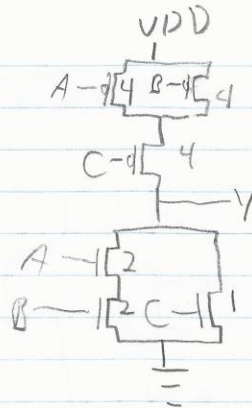
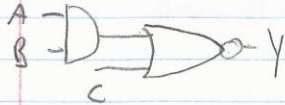


1. AOI21



$$Y = \overline{AB + C}$$

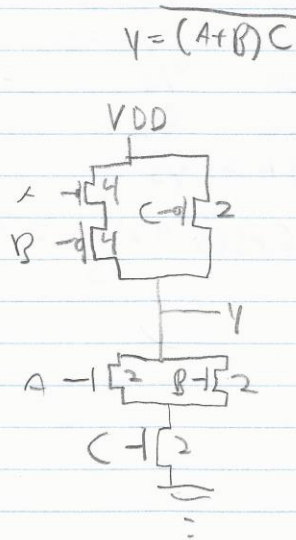
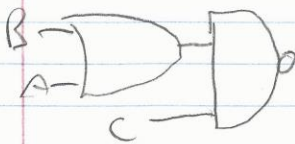
$$g_A = \frac{4+2}{3} = 2$$

$$g_B = \frac{4+2}{3} = 2$$

$$g_C = \frac{4+1}{3} = \frac{5}{3}$$

$$P = \frac{4+2+1}{3} = \frac{7}{3}$$

OAI21



$$Y = \overline{(A+B)C}$$

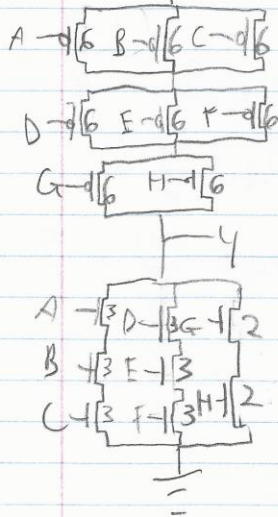
$$g_A = \frac{6}{3} = 2$$

$$g_B = \frac{6}{3} = 2$$

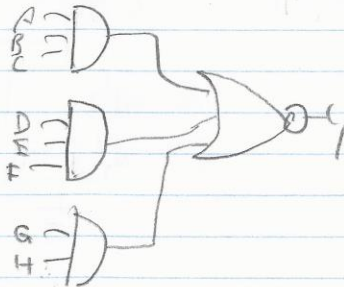
$$g_C = \frac{4}{3}$$

$$P = \frac{(4+2+2+2)}{3} = \frac{10}{3}$$

AOI 3 3 2
VDD



$$Y = ABC + DEF + GH$$

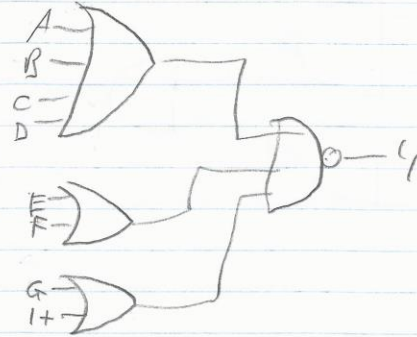
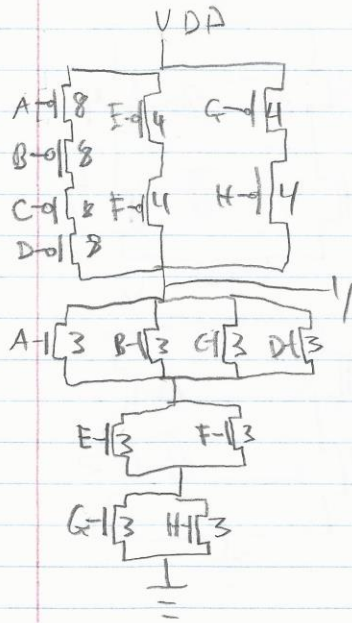


