Traversing Graphs

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Computer Science Department, Higher School of Economics

Outline

Traversing Trees

Traversing Graphs: DFS and BFS

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- We start with an easier case of traversing trees, starting from the root node.

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- This is the *depth-first* traversing algorithm for trees.
- The only question is when to visit the root.

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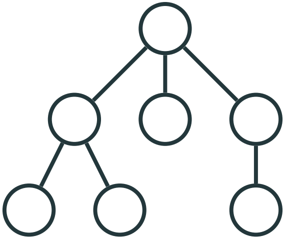

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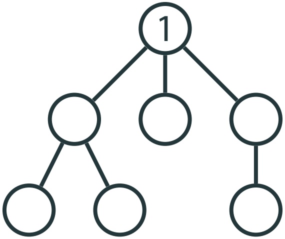

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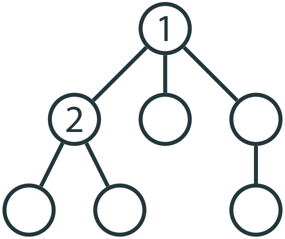

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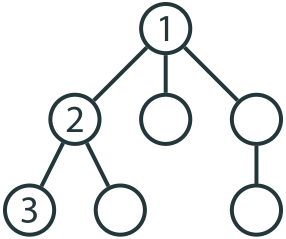

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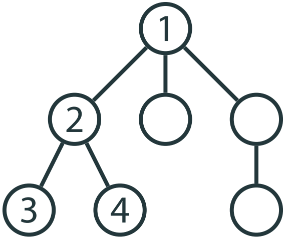

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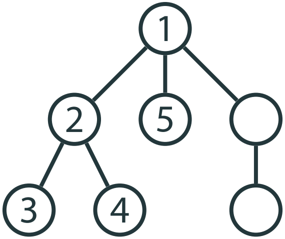

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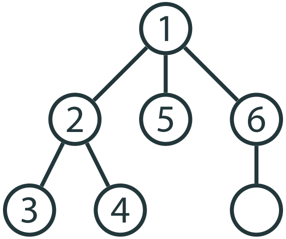

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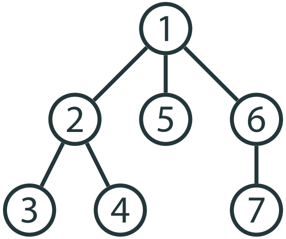

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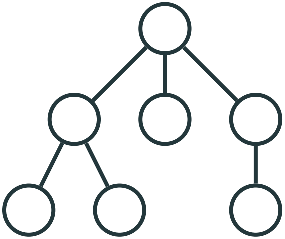

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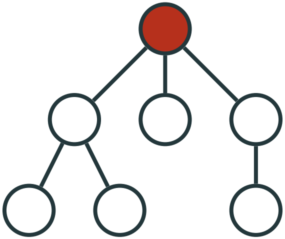

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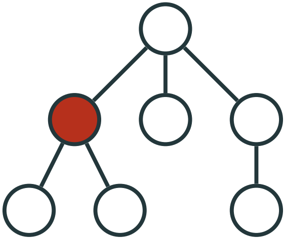

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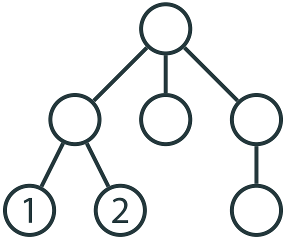

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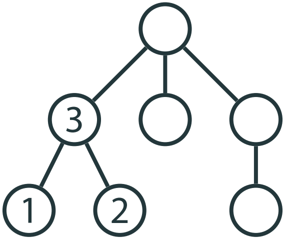

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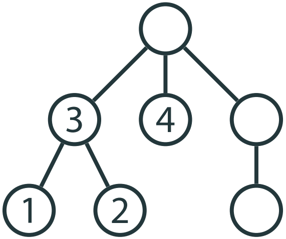

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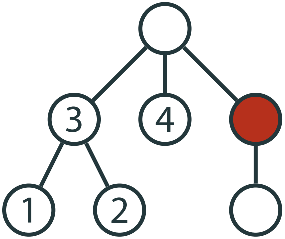

