Wave phenomena: ripple tank experiments

References:

PASCO Instruction Manual and Experiment Guide for Ripple Generator and Ripple Tank System
G. Kuwabara, T. Hasegawa, K. Kono: Water waves in a ripple tank, Am. J. Phys. 54 (11) 1986
A.P. French: Vibrations and Waves, Norton 1971, Ch. 7 and 8 (Waves in two dimensions)
R.D. Knight: Physics for Scientists and Engineers (With Modern Physics): A Strategic Approach, Pearson Education Inc. 2004
1 weight: Exercises 1-3; 2 weights: all

Introduction

Two-dimensional waves in water have been intensively studied in theoretical hydrodynamics. They display more complicated behaviors than acoustic or electromagnetic waves
Water surface waves travel along the boundary between air and water. The restoring forces of the wave motion are surface tension and gravity. At different water depths, these two forces play different roles.
The ripple tank can be used to study almost all the wave properties: reflection, refraction, interference and diffraction. In addition to this, the wave phase velocity can be investigated at different water depths and in the presence of obstacles of various shapes.

Wave properties

The wave speed

A two-dimensional traveling wave is a disturbance of a medium, which can be expressed as a function: $\psi(ax + by - vt)$

Here v is the wave speed: $v = \dfrac{\omega}{k} = f\lambda$ (1)

ω is the angular frequency, f is frequency, λ is the wavelength and k is the wave number.

v is also called *phase velocity*. A non-dispersive wave has a constant v; dispersion is characterized by a dependence of v on λ.
Three different regions are indicated on the dispersion curve from Figure 1:

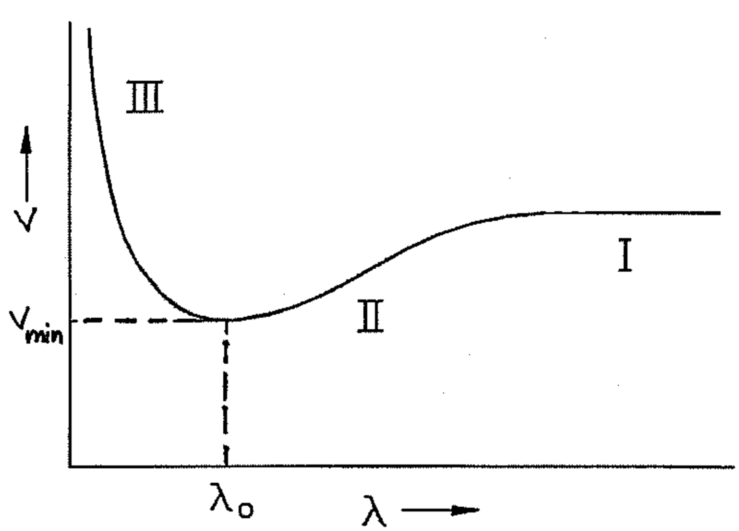

Figure 1: Dispersion curve for water waves

Region I: Shallow water gravity waves. Gravity is the restoring force, which forces the motion to be largely longitudinal and quasi independent of λ.

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$$v = \sqrt{gd} \qquad \text{where d is the depth.} \qquad (2)$$

This is the region in which the ripple tank is supposed to operate.

Region II: Deep water gravity waves. In a surface wave, the disturbance on the surface dies away in the interior of the medium in a depth of about one wavelength.

$$v = \sqrt{\frac{g\lambda}{2\pi}} \qquad (3)$$

If the distance to the bottom d is greater than λ, the bottom will not influence the motion. This is the dispersion relation for deep water ocean waves. Disturbances raised by the wind in tropical storms get sorted out by wavelength. The long waves, outdistancing the rest, appear in the temperate zones as the great ocean rollers.

Region III is the regime of true "ripples", where the restoring force is not gravity but surface tension σ.

