Binomial Theorem

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Outline

Pascal's Triangle

Binomial Theorem

Practice Counting

Combinations

Problem

Suppose we have a dataset of size n to train our ML model. We want to separate a testing dataset of size k from it. How many ways do we have to do it?

Combinations

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- We want to pick a subset of size k from n element set
- We know the answer:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Combinations

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- Consider one element A in the dataset
- There are two types of testing datasets:
  1. Datasets containing A
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- There are $\binom{n-1}{k-1}$ testing sets of the first type
- There are $\binom{n-1}{k}$ testing sets of the second type
- By the rule of sum we have $\binom{n-1}{k-1} + \binom{n-1}{k}$ testing sets in total

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- On one hand the answer is $\binom{n}{k}$
- On the other hand the answer is $\binom{n-1}{k-1} + \binom{n-1}{k}$
- Thus, we have

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

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- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
- $\binom{n-1}{k-1} = \frac{(n-1)!}{(k-1)!(n-k)!}, \ \binom{n-1}{k} = \frac{(n-1)!}{k!(n-k-1)!}$

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- We get

$$\frac{(n-1)!}{(k-1)!(n-k)!} + \frac{(n-1)!}{k!(n-k-1)!} =$$

$$\frac{(n-1)!}{(k-1)!(n-k-1)!}\left(\frac{1}{n-k} + \frac{1}{k}\right) =$$

$$\frac{(n-1)!}{(k-1)!(n-k-1)!}\left(\frac{k+(n-k)}{(n-k)k}\right) = \frac{n!}{k!(n-k)!}$$

Pascal's Triangle

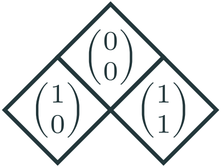
$n = 0$

$$\binom{0}{0}$$

Pascal's Triangle

$n = 0$

$n = 1$

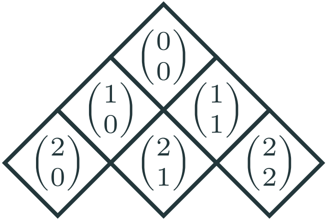


Pascal’s Triangle

$n = 0$

$n = 1$

$n = 2$



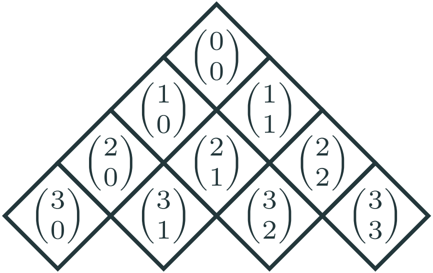
Pascal’s Triangle

$n = 0$

$n = 1$

$n = 2$

$n = 3$



Pascal's Triangle

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Pascal's Triangle

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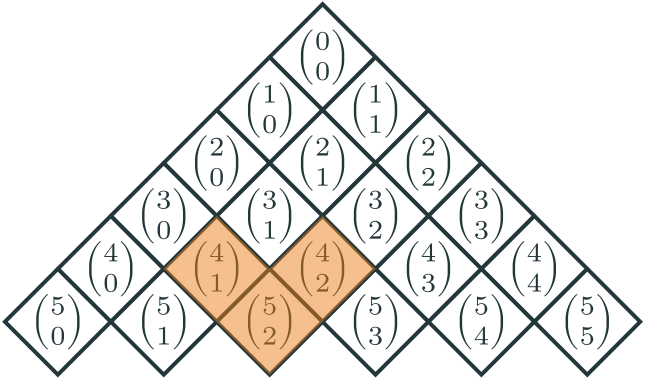

