When the markov process is an AR(1), $h(\xi) = \delta_0 + \delta_1 \xi$

$$W_t = \alpha k_t + \delta_0 + \delta_1 \left(W_{t-1} - \alpha k_{t-1}\right) + (1-\beta)\varepsilon_t - \delta_1 (1-\beta)\varepsilon_{t-1} + \eta_t$$

We can use instruments $Z_t = \{k_t, k_{t-1}, m_{t-1}, W_{t-2}\}$ (plus the constant) such that

$$E[(1-\beta)\varepsilon_t - \delta_1(1-\beta)\varepsilon_{t-1} + \eta_t | Z_t] = 0$$

Indeed,

$$E[W_t | Z_t] = \alpha k_t + \delta_0 + \delta_1 E[W_{t-1} - \alpha k_{t-1} | Z_t]$$

Hence, all the parameters of $h(\cdot)$ and α are identified.

When the Markov process is a second degree polynomial,$h(\xi) = \delta_0 + \delta_1 \xi + \delta_2 \xi^2$

$$\begin{aligned}
W_t =& \alpha k_t + \delta_0 + \delta_1 \left(W_{t-1} - \alpha k_{t-1}\right) + \delta_2 \left(W_{t-1} - \alpha k_{t-1} - (1-\beta)\varepsilon_{t-1}\right)^2 \\
&+ (1-\beta)\varepsilon_t - \delta_1(1-\beta)\varepsilon_{t-1} + \eta_t \\
=& \alpha k_t + \delta_0 + \delta_1 \left(W_{t-1} - \alpha k_{t-1}\right) + \delta_2 \left(W_{t-1} - \alpha k_{t-1}\right)^2 \\
&+ \delta_2 (1-\beta)^2 \varepsilon_{t-1}^2 - 2\delta_2 (1-\beta) \left(W_{t-1} - \alpha k_{t-1}\right) \varepsilon_{t-1} \\
&+ (1-\beta)\varepsilon_t - \delta_1(1-\beta)\varepsilon_{t-1} + \eta_t
\end{aligned}$$

When $h(\cdot)$ is a second degree polynomial, I have two additional structural terms.

Using the same instruments as before, $Z_t = \{k_t, k_{t-1}, m_{t-1}, W_{t-2}\}$

$$
\begin{aligned}
E[W_t|Z_t] =& \alpha k_t + \delta_0 + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] + \delta_2 E[(W_{t-1} - \alpha k_{t-1})^2 |Z_t] \\
& + \delta_2(1-\beta)^2 E[\varepsilon_{t-1}^2|Z_t] - 2\delta_2(1-\beta) \underbrace{E[(W_{t-1} - \alpha k_{t-1})\, \varepsilon_{t-1}|Z_t]}_{\substack{\text{I can deduct the} \\ \text{structural conditional} \\ \text{covariance of this term}}} \\
=& \alpha k_t + \delta_0 + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] + \delta_2 E[(W_{t-1} - \alpha k_{t-1})^2 |Z_t] \\
& + \delta_2(1-\beta)^2 E[\varepsilon_{t-1}^2] - 2\delta_2(1-\beta) E[(\omega_{t-1} + (1-\beta)\varepsilon_{t-1} + \alpha k_t - \alpha k_{t-1})\, \varepsilon_{t-1}|Z_t] \\
=& \alpha k_t + \delta_0 + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] + \delta_2 E[(W_{t-1} - \alpha k_{t-1})^2 |Z_t] \\
& + \delta_2(1-\beta)^2 \sigma_{\varepsilon_{t-1}}^2 - 2\delta_2(1-\beta) E[\omega_{t-1}\varepsilon_{t-1} + (1-\beta)\varepsilon_{t-1}^2 + \alpha k_t \varepsilon_{t-1} - \alpha k_{t-1}\varepsilon_{t-1}|Z_t] \\
=& \alpha k_t + \delta_0 + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] + \delta_2 E[(W_{t-1} - \alpha k_{t-1})^2 |Z_t] \\
& + \delta_2(1-\beta)^2 \sigma_{\varepsilon_{t-1}}^2 - 2\delta_2(1-\beta)^2 E[\varepsilon_{t-1}^2] \\
=& \alpha k_t + \delta_0 + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] + \delta_2 E[(W_{t-1} - \alpha k_{t-1})^2 |Z_t] \\
& - \delta_2(1-\beta)^2 \sigma_{\varepsilon_{t-1}}^2 \\
=& \alpha k_t + \delta_0 + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] \\
& + \delta_2 \left(E[(W_{t-1} - \alpha k_{t-1})^2 |Z_t] - (1-\beta)^2 \sigma_{\varepsilon_{t-1}}^2 \right) \\
=& \alpha k_t + \underbrace{\delta_0 - \delta_2(1-\beta)^2 \sigma_{\varepsilon_{t-1}}^2}_{\tilde{\delta_0}} \\
& + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] + \delta_2 E[(W_{t-1} - \alpha k_{t-1})^2 |Z_t]
\end{aligned}
$$

Since I know β and σ_ε, and δ_2 is identified from $(W_{t-1} - \alpha k_{t-1})^2$, I can identify δ_0, and hence, all the parameters of $h(\cdot)$ and α.