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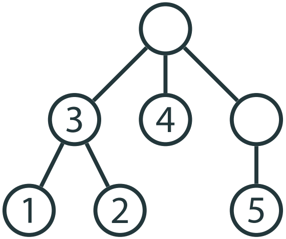

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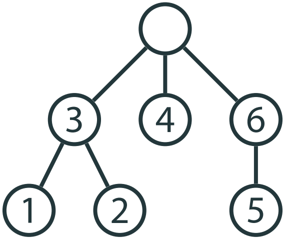

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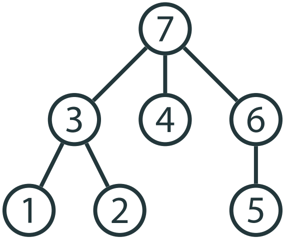

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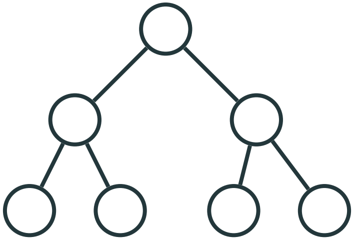

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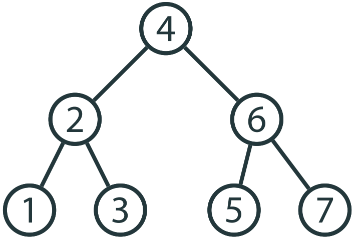
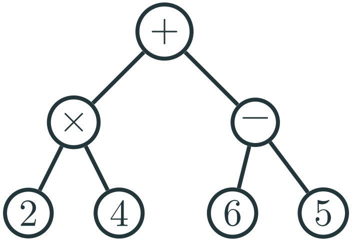


Example: Syntax Trees



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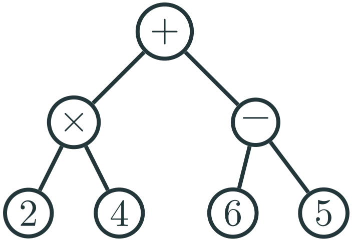

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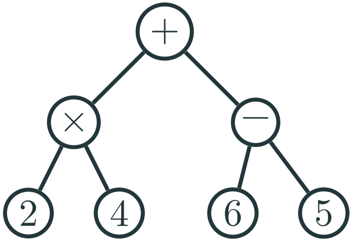

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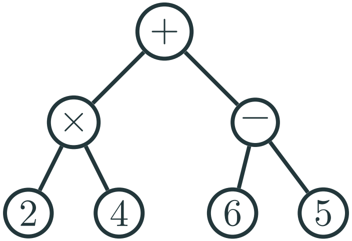

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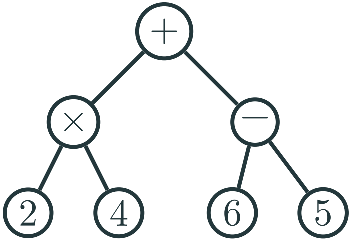

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- In-order: infix notation. $(2 \times 4) + (6 - 5)$

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- Difference from trees: now we have to remember which vertices were already visited (via another route).

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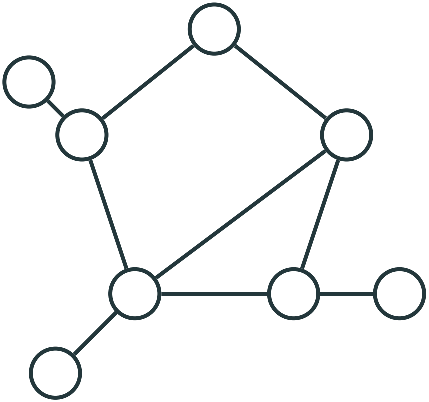

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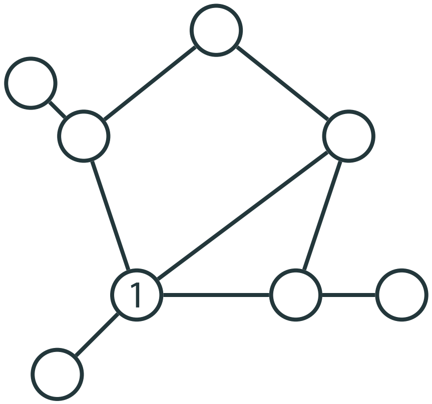

