Higher dim Osc

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1 Introduction

We make the metric Ansatz:

$$ds^2 = \alpha^2 dt^2 + a^2 dr^2 + r^2 d\Omega_3 \tag{1}$$

where $A(t,r) = a^2$ and $C(t,r) = (a/\alpha)^2$.
The Einstein equation t,t component in 3+1D

$$\begin{aligned}A^{(0,1)}(t,r) = {} & \frac{1}{2} r A(t,r) \phi^{(1,0)}(t,r)^2 C(t,r) + \frac{1}{2} r A(t,r) \phi^{(0,1)}(t,r)^2 + \frac{1}{2} r A(t,r)^2 \phi(t,r)^2 \\ & - \frac{1.A(t,r)^2}{r} + \frac{1.A(t,r)}{r}\end{aligned} \tag{2}$$

The Einstein equation t,t component in 4+1D

$$\begin{aligned}A^{(0,1)}(t,r) = {} & \frac{1}{3} r A(t,r) \phi^{(1,0)}(t,r)^2 C(t,r) + \frac{1}{3} r A(t,r) \phi^{(0,1)}(t,r)^2 + \frac{1}{3} r A(t,r)^2 \phi(t,r)^2 \\ & - \frac{2.A(t,r)^2}{r} + \frac{2.A(t,r)}{r}\end{aligned} \tag{3}$$

The Einstein equation t,r component in 3+1D

$$A^{(1,0)}(t,r) = r A(t,r) \phi^{(0,1)}(t,r) \phi^{(1,0)}(t,r) \tag{4}$$

The Einstein equation t,r component in 4+1D

$$A^{(1,0)}(t,r) = \frac{2}{3} r A(t,r) \phi^{(0,1)}(t,r) \phi^{(1,0)}(t,r) \tag{5}$$

The Einstein equation r,r component in 3+1D

$$C^{(0,1)}(t,r) = r A(t,r) \phi(t,r)^2 C(t,r) - \frac{2.A(t,r) C(t,r)}{r} + \frac{2.C(t,r)}{r} \tag{6}$$

The Einstein equation r,r component in 4+1D

$$C^{(0,1)}(t,r) = \frac{2}{3} r A(t,r) \phi(t,r)^2 C(t,r) - \frac{4.A(t,r) C(t,r)}{r} + \frac{4.C(t,r)}{r} \tag{7}$$

The Klein Gordon equation in 3+1D in as similar form as the paper

$$\begin{aligned}\phi^{(2,0)}(t,r)C(t,r) = {} & -\frac{C^{(0,1)}(t,r)\phi^{(0,1)}(t,r)}{2C(t,r)} - A(t,r)\phi(t,r) \\ & -\frac{1}{2}C^{(1,0)}(t,r)\phi^{(1,0)}(t,r) + \frac{2\phi^{(0,1)}(t,r)}{r} + \phi^{(0,2)}(t,r)\end{aligned} \tag{8}$$

The Klein Gordon equation in 4+1D in as similar form as the paper

$$\begin{aligned}\phi^{(2,0)}(t,r)C(t,r) = {} & -\frac{C^{(0,1)}(t,r)\phi^{(0,1)}(t,r)}{2C(t,r)} - A(t,r)\phi(t,r) \\ & -\frac{1}{2}C^{(1,0)}(t,r)\phi^{(1,0)}(t,r) + \frac{3\phi^{(0,1)}(t,r)}{r} + \phi^{(0,2)}(t,r)\end{aligned} \tag{9}$$

The Klein Gordon equation in 3+1D in as similar form as the Code

$$\begin{aligned}\phi^{(2,0)}(t,r)C(t,r) = {} & -0.5rA(t,r)\phi^{(0,1)}(t,r)\phi(t,r)^2 \\ & + \frac{1.A(t,r)\phi^{(0,1)}(t,r)}{r} - A(t,r)\phi(t,r) - \frac{1}{2}C^{(1,0)}(t,r)\phi^{(1,0)}(t,r) \\ & + \frac{1.\phi^{(0,1)}(t,r)}{r} + \phi^{(0,2)}(t,r) + 0.\end{aligned} \tag{10}$$

The Klein Gordon equation in 4+1D in as similar form as the Code

$$\begin{aligned}\phi^{(2,0)}(t,r)C(t,r) = {} & -\frac{1}{3}rA(t,r)\phi^{(0,1)}(t,r)\phi(t,r)^2 \\ & + \frac{2.A(t,r)\phi^{(0,1)}(t,r)}{r} - A(t,r)\phi(t,r) - \frac{1}{2}C^{(1,0)}(t,r)\phi^{(1,0)}(t,r) \\ & + \frac{1.\phi^{(0,1)}(t,r)}{r} + \phi^{(0,2)}(t,r) + 0.\end{aligned} \tag{11}$$

