CPSC 320 2024W1: Spanning Tree Tutorial Problem

A spanning algorithm

Let $G = (V, E)$ denote a connected, undirected graph with $n \geq 2$ nodes and m weighted edges. Let $\text{wt}(e)$ denote the weight of edge e of G. The following algorithm is similar but not identical to Kruskal's minimum spanning tree algorithm. (This version of the algorithm is interesting because it can be implemented efficiently on a multi-processor computer. Roughly this is because the steps for each connected component C can all be handled by different processors.)

Algorithm Spanning($G = (V, E)$, wt())

Let $G' = (V, E')$ where $E' = \emptyset$
While G' is not connected
  E-new $= \emptyset$
  For each connected component C of $G' = (V, E')$
    Find an edge $e = (u, v) \in E$ of minimum weight $\text{wt}(e)$ that
    connects a node u in C to a node v that is *not* in C
    E-new $= E$-new $\cup \{e\}$
  $E' = E' \cup E$-new
Return G'

1. How many iterations of the While loop will be executed in the *worst* case?
   ◯ $\Theta(1)$ ◯ $\Theta(\log n)$ ◯ $\Theta(n)$
2. Describe an input graph with n nodes on which this worst case behaviour can happen, and explain briefly what edge choices would lead to the worst case behaviour.
3. Which of the following statements is true? Choose one, and justify your answer briefly.
   ◯ The algorithm always returns a tree on *all* inputs G.
   ◯ The algorithm returns a tree on *some* inputs G but may not return a tree on other inputs.
   ◯ The algorithm /never/ returns a tree on any input G.
4. How many iterations of the While loop will be executed in the *best* case?
   ◯ $\Theta(1)$ ◯ $\Theta(\log n)$ ◯ $\Theta(n)$
5. Describe an input graph with n nodes on which this best case behaviour can happen, and explain briefly what edge choices would lead to the best case behaviour.
6. Explain why the algorithm always returns a tree on all inputs $G = (V, E)$ where all edges of E have different weights.
7. Explain why the tree returned by the algorithm is a minimum spanning tree on all inputs $G = (V, E)$ where all edges of E have different weights.