Here: $$v = \sqrt{\frac{\sigma\, 2\pi}{\rho\lambda}} \qquad \text{where } \rho \text{ is the density} \qquad (4)$$

At the minimum speed v_{min}, when $\lambda = \lambda_o$ surface tension and gravity are equally important and:

$$\frac{\sigma}{\rho}\frac{2\pi}{\lambda_o} = \frac{g\,\lambda_o}{2\pi} \quad \text{or:} \quad \lambda_o = 2\pi\sqrt{\frac{\sigma}{\rho g}}$$

This distance λ_o = 1.7 cm for water, with v_{min} = 23 cm/sec. This region, with a negative slope of *v* vs λ, is an example of *anomalous dispersion* because the energy in the wave moves *faster* than the wave phase (group velocity is larger than phase velocity). Have you ever noticed the motion of a patch of waves on a puddle blown by the wind?

The wave equation

The wave equation in rectangular coordinates x, y has the typical form:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \qquad (5)$$

Plane waves are generated by oscillations of a straight (flat) object whose dimensions are large compared to the wavelength.

Equation (5) can be re-written in spherical or cylindrical coordinates, depending on the particular geometry of the experimental setup.

In spherical symmetry (coordinates r, θ) the wave equation will be particularly useful in interpreting the circular wave pattern from diffraction and interference experiments:

For a circular wave: $$\frac{\partial\psi}{\partial\theta} = 0 \quad and \quad \frac{\partial^2\psi}{\partial t^2} = v^2\,\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) \qquad (6)$$

Circular waves are generated by objects whose linear dimensions are small compared to the wavelength.

The circular wave has the mathematical form: $\psi(r,t) = a\cos(kr - \omega t + \varphi_0)$ (7)

Huygens' Principle

Huygens postulated a principle in two steps:

- Each point on a wave front is the source of a spherical wavelet that spreads out at the wave speed;

- At a later time, the shape of the wave front is the tangent to all the wavelets.

Huygens' principle is not a theoretical formulation, but more an intuitive tool useful to understand the physics of wave propagation.

Reflection and refraction of plane waves

Using the Huygens' construction, the basic laws of reflection and refraction (Snell's Laws) can be demonstrated:

$$\theta = \theta' \qquad (8)$$

$$\frac{1}{v_1}\sin\theta_i = \frac{1}{v_2}\sin\theta_r \qquad (9)$$

Here v_1 and v_2 are velocities of the wave in the two media, θ and θ' are incidence and reflection angles, θ_i and θ_r are incidence and refraction angles.

Interference

When a wave front passes through two narrow openings in a barrier, each unobstructed point on the original wave front will act as a secondary source of waves. The two "new" waves resulting beyond the barrier will superimpose. There will be a well-defined phase relationship among the primary and secondary waves, called *coherence.*

In turn, there will be a systematic phase relation among the secondary waves arriving at distant points.

Superposition of the two secondary waves leads to the resulting wave form at a distant point P (see Figure 2):

$$\psi_P(t) = a\left[\cos\omega\left(t-\frac{r_1}{v}\right)+\cos\omega\left(t-\frac{r_2}{v}\right)\right] = 2a\cos\omega t\,\cos\left[\frac{\pi(r_2-r_1)}{\lambda}\right] \qquad (10)$$

The interference maxima are set by the condition:

$$r_2 - r_1 = d\sin\theta_m = m\lambda \qquad (11)$$

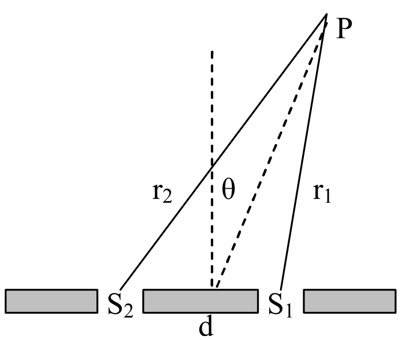


Figure 2 Double slit interference

The amplitude of the resulting wave at some arbitrary direction θ is given by:

$$A(\theta) = 2a\cos\left(\frac{\pi\, d\sin\theta}{\lambda}\right) = 2a\cos\left(\frac{kd\sin\theta}{2}\right) \qquad (12)$$