2 Axion equation

The Einstein equation t,t component in 3+1D

$$\begin{aligned}A^{(0,1)}(t,r) = {} & \frac{1}{2}rA(t,r)\phi^{(1,0)}(t,r)^2C(t,r) + \frac{1}{2}rA(t,r)\phi^{(0,1)}(t,r)^2 + rA(t,r)^2V(\phi(t,r)) \\ & - \frac{A(t,r)^2}{r} + \frac{A(t,r)}{r}\end{aligned} \tag{12}$$

The Einstein equation t,r component in 3+1D

$$A^{(1,0)}(t,r) = rA(t,r)\phi^{(0,1)}(t,r)\phi^{(1,0)}(t,r) \tag{13}$$

The Einstein equation r,r component in 3+1D

$$C^{(0,1)}(t,r) = 2rA(t,r)V(\phi(t,r))C(t,r) - \frac{2A(t,r)C(t,r)}{r} + \frac{2C(t,r)}{r} \tag{14}$$

The Klein Gordon equation in 3+1D as in Paper

$$\begin{aligned} \phi^{(2,0)}(t,r)C(t,r) = {} & -\frac{C^{(0,1)}(t,r)\phi^{(0,1)}(t,r)}{2C(t,r)} - A(t,r)V'(\phi(t,r)) \\ & - \frac{1}{2}C^{(1,0)}(t,r)\phi^{(1,0)}(t,r) + \frac{2\phi^{(0,1)}(t,r)}{r} + \phi^{(0,2)}(t,r) \end{aligned} \tag{15}$$

The Klein Gordon equation in 3+1D as in Code

$$\begin{aligned} \phi^{(2,0)}(t,r)C(t,r) = {} & -rA(t,r)\phi^{(0,1)}(t,r)V(\phi(t,r)) \\ & + \frac{A(t,r)\phi^{(0,1)}(t,r)}{r} - A(t,r)V'(\phi(t,r)) - \frac{1}{2}C^{(1,0)}(t,r)\phi^{(1,0)}(t,r) \\ & + \frac{\phi^{(0,1)}(t,r)}{r} + \phi^{(0,2)}(t,r) \end{aligned} \tag{16}$$

3 Fourier Space

$$\phi(t,x) = \sum_{j=0}^{\infty} \hat{\phi}_j(x)\cos(j\omega t) \tag{17}$$

$$A(t,x) = \sum_{j=0}^{\infty} \hat{A}_j(x)\cos(j\omega t) \tag{18}$$

$$C(t,x) = \sum_{j=0}^{\infty} \hat{C}_j(x)\cos(j\omega t) \tag{19}$$

$$\begin{aligned} \phi(t,x)A(t,x) &= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \hat{\phi}_j(x)\hat{A}_i(x)\cos(i\omega t)\cos(j\omega t) \\ &= \sum_{n=0}^{\infty}\sum_{i=0}^{\infty} \hat{\phi}_{n-i}(x)A_i(x)\frac{1}{2}(\cos((j+i)\omega t) + \cos((j-i)\omega t)) \\ &\underbrace{=}_{n=i+j} \sum_{n=0}^{\infty}\sum_{i=0}^{\infty} \hat{\phi}_{n-i}(x)\hat{A}_i(x)\frac{1}{2}(\cos(n\omega t) + \cos((j-i)\omega t)) \\ &= \sum_{n=0}^{\infty}\sum_{i=0}^{\infty} \hat{\phi}_{n-i}(x)\hat{A}_i(x)\frac{1}{2}(\cos(n\omega t)) + \sum_{n=0}^{\infty}\sum_{i=0}^{-\infty} \hat{\phi}_{n+i}(x)\hat{A}_{-i}(x)\frac{1}{2}(\cos(n\omega t)) \\ &= \sum_{n=0}^{\infty}\sum_{i=-\infty}^{\infty} \hat{\phi}_{n-|i|}(x)\hat{A}_{|i|}(x)\cos(n\omega t) \end{aligned} \tag{20}$$

will be written as

$$A(t,x) = \sum_{j=0}^{\infty} \hat{A}_j(x)\partial_t \cos(j\omega t) = \sum_{j=0}^{\infty} \hat{A}_j(x) j\omega \cos(j\omega t) \tag{21}$$

$$\begin{aligned} A^{(0,1)}(t,r) = {}& \frac{1}{2} r A(t,r)\phi^{(1,0)}(t,r)^2 C(t,r) + \frac{1}{2} r A(t,r)\phi^{(0,1)}(t,r)^2 + \frac{1}{2} r A(t,r)^2 \phi(t,r)^2 \\ & - \frac{1.A(t,r)^2}{r} + \frac{1.A(t,r)}{r} \end{aligned} \tag{22}$$

