials	0.39	1	0.37	0.43	1	0.41
	Deductibles	0.43	1	0.33	0.47	1	0.37
322	Intermediates	0.4	1	0.82	0.51	1	0.41
	Materials	0.35	1	0.41	0.42	1	0.45
	Deductibles	0.37	1	0.38	0.43	1	0.45
331	Intermediates	0.51	1	0.24	0.51	1	0.38
	Materials	0.32	1	0.42	0.4	1	0.48
	Deductibles	0.39	1	0.3	0.44	1	0.43
381	Intermediates	0.56	1	0.27	0.53	1	0.29
	Materials	0.41	1	0.4	0.41	1	0.39
	Deductibles	0.44	1	0.37	0.44	1	0.35

Table 44: First State of Cobb-Douglas production function intermediate elasticities estimates using GNR Stata estimator and R estimator with trimmed data at 0.05. **Corporations vs. All.** ε is the **measurement error**. Top evading industries.

Industry	Inter.	Working Data (R)					
		Corps.			All		
		m	\mathcal{E}	err sd	m	\mathcal{E}	err sd
322	Intermediates	0.4	1	0.82	0.51	1	0.41
	Materials	0.35	1	0.41	0.42	1	0.45
	Deductibles	0.37	1	0.38	0.43	1	0.45
369	Intermediates	0.41	1	0.41	0.37	1	0.54
	Materials	0.24	1	0.61	0.28	1	0.62
	Deductibles	0.35	1	0.41	0.34	1	0.46
331	Intermediates	0.51	1	0.24	0.51	1	0.38
	Materials	0.32	1	0.42	0.4	1	0.48
	Deductibles	0.39	1	0.3	0.44	1	0.43
321	Intermediates	0.53	1	0.24	0.55	1	0.3
	Materials	0.39	1	0.37	0.43	1	0.41
	Deductibles	0.43	1	0.33	0.47	1	0.37
351	Intermediates	0.55	1	0.3	0.55	1	0.32
	Materials	0.34	1	0.63	0.35	1	0.62
	Deductibles	0.44	1	0.4	0.44	1	0.41

21 References

22 Fortran GNR

22.1 Non-Parametric Production Function

22.2 Cobb-Douglas Production Function

22.3 CD GNR + trimming

22.4 CD GNR + Trimming + Corporations

22.5 CD GNR + Trimming + Measurement Error

22.6 Industry Characteristics

23 CD GNR Intermediates

Observations with value of zero for some intermediates were driving down the estimates of the output elasticity of intermediates. This is a common problem when using data from surveys. To avoid this problem, I trimmed the observations with a share of intermediates below 0.05.

These observations increase the variance of the error term, which in turn increases the value of \mathcal{E} . The higher the value of \mathcal{E} , the lower the value of the elasticity of intermediates.

Figure ?? show that at when trimming observations with a share of intermediates below 0.05, the estimates of \mathcal{E} and the output elasticity of intermediates start to stabilize. That is the changes in the values of these variables are small. In addition, Figure ?? shows that at

Table 45: Intermediate elasticities for different groups of firms using GNR method assuming a CD functional form. Data trimmed at 0.05 of the intermediates share cost of gross output. ε is defined as **measurement error**.

Inds.	Intermediate	m			
		Corps.	Exp.	Imp.	All
311	Materials	0.6	0.61	0.35	0.61
312	Materials	0.58	0.57	0.49	0.58
313	Materials	0.29	0.35	0.36	0.31
321	Materials	0.39	0.44	0.37	0.43
322	Materials	0.35	0.42	0.41	0.42
323	Materials	0.56	0.47	0.22	0.48
324	Materials	0.43	0.46	0.22	0.46
331	Materials	0.32	0.41	0.42	0.4
332	Materials	0.36	0.35	0.42	0.35

Table 46: One side test of tax evasion through inputs overreporting. Different non-evaders groups used as reference.