$$\begin{aligned} g_A &= 9/3 = 3 \\ g_B &= 9/3 = 3 \\ g_C &= 9/3 = 3 \\ g_D &= 9/3 = 3 \\ g_E &= 9/3 = 3 \\ g_F &= 9/3 = 3 \\ g_G &= 8/3 \\ g_H &= 8/3 \end{aligned}$$

$$P = (6 + 6 + 3 + 3 + 2)/3 = 20/3$$

0A F 4 2 2

$$Y = (A + B + C + D)(E + F)(G + H)$$

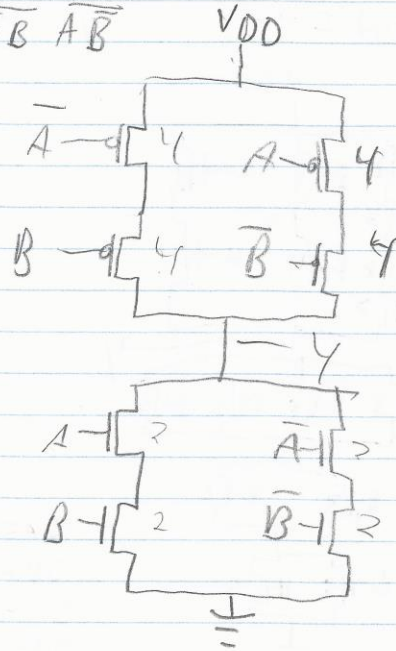
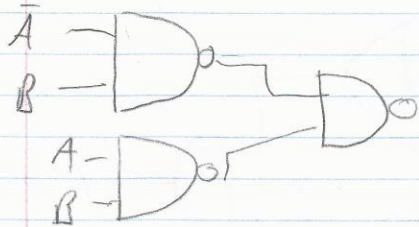


$$\begin{aligned} g_A &= \frac{1}{3} \\ g_B &= \frac{1}{3} \\ g_C &= \frac{1}{3} \\ g_D &= \frac{1}{3} \\ g_E &= \frac{7}{3} \\ g_F &= \frac{7}{3} \\ g_G &= \frac{7}{3} \\ g_H &= \frac{7}{3} \end{aligned}$$

$$P = (8 + 4 + 4 + 3 + 3 + 3 + 3) \cdot \frac{1}{3} = \frac{28}{3}$$

Xor

$$A \oplus B = \bar{A}B + A\bar{B} \quad Y = \overline{\bar{A}B} \overline{A\bar{B}}$$



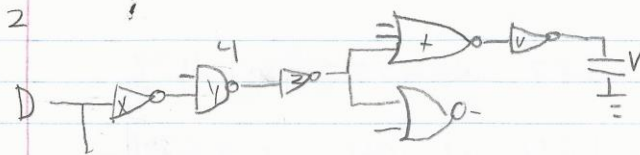
$$g_A = \frac{6}{3} = 2$$

$$g_{\bar{A}} = \frac{6}{3} = 2$$

$$g_B = \frac{6}{3} = 2$$

$$g_{\bar{B}} = \frac{6}{3} = 2$$

$$p = \frac{12}{3} = 4$$



$$G = 1 \cdot \frac{4}{3} \cdot 1 \cdot \frac{2}{3} \cdot 1 = \frac{28}{9}$$

$$B = \frac{C_{on} + C_{off}}{C_{on}} = \frac{2+5}{2} = \frac{17}{2} \quad 14 = \frac{10000}{9}$$

$$F = G \cdot B \cdot t = \frac{28}{9} \cdot \frac{17}{2} \cdot \frac{10000}{9} = 5925.9$$