The amplitude will vary as:

$$A(\theta) = A_{\max} \ \ when \ \frac{kd\sin\theta}{2} = 0$$
$$A(\theta) = A_{\min} \ \ when \ \frac{kd\sin\theta}{2} = \frac{\pi}{2} \qquad (13)$$

Diffraction

Plane waves spread out behind a narrow opening, taking circular shapes. Each point from the opening becomes a secondary source of waves. Only at large distances from the opening, the wave fronts become plane again. Using Huygens' Principle, a condition similar to Equation (11) can be demonstrated:

$$a\sin\theta = \lambda \qquad (14)$$

Here a is the slit opening and θ is the angular spread of the first diffraction maximum.

Apparatus notes

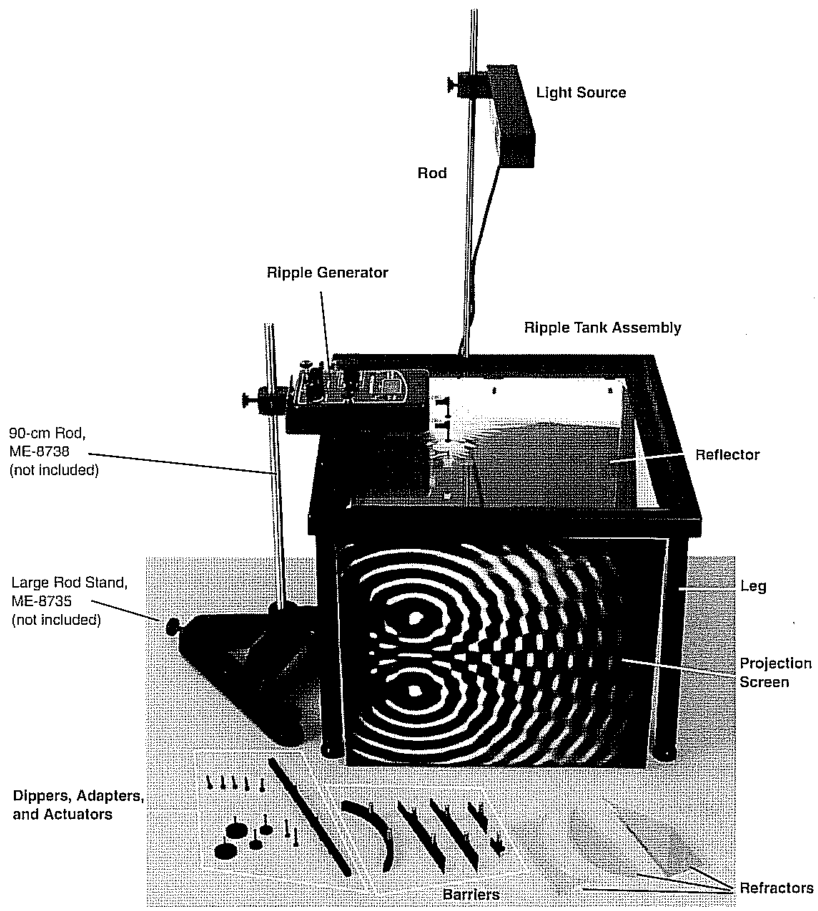
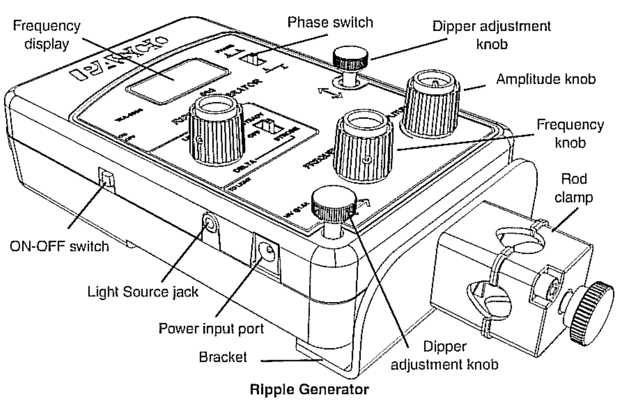


