Clicker Question #1

How can we best describe the runtime of BruteForceChange?

A. Linear
B. Quadratic
C. Polynomial (but not linear or quadratic)
D. Exponential
E. Factorial

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Dynamic Programming

CPSC 320 2024S

Optimization problems

- Just as with Greedy algorithms, this unit primarily deals with **optimization problems**:
  - We have a problem with several valid solutions
  - There is a score, or objective function, that tells us how good (or bad) each solution is
  - We are looking for the valid solution that minimizes or maximizes the score

When to use dynamic programming

- A ~~greedy~~ dynamic programming algorithm proceeds by:
  - ~~Making a single choice based on a simple, local criterion~~
  - Considering a (polynomial) number of possible choices
  - Solving one or more subproblem(s) that result from each choice
  - Combining the choice and its associated subproblem(s) (e.g., by calculating the total score)
  - Selecting the best choice (with maximum or minimum score)
- This technique is useful when the possible choices lead to **repeated or overlapping** subproblems

When to use dynamic programming?

- **Repeated/overlapping subproblems:**
  - Meaning that if two of the subproblems are **A** and **B** then smaller subproblems of **A** are also smaller subproblems of **B**
- Choosing an algorithm design technique:
  - If you have a simple, local criterion to make a choice that leads to an optimal (or good enough!) solution → **Greedy**
  - If not, and subproblems overlap → **Dynamic programming**
  - If subproblems don't overlap → **Divide and conquer**

Dynamic programming versus memoization?

- These are variants of the same problem-solving approach
- In this class we use memoization to refer to to a *recursive* implementation (top-down approach)
- We use dynamic programming to refer to an *iterative* implementation (bottom-up approach)

An interesting question is, "Where did the name, dynamic programming, come from?" The 1950s were not good years for mathematical research. […] Wilson […] was Secretary of Defense […] I felt I had to do something to shield Wilson and the Air Force from the fact that I was really doing mathematics […] I was interested in planning, in decision making, in thinking. But planning, is not a good word for various reasons. I decided therefore to use the word "programming". I wanted to get across the idea that this was dynamic, this was multistage, this was time-varying. I thought […] it's impossible to use the word dynamic in a pejorative sense. […] Thus, I thought dynamic programming was a good name. It was something not even a Congressman could object to.

—Richard Bellman, Eye of the Hurricane: An Autobiography (1984, page 159)

See https://en.wikipedia.org/wiki/Dynamic_programming

Designing a DP or Memoization algorithm

- Step 1: determine the possible choices (and associated subproblems)
  - E.g., which intervals from **1** to **j** to choose in the weighted interval scheduling problem
  - E.g., which coins to give in the change-making problem (current worksheet!)
  - E.g., which items to choose for the knapsack problem

Designing a DP or Memoization algorithm

- Step 2: define a recurrence relation for **the value of the objective function** in terms of **its value(s) for one of more subproblems**
- E.g., OPT(j) = min{OPT(i-25), OPT(i-10), OPT(i-1)}
- E.g., HP[i,j] = max{1, min{HP[i+1,j], HP[i,j+1]- G[i,j]} (in upcoming tutorial and assignment)

  (base cases omitted here)
- A **table** is used to store values of the objective function for different subproblems

Designing a DP or Memoization algorithm

- Step 3: determine the “shape” of the table that stores solutions to subproblems
  - Dimension will depend on the number and range of parameters in the subproblem
  - It’s usually an array with as many dimensions as there are parameters to the subproblem
    - E.g., M[1, …, n] for weighted interval scheduling
    - E.g., M[0, …, n][0, …, W] for knapsack

Designing a DP or Memoization algorithm

- Step 4: implement the algorithm
  - Approach 1: memoization (recursion)
    - Like a straightforward recursive approach to compute the value given by the recurrence relation
    - But first look at the table to see if this solution has already been computed
    - If so, return it immediately

Designing a DP or Memoization algorithm

- Approach 2: iteration (sometimes only called “dynamic programming”)
  - Loop over the problem, starting with the base cases
  - Then solve later subproblems
  - Make sure that by the time subproblem S needs the solution to S’, the solution to S’ has already been computed
- For each subproblem:
  - Compute the optimal solution to the subproblem, using table lookups instead of recursive calls

Designing a DP or Memoization algorithm

- Step 5: retrieve the optimal solution
  - Step 4 gives us the value of the objective function for the optimal solution, but not the solution
  - To retrieve the solution:
    - Start with the original problem and determine the last choice we made
    - Insert this choice into the solution (usually at the end)
    - Then repeat starting from the subproblem we get after making that choice
- We don’t want to store solutions for every subproblem because it would increase the space

A non-optimization example: Fibonacci numbers

- Each Fibonacci number is the sum of the two previous Fibonacci numbers
  - The sequence: 1, 1, 2, 3, 5, 8, 13, 21, …
- Recurrence:
  - FIB(0) = 0, FIB(1) = 1 // base cases
  - FIB(n) = FIB(n-1) + FIB(n-2) // recursive cases

A straightforward recursive implementation

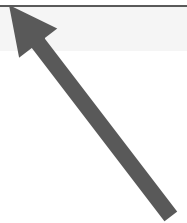
```
def fibonacci(n):
    if n <= 1:
        return n
    else:
        return fibonacci(n-1) + fibonacci(n-2)
```

A memoized implementation

```
# memoized version
def fibonacci_mem(n, memo={}):
    if n in memo:
        return memo[n]
    elif n <= 1:
        return n
    else:
        memo[n] = fibonacci_mem(n-1, memo) + fibonacci_mem(n-2, memo)
        return memo[n]
```

An iterative implementation – clicker question

```
# iterative dynamic programming version
def fibonacci_iter(n):
    fib = [0, 1]  # Initialize Fibonacci sequence with the first two numbers
    for i in range(2, n + 1):
        fib.append(                    )
    return fib[n]
```



Fill in the blank!

An iterative implementation

```python
# iterative dynamic programming version
def fibonacci_iter(n):
    fib = [0, 1]  # Initialize Fibonacci sequence with the first two numbers
    for i in range(2, n + 1):
        fib.append(fib[i - 1] + fib[i - 2])
    return fib[n]
```