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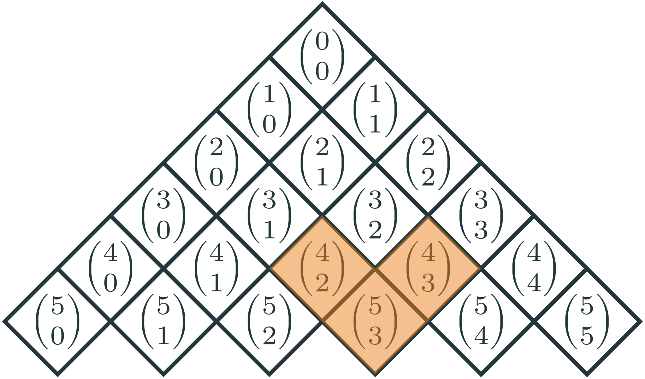

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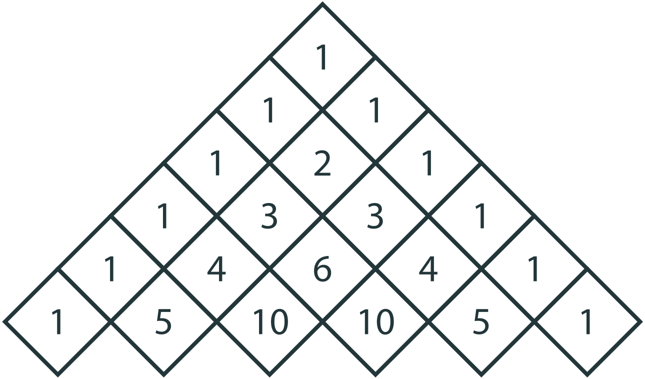

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Code

```
C = dict() # C[n,k] is equal to n choose k

for n in range(8):
    C[n, 0] = 1
    C[n, n] = 1

    for k in range(1, n):
        C[n, k] = C[n - 1, k - 1] + C[n - 1, k]

print(C[7, 4])
```

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The output:

```
35
```

Computing Binomials

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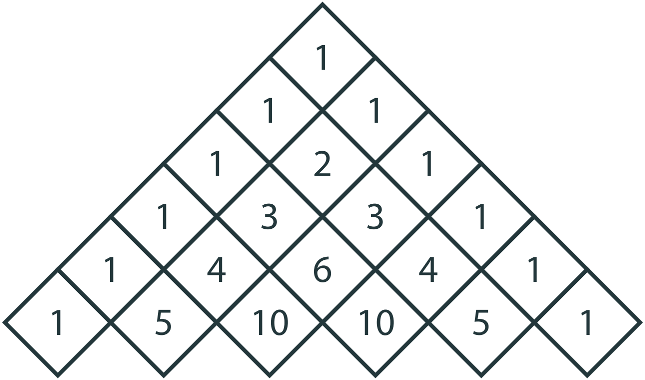
- Much better to compute binomials
- Another good option: $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$

Symmetry

Pascal’s Triangle is Symmetric:



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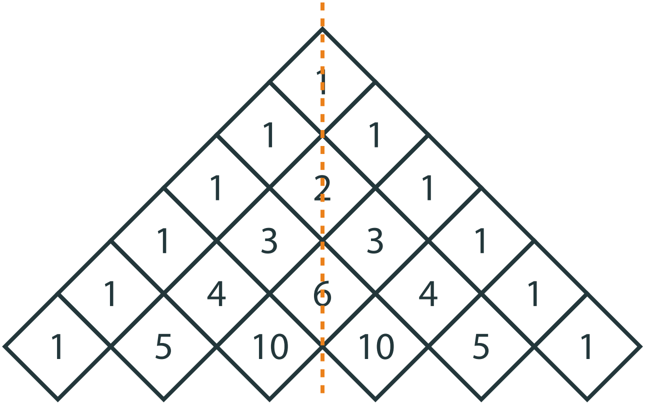