$$F' = \sqrt[5]{5925.93} = 5.68$$

$$u = \frac{V \cdot g \cdot t}{F} = \frac{10000}{5.68} = 1760.6$$

$$t = \frac{u \cdot g \cdot t}{F} = \frac{(1760.6)(7.3)}{5.68} = 723.3$$

$$2 = \frac{t \cdot b \cdot g \cdot t}{F} = \frac{723.3 \left(\frac{17}{2} \right) \cdot 1}{5.68} = 218.3$$

$$4 = \frac{2 \cdot g \cdot t}{F} = \frac{218.3 \cdot \frac{4}{3}}{5.68} = 51.2$$

$$x = \frac{51.2 \cdot 1}{5.68} \approx 9$$

We do not include the branching when the D input goes to the three input and because it is not driven from the output of a gate and the capacitance starts at the inverter after D

$$V: 3s = 1760 \quad S = 587 \quad nV = 587 \quad pV = 1179$$

$$T: 7s = 723 \quad S = 104 \quad nT = 104 \quad pT = 624$$

$$Z: 3s = 218 \quad S = 73 \quad nZ = 73 \quad pZ = 146$$

$$Y: 4s = 512 \quad S = 13 \quad nY = 26 \quad pY = 26$$

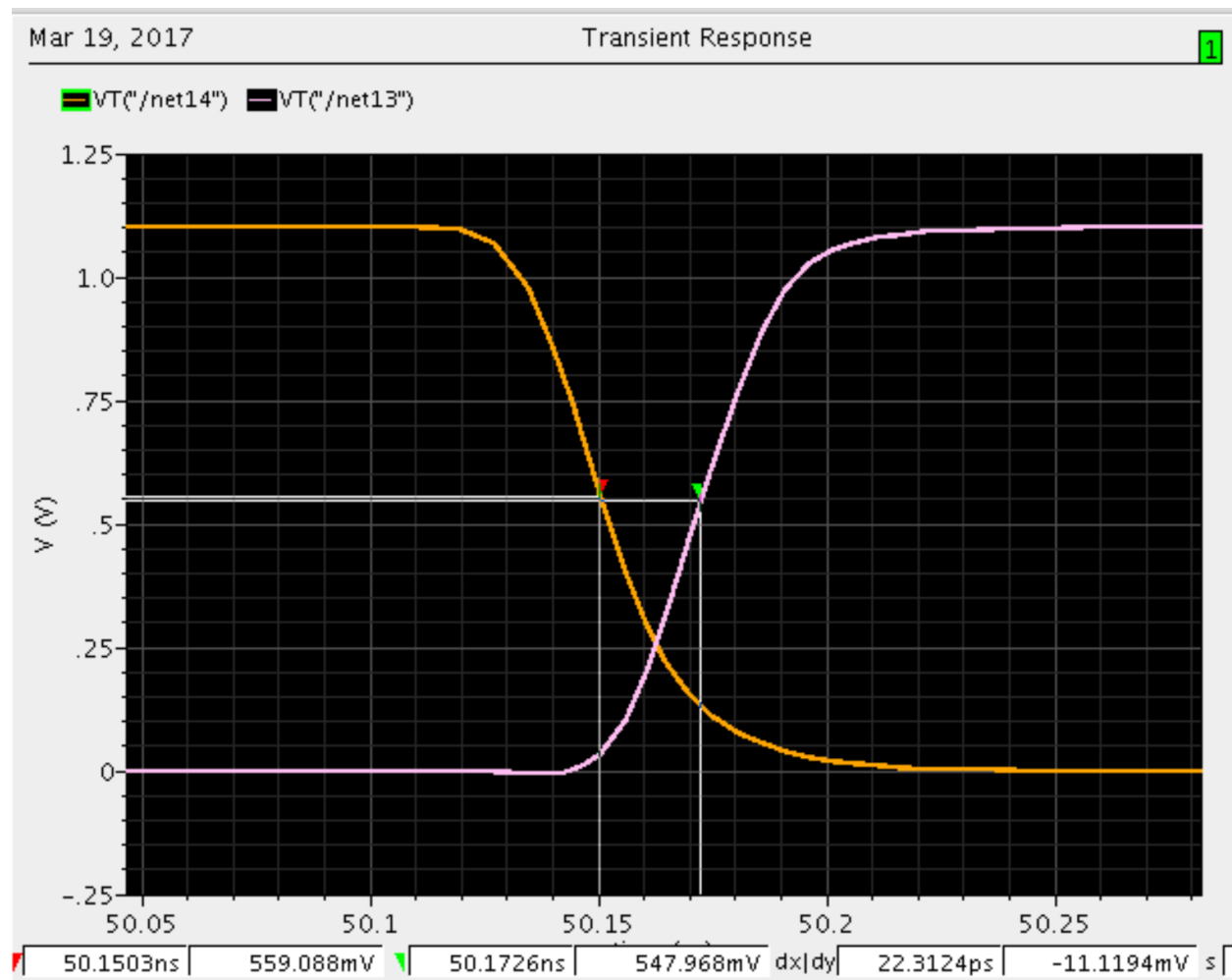
$$X: 3s = 9 \quad S = 3 \quad nX = 3 \quad nY = 6$$