Figure 3 Ripple tank setup

The experimental setup (Figure 3) includes three main parts: ripple tank and light reflector, ripple generator and light source. You will receive a kit box with: dippers, attachments, reflectors, refractors, barriers, surfactant, pipette and ruler.

To open the data acquisition program, click on the 'Ripple' shortcut, located on the desktop. A set of buttons at the top left allow you to begin/pause/end a recording.
The calibration grid image was already recorded.
Level the ripple tank. Close the evacuation tube clamp and add a small amount of water to the center of the tank. Adjust the legs until the water circle stays in the middle of the tank.
After you added the full amount of water (~800ml), dampen the beaches by pressing and releasing the foam so that water enters the pores of the foam rubber.
After finishing the experiment, clean the tank. First carefully remove the projection screen and reflector; fold the screen over the reflector to protect it. Empty the tank in the pail, gently press the foam beaches to release the water, press a rolled towel onto the foam to absorb the remaining water and completely dry the glass plate with a soft cloth.
The ripple generator has two dipper adjustment knobs for making fine changes to the height and orientation of the generator (Figure 4).



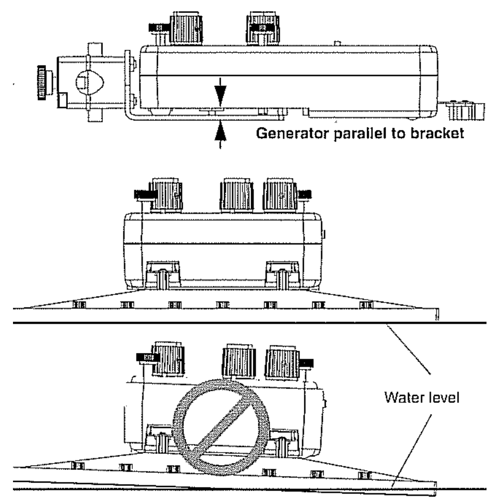


Figure 4

Figure 5 Leveling the ripple generator

Check if ripple generator is parallel to the bracket (see Figure 5). It is very important that the dippers barely touch the surface of the water in the tank. For the plane wave dipper it is also important that the dipper has equal contact with the surface over its entire length (Figure 5).
Ripple generator settings (see Figure 4):

- Amplitude: adjust it to get a clear wave pattern without distortions.
- Frequency: adjustment can be done in 0.1 Hz increments. The range is 1.0 Hz to 50.0 Hz.
- Phase: two settings are available: in phase (0°) and out of phase (180°). Settings can be changed while the ripple generator is in operation.
- Light source controls: the light source can be used as a strobe or as a steady source. The DELTA knob adjusts the frequency of the strobe light source with respect to the ripple generator frequency (by default, the frequency of the light source is the same as the frequency of the generator).

Image acquisition

A set of buttons at the top left of the acquisition window allows you to begin/pause/end a recording.
The light should be setup on STROBE.
Clicking 'View Tank in Real Time' will show camera's view in real time
Clicking 'Freeze Tank Image' will store the screen image. You may store it with Filter ON (rough edges) or OFF (softer edges).
The program allows distances and angles to be independently measured. Perfectly horizontal or vertical lines can be drawn by holding the Shift key. Angles are always measured as if the vectors on the screen were placed tail to tail, counter-clockwise from the higher numbered to the lower numbered line.
It is possible to zoom in (and not out)
Note: start by getting used with uncertainties in ripple tank measurements; evaluate the main sources of error.

