Tax Evasion and Productivity

Hans Martinez*
Department of Economics, University of Western Ontario

April 13, 2024

Abstract

Corporate tax evasion through cost overreporting spreads internationally causing governments significant tax revenue losses. Detecting and measuring the magnitude of tax evasion remains a challenge, even for the few studies on overreporting where researchers can exploit administrative data. Moreover, if this evasion strategy accounts for economic losses as large as reported, then cost overreporting might bias estimates of production functions, especially productivity. This paper addresses both issues. I first provide a novel strategy to estimate cost overreporting using commonly available firm-level data. I then formally show that ignoring cost overreporting leads to downward biased productivity estimates. Finally, I demonstrate how to recover productivity in the presence of tax evasion.

Keywords: Tax Evasion, Cost Overreporting, Production Function Estimation, Productivity

*email: hmarti33@uwo.ca. I thank my supervisors Salvador Navarro, David Rivers, and Victor Aguiar for their guidance.

Introduction

Corporate tax evasion through cost overreporting spreads globally causing governments significant tax revenue losses. Detecting and measuring the magnitude of cost overreporting remains a challenge, even for researchers with access to government administrative data. Moreover, if this evasion strategy accounts for economic losses as large as reported, then cost overreporting might bias estimates of production functions, including productivity. This paper addresses both issues. I first provide a novel strategy to estimate cost overreporting using more commonly available firm-level data. I then formally show that ignoring cost overreporting leads to downward biased productivity estimates. Finally, I demonstrate how to recover unbiased estimates of the production function and productivity in the presence of tax evasion.

Cost overreporting arises when firms acquire false invoices to claim additional tax deductions on value-added (VAT) and corporate income taxes (CIT). According to the OECD's document *Technology Tools to Tackle Tax Evasion and Tax Fraud* (2017), cost overreporting — also known as "fake invoicing", "ghost firms", "invoice mills", or "missing traders"— permeates internationally. Reports from Latin America, Eastern Europe, Asia, and Africa claim cost overreporting led to annual tax revenue losses as large as 5.6% of the GDP, for example, in Poland, 2016 (Poland's Minister of Finance, 2018)[1]

Despite its relevance, cost overreporting has been mostly overlooked by the literature. On one hand, the few studies on this evasion strategy exploit detailed administrative data (Zumaya et al. 2021, Carrillo et al. 2022). Government tax authorities restrict access to

[1] Other reports show that cost overreporting led to revenue losses of 0.2% of Chile's GDP in 2004 (Gonzalez and Velasquez, 2013; Jorrat, 2001; CIAT, 2008); 0.2% of Colombia's GDP (Portafolio, 2019); and 0.03% of Mexico's GDP in 2018 (Senado de la Republica, 2019).

administrative data because of firms' confidentiality concerns. On the other hand, to the best of my knowledge, no study has attempted to structurally identify cost overreporting. Unlike the case of individuals (Pissarides & Weber 1989, Paulus 2015), when it comes to corporate tax evasion, researchers have to account for an additional source of unobserved heterogeneity, productivity. Why? Because cost overreporting might be naively quantified as low productivity. Intuitively, for a given output level, high input utilization by a firm could be explained by either the amount of input the firm overreports to evade taxes or by a negative productivity shock.

To address this gap in the literature, first I formally show that ignoring tax evasion leads to downwardly biased productivity estimates. I then provide a new estimation strategy, requiring only commonly available firm-level data, to jointly recover the densities of tax evasion and productivity. The intuition works as follows. In the absence of tax evasion, the first-order conditions of the firms' cost-minimization problem inform about a common technology, the production function. Consequently, in the presence of cost overreporting, deviations from this common technology identify tax evasion up to the current-period output shock. Then, from the subset of non-overreporting firms, the strategy identifies the production function parameters and the density of the output shock. Finally, using deconvolution techniques, I can jointly recover the distributions of tax evasion and productivity.

I am currently working on an application of the method using firm-level data from Colombia between 1981 and 1991. Colombia is an interesting case because two major fiscal reforms in this period will allow me to show how to apply the method to evaluate a policy change. I include a description of the Colombian Tax System of the time. The plan is to also have estimates for Ecuador and Mexico. Both of these countries have administrative data which I plan to use to evaluate the method's estimates.

1 A parsimonious model of tax evasion through input overreporting

Price-taking firms maximize expected after-tax profits. Firms choose the flexible input M_{it} to produce output Y_{it} given output and input prices $\{P_t, \rho_t\}$, a common technology, the production function (Equation 1), and their productivity ω_{it}.

$$Y_{it} = G(M_{it}) \exp(\omega_{it} + \varepsilon_{it}) \tag{1}$$

As standard in the literature, productivity ω_{it} is known to firms when they take input decisions. This is the well-known endogeneity problem of simultaneity. On the other hand, firms face output shocks. The output shock ε_{it} is not part of the firms' information set.

The model departs from the literature by allowing firms to overreport their inputs e_{it} to reduce their tax burden and optimize after-tax profits. Firms, then, consider in their optimization problem the profit tax τ, the evasion penalty/cost $\kappa(e)$, and the probability of detection $q(e_{it}|\theta_{it})$.

Firms solve Equation 2

$$\begin{aligned} \max_{M_{it}, e_{it} \in [0,\infty),} \; & [1 - q(e_{it}|\theta_{it})] \left[(P_t \mathbb{E}[Y_{it}] - \rho_t M_{it}) - \tau \left(P_t \mathbb{E}[Y_{it}] - \rho_t (M_{it} + e_{it}) \right) \right] \\ & + q(e_{it}|\theta_{it}) \left[(1-\tau)(P_t \mathbb{E}[Y_{it}] - \rho_t M_{it}) - \kappa(e) \right] \end{aligned} \tag{2}$$

$$\text{s.t.} \quad Y_{it} = G(M_{it}) \exp(\omega_{it} + \varepsilon_{it})$$

The probability of detection $q(e_{it}|\theta_{it})$ is monotonically increasing in the amount evaded e_{it}, conditional on the type of the firm θ_{it}. Intuitively, for a given type, firms that evade more are more likely to get caught.

The type of the firm θ_{it} might be discrete, like the type of juridical organization, or continuous, like the level of revenue[2]. Some types might be more likely to be detected if the firm engages in tax evasion. For example, in contrast to other types of juridical organizations in Colombia, corporations are closely supervised and are required to have an auditor. That is, for a given level of tax evasion e_0 and two different types $\theta' \neq \theta \in \Theta$, then $q(e_0|\theta') \geq q(e_0|\theta)$.

If the type θ is continuous, it might be a function of inputs; for example, level of revenue. Firms will then affect their probability of detection $q(e|\theta)$ in two ways: directly, by choosing how much they evade e; and indirectly, when choosing inputs M.

The optimal decision of the firm will depend on the fiscal environment $\Gamma = \{\tau, \kappa, q\}$, namely the tax rates, the penalty/cost of detection, and the probability of detection.

The firms' problem (Equation 2) can be rewritten as follows,

$$\begin{aligned}\max_{M_{it}, e_{it}} \mathbb{E}[\pi_{it}|\Gamma] =& (1-\tau)\left(\mathbb{E}[Y_{it}] - \frac{\rho_t}{P_t}M_{it}\right) + [1 - q(e_{it}|\theta_{it})]\left(\frac{\rho_t}{P_t}e_{it}\tau\right) - q(e_{it}|\theta_{it})\kappa(e_{it}) \\ \text{s.t.} \quad & Y_{it} = G(M_{it})\exp(\omega_{it} + \varepsilon_{it})\end{aligned}$$

Intuitively, if the firm overreports her inputs' cost, she will get the share of the value she overreported with probability $(1-q)$ and she will be penalized with probability q.