Inds.	Corporations		Exporters		Materials
	Materials	Deductibles	Materials	Deductibles	
311	0.1111 (0.0045)***	0.0294 (0.005)***	-0.1489 (0.0124)	-0.0416 (0.0056)	-0.2007 (0.005)
312	-0.0098 (0.0158)	0.0251 (0.0115)**	0.0516 (0.0257)**	0.0648 (0.0127)***	-0.1104 (0.005)
313	0.0659 (0.0123)***	0.0377 (0.0111)***	0.2794 (0.0123)***	0.3439 (0.0111)***	0.1218 (0.011)
321	0.1262 (0.0078)***	0.0885 (0.007)***	0.0604 (0.0111)***	0.115 (0.0079)***	0.016 (0.011)
322	1.27 (0.0068)***	0.6404 (0.0068)***	0.1215 (0.0145)***	0.1103 (0.009)***	-0.1187 (0.005)
323	-0.1534 (0.0133)	-0.1635 (0.0137)	-0.181 (0.0179)	-0.1646 (0.0152)	-0.1544 (0.005)
324	0.0664 (0.0086)***	0.0578 (0.0084)***	-0.1202 (0.0188)	-0.098 (0.0099)	-0.0588 (0.005)
331	0.2177 (0.0154)***	0.1205 (0.0139)***	0.086 (0.0198)***	0.1038 (0.0148)***	-0.0852 (0.005)
332	-0.028 (0.0136)	-0.0284 (0.0127)	-0.1286 (0.0142)	-0.1522 (0.0133)	-0.2887 (0.005)

this threshold, the change in percentage of observations dropped is also small.

A more detailed picture of this is displayed in `?@tbl-cd-gnr-inter-trim`. The table compares the estimates of the output elasticity of intermediates and other statistics of the first stage of GNR(2020) assuming a Cobb-Douglas Production Function for different definitions of intermediates and different thresholds for trimming observations.

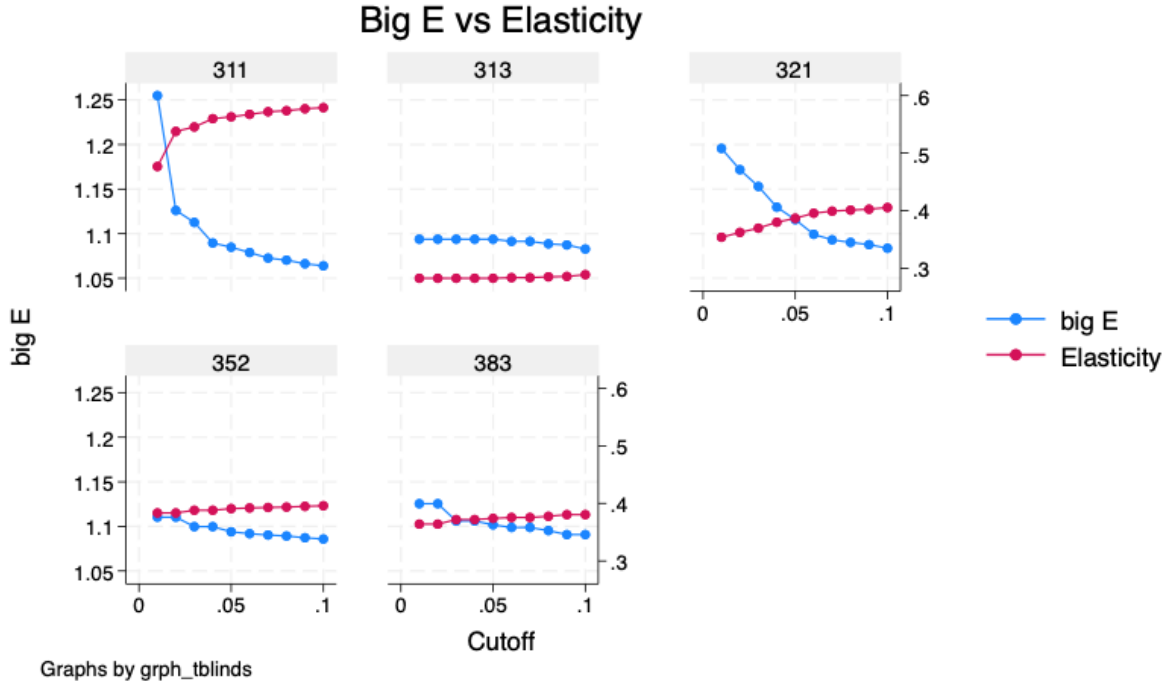


Figure 11: Estimates of \mathcal{E} and the output elasticity of intermediates (raw materials) trimming observations with a share of intermediates below different thresholds going from 0.01 to 0.1 . The elasticities of intermediates were estimated assuming a Cobb-Douglas production function using the first stage of GNR(2020).

