# CPSC 320 2024W1: Dynamic Programming Tutorial Problems

## 1 Legend of Zelda

[Thanks to Denis Lalaj for contributing this problem, which is also on assignment 4.]

Your friend is a big fan of the Legend of Zelda games and so decides to make his own version of them. He has been struggling to understand dynamic programming however, so he asks you to help him out with the knowledge you have learned from your CPSC 320 class. He seeks to set up the game in a 2D maze where Link starts his quest by entering the room with coordinates $(1, 1)$ in the maze, with the goal of saving princess Zelda who is trapped at the $(m, n)$ corner. From each room, Link move to the neighbour room immediately to the right or down, but there is a caveat. Some rooms hide memories of Zelda (denoted by non-negative integers) that Link can collect, and these increase his health points. Other rooms hide monsters (denoted by negative integers), which damage Link's health points. To save princess Zelda, Link must ensure his health points do not drop to zero (or worse... negative) at any point. Your friend wants to add a super hard mode for the game, in which Link starts with the minimum possible health points that barely allows him to rescue Zelda, even if he chooses the best possible path.

Formally an instance of the problem is $G[1..m][1..n]$—a 2D array of size $m \times n$ where each entry is an integer value (representing the points in a room). For $1 \leq i \leq m$ and $1 \leq j \leq m$, let $HP[i, j]$ denote the minimum number of health points that Link needs to have, when starting by entering the room with coordinates $(i, j)$. The problem is to compute $HP[1, 1]$.

**Example:** Two example grids are as follows, the first is $2 \times 2$ and the second is $3 \times 2$:

| | |
|---|---|
| 1 | 3 |
| 2 | -4 |

| | |
|---|---|
| -1 | -1 |
| 3 | 2 |
| 1 | 4 |

For the grid on the left,

- $HP[2, 2] = 5$, since Link must have one point left after losing 4 points when entering room (2,2).
- $HP[2, 1] = 3$, since in room (2,1), Link will first acquire 2 points and needs a total of 5 points when moving on to (2,2).
- $HP[1, 1] = 1$, since if Link starts with 1 point, accumulates 1 point in room (1,1) and another 3 points by entering room (1,2), then Link has a total of 5 points when entering room (2,2), which is just sufficient.
- Link should take the path (1,1), (1,2), (2,2). This is better than the alternative, (1,1), (2,1), (2,2), since in this case Link would need two points initially.

1. For the grid on the right above, what is $H[3, 2]$?
2. Next we'll develop a recurrence for $HP[i, j]$. Start with a base case when $i = m$ and $j = n$. What is $H[m, n]$ when $G[m, n]$ is negative? When $G[m, n]$ is positive?

3. Here is a recurrence for $HP[i,j]$, when $i < m$ and $j < n$. Explain why this is correct.

$$HP[i,j] = \begin{cases} \max(1, \min(HP[i+1,j], HP[i,j+1]) - G[i,j]), & \text{if } 1 \leq i \leq m-1 \text{ or } 1 \leq j \leq n-1, \\ \infty, & \text{if } i = m+1 \text{ or } j = n+1. \end{cases}$$

4. Use the recurrence from the previous two parts (base case plus general case) to write a memoized algorithm to compute $HP[1,1]$.