Assuming well-behaved functions and no corner solutions, the first-order conditions lead to the following system of differential equations,

$$G_M(M_{it})\exp(\omega_{it})\mathcal{E} - \frac{\rho_t}{P_t} = \frac{1}{(1-\tau)}\frac{\partial q(e_{it}|\theta_{it})}{\partial \theta_{it}}\frac{\partial \theta_{it}}{\partial M}\left[\frac{\rho_t}{P_t}e_{it}\tau + \kappa(e_{it})\right] \tag{3}$$

[2]Level of revenue is a common measure for fiscal authorities to determine a firm's taxes and/or level of scrutiny, e.g., Mexico, Spain, Colombia, and Ecuador.

$$
[1-q(e_{it}|\theta_{it})]\frac{\rho_t}{P_t}\tau - q(e_{it}|\theta_{it})\kappa'(e_{it}) = q'(e_{it}|\theta_{it})\left[\frac{\rho_t}{P_t}\tau e_{it} + \kappa(e_{it})\right] \tag{4}
$$

where $\boldsymbol{\mathcal{E}} = \mathbb{E}[\exp(\varepsilon_{it})]$. The type of firms is continuous and increasing on the input. The probability of detection is increasing in the type continuum. In particular, $\frac{\partial q(e_{it}|\theta_{it})}{\partial \theta_{it}}\frac{\partial \theta_{it}}{\partial M} \geq 0$.

The left-hand side of Equation 3 is the familiar marginal output of inputs and the price ratio. In the absence of incentives' distortions induced by the fiscal environment, they are equal. But now, the equality holds no more. There's a wedge arising from the fiscal environment. The right-hand side of the equation is positive by the assumptions of the model.

Equation 4 solves the optimal evasion decision. The left-hand side is the marginal benefit net of the marginal cost of evasion. The right-hand side is the rate of change of the probability of detection due to a change in evasion weighted by the benefit and cost of evading.

1.1 Case 1 (Independence): $q(e|\theta) = q(e)$ and $\kappa(e) = \kappa_0$

Consider the case when the probability of detection is independent of type, $q(e|\theta) = q(e)$. This could be the case if the type is the juridical organization of the firm. Hence, the type of the firm, and thus the probability of detection, does not change with the firm's input decisions, $\frac{\partial q(e_{it}|\theta_{it})}{\partial \theta_{it}}\frac{\partial \theta_{it}}{\partial M} = 0$. In addition, assume the evasion cost is constant, $\kappa(e) = \kappa_0$, for simplicity.

In this case, the first-order conditions of Equation 2 with respect to the input M_{it} and the tax evasion e_{it} yield the following

$$G_M(M_{it}) \exp(\omega_{it}) \mathcal{E} = \frac{\rho_t}{P_t} \tag{5}$$

$$e_{it} = \frac{1 - q(e_{it})}{q'(e_{it})} - \frac{\kappa_0}{\frac{\rho_t}{P_t}\tau} \tag{6}$$

Equation 5, the well-known optimality condition, says that the price ratio is equal to the marginal product of the inputs.

Likewise, Equation 6 reveals the firms' optimal tax evasion decision decreases if the probability of detection $q(e_{it})$ or the penalty of evading κ increases. Tax evasion also depends on how sensitive the probability of detection is to the level of evasion $q'(e)$. In particular, greater sensibility will result in lower levels of evasion.

Note that the net change of tax evasion due to an increase in the relative prices $\frac{\rho_t}{P_t}$ or the tax rate τ is not evident at first sight. The net effect will also depend on the change in the detection probability induced by the changes in the relative prices or the tax rate. In particular, an increase in relative prices $\frac{\rho_t}{P_t}$ or the tax rate τ will incentivize a higher tax evasion level, however, a higher tax evasion level will increase the probability of detection —depending on the shape of the probability as a function of e—, so it will deter higher levels of evasion. An increase in the tax rate, for instance, will only increase tax evasion if the change in the tax rates increases the incentives to evade more than the decrease in the incentives due to the changes in the detection probability.

Formally, suppose a firm increases its tax evasion, $e_1 - e_0 > 0$ because of an increase in taxes $\tau_1 > \tau_0$. Then, it follows that

$$\left(\frac{\tau_1 - \tau_0}{\tau_1 \tau_0}\right) \frac{P\kappa}{\rho} > \left(\frac{1 - q(e_1)}{q'(e_1)} - \frac{1 - q(e_0)}{q'(e_0)}\right)$$

The change in the probability of detection weighted by the slope of the probability function should be less than the change in the tax rate weighted by the penalty of evading and the relative prices[3].

1.2 Case 2 (Spain): Discrete increase in the probability of detection after a certain threshold of revenue

In Spain, the Large Taxpayers Unit (LTU) of the tax authority focuses exclusively on firms with total operating revenue above 6 million euros. The LTU has more auditors per taxpayer than the rest of the tax authority, and these auditors are on average more experienced and better trained to deal with the most complex taxpayers. This LTU creates a discontinuity in the monitoring effort of the tax authority. Consequently, at this arbitrary revenue level, the probability of detection increases discretely (Almunia & Lopez-Rodriguez 2018).

In this scenario, depending on the productivity shock, the firm might be better off choosing not to produce past the revenue threshold. Indeed, for a relevant range of productivity draws $\Omega^B = [\omega^L, \omega^H]$, the firms will not choose to grow past the revenue threshold if the expected after-tax profits of staying small are greater than the expected after-tax profits of growing.

In the model, there is now a threshold of revenue θ^L after which the probability of detec-

[3] An analogous condition for an increase in relative prices leading to higher levels of tax evasion exists. Under this condition, the model is consistent with the literature that macroeconomic downturns lead to higher evasion.

tion increases discretely. To make things simpler, assume that before the threshold, the probability changes as a function of evasion but does not vary conditional on size. After the threshold, the probability increases for every level of evasion but does not vary conditional on size.

Formally, let $\Theta_L = \{\theta_i : \theta_i < \theta^L\}$ and $\Theta_H = \{\theta_i : \theta_i \geq \theta^L\}$, then for all e_0 and $\theta_i' \neq \theta_i$, $q(e_0|\theta_i \in \Theta_k) = q(e_0|\theta_i' \in \Theta_k)$ with $k = \{L, H\}$, but $q(e_0|\theta_i' \in \Theta_H) \geq q(e_0|\theta_i \in \Theta_L)$.

Firms' revenue with productivity draw ω^L corresponds exactly to the enforcement threshold θ^L. Production and reporting decisions of firms with productivity draws below ω^L are not affected by the change in the probability of detection. Firms choose their inputs according to Equation 5 and their evasion decision according to Equation 4. Firms with productivity draws above ω^U

Firms with productivity $\omega_i \in \Omega^B$ will choose the input level $\tilde{M}_i$ resulting in an expected revenue below the threshold $\theta_i < \theta^L$, if the expected after-tax profit of staying small are greater than growing, $\mathbb{E}[\pi_i|\Theta_L, \Omega^B] - \mathbb{E}[\pi_i|\Theta_H, \Omega^B] \geq 0$.