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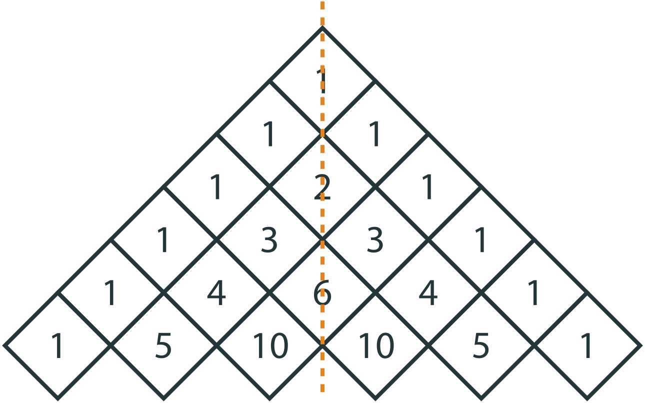

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Theorem

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Proof

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n!}{(n-k)!k!} = \binom{n}{n-k}$$

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If $k \leq n/2$

$$\binom{n}{k-1} < \binom{n}{k}$$

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- Indeed,

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- The last inequality follows from $\frac{k}{n-k+1} < 1$

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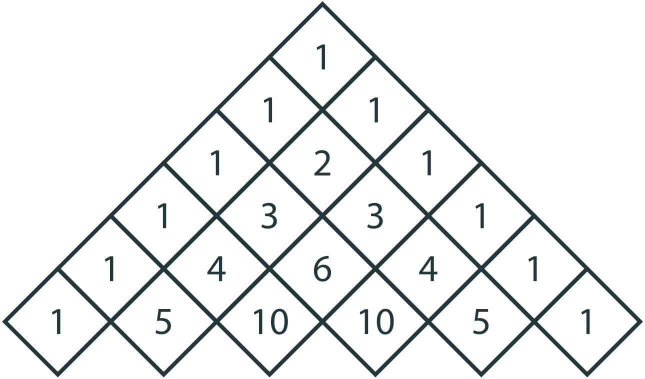
Corollary

If $k \geq n/2$

$$\binom{n}{k} > \binom{n}{k+1}$$

Comparing Binomials

Binomial coefficients grow in the middle:



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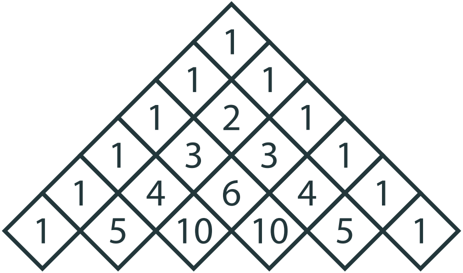

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Binomial Theorem

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \cdots + \binom{n}{k} a^{n-k} b^k + \cdots + \binom{n}{n} b^n$$

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Equivalently,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Binomial Theorem, Proof

$$\overbrace{(a+b)\cdot(a+b)\cdot(a+b)\cdot\ldots\cdot(a+b)}^{n}$$

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- Let's open the brackets
- In each bracket there are two options to pick a summand
- 2^n summands in total
- How many summands of the form $a^{n-k}b^k$?

Binomial Theorem, Proof

$$\overbrace{(a+b)\cdot(a+b)\cdot(a+b)\cdot\ldots\cdot(a+b)}^{n}$$

- To get $a^{n-k}b^k$ we have to pick b in exactly k brackets

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- This is exactly $\binom{n}{k}$
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- We have $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^k$

Consequences

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{k}a^{n-k}b^k + \cdots + \binom{n}{n}b^n$$

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- The number of subsets is equal to 2^n

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$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n}$$

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- Or equivalently, $0 = \sum_{k=0}^{n}(-1)^k\binom{n}{k}$
- The number of odd size subsets is the same as the number of even size subsets.

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Number of Hands

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What is the number of 5-card hands dealt off of a standard 52-card deck?

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A♣ 2♢ 7♠ 5♣ 4♡

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What is the number of 5-card hands dealt off of a standard 52-card deck?

- Cards in the hand are unordered, so we pick a subset
- The answer is $\binom{52}{5} = 2\,598\,960$

Two Hearts and Three Spades

Problem

What is the number of 5-card hands with two hearts and three spades?

