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BAYESIAN STATISTICS – APRIL6

Bayesian Inference with Uniform Probability Distribution

- In order to obtain a posterior distribution, which represents the updated knowledge about the parameter, Bayesian inference uses previous knowledge (prior distribution) about a parameter and updates it with new data (likelihood).

Example with Data

- Let us play out a scenario in which you think the parameter of a uniform distribution is somewhere between 0 and 10 (your prior), but you don't know for sure. At this point, you notice that x = 6.

    - **Prior** - Uniform distribution between 0 and 10.
    - **Likelihood** - A simple likelihood function, such as a normal distribution with some variance and centered around the observed data point x = 6, can be selected. This suggests that values nearer 6 have a higher probability than values further away.
    - **Posterior** - The likelihood function will have an impact on the resulting posterior distribution, which will be different from a flat uniform prior in that it will taper off towards the edges (0 and 10), with a higher value around 6. This represents the most recent understanding that, in light of the observed data point, the parameter is more likely to be near 6.

- Here's some Python code to generate samples from the prior and posterior distribution:

```
import numpy as np

# Prior (uniform distribution between 0 and 10)
prior = np.random.uniform(0, 10, size=1000)

# Likelihood function (example: observing value 6 with some noise)
likelihood = np.exp(-(prior - 6)**2 / (2 * 2**2))

# Evidence (sum of likelihoods)
evidence = np.sum(likelihood)

# Posterior probability
posterior = likelihood / evidence

# Sample from prior and posterior
prior_sample = prior[0]
posterior_sample = posterior.argmax()

print("Sample from prior distribution:", prior_sample)
print("Sample from posterior distribution:", posterior_sample)
```