The optimal input choice M_i^* for firms with productivity $\omega_i \in \Omega^B$ implies an expected revenue greater than or equal to the threshold $\theta_i^* \geq \theta^L$. Let the expected profits given M_i^* and the optimal tax evasion in the range of size θ_l, e_{it}^*, is $\pi_l \equiv \mathbb{E}[\pi(M_{it}^*, e_{it}^*)|\theta_l]$. Let $\tilde{M}_{it}$ be the input level such that the expected revenue is below the threshold $\tilde{s}_{it} < \theta^L$ and $\tilde{e}_{it}$ be the optimal tax evasion in the range of size θ_s. Let also the expected profits of staying small are $\pi_s \equiv \mathbb{E}[\pi(\tilde{M}_{it}, \tilde{e}_{it})|\theta_s]$.

In this second case, therefore, firms might optimally choose to remain small if, for a low productivity shock, the expected profits of not growing are greater than the expected profits of growing $\pi_l < \pi_s$. Firms choosing to remain small will lead to a bunching below the threshold in the size distribution of firms.

Besides the higher levels of evasion before the threshold —simply because of the higher probability of detection—, we can also expect bunching firms to evade more than their similar-sized peers. At $\tilde{M}_{it}$, the optimization condition of Equation 5 no longer holds, hence, the marginal product of the input is now greater than the relative prices. Therefore, according to Equation 6, bunching firms would compensate for their *higher* costs by increasing overreporting.

1.2.1 Discussion

What is new in this paper relative to the literature is that it focuses on the production function framework using public data whereas Almunia & Lopez-Rodriguez (2018) and other papers use a bunching estimator with government administrative data which is difficult to access. Second, the paper focuses on input overreporting rather than on revenue underreporting, which is the relevant margin of evasion for manufacturing firms. More on this point in the revenue underreporting section. Finally, in contrast to Almunia & Lopez-Rodriguez (2018) where the authors conclude that misreporting does not imply real losses in production but only fictitious reduction of the real sales, firms might optimally forgo higher revenue levels if the expected profits of staying small and evade taxes by misreporting are greater than the expected profits of growing and avoid misreporting.

1.3 Case 3 (Colombia & Mexico): Discrete increase in the tax rate after a revenue threshold

1.3.1 Colombia, Individual Proprietorships

In Colombia between 1981 and 1991, individual firm proprietors were subject to the individual income tax schedule. Individuals had incentives to not form juridical organizations to avoid double taxation. The tax authority suffered from severe limitations and inefficiencies at the time.

In this case, after the revenue threshold, the tax rate increases discretely but the probability of detection does not. The jump in the tax rate generates the incentive to increase evasion. However, a higher level of evasion increases the cost of evading by increasing the probability of detection. If the cost of an increased evasion outweighs the benefits of growing past the revenue threshold, the firms would bunch below the cutoff.

1.3.2 Mexico, Irreversible Change in Tax Regime after a Revenue Threshold

In Mexico, firms with annual revenues below 2 million pesos are taxed under the REPECO (*Regime de Pequeños Contribuyentes*) regime of small contributors at 2 percent of annual revenues, while firms above that threshold are taxed under the general regime at 30 percent. Firms must transition to the general regime if revenues increase beyond the threshold. Once in the general regime, firms cannot revert to the REPECO regime.

Firms' decision is now dynamic. Firms will maximize the sum of current and future after-tax profits. The discrete jump in the tax rate will lead to a bunching below the threshold. Moreover, the bunching will be exacerbated because firms will choose to grow past the cutoff only if the future productivity shocks allow the firm to continue to be profitable.

1.4 Case 4 (Colombia): Firms first choose type, input decisions do not affect the probability of detection

In Colombia between 1981 and 1991, Corporations were closely supervised by the Superintendent of Corporations and were required to have an auditor. All other firms were subject to the regular monitoring efforts of the tax authority, which suffered from severe limitations and inefficiencies at the time.

In the model, firms first choose their type. Input decisions do not affect the probability of detection. However, if the type is *Corporation* the probability of detection is higher than *Partnership*. Firms maximize the sum of their expected profits. In their optimization problem, firms will consider the sum of expected productivity shocks and their corresponding probability of detection. High-productivity firms will self-select into *Corporations*.

2 The empirical framework

Suppose we have access to panel data, where we observe output Y_{it}, intermediate inputs M_{it}, capital K_{it}, labor L_{it}, and output P_t and intermediate input prices ρ_t for I firms over T periods. Then our set of observations is $\mathcal{O} = \{Y_{it}, M_{it}, K_{it}, L_{it}, P_t, \rho_t\}_{i \in I, t \in T}$. As is standard in the literature, firms are price-takers and the intermediates are flexible.

The objects of interest are the production function (PF), $Y_{it} = G(M_{it}, K_{it}, L_{it})e^{\omega_{it}+\varepsilon^Y_{it}}$, and productivity ω_{it}. ε^Y_{it} is the current period output shock. We are also interested in the Markov process of productivity, which we assume is AR(1), $\omega_{it} = h(\omega_{it-1}) + \eta_{it}$, with $\mathbb{E}[\eta_{it}|\omega_{it-1}] = 0$.

2.1 Tax evasion and the productivity bias

Firms overreport their true intermediate inputs M_{it}^* by $e^{\varepsilon_{it}^M}$ to evade taxes. Then, reported inputs are

$$M_{it} = M_{it}^* e^{\varepsilon_{it}^M}$$

with $\varepsilon_{it}^M \geq 0$ and $\varepsilon_{it}^M \not\perp M_{it}^*$.

It is fairly easy to see that the productivity bias, the difference between the naively estimated $\tilde{\omega}_{it}$ and true productivity ω_{it} is as follows:

$$\mathbb{E}[\tilde{\omega}_{it}|\mathcal{I}_{it}] - \mathbb{E}[\omega_{it}|\mathcal{I}_{it}] \leq \ln \mathbb{E}\left[\left.\frac{G(M_{it}^*, K_{it}, L_{it})}{G(M_{it}^* e^{\varepsilon_{it}^M}, K_{it}, L_{it})}\right| \mathcal{I}_{it}\right] \leq 0$$

Where $\mathcal{I}$ stands for the information set of firm i in time t.

The previous result holds because of Jensen's inequality and because $G(\cdot)$ is monotonically increasing in its arguments.

3 Colombia 1981-1991

3.1 Colombian Corporate Tax System

The relevant corporate taxes for input overreporting in Colombia during this period are the Corporate Income Tax (CIT) and the Sales Tax. The Sales Tax gradually transformed into a kind of Value-Added Tax (VAT). Also relevant for the CIT are the different juridical organizations that exist in Colombia.

This period was characterized by high levels of overall tax evasion (Torres & Sourdis 1994).

The fiscal rules had a system of penalties and interest that encouraged false and delinquent returns (Mclure 1989). The fiscal authority was characterized by having an inefficient auditing system, being overburdened, and legal loopholes (Perry & de Triana 1990).

3.1.1 Juridical Organizations

In Colombia, there are five types of juridical organizations: Corporations, Partnerships, Limited Liability Companies, and Individual Proprietorships.

Corporations (*sociedad anónima*) are the typical associations of capital. They are the counterpart of the US corporation. The capital of a corporation is provided by the shareholders (no less than 5) and is divided into tradable shares of equal value. Shareholders' liability is limited to the capital contributed. Corporations are subject to the Superintendent of Corporations and are closely supervised, being required to have an auditor.