Note: *LabVIEW is a powerful graphical development environment, used for signal acquisition, measurement analysis, and data presentation. If you want to find out more about it, use the LabView 8.2 shortcut from the desktop, which allows you to browse Resources and Documentation. By accessing the National Instruments web page, you may also get more facts about LabVIEW: http://www.ni.com/labview/*

Exercise 1: Reflection
Straight Barrier

Place a long straight barrier at an angle in the middle of the tank and add enough water to the tank so that the water level is halfway up the long barrier.
Position the ripple generator over the midpoint of one side of the tank; connect the plane wave dipper to the ripple arms. Adjust the ripple generator until the bottom of the plane dipper is barely in contact with the surface of the water. Set the frequency to 20 Hz and amplitude to less than half maximum
Adjust the light source such as to get the best possible viewing on the screen. Set the light source to STROBE.
Calibrate the setup for magnification: use a measured refractor; take measurements on the screen image.
Adjust the amplitude as necessary to make a clear pattern of plane waves.
Capture the image. Take measurements of incidence and reflection angles, verify Equation (8).

Curved Barrier

Position a curved barrier in front of the plane wave dipper, so that it curves towards the plane wave generator.
Record the reflected wave pattern. Estimate the focal distance and radius of the curved barrier. Turn the barrier around by 180 degrees, repeat the measurements.

Exercise 2: The wave speed

a) Wave speed and frequency

Using the same setup as in Exercise 1, without any barrier; set the frequency to 5 Hz and amplitude to less than half maximum.
Measure and record the distance of five wavelengths. Calculate $\lambda_{average}$.
Repeat the measurements at 4-5 other frequencies, graph λ_{av} vs. f. Using Equation (1), calculate v.

b) Wave speed and water depth

Repeat the wavelength measurements at a fixed frequency (5–10 Hz), but at different water depths between 2 mm and 10 mm.
Calculate the wave speed for each water depth. Graph v vs. λ. Verify Equation 2.
Discuss the sources of error.

Exercise 3: Refraction with a straight barrier

Place the trapezoidal refractor in the middle of the tank so that the triangular end points to the plane wave dipper. Adjust the amount of water so that the water level covers the refractor by 1-2 mm.
Set the light source to STROBE and adjust the frequency to 15 Hz or less. Adjust the amplitude to get a clear pattern.
Capture the refraction pattern. Explain how refraction takes place in the ripple tank setup. Measure the angles of incidence and refraction, verify Equation (9).

Exercise 4: Diffraction

Setup the ripple tank with the plane wave dipper and two straight barriers arranged to form a 3 cm “slit”. Set the generator to ~20 Hz and amplitude to ~ half maximum. Set the light source to STROBE. Vary the amplitude to get a clear pattern.
Capture the diffraction pattern; determine the angular spread (θ) of the circular waves past the slit.
Repeat for 4-5 different slit openings (a). Graph the width of the spread (sinθ) vs.1/a. Verify Equation (14).

Exercise 5: Interference

Turn off the ripple generator; replace the plane wave dipper with two standard dippers. Adjust the generator so that the two standard dippers barely touch the surface of the water. Adjust the generator to 20Hz, ~ half amplitude and STROBE function.
Capture the pattern image.
The intersections of wave fronts appear as bright or dark spots. To identify the order of interference m (Equation 11) you may use the geometry from Young’s interference apparatus schematics.
Measure angles θ corresponding to $m = \pm 1, \pm 2$, confirm Equation (13).
Vary the distance d between the two point dippers by attaching them to the plane wave dipper. Repeat the experiment. Explain the changes in the interference pattern with variation of distance between dippers.
Set one dipper in opposition of phase. Explain the pattern. Reformulate Equation 10.
Discuss the sources of error.

Additional suggested experiments

Image formed by a plane mirror: use one point dipper and a straight barrier.
Interference with multiple sources: attach 3 or 5 dippers to the plane wave dipper. Explain the patterns formed.
Refraction by curved refractors: use the concave/convex refractors. Get the focal distance of the “lenses”.

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