Big Bang Nucleosynthesis

Big Bang Kernesyntese

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Colophon

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— Big Bang Kernesyntese

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Abstract (English)

Nucleosyntehesis is wack

Resumé (Dansk)

Kernesyntese er spøjs

List of Corrections

Preface

This thesis concludes my Master's degree in/at .....

Contents

Introduction

This thesis is about nucleosysthesis

Chapter 1

BBN physics and cosmology

To understand the process of Big Bang nucleosynthesis, we must examine the intersection between Cosmology, thermodynamics, particle, and nuclear physics. Though this might seem daunting, it turns out that the unique conditions during this epoch allow for extensive simplifications of this otherwise monumental task. Throughout this section we use $\hbar = c = k_B = 1$.

1.1 Determining background parameters

1.1.1 Temperature and scale factor

BBN takes place after inflation while the universe is still radiation dominated. This can be described by the Friedman equation, which can be further simplified with the reasonable approximation, that both curvature and the cosmological constant are zero.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{tot} \tag{1.1}$$

With ρ_{tot} referring to the total energy density of photons, leptons and baryons.

$$\rho_{tot} = \rho_\gamma + \rho_\nu + (\rho_{e^-} + \rho_{e^+}) + \rho_b \tag{1.2}$$

To find an expression for the temperature evolution, we use energy conservation. We can consider the neutrinos as decoupled during BBN, and so the photon temperature will be determined by the remaining components. Since this point the universe is very much homogeneous and isotropic, we utilize the fluid equation for adiabatic expansion.

$$\dot{\rho_{set}} + 3\frac{\dot{a}}{a}(\rho_{set} + P_{set}) = 0 \tag{1.3}$$

With ρ_{set} being the density of none-decoupled components and P_{set} being their pressures.

$$\rho_{set} = \rho_\gamma + (\rho_{e^-} + \rho_{e^+}) + \rho_b \quad , \quad P_{set} = P_\gamma + (P_{e^-} + P_{e^+}) + P_b \tag{1.4}$$

With this we can set up differential equations describing the time evolution of the scale factor and photon temperature.

$$\frac{dT}{dt} = -3H\frac{\rho_{set}(T,a) + P_{set}(T,a)}{\frac{d\rho_{set}(T,a)}{dT}} \quad , \quad \frac{da}{dt} = a\sqrt{\frac{8\pi G}{3}\rho_{tot}(T,a)} \tag{1.5}$$

1.1.2 Additional parameters

Most BBN codes are based on the original code by Wagoner described in section 2.1. These don't track the scale factor, but instead use the quantity h.

$$h = M_u \frac{n_b}{T_9^3} \tag{1.6}$$

M_u being atomic mass units, n_b the baryon number density, and T_9 the temperature in 10^9 Kelvin. This quantity was useful since it stays approximately constant throughout BBN, while being easy to directly convert to baryon density. However, with modern computers this numerical simplicity is inconsequential, and as such it is more reasonable to track the scale factor. The electron chemical potential ϕ was also tracked by the Wagoner code and its successors. The only effect of this is ensuring a non-zero electron density after reheating. We can easily set this to 0, as the impact will be 3 orders of magnitude lower than the already miniscule impact of the baryon density.

Neutrino temperature?

1.2 Energy densities and pressure

In the very early universe most particles were in thermal equilibrium, and can be described by the rules of statistical physics. The average number of particles in a given state is governed by the Fermi-Dirac distribution for fermions, and the Bose-Einstein distribution for bosons.

$$\bar{n}_{FD} = \frac{1}{e^{(E-\phi)/T} + 1} \quad , \quad \bar{n}_{BE} = \frac{1}{e^{(E-\phi)/T} - 1} \tag{1.7}$$

With E being the total energy each particle in the state and ϕ the chemical potential. The number density can be found generally by integrating over all possible momentum states.

$$n(T) = \frac{g}{(2\pi)^3}\int_0^\infty \bar{n}(p,T)dp^3 = \frac{g}{2\pi^2}\int_0^\infty \bar{n}(p,T)dp \tag{1.8}$$

With g being the degeneracy parameter. We can similarly find an expression for the energy density by multiplying the integrand by the relativistic energy $E^2 = m^2 + p^2$.

$$\rho(T) = \frac{g}{2\pi^2}\int_0^\infty \bar{n}(p,T)\sqrt{m^2 + p^2}dp \tag{1.9}$$

Pressure is defined as the force exerted per unit area. Consider a relativistic particle confined to a sphere of radius r. Whenever it collides with the surface, it will exert a force proportional to the change in momentum.

$$F = \frac{dp}{dt} = \frac{\Delta p}{\Delta t} \quad , \quad \Delta p = 2p\cos\theta \tag{1.10}$$

With θ being the incident angle.

The time between collisions can be deduced based on the distance traveled.

$$\Delta t = \frac{L}{\mathrm{v}} = L\frac{\sqrt{m^2 + p^2}}{p} \tag{1.11}$$

With distance between collisions L and velocity v.