Joint Stock Companies (*sociedad en comandita por acciones*) comprises two or more managing partners who are jointly and severally liable, and five or more limited partners whose liability is limited to their respective contributions. Joint Stock Companies are taxed as Corporations. Its shares are tradable, like the shares of Corporations.

Partnerships are associations of two or more persons. Partners are jointly and severally liable for the partnership's operations. Partnerships include general partnerships (*sociedad colectiva*), de Facto partnerships (*sociedades de hecho*), and ordinary limited partnerships (*sociedad en comandita simple*).

A limited liability company (*sociedad de responsabilidad limitada*) is an association of two or more persons —not exceeding 20 (Fiscal Survey) or 25 (1992 *EAM* survey documents)—, whose shares cannot be traded. The personal liability of the partners is limited to the capital contributed. The Limited Liability Company is quite important in Colombia (Fiscal

Survey).

Finally, proprietorships are individuals (natural persons) who allocate part of their assets to conduct commercial activities.

There are other juridical organizations in the data that will be excluded from the final analysis. These organizations are non-profit, like cooperatives and community enterprises, or state industrial enterprises, the proceeds of which come from taxes, fees, or special contributions.

3.1.2 Corporate Income tax

The juridical organizations were subject to different Corporate Income Tax rates. Corporations were taxed at a fixed rate of 40%, while Partnerships and Limited Liability companies at 20%. Individual proprietors were subject to the graduated Individual Tax Schedule consisting of 56 rates, ranging from 0.50 to 51 percent.

Corporations were taxed on their distributed dividends, while partnerships and limited liability companies were taxed on their profits, whether or not distributed. Owners of juridical organizations were double taxed, at the firm and the individual level, whereas the income of proprietorships was taxed only once, at the individual level.

Since 1974, individuals and juridical organizations, except for limited liability companies, were subject to the minimum presumptive income. Rent (income and profits) was presumed to be no less than 8 percent of net wealth (assets less depreciation, real estate, livestock, securities).

Certain industries like airlines, publishing, and reforestation sectors, and various activities in selected regions (primarily "frontier" and other less developed ones) had their income tax exempted, limited, or reduced.

Table 1: Juridical Organizations in Colombia (1980s), A Summary

Organization	Corporate Income Tax	Liability	Capital	Owners
Corporation	40% (on distributed dividends)	Limited to capital participation	Tradable capital shares	$N \geq 5$
Limited Co.	20% (on profits)	Limited to capital participation	Non-tradable capital shares	$2 \leq N \leq 20(25)$
Partnership	20% (on profits)	Full	Not a capital association	$N \geq 2$
Proprietorship	Individual Income Tax	Full	Owner	$N = 1$

3.1.3 Sales taxes

Sales taxes were originally targeted at the manufacturing sector on finished goods and imports. Since 1974, manufacturers were allowed to credit taxes paid on any purchase made by the firm, except the acquisition of capital goods (Perry & de Triana 1990). The credits worked through a system of refunds. Consequently, the tax became a kind of value-added tax (VAT).

The basic rate was 15 percent. There was also a preferential rate of 6 percent for certain industries like clothing, footwear, and major inputs used for building popular housing, and a rate of 35 percent for luxury goods. Exports, foodstuff, drugs, and textbooks were excluded from the beginning. Also excluded were inputs, transportation equipment, agricultural machinery, and equipment.

3.1.4 Discussion

From Colombia's tax system, we can conclude that corporations are the least likely to evade taxes by misreporting because of at least three reasons. One, the Superintendent of Corporations closely monitored corporations by requiring them to have an auditor. In the model, this implies that the probability of detection is higher for them. Second, free tradable shares impose an incentive to be profitable because the market value of the shares might be negatively affected otherwise. In other words, if a corporation fakely reports lower profits, the value of its shares would likely decrease scaring away shareholders and potential investors. Joint stock companies have freely tradable shares too. Three, corporations pay CIT on distributed dividends, not on profits as Partnerships and LLCs did. Corporations have an additional margin regarding the income tax they pay because they decide when to pay dividends. This might generate other types of incentives that might be influenced by the corporation's policy regarding their dividends and the demands of their shareholders. On the other hand, Proprietorships and LLCs are subject to the incentive to evade CIT by artificially reducing their profits.

Moreover, Proprietorships, Partnerships, and Limited Companies had incentives to overreport inputs to evade VAT and CIT. The incentives to evade varied across industries because the sales tax rate differed between industries. The incentives to evade also varied within industry sectors because juridical organizations within an industry were subject to different CIT rates. There were additional sources of variation depending on the firm's location due to local exemptions and sales composition (inputs to other firms, to consumers, to the foreign market).

Lastly, Individual proprietorships were likely to bunch at the individual income thresholds because they were subject to individual income tax which was increasing by brackets.

3.2 Fiscal Reforms

During this period, Colombia went through three major fiscal reforms (1983, 1986, 1990).

3.2.1 1983

The 1983 reform tried to alleviate the double taxation by introducing a tax credit of 10% of dividends received for individuals.

In addition, Law 9 of 1983 instituted a measure of presumptive income equal to 2 percent of gross receipts to supplement the measure based on net wealth. This reform was aimed specifically at the commerce and service sectors; the former was thought to evade the wealth-based presumptive tax by systematically understating inventories. In addition, it extended the presumptive income tax to limited liability companies.

In 1983, the value-added tax (VAT) was extended to the retail level, with a *simplified system* being made available to small retailers to ease compliance costs and the administrative burden.

The 1983 reform relatively unified the value-added tax (VAT) rates by combining previously taxed goods at 6% and 15 percent into 10%. The number of the products and services that were levied expanded.

In 1984, VAT exemptions for agricultural machinery, transportation equipment, and certain other goods were eliminated.

3.2.2 1986

The 1986 reforms unified the taxation of corporations and limited liability companies by taxing both at a rate of 30%. The top tax rate applied to individual income was reduced from 49 to 30%.

Double taxation was eliminated. The reform exempted corporate dividends and participation in profits of limited liability companies from tax at the individual shareholder/owner level.

Lastly, the 1986 reform relocated the tax collection and reception of tax reports to the banking system, among other things.

3.2.3 1990

The 1990 reform increased the VAT from 10% to 12%.

In addition, the individual income obtained from the sale of shares through the stock market was exempted from taxation. Income tax was waived for investment funds, mutual funds, and securities, and the rates for remittances and income for foreign investment were reduced. Tax amnesties were granted, and the sanitation tax was reduced to encourage the repatriation of capital.

3.2.4 Discussion

Increases in the VAT would increase the incentives to evade. Decreases in CIT would decrease them. Intuitively, we expect higher levels of tax evasion if tax rates increase.

The CIT homogenization between Corporations and LLCs in 1986 would have motivated LLCs to incorporate. Likewise, the elimination of double taxation also in 1986 would have motivated proprietorships to become LLCs, Corporations, or Partnerships.

On the other hand, reporting more information to the tax authority — like the banking system being responsible for the collection and reception of tax reports in 1986— would decrease tax evasion.

3.3 Data

The Colombian data is a well-known firm-level panel data set that has been used in the estimation of production functions and productivity before. The Colombian dataset comes from the Annual Survey of Manufacturing (EAM) and contains information about manufacturing firms with more than 10 employees from 1981 to 1991.

Besides the information on output, intermediates, capital, and labor, the dataset includes the type of juridical organization and the sales taxes. Table 2 offers some summary statistics.