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- We need two cards from the set of 13 and 3 cards among other 13

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- We need two cards from the set of 13 and 3 cards among other 13
- The answer is $\binom{13}{2}\binom{13}{3} = 22\,308$

4-Digit Numbers Containing 7

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What is the number of non-negative integers with at most four digits at least one of which is equal to 7?

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- The answer is $10^4 - 9^4 = 3\,439$

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- If we try to count options for each position and apply the product rule, there are problems
- 10 options for the first position, but for the second the number of options depends on the first number
- Idea: look from the other side

Numbers with Decreasing Digits

$$* \quad * \quad * \quad *$$

- We pick digits from 0 to 9 to be in our number

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Picked 3, 4 ,2, 7

- We pick digits from 0 to 9 to be in our number
- Once we picked four distinct digits, our number is uniquely determined

Numbers with Decreasing Digits

7 4 3 2

Picked 3, 4 ,2, 7

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- We pick digits from 0 to 9 to be in our number
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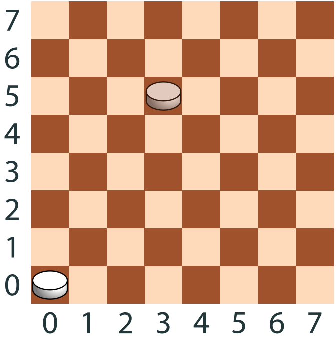
- Order of picks does not matter
- We have combinations of size 4 from 10 options
- The answer is $\binom{10}{4} = 210$

Piece on a Chessboard



A piece can move one step up or one step to the right. What is the number of ways of getting from the cell $[0, 0]$ (bottom left corner) to the cell $[3, 5]$?

Solution

- There are exactly eight moves

Solution

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- Three of them should be to the right, while the remaining five should go up

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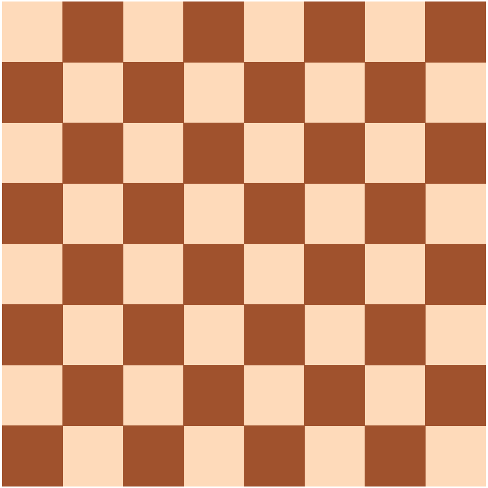
- Moreover, any such combination of three moves to the right and five moves up is a valid way of getting to the cell $[3, 5]$
- We need to pick a subset of 3 moves among 8 moves
- The answer is $\binom{8}{3} = 56$

Solution Using Pascal’s Triangle



Solution Using Pascal's Triangle

1							
1							
1							
1							
1							
1							
1	2						
1	1	1	1	1	1	1	1

Solution Using Pascal's Triangle

1							
1							
1							
1							
1							
1							
1	2	3					
1	1	1	1	1	1	1	1

Solution Using Pascal's Triangle

1							
1							
1							
1							
1							
1	3						
1	2	3					
1	1	1	1	1	1	1	1

Solution Using Pascal's Triangle

1							
1							
1							
1							
1							
1	3						
1	2	3	4				
1	1	1	1	1	1	1	1

Solution Using Pascal's Triangle

1							
1							
1							
1							
1							
1	3	6					
1	2	3	4				
1	1	1	1	1	1	1	1

Solution Using Pascal's Triangle

1							
1							
1							
1							
1	4						
1	3	6					
1	2	3	4				
1	1	1	1	1	1	1	1

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- We studied binomial coefficients extensively
- They have several mathematical and combinatorial interpretations
- We have practiced to apply our knowledge
- In the next lesson we will see one more standard combinatorial setting