Network Analysis

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Outline

Watts – Strogatz Random Graphs

Clustering Coefficient

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- Edge probability, however, is the same.

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- Indeed, the existence of edges A – B and A – C increases the probability of B – C.
- This is called the *small-world property*.
- Thus, we need better models for social network graphs, than Erdős – Rényi.

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- Published in *Nature*, 1998.

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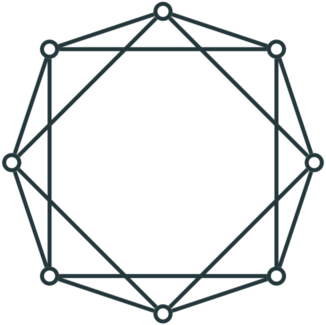
  - $\beta \in [0, 1]$ (constant)
- Step 1: regular ring lattice. Put n vertices in a circle and connect each vertex with $k/2$ neighbours on the left and $k/2$ neighbours on the right.

Regular Ring Lattice Example



$n = 8, k = 4$

Watts – Strogatz Model

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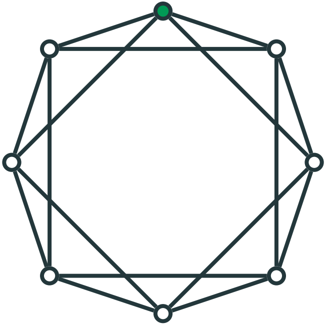
- Now we add some randomness.
- Each edge going to the right from a vertex gets *rewired* with probability β to another random point, while avoiding loops and parallel edges.
- This gives the random graph $WS(n, k, \beta)$.

Rewiring Example



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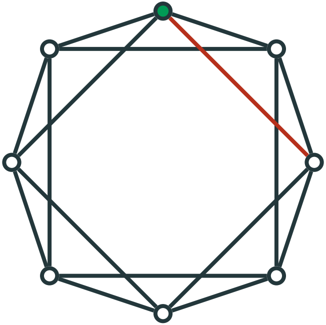

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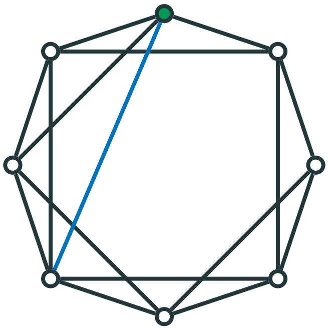

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- Average distance falls rapidly when $\beta \to 1$.

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- There are also other, more sophisticated models, like Barabási – Albert, for example.
- Watts – Strogatz is implemented in NetworkX (just invoke `watts_strogatz_graph`).