

# BIG BANG NUCLEOSYNTHESIS

BIG BANG KERNESYNTSE



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MASTER'S THESIS IN COSMOLOGY

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## Colophon

*Big Bang Nucleosynthesis*

— *Big Bang Kernesyntese*

Master's thesis by Hans. Written under supervision by Asc.Prof. Thomas Tram, Department of Physics and Astronomy, Aarhus University.

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### **Abstract (English)**

Nucleosyntehesis is wack

### **Resumé (Dansk)**

Kernesyntese er spøjs

## List of Corrections

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| This is a warning! . . . . . | 4 |
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# Preface

This thesis concludes my Master's degree in/at .....

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# Introduction

This thesis is about nucleosynthesis





# BBN physics and cosmology

To understand the process of Big Bang nucleosynthesis, we must examine the intersection between Cosmology, thermodynamics, particle, and nuclear physics. Though this might seem daunting, it turns out that the unique conditions during this epoch allow for extensive simplifications of this otherwise monumental task.

## 1.1 Determining background parameters

### 1.1.1 Temperature and scale factor

BBN takes place after inflation while the universe is still radiation dominated. This can be described by the Friedman equation, which can be further simplified with the reasonable approximation, that both curvature and the cosmological constant are zero.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{tot} \quad (1.1)$$

With  $\rho_{tot}$  referring to the total energy density of photons, leptons and baryons.

$$\rho_{tot} = \rho_\gamma + \rho_\nu + (\rho_{e^-} + \rho_{e^+}) + \rho_b \quad (1.2)$$

To find an expression for the temperature evolution, we use energy conservation. We can consider the neutrinos as decoupled during BBN, and so the photon temperature will be determined by the remaining components. Since this point the universe is very much homogeneous and isotropic, we utilize the fluid equation for adiabatic expansion.

$$\dot{\rho}_{set} + 3\frac{\dot{a}}{a}(\rho_{set} + P_{set}) = 0 \quad (1.3)$$

With  $\rho_{set}$  being the density of none-decoupled components and  $P_{set}$  being their pressures.

$$\rho_{set} = \rho_Y + (\rho_{e^-} + \rho_{e^+}) + \rho_b \quad , \quad P_{set} = P_Y + (P_{e^-} + P_{e^+}) + P_b \quad (1.4)$$

With this we can set up differential equations describing the time evolution of the scale factor and photon temperature.

$$\frac{dT}{dt} = -3H \frac{\rho_{set}(T, a) + P_{set}(T, a)}{\frac{d\rho_{set}(T, a)}{dT}} \quad , \quad \frac{da}{dt} = a \sqrt{\frac{8\pi G}{3} \rho_{tot}(T, a)} \quad (1.5)$$

### 1.1.2 Additional parameters

Most BBN codes are based on the original code by Wagoner described in section 2.1. These don't track the scale factor, but instead use the quantity  $h$ .

$$h = M_u \frac{n_b}{T_9^3} \quad (1.6)$$

$M_u$  being atomic mass units,  $n_b$  the baryon number density, and  $T_9$  the temperature in  $10^9$  Kelvin. This quantity was useful since it stays approximately constant throughout BBN, while being easy to directly convert to baryon density. However, with modern computers this numerical simplicity is inconsequential, and as such it is more reasonable to track the scale factor. The electron chemical potential  $\phi$  was also tracked by the Wagoner code and its successors. The only effect of this is ensuring a non-zero electron density after reheating. We can easily set this to 0, as the impact will be 3 orders of magnitude lower than the already miniscule impact of the baryon density.

Neutrino temperature?

## 1.2 Energy densities and pressure

### 1.2.1 Photons

The energy density of photons as a function of temperature can be found by integrating over the energy distribution given by Planck's law.

$$\rho_Y(T) = \int_0^\infty \frac{\omega^3}{\pi^2} \frac{1}{e^{\omega/T} - 1} d\omega = \frac{T^4}{\pi^2} \int_0^\infty \frac{u^3}{e^u - 1} du \quad (1.7)$$

This integral is a well know representation of the Riemann Zeta function [6, (25.5.1)].

$$\rho_Y(T) = \frac{T^4}{\pi^2} \Gamma(4) \zeta(4) = \frac{\pi^2}{15} T^4 \quad (1.8)$$

From this we can easily define the pressure and temperature derivative.

$$\frac{d\rho_Y(T)}{dT} = \frac{4}{15} \pi^2 T^3 \quad , \quad P_Y(T) = \frac{\rho_Y(T)}{3} \quad (1.9)$$

### 1.2.2 Neutrinos

### 1.2.3 Electrons and positrons

### 1.2.4 Baryons

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## 1.3 Nuclear reactions

### 1.3.1 Proton $\rightleftharpoons$ neutron rate

Warning test

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## 1.4 Initial conditions



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## BBN code

### 2.1 History of BBN codes

The concept of Big Bang nucleosynthesis is almost as old as the Big Bang theory itself, with the with it first being proposed in the paper by Alpher, Bethe, and Gamow [1]. This early model used neutron capture and subsequent beta decay as the mechanism for BBN, though its greatest problem was the inability to explain the unusually high abundance of oxygen and carbon in the present universe. And so, it was in large part supplanted by the new theory of stellar nucleosynthesis, as the main explanation for the origin of elements.

During the next decades it became clear that stars could not be the only explanation for the present element abundances, and with the discovery of the CMB in 1965, new attention was brought to the early universe. Only a year later Peebles showed how simple BBN physics could be used to explain the high helium abundance, unaccounted for by stellar nucleosynthesis [2].

In the following years Wagoner created and refined the first proper BBN code, described in a series of defining papers[3][4][5]. With the legacy of this code still heavily influencing the way BBN calculations are performed today.

By the late 80s the Wagoner code was severely outdated. With multiple inefficiencies due to among other things, the fact that it was originally designed to run on punch cards. This inspired Lawrence Kawano to create the now ubiquitous NUC123, colloquially know as the Kawano code. Which set the gold Standard for all future BBN codes.

In current day and age, there exists multiple publicly available BBN codes, and a countless number of private codes. The most well know of these are PARthENoPE spiritual successor to NUC123, AlterBBN, and PRIMAT.

### 2.2 Structure of BBN codes

The objective of any BBN code is to solve the system of differential equations described in chapter 1.

### **2.2.1 Wagoner**

### **2.2.2 Kawano**

### **2.2.3 Modern codes**

PArthENoPE, AlterBBN, PRIMAT

### **2.2.4 AlterBBN**

AlterBBN is written in c and based on Kawano's NUC123. It maintains the same basic structure and integration method, though it uses natural units for everything but the reaction network. However, they define energy in GeV rather than MeV. What separates AlterBBN for other codes is that as the name implies, it allows the use of alternate cosmological models and parameters. Therefore, this code is especially well suited for testing the effects these alterations have on final abundances. Wot



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## Comparison with AlterBBN



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## Comparison with AlterBBN



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