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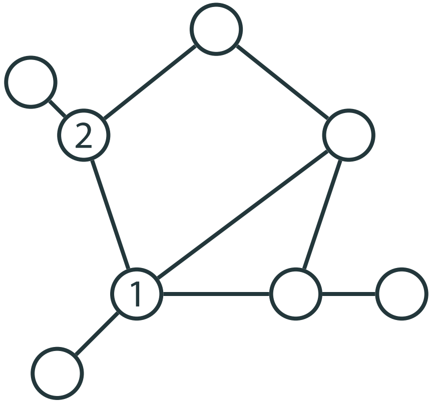

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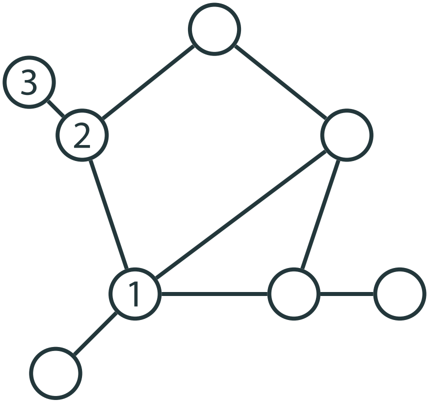

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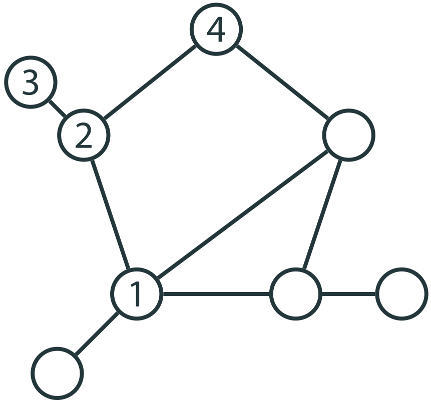

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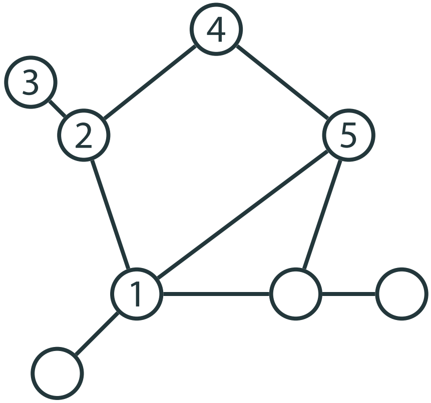

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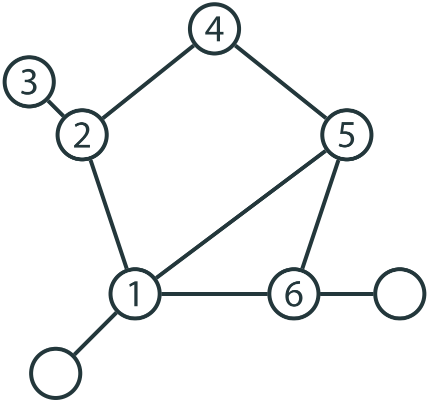

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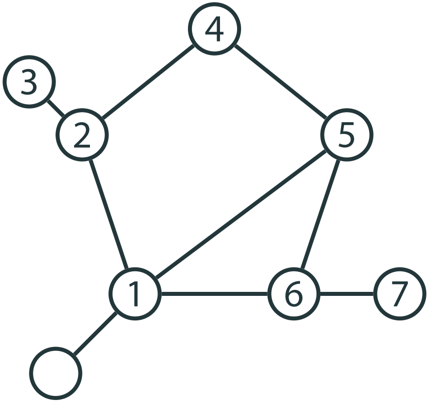

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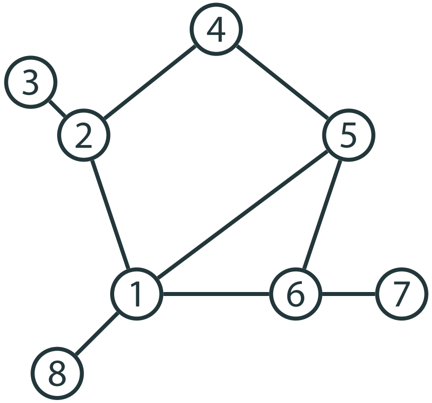

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- At each step, the BFS algorithm visits **all** new vertices which are adjacent to vertices visited at the previous step.