24 Density

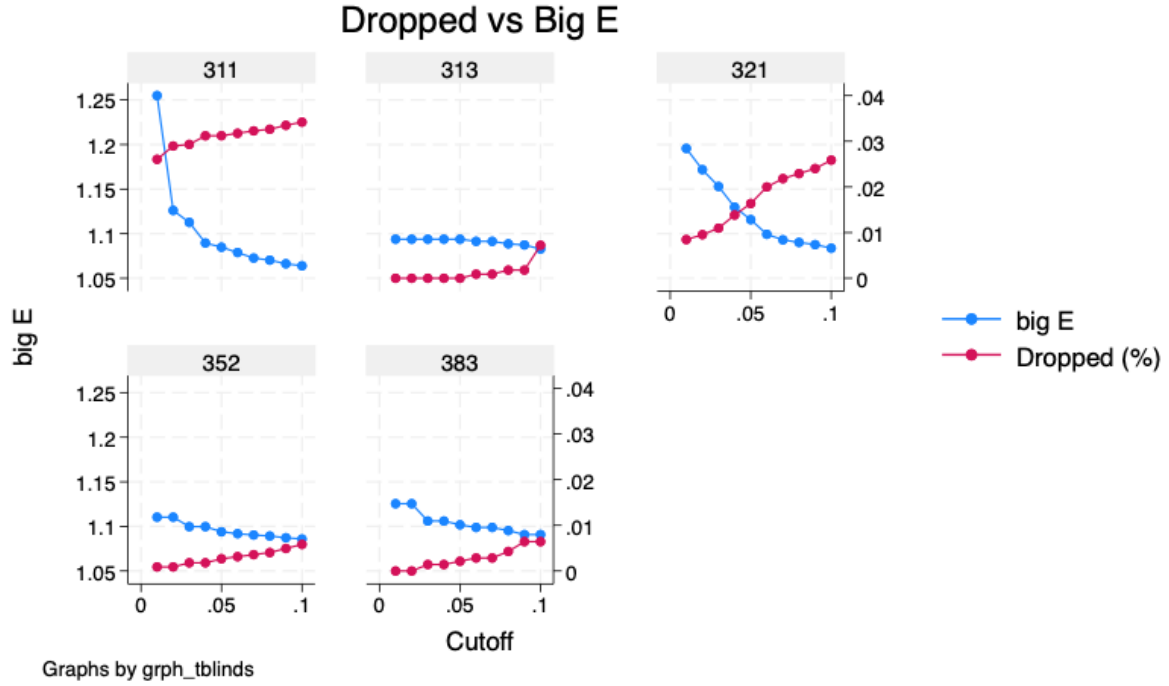


Figure 12: Estimates of \mathcal{E} and percentage of observations dropped by trimming observations with a share of intermediates (raw materials) below different thresholds going from 0.01 to 0.1 . The elasticities of intermediates were estimated assuming a Cobb-Douglas production function using the first stage of GNR(2020).

Table 47: Estimates of Cobb-Douglas Production Function parameters using GNR(2020) and statistics of the first stage for different definitions of intermediates. *m s e* stands for raw materials, services, and energy, *mats* for raw materials, and *ded* for deductible intermediates (raw materials, electricity, fuels, and repair and maintenance services). Observations were trimmed below a share of intermediates of 0.05.

Ind.	Inter.	m	l	k	bigE	si_mean	err_mean	err_sd
311	m_s_e	0.6488	0.1827	0.1832	1.0652	0.7219	0	0.3091
	mats	0.5630	0.2084	0.2443	1.0848	0.6416	0	0.3470
	ded	0.5567	0.2194	0.2337	1.1232	0.6631	0	0.3948
321	m_s_e	0.5183	0.2658	0.2088	1.0557	0.5689	0	0.2996
	mats	0.3869	0.3412	0.2604	1.1158	0.4633	0	0.4136
	ded	0.4273	0.3237	0.2470	1.0934	0.4949	0	0.3720
352	m_s_e	0.5749	0.2133	0.2415	1.0437	0.6201	0	0.2718
	mats	0.3909	0.2858	0.3527	1.0940	0.4574	0	0.3922
	ded	0.4114	0.2797	0.3360	1.0877	0.4756	0	0.3746
313	m_s_e	0.4324	0.1658	0.3086	1.0721	0.4893	0	0.3479
	mats	0.2826	0.2943	0.3395	1.0938	0.3347	0	0.4104
	ded	0.3216	0.2912	0.3203	1.0767	0.3699	0	0.3726
383	m_s_e	0.5261	0.3666	0.1892	1.0445	0.5694	0	0.2786
	mats	0.3739	0.4889	0.2523	1.1018	0.4427	0	0.4069
	ded	0.4015	0.4532	0.2427	1.0891	0.4655	0	0.3798

Industry 322