When the Markov process is a third degree polynomial, $h(\xi) = \delta_0 + \delta_1\xi + \delta_2\xi^2 + \delta_3\xi^3$

$$
\begin{aligned}
W_t =& \alpha k_t + \delta_0 + \delta_1 (W_{t-1} - \alpha k_{t-1}) + \delta_2 (W_{t-1} - \alpha k_{t-1} - (1-\beta)\varepsilon_{t-1})^2 \\
& + \delta_3 (W_{t-1} - \alpha k_{t-1} - (1-\beta)\varepsilon_{t-1})^3 + (1-\beta)\varepsilon_t - \delta_1(1-\beta)\varepsilon_{t-1} + \eta_t \\
=& \alpha k_t + \delta_0 + \delta_1 (W_{t-1} - \alpha k_{t-1}) + \delta_2 (W_{t-1} - \alpha k_{t-1})^2 \\
& + \delta_3 (W_{t-1} - \alpha k_{t-1})^3 + \delta_2(1-\beta)^2\varepsilon_{t-1}^2 - 2\delta_2(1-\beta)(W_{t-1} - \alpha k_{t-1})\,\varepsilon_{t-1} \\
& + \delta_3(1-\beta)^3\varepsilon_{t-1}^3 - 3\delta_3(1-\beta)^2 (W_{t-1} - \alpha k_{t-1})\,\varepsilon_{t-1}^2 + 3\delta_3(1-\beta)(W_{t-1} - \alpha k_{t-1})^2 \varepsilon_{t-1} \\
& + (1-\beta)\varepsilon_t - \delta_1(1-\beta)\varepsilon_{t-1} + \eta_t
\end{aligned}
$$

Using the same instruments as before, $Z_t = \{k_t, k_{t-1}, m_{t-1}, W_{t-2}\}$

$$
\begin{aligned}
E[W_t|Z_t] =& \alpha k_t + \delta_0 + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] + \delta_2 E[(W_{t-1} - \alpha k_{t-1})^2 \,|Z_t] \\
& + \delta_3 E[(W_{t-1} - \alpha k_{t-1})^3 \,|Z_t] + \delta_2 (1-\beta)^2 E[\varepsilon_{t-1}^2|Z_t] \\
& - 2\delta_2 (1-\beta) E[(W_{t-1} - \alpha k_{t-1})\, \varepsilon_{t-1}|Z_t] + \delta_3 (1-\beta)^3 E[\varepsilon_{t-1}^3|Z_t] \\
& - 3\delta_3 (1-\beta)^2 E[(W_{t-1} - \alpha k_{t-1})\, \varepsilon_{t-1}^2|Z_t] + 3\delta_3 (1-\beta) E[(W_{t-1} - \alpha k_{t-1})^2\, \varepsilon_{t-1}|Z_t] \\
=& \alpha k_t + \delta_0 + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] + \delta_2 E[(W_{t-1} - \alpha k_{t-1})^2 \,|Z_t] \\
& + \delta_3 E[(W_{t-1} - \alpha k_{t-1})^3 \,|Z_t] - \delta_2 (1-\beta)^2 \sigma_{\varepsilon_{t-1}}^2 \\
& - \delta_3 (1-\beta)^3 E[\varepsilon_{t-1}^3] \\
& - 3\delta_3 (1-\beta)^2 \sigma_{\varepsilon_{t-1}}^2 E[\omega_{t-1}|Z_t] \\
=& \alpha k_t + \underbrace{\delta_0 - \delta_2 (1-\beta)^2 \sigma_{\varepsilon_{t-1}}^2 - \delta_3 (1-\beta)^3 E[\varepsilon_{t-1}^3] - 3\delta_3 (1-\beta)^2 \sigma_{\varepsilon_{t-1}}^2 E[\omega_{t-1}|Z_t]}_{\tilde{\delta}_0} \\
& + \delta_1 E[W_{t-1} - \alpha k_{t-1}|Z_t] + \delta_2 E[(W_{t-1} - \alpha k_{t-1})^2 \,|Z_t] \\
& + \delta_3 E[(W_{t-1} - \alpha k_{t-1})^3 \,|Z_t]
\end{aligned}
$$

Now, even though I know β and σ_ε, and δ_2 and δ_3 are identified from $(W_{t-1} - \alpha k_{t-1})^2$ and $(W_{t-1} - \alpha k_{t-1})^3$, I cannot identify δ_0 because $E[\omega_{t-1}|Z]$ is unknown.