A simple graphical analysis shows that the average (of the log) intermediates cost share of

Table 2: Summary Statistics, Manufacturing Firms in Colombia (1981-1991)

	Missing	Mean	SD	Min	Max
Capital	0	81450.0	1396998.4	1.0	270221640.5
Materials & Services	0	144808.3	714091.1	13.0	35370843.8
Labor (Skilled)	0	30.1	82.9	1.0	7389.0
Labor (Unskilled)	0	67.0	149.2	1.0	5174.0
Revenue	0	219548.2	908964.3	82.2	44236868.0
Sales Taxes	0	13449.8	124432.7	0.0	16441078.0
JO_class	N	%			
Corporation	8805	21.0			
Ltd. Co.	25643	61.3			
Other	541	1.3			
Partnership	1462	3.5			
Proprietorship	5380	12.9			

sales started growing after 1983 and that it stabilized in 1988 after the policy changes of the 1986 reform settled in (Figure 1). The dataset does not capture any changes after the 1990 reform, although there is only one more year of data.

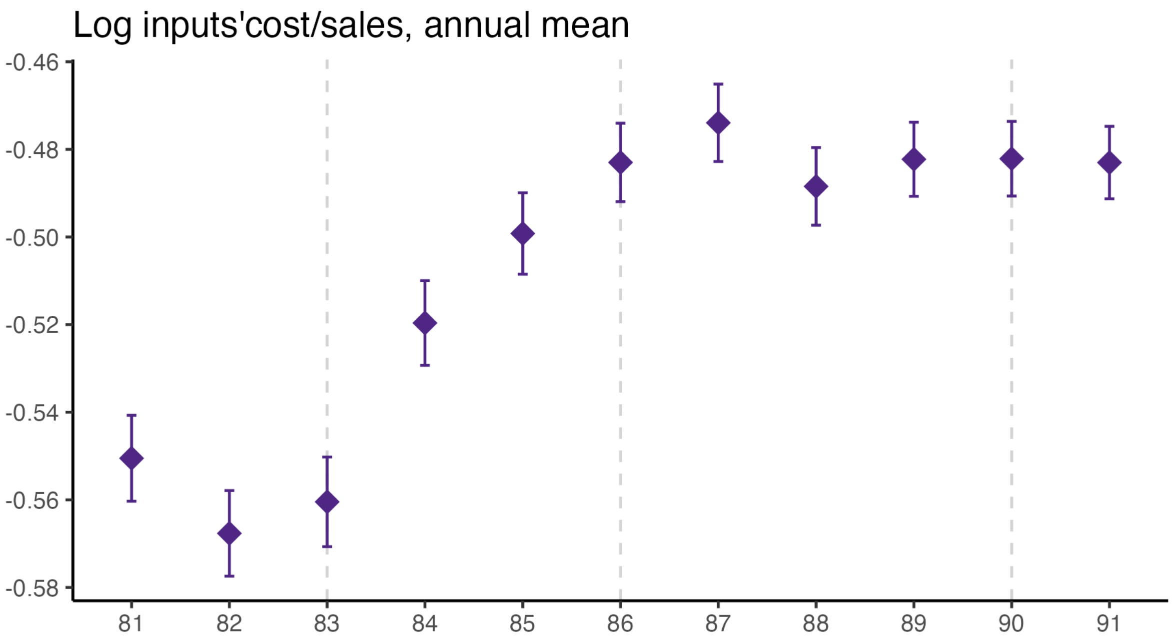

Figure 1: Input's cost share of sales, average by year of the logs.

As a validation exercise, we can see that the VAT changes induced by the three fiscal reforms are captured in the dataset. Figure 2 shows that the sales tax increased to 10% after the 1983 reform, and then around 12% after the 1990 reform.

Just as an exercise to see if other economic changes in this period were driving the apparent changes in overreporting, Figure 3 shows that sales, for instance, were not exactly following the changes in fiscal policy. Sales started to grow during 1983, the year of the reform, whereas the cost share of sales started to grow the year after. Likewise, sales fell in 1986, while the cost share seems to reduce its growth after 1986.

Finally, in a preliminary empirical assessment, I observe that the sales tax rate is a signif-

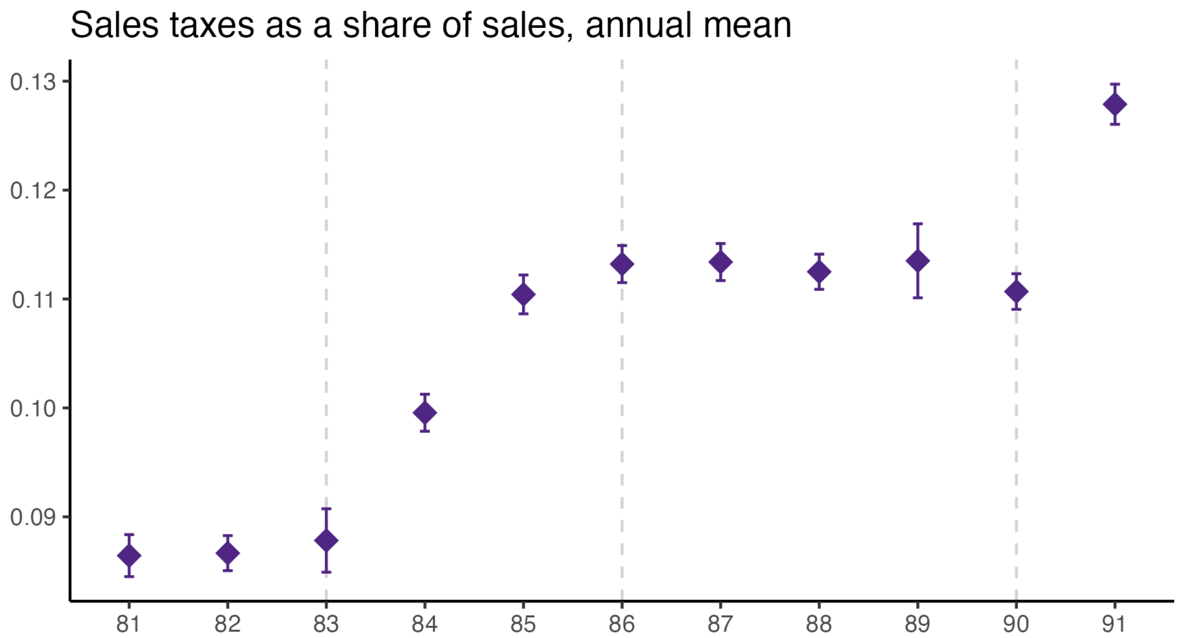

Figure 2: Sale taxes paid as share of sales, average by year.