$$D = N \hat{f} \cdot t_p \quad P = 1+1+1+2+3 = 8$$

$$D = 5(5.68) + 8 = 36.4$$

One way to lower the delay is to reduce the output cap from 10000 to 1000 which makes $\hat{f} = 3.58$ and $D = 5 \times 3.58 + 8 = 25.9$

3. For all the simulations, I took the delay between the two curves a 0.55V. All of the simulations looked similar to the Picture below.



FO4 Delay = 4g+p

F01 Delay = g+p

Solving systems of equations for each case

Given that FO4 inverter Delay is 15 ps. Which makes $15 = 4 * 1 + P$ $P=11$ for parasitic delay for inverter

Inverter

F04 D=18.21ps F01 D=14.96ps

G=1.08 This value is very close where the theoretical is 1

P=13.87ps This is very close to $1 * 11ps=11ps$ which is the theoretical delay

2-input nand

F04 D=26.326ps F01 D=22.312ps

G=1.336 This value is very close where the theoretical is $4/3$

P=20.97ps This is very close to $2 * 11\text{ps}=22\text{ps}$ which is the theoretical delay

3-input nand

F04 D=34.52ps F01 D=29.63ps

G=1.63 This value is fairly close to the theoretical value of $5/3$

P=27.9ps This value is close to the theoretical value of $3*11\text{ps}=33\text{ps}$

2-input nor

F04 D=31.545ps F01 D=26.22ps

G=1.775 This value is fairly close to the theoretical value of $5/3$

P=24.45ps This value is close to the theoretical value of $2*11\text{ps}=22\text{ps}$

3-input nor

F04 D= 39.82ps F01 D=32.92ps

G=2.304 This value is fairly close to the theoretical value of $7/3$

P=30.62ps This value is close to the theoretical value of $3*11\text{ps}=33\text{ps}$

OAI21

F04 D=36.57ps F01 D=31.14ps

G=1.81 This value is fairly close to the theoretical value of $5/3$

P=29.34ps This value is close to the theoretical value of $(7/3)*11\text{ps}=25.7\text{ps}$

4.

Critical path for circuit from problem 2

I used 1.2pF for output cap since a 90nm cmos had capacitance of 0.12fF. The input inverter is 9x the 90nm cap so the capacitance is 1.08fF. The output cap is $(10000/90)*1.08\text{fF}=1.2\text{pF}$

I found the delay to be: D=108.64ps

Our theoretical normalized delay was 36.4 and the normalized FO4 was 5.

So the Theoretical delay in ps is $(36.4/5)*\text{FO4 inverter delay} = 7.28 * 18.21\text{ps} = 132.569\text{ps}$

132.6ps is fairly close to the 108.64ps simulated delay.