Next, consider the triangle created by the center of the sphere and two consecutive collision points. Using the law of cosines we can determine L.

$$r^2 = L^2 + r^2 + 2Lr\cos\theta \Rightarrow L = 2r\cos\theta \tag{1.12}$$

We can then determine the force and pressure exerted on the sphere by each particle.

$$F = \frac{p^2}{r\sqrt{m^2 + p^2}} \quad , \quad P = \frac{p^2}{4\pi r^3\sqrt{m^2 + p^2}} \tag{1.13}$$

Generalizing this for any volume, we get the integral for the total pressure of a relativistic gas.

$$PV = \frac{p^2}{3\sqrt{m^2 + p^2}} \tag{1.14}$$

$$P(T) = \frac{g}{6\pi^2}\int_0^\infty \bar{n}(p,T)\frac{p^2}{\sqrt{m^2 + p^2}}dp \tag{1.15}$$

Additionally, we see that the pressure of an ultra-relativistic gas follows a simple relation.

$$P(T) = \frac{\rho(T)}{3} \quad (\text{for } m \ll p) \tag{1.16}$$

1.2.1 Photons

Photons are massless bosons with 2 distinct polarizations, for each momentum state. With $g = 2$, we use 1.9 to determine the energy density.

$$\rho_\gamma(T) = \int_0^\infty \frac{p^3}{\pi^2}\frac{1}{e^{p/T} - 1}dp = \frac{T^4}{\pi^2}\int_0^\infty \frac{u^3}{e^u - 1}du \tag{1.17}$$

This integral is a well know representation of the Riemann Zeta function [7, (25.5.1)].

$$\rho_\gamma(T) = \frac{T^4}{\pi^2}\Gamma(4)\zeta(4) = \frac{\pi^2}{15}T^4 \tag{1.18}$$

Now we can easily define the temperature derivative and pressure.

$$\frac{d\rho_\gamma(T)}{dT} = \frac{4}{15}\pi^2 T^3 \quad , \quad P_\gamma(T) = \frac{\rho_\gamma(T)}{3} \tag{1.19}$$

1.2.2 Neutrinos

Neutrinos are massless fermions and so have 2 distinct spin states, as well as 3 flavors.

$$\rho_\nu(T_\nu) = N_\nu \int_0^\infty \frac{p^3}{\pi^2} \frac{1}{e^{p/T_\nu} + 1} dp = N_\nu \frac{T_\nu^4}{\pi^2} \int_0^\infty \frac{u^3}{e^u + 1} du \tag{1.20}$$

This is also an integral representation of the Riemann Zeta function [7, (25.5.3)].

$$\rho_\nu(T_\nu) = N_\nu \frac{T_\nu^4}{\pi^2} \Gamma(4)\zeta(4)(1 - 2^3) = N_\nu \frac{7}{8} \frac{\pi^2}{15} T_\nu^4 \tag{1.21}$$

The effective neutrino number is $N_\nu = 3.046$ to take into account various QFT corrections[6].

Tracking the neutrino temperature separately is quite troublesome, luckily we don't have to. Since neutrinos decouple very early, the only significant change in their energy density will be due to the expansion of the universe. Therefore, we can track the later evolution using the scale factor.

$$\rho_\nu(t) = \frac{\rho_\nu(T_0)}{a(t)^4} \tag{1.22}$$

1.2.3 Electrons and positrons

Electrons and positrons unfortunately have mass, which makes soling for their density and pressure much more troublesome.

1.2.4 Baryons

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1.3 Nuclear reactions

1.3.1 Proton $\leftrightharpoons$ neutron rate

Warning test

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1.4 Initial conditions

Chapter 2

BBN code

2.1 History of BBN codes

The concept of Big Bang nucleosynthesis is almost as old as the Big Bang theory itself, with the with it first being proposed in the paper by Alpher, Bethe, and Gamow [1]. This early model used neutron capture and subsequent beta decay as the mechanism for BBN, though its greatest problem was the inability to explain the unusually high abundance of oxygen and carbon in the present universe. And so, it was in large part supplanted by the new theory of stellar nucleosynthesis, as the main explanation for the origin of elements.

During the next decades it became clear that stars could not be the only explanation for the present element abundances, and with the discovery of the CMB in 1965, new attention was brought to the early universe. Only a year later Peebles showed how simple BBN physics could be used to explain the high helium abundance, unaccounted for by stellar nucleosynthesis [2].

In the following years Wagoner created and refined the first proper BBN code, described in a series of defining papers[3][4][5]. With the legacy of this code still heavily influencing the way BBN calculations are performed today.

By the late 80s the Wagoner code was severely outdated. With multiple inefficiencies due to among other things, the fact that it was originally designed to run on punch cards. This inspired Lawrence Kawano to create the now ubiquitous NUC123, colloquially know as the Kawano code. Which set the gold Standard for all future BBN codes.

In current day and age, there exists multiple publicly available BBN codes, and a countless number of private codes. The most well know of these are PArthENoPE spiritual successor to NUC123, AlterBBN, and PRIMAT.

2.2 Structure of BBN codes

The objective of any BBN code is to solve the system of differential equations described in chapter 1.

2.2.1 Wagoner

2.2.2 Kawano

2.2.3 Modern codes

PArthENoPE, AlterBBN, PRIMAT

2.2.4 AlterBBN

AlterBBN is written in c and based on Kawano's NUC123. It maintains the same basic structure and integration method, though it uses natural units for everything but the reaction network. However, they define energy in GeV rather than MeV. What separates AlterBBN for other codes is that as the name implies, it allows the use of alternate cosmological models and parameters. Therefore, this code is especially well suited for testing the effects these alterations have on final abundances. Wot

Chapter 3

Comparison with AlterBBN

Chapter 4

Comparison with AlterBBN

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