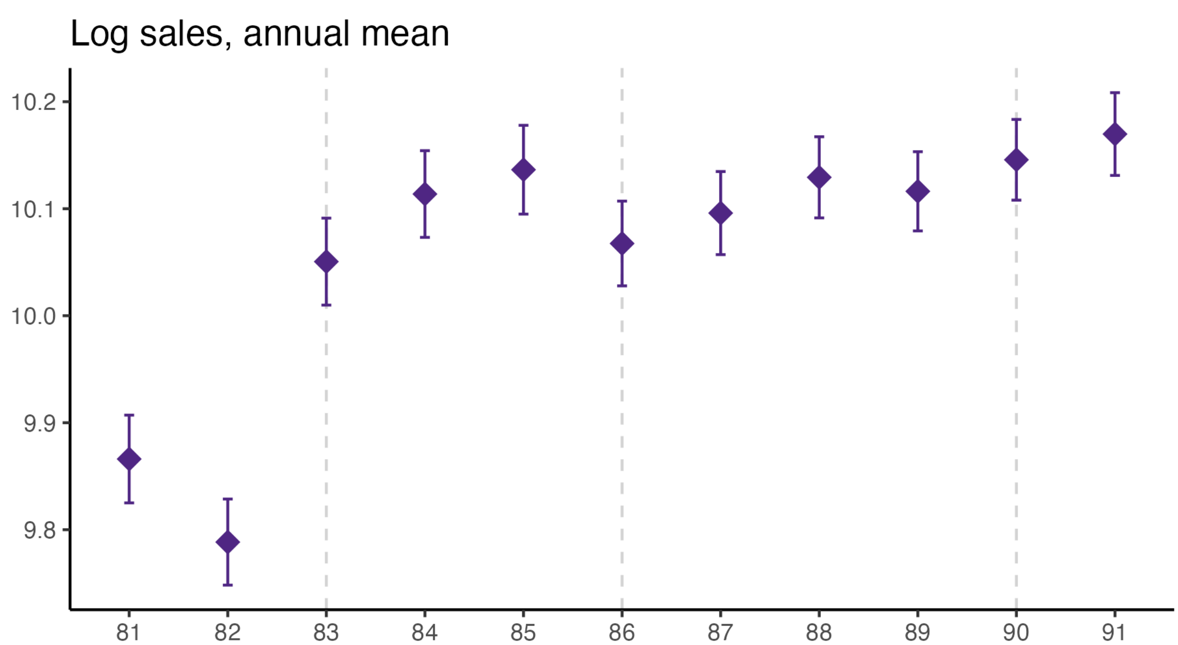

Figure 3: Sales in logs, annual mean.

icant determinant of the log share of revenue and that non-corporations consistently use 13-17 percent more intermediates than Corporations for a rich set of controls Table 3. The results were estimated following Equation 7

$$log(s_{it}) = \alpha_1 Tax_{it} + {}'_1 JurOrg_i + \beta'_2 JurOrg_i \times \gamma_t + \gamma_t + \gamma_{ind} + \gamma_{metro} + \beta' Z + \varepsilon_{it} \quad (7)$$

Although this is not deterministic evidence, it does not contradict the hypothesis that firms other than corporations have incentives to overreport intermediates to evade taxes and that the higher the taxes the higher the incentives to evade by misreporting.

4 Identification Strategy

Because the firms' optimization decisions depend on the fiscal environment, the identification strategy should be motivated by the fiscal environment Γ. In particular, the identification strategy will be as good as how well we can tell apart a subset of firms that have the highest incentive to not evade. For example, in the case of Spain, the firms above the revenue LTU threshold. In the case of Colombia, the corporations.

Assumption 4.1: Non-Evaders

Based on the fiscal environment Γ, the researcher can identify a subset of firms $\theta_i \in \Theta^{NE} \subset \Theta$ that does not evade taxes by overreporting inputs.

For those firms, then $\mathbb{E}[e_{it}|\theta_i \in \Theta^{NE}] = 0$

In addition, I impose the following timing assumption.

Table 3: Effect of the Juridical Organization Type and Sales Tax on the Log Share of Intermediate Inputs.

Dependent Variable:		$log(s)$	
Model:	(1)	(2)	(3)
Variables			
Sales Tax Rate	0.2776**	0.4019***	0.3798***
	(0.1158)	(0.0907)	(0.0917)
Proprietorship	0.1502***	0.1236***	0.1203***
	(0.0252)	(0.0200)	(0.0193)
Ltd. Co.	0.1647***	0.1403***	0.1352***
	(0.0265)	(0.0210)	(0.0210)
Partnership	0.1830***	0.1588***	0.1577***
	(0.0297)	(0.0239)	(0.0234)
Fixed-effects			
Industry		Yes	Yes
Metro Area			Yes
Fit statistics			
Observations	41,830	41,830	41,830
R^2	0.54871	0.56214	0.56715
Within R^2		0.51571	0.51776

Clustered (Industry & Year) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Assumption 4.2: Independence

Firms choose overreporting e_{it} *before* the output shock ε_{it}

Assumption 4.2 implies that input overreporting is independent of the current period output shock, $e_{it} \perp \varepsilon_{it}$. In the literature is not rare to assume that the output shock is not part of the information set of the firms, $\varepsilon_{it} \notin \mathcal{I}_t$ (Gandhi et al. 2020). Timing and information set assumptions are not uncommon for identification strategies in production functions and demand estimation (Ackerberg et al. 2021, Ackerberg 2019).

4.1 Identifying the production function parameters

The econometrician observes then the overreported inputs in the data, $M_{it} = M^*_{it} \exp(e_{it})$[4]. Assume the production function is Cobb-Douglas, $G(M^*_{it}, K_{it}, L_{it}) \exp(\omega_{it} + \varepsilon_{it}) = M^{*\beta}_{it} K^{\alpha_K}_{it} L^{\alpha_L}_{it} \exp(\omega_{it} + \varepsilon_{it})$. Then, for the case of Colombia, Equation 5 applies since the type of firms is the juridical organization and the non-evaders are Corporations. By multiplying both sides by the intermediate inputs and dividing by the output, we get

$$\begin{aligned} \ln\left(\frac{\rho_t M^*_{it}}{P_t Y_{it}}\right) + e_{it} &= \ln \beta + \ln \mathcal{E} - \varepsilon_{it} \\ &\equiv \ln D^{\mathcal{E}} - \varepsilon_{it} \end{aligned} \tag{8}$$

where, $\mathcal{E} \equiv \mathbb{E}[\exp(\varepsilon_{it})|\mathcal{I}_{it}] = \mathbb{E}[\exp(\varepsilon_{it})]$.

We can use Equation 8 and assumption 4.1 to recover the production function parameter β

$$\mathbb{E}\left[\ln\left(\frac{\rho_t M^*_{it}}{P_t Y_{it}}\right)\middle|\Theta^{NE}\right] = \ln D^{\mathcal{E}} \tag{9}$$

[4]Note we can always rewrite $M^* + e = M^* \exp\{a\}$, then $\exp\{a\} = \frac{e}{M^*} + 1$.

The constant $\mathcal{E}$ is also identified (Gandhi et al. 2020),

$$\mathcal{E} = \mathbb{E}\left[\exp\left(\ln D^{\mathcal{E}} - \ln\left(\frac{\rho_t M_{it}^*}{P_t Y_{it}}\right)\right)|\theta^{NE}\right] = \mathbb{E}\left[\exp(\varepsilon_{it})|\theta^{NE}\right] = \mathbb{E}[\exp(\varepsilon_{it})] \tag{10}$$

and, thus, the output elasticity of input, β, is also identified,

$$\beta = \exp\left(\ln D^{\mathcal{E}} - \ln \mathcal{E}\right). \tag{11}$$

4.2 Identifying Tax Evasion

Having recovered both the flexible input elasticity, β, and the constant $\mathcal{E}$, for all firms, I can form the following variable using observed data.

$$\begin{aligned}\mathcal{V}_{it} &\equiv \ln\left(\frac{\rho_t M_{it}}{P_t Y_{it}}\right) - \ln\beta - \ln\mathcal{E} \\ &= \ln\left(\frac{\rho_t M_{it}^*}{P_t Y_{it}}\right) - \ln\beta - \ln\mathcal{E} + e_{it} \\ &= -\varepsilon_{it} + e_{it}\end{aligned}$$

By assumption 4.2, the tax evasion, ε_{it} ,is independent of e_{it}. Note that, from Equation 8, we also recovered $f_\varepsilon(\varepsilon)$ the distribution of ε. Tax evasion therefore can be recovered up to an independent random variable by using deconvolution methods.