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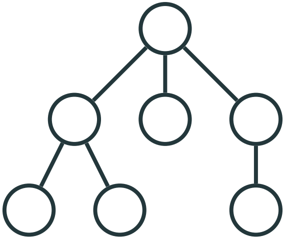

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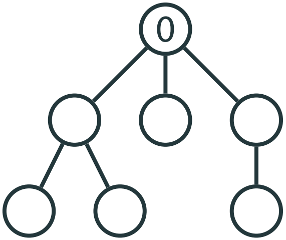

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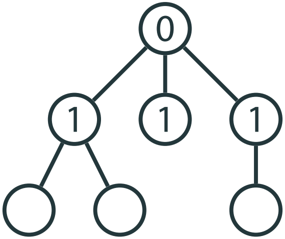

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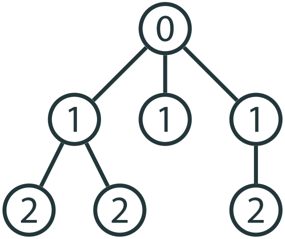

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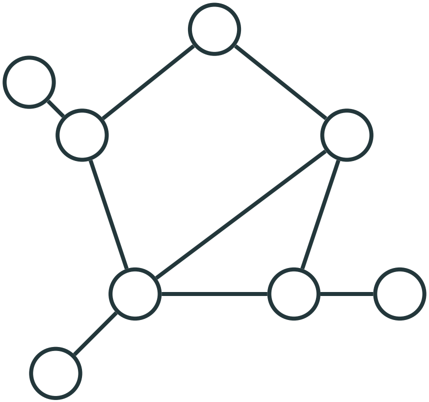

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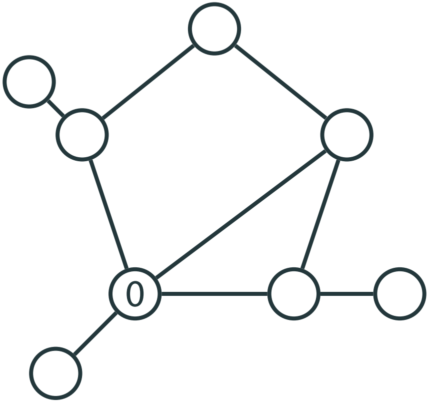

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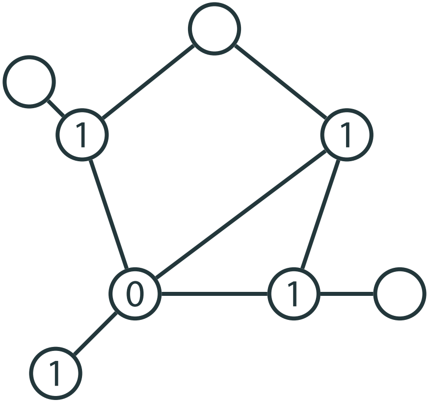

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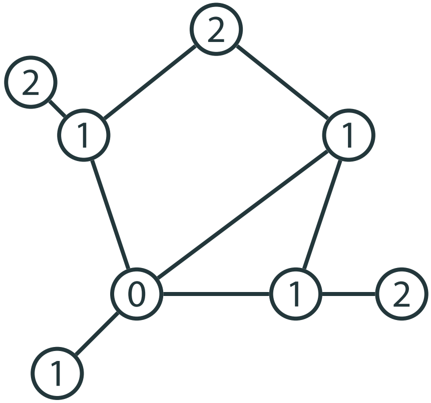

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