In Fourier space

$$\begin{aligned} \hat{A}_j^{(0,1)}(t) = {}& \frac{1}{2} r j^2 \omega^2 (\hat{A} * \hat{\phi} * \hat{\phi} * \hat{C})_j + \frac{1}{2} r (\hat{A} * \hat{\phi}^{(0,1)} * \hat{\phi}^{(0,1)})_j + \frac{1}{2} r (\hat{A} * \hat{A} * \hat{\phi} * \hat{\phi})_j \\ & - \frac{(\hat{A} * \hat{A})_j}{r} + \frac{A_j}{r} \end{aligned} \tag{23}$$

4 ADM mass

Asymptotic behaviour of Metric

$$ds^2 = -\left(A - \frac{2M}{r^{d-3}}\right) dt^2 + \left(C - \frac{2M}{r^{d-3}}\right)^{-1} + r^2 d\Omega_{d-2}^2 \tag{24}$$

where $C, B \in R$ we can find the asymptomatic mass by :

$$\begin{aligned} A &= \left(C - \frac{2M}{r^{d-3}}\right)^{-1} \\ M &= \frac{1}{2}\left(C - \frac{1}{A}\right) r^{-3+d} \end{aligned} \tag{25}$$

5 Non-relativistic limit

$$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + mU\psi \tag{26}$$

$$\nabla^2 U = 4\pi G |\psi|^2 \tag{27}$$

we Fourier transform $\nabla = 1/L$ where L is the characteristic length of the Osc.

$$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + mU\psi \tag{28}$$

$$\nabla^2 U = 4\pi G |\psi|^2 \tag{29}$$

6 Conclusion

3+1D $\neq$ 4+1D

7 AdS Solitons

7.1 Ansatz

We make the metric Ansatz:

$$ds^2 = \left(\alpha^2 - \frac{r^2\Lambda}{6}\right) dt^2 + a^2 dr^2 + r^2 d\Omega_3 \tag{30}$$

where $A(t,r) = a^2$ and $C(t,r) = (a/\alpha)^2$.

compared to the ansatz in BRITO PROCA Star

$$ds^2 = \sigma^2(r)F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\Omega \tag{31}$$

where $F(r) = 1 - \frac{2m(r)}{r^2} - \Lambda r^2/6$

7.2 Stability

In this section we discuss solution for higher-dimensional scalar and vector Boson stars [2]. One finds that the maximum mass for Boson Stars in 5 dimension diverges for $\phi_0(0) \to 0$. The biding energy seems to be negative for all values, indicating that such solution are unstable (analogous for Boson Stars: [1]). In [2] they hypothise that for $\Lambda < 0$ there are stable solutions.

References

[1] Yves Brihaye and Betti Hartmann. Minimal boson stars in 5 dimensions: classical instability and existence of ergoregions. *Class. Quant. Grav.*, 33(6):065002, 2016.

[2] Miguel Duarte and Richard Brito. Asymptotically anti-de Sitter Proca Stars. *Phys. Rev.*, D94(6):064055, 2016.

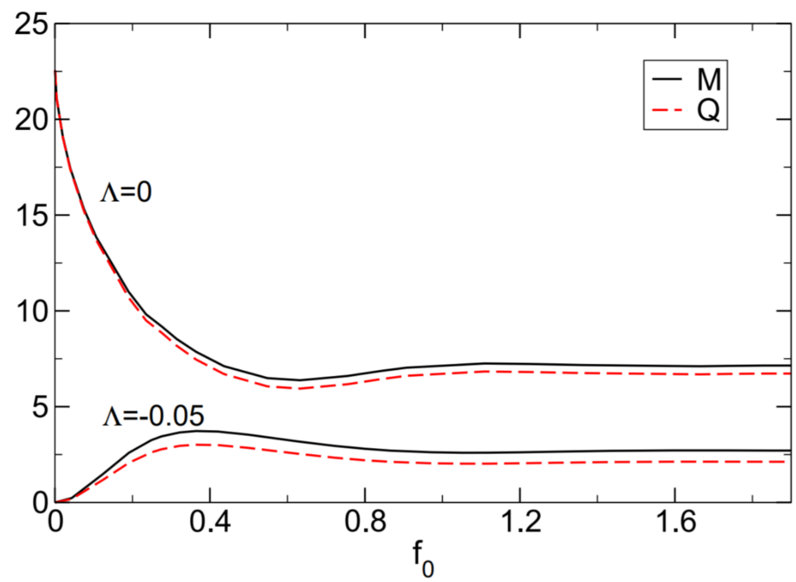

Figure 1: **Stability of vector Boson stars in 5 dimension:** Where f_0 is the parameter defining Proca stars. Adding a negative cosmological constant gives the upper mass a bound.