In particular, from probability theory,

Definition 4.1.

Definition: Convolution

The density of the sum of two **independent** random variables is equal to the **convolution** of the densities of both addends; hence

$$h = f * m = \int f(\mathcal{Z} - \varepsilon) m(\varepsilon) d\varepsilon$$

where f is the density of $\mathcal{Z}$ (Meister, 2009)

4.3 Identifying Productivity

Here I show how to recover the rest of the parameters of the production function, including productivity, and the Markov process of productivity. We can do it in several ways, depending on our object of interest.

In the literature, it is not uncommon to assume that productivity follows a Markov process. That is,

$$\omega_{it} = h(\omega_{it-1}) + \eta_{it} \tag{12}$$

We can form the following observed variable for a guess of $\alpha = (\alpha_K, \alpha_L)$,

$$\begin{aligned} \mathcal{W}_{it}(\alpha) &\equiv \ln Y_{it} - \beta M_{it} - \alpha_K \ln K_{it} - \alpha_L \ln L_{it} + \beta \mathcal{V}_{it} \\ &= \omega_{it}(\alpha) + (1 - \beta)\varepsilon_{it} \end{aligned}$$

If we are interested only in productivity, we could use deconvolution to learn α and the distribution of productivity.

Alternatively, if we are interested in also identifying the Markov process of productivity, we can use the orthogonality we get from Equation 12, $\mathbb{E}[\eta_{it} | k_{it}, l_{it}, k_{it-1}, l_{it-1}, \mathcal{W}_{it-1}] = 0$. Orthoganility follows from $(k_{it-1}, l_{it-1}, \mathcal{W}_{it-1})$ being known to the firm at period $t-1$, and (k_{it-1}, l_{it-1}) being predetermined.

Namely, substituting ω_{it} in Equation 12, we have

$$\mathbb{E}[\mathcal{W}_{it}(\alpha)|k_{it}, l_{it}, , k_{it-1}, l_{it-1}, \mathcal{W}_{it-1}] = h(\mathcal{W}_{it-1}(\alpha))$$

Thus, α and h are identified.

4.4 Translog Production Function

To relax the assumption of a CD production function, we would need a flexible input that firms do not overreport to identify tax evasion. Firms might face flexible inputs that they cannot deduct from their VAT or CIT, for example. If this is the case, then firms would have no incentives to overreport non-deductible flexible inputs.

Assume now that L_{it} is a non-deductible flexible input and, without loss of generality, there are only two inputs (L_{it}, M_{it}). Let's assume the production function is now *translog* and let $\ln X_{it} = x_{it}$. We have,

$$\ln G(l, m) = \beta_0 m_{it} + \beta_1 m_{it} l_{it} + \beta_2 m_{it} m_{it} + \beta_3 l_{it} + \beta_4 l_{it} l_{it}$$

Then, equation Equation 8 becomes

$$\begin{aligned} s_{it}^L &= \ln\left(\beta_0 + \beta_1 l_{it} + \beta_2 (m_{it}^* + e_{it})\right) + \ln \mathcal{E} - \varepsilon_{it} \\ &\equiv \ln D^{\mathcal{E}}(l_{it}, m_{it}^* + e_{it}) - \varepsilon_{it} \end{aligned}$$

where $s_{it}^L \equiv \ln\left(\frac{\rho_t^L L_{it}}{P_t Y_{it}}\right)$.

Note that by assumption 4.1, $D^{\mathcal{E}}$ and the density of ε are still identified. To identify tax evasion, we can form the analog of Equation 12,

$$
\begin{aligned}
\mathcal{V}_{it}^{TL} &= \ln\left(\frac{\rho L}{PY}\right) - \ln D^{\mathcal{E}}(l, m^* + e) \\
&= \ln\left(\frac{\rho L}{PY}\right) - \ln D^{\mathcal{E}}(l, m^* + e) \\
&+ \left[\ln D^{\mathcal{E}}(l, m^*)\right] \\
&- \left[\ln D^{\mathcal{E}}(l, m^*)\right] \\
&= \ln\left(\frac{\rho L}{PY}\right) - \ln D^{\mathcal{E}}(l, m^*) \\
&- \left[\ln D^{\mathcal{E}}(l, m^* + e) - \ln D^{\mathcal{E}}(l, m^*)\right] \\
&= -\varepsilon(l, m^*) - \delta(l, m^*, m^* + e)
\end{aligned}
$$

where in the case of the translog production function $\delta(l, m^*, m^*+e) \equiv \ln\left(\beta_0 + \beta_1 l_{it} + \beta_2(m_{it}^* + e_{it})\right) - \ln\left(\beta_0 + \beta_1 l_{it} + \beta_2 m_{it}^*\right)$. Note that is always positive $\delta(l, m^*, m^* + e) \geq 0$ because $e_{it} \geq 0$.

Because firms cannot use l to deduct taxes, l is orthogonal to e. Hence, conditional on m^*, $\varepsilon(l, m^*)$ and $\delta(l, m^*, m^*+e)$ are independent. Thus, we can apply deconvolution techniques again.

4.5 Non-Parametric Identification

The previous result also suggests a non-parametric identification strategy, as long as $\delta(l, m^*, m^* + e)$ is monotonic in its $m^* + e$ argument. This identification strategy is analogous to Hu et al. (2022), where the authors also require monotonicity and independence to recover a nonparametric function of m^* with nonclassical measurement error. In our case, intuitively, if we know the density of ε and the function $D^{\mathcal{E}}$, the variation left is due to e, which can be recovered as long as we can vary δ by moving e.

To see why the non-deductible flexible input is needed to identify tax evasion consider the following. Suppose that only the input M is flexible and deductible.

$$\ln\left(\frac{\rho M}{PY}\right) = \ln D^{\mathcal{E}}(K, L, M) - \varepsilon$$

$D^{\mathcal{E}}(K, L, M)$ is still identified by assumption 4.1, however, when we form the analogous of Equation 12, we now have

$$\begin{aligned}
\ln\left(\frac{\rho M}{PY}\right) - \ln D^{\mathcal{E}}(K, L, M) =& \ln\left(\frac{\rho(M^* + e)}{PY}\right) - \ln D^{\mathcal{E}}(K, L, M^* + e) \\
=& \ln\left(\frac{\rho(M^* + e)}{PY}\right) - \ln D^{\mathcal{E}}(K, L, M^* + e) \\
& + \left[\ln\left(\frac{\rho M^*}{PY}\right) - \ln D^{\mathcal{E}}(K, L, M^*)\right] \\
& - \left[\ln\left(\frac{\rho M^*}{PY}\right) - \ln D^{\mathcal{E}}(K, L, M^*)\right] \\
=& \ln\left(\frac{\rho M^*}{PY}\right) - \ln D^{\mathcal{E}}(K, L, M^*) \\
& + \left[\ln\left(\frac{\rho(M^* + e)}{PY}\right) - \ln\left(\frac{\rho(M^*)}{PY}\right)\right] \\
& - \left[\ln D^{\mathcal{E}}(K, L, M^* + e) - \ln D^{\mathcal{E}}(K, L, M^*)\right] \\
=& - \varepsilon \\
& + \left[\ln D^{\mathcal{E}}(K, L, M^* + e) - \varepsilon - \ln D^{\mathcal{E}}(K, L, M^*) + \varepsilon\right] \\
& - \left[\ln D^{\mathcal{E}}(K, L, M^* + e) - \ln D^{\mathcal{E}}(K, L, M^*)\right] \\
& = -\varepsilon(K, L, M^*)
\end{aligned}$$

Now, we are not able to separate the variation of ε from e.

5 Implementation

We are interested in the distribution of tax evasion e but it cannot be observed. What is observed is the contaminated version $\mathcal{V}$ Equation 12. Evasion e and the output shock ε are independent [4.2] with probability density distributions f_e and f_ε. Then, from Definition 4.1

$$f_{\mathcal{V}}(\mathcal{V}) = \int f_\varepsilon(e - \mathcal{V}) f_e(e) \mathrm{d}e$$

where $f_{\mathcal{V}}$ denotes the density of $\mathcal{V}$.

5.1 Parametric MLE

Assume a functional form for $f_\varepsilon(\cdot;\gamma)$ that depends on known parameters γ. Assume a known functional form for the density $f_e(\cdot;\lambda)$ that depends on unknown parameters λ. We can estimate parameters γ, λ by

$$\arg\max_{\gamma,\lambda} = \frac{1}{n} \sum_{i=1}^{n} \log \left(\int f_\varepsilon(e - \mathcal{V};\gamma) f_e(e;\lambda) \mathrm{d}e \right)$$

Properties of MLE with unobserved scalar heterogeneity have been derived elsewhere before (Chen 2007, Yi 2021).

5.2 Non-Parametric MLE

Consider the following log-density model:

$$f_{e|\Theta}(e) = \exp(s(e;\theta) - C(\theta))$$

where,

$$s(e;\theta) = \sum_{j=1}^{k_n} \theta_j B_j(e),$$

$\{B_j(E), j = 1, 2, ...\}$ is a sequence of known basis functions, and

$$C(\theta) = \log\left(\int \exp(s(e;\theta))\mathrm{d}e\right)$$

The log likelihood of the observed variable $\mathcal{V}$ is

$$\begin{aligned} l_{\mathcal{V}}(\ ) &= \sum_{i=1}^{n} \log\left(\int f_{\varepsilon}(e-\mathcal{V}) \exp(s(e;\theta) - C(\theta))\mathrm{d}e\right) \\ &= \sum_{i=1}^{n} \log\left(\int f_{\varepsilon}(e-\mathcal{V}) \exp(s(e;\theta))\mathrm{d}e\right) - nC(\theta) \end{aligned}$$

The usual maximum likelihood estimate $\widehat{\theta}$ is the maximizer of $l_{\mathcal{V}}(\theta)$.

Laguerre polynomials can be used to approximate any function $L_2([0,\infty), leb)$ L_2 norm relative to the Lebesgue measure and domain $[0,\infty)$ (Chen 2007).

The EM algorithm (Kang & Qiu 2021) starts with an initial estimate $\hat{\ }^{0}$ and iteratively updates the estimate as follows.

Expectation-Step: Given the current estimate $\hat{\ }^{(k)}$ of $\hat{\ }$, calculate

$$b_j\left(\hat{\ }^{(k)}\right) = \sum_{i=1}^{n} \int B_j(e) f_{e|\mathcal{V},\hat{\theta}^{(k)}}(e|\mathcal{V})\mathrm{d}e$$

where,

$$f_{e|\mathcal{V},\hat{\theta}}(e|\mathcal{V}) = f_{\varepsilon}(e-\mathcal{V}) \exp(s(e;\theta) - C(\theta|\mathcal{V}))$$

$$C(\theta|\mathcal{V}) = \log\left(\int f_{\varepsilon}(e-\mathcal{V}) \exp(s(e;\theta))\mathrm{d}e\right)$$

Maximization-Step: Determine the updated estimate $\hat{}^{(k+1)}$ by maximizing

$$Q(\ |\ ^{(k)}) = \sum_{j=1}^{k_n} \theta_j b_j \left(\hat{}^{(k)}\right) - nC(\ )$$

The EM algorithm stops when $l_{\mathcal{V}}(\ ^{(k+1)}) - l_{\mathcal{V}}(\ ^{(k)}) < 10^{-6}$.

References

Ackerberg, D. A. (2019), 'Timing assumptions and e¢ ciency: Empirical evidence in a production function context'.

Ackerberg, D., Frazer, G., Luo, Y. & Su, Y. (2021), 'Under-identification of structural models based on timing and information set assumptions'.
URL: *https://ssrn.com/abstract=3717757*

Almunia, M. & Lopez-Rodriguez, D. (2018), 'Under the radar: The effects of monitoring firms on tax compliance †', *American Economic Journal: Economic Policy* **10**, 1–38.
URL: *https://doi.org/10.1257/pol.20160229*

Carrillo, P., Donaldson, D., Pomeranz, D. & Singhal, M. (2022), 'Ghosting the tax authority: Fake firms and tax fraud'.
URL: *http://www.nber.org/papers/w30242*

Chen, X. (2007), 'Large sample sieve estimation of semi-nonparametric models *'.

Gandhi, A., Navarro, S. & Rivers, D. A. (2020), 'On the identification of gross output production functions'.

Hu, Y., Schennach, S. & Shiu, J. L. (2022), 'Identification of nonparametric monotonic regression models with continuous nonclassical measurement errors', *Journal of Econometrics* **226**, 269–294.

Kang, Y. & Qiu, P. (2021), 'Nonparametric deconvolution by fourier transformation and other related approaches', *Handbook of Measurement Error Models* pp. 221–244.
URL: *https://www-taylorfrancis-com.proxy1.lib.uwo.ca/chapters/edit/10.1201/9781315101279-11/nonparametric-deconvolution-fourier-transformation-related-approaches-yicheng-kang-peihua-qiu*

Mclure, C. E. (1989), Analysis and reform of the colombian tax system, *in* M. Gillis, ed., 'Tax Reform in Developing Countries', Duke University Press, Durham and London.
URL: *https://pdf.usaid.gov/pdf_docs/pnabp729.pdf*

Paulus, A. (2015), 'Tax evasion and measurement error: An econometric analysis of survey data linked with tax records'.
URL: *http://hdl.handle.net/10419/126489*

Perry, G. & de Triana, L. O. (1990), 'The vat in colombia'.

Pissarides, C. A. & Weber, G. (1989), 'An expenditure-based estimate of britain's black economy', *Journal of Public Economics* **39**, 17–32.

Technology Tools to Tackle Tax Evasion and Tax Fraud (2017).
URL: *www.oecd.org*

Torres, F. S. & Sourdis, C. G. (1994), Reformas tributarias en colombia 1980-1992. aspectos de equidad, eficiencia, y simplificación administrativa, *in* 'VI Seminario Regional de Política Fiscal'.

Yi, G. Y. (2021), 'Likelihood methods with measurement error and misclassification', *Handbook of Measurement Error Models* pp. 99–126.

URL: *https://www-taylorfrancis-com.proxy1.lib.uwo.ca/chapters/edit/10.1201/9781315101279-6/likelihood-methods-measurement-error-misclassification-grace-yi*

Zumaya, M., Guerrero, R., Islas, E., Pineda, O., Gershenson, C., Iñiguez, G. & Pineda, C. (2021), 'Identifying tax evasion in mexico with tools from network science and machine learning'.

URL: *http://arxiv.org